

Quark mass effects in the thermodynamical properties of an extended (P)NJL model

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Outline

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- Extended Nambu–Jona-Lasinio Model: multi-quark interactions
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- Polyakov potentials

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- Extended NJL
- Extended NJL with Log. Polyakov potential
- Extended NJL with Exp. K-Log. Polyakov potential
- Correlations in the uds base

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Introduction

QCD: the Theory of **Strong Interactions**

- Very successful pQCD at high energy
- Non-perturbative low energy regime requires the use of other tools for instance:
 - IQCD
 - AdS/QCD
 - Dyson-Schwinger
 - FRG
 - Chiral perturbation theory
 - Effective models
 - **Dynamical/Explicit Chiral Symmetry Breaking** plays a big role in low energy phenomenology

Nambu–Jona-Lasinio Model

NJL: effective model for the non-perturbative low energy regime of **QCD** with Dynamical Chiral Symmetry Breaking (**$D_\chi SB$**)

- **NJL** shares the global symmetries with **QCD**
- Dynamical generation of the **constituent mass**
- Light pseudoscalar as (quasi) **Nambu-Goldstone boson**
- **Quark condensates** as order parameter
- No gluons (no confinement/deconfinement)
- Local and non renormalizable

Multi-quark interactions (u , d and s)¹

$$\mathcal{L}_{\text{eff}} = \bar{q}i\not{\partial}q + \mathcal{L}_m$$

¹ $\Sigma = (s_a - ip_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a i\gamma_5 q$, and $a = 0, 1, 3, 8$

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$$\mathcal{L}_{\text{eff}} = \bar{q}i\not{\partial}q + \mathcal{L}_m$$

■ Explicit Chiral symmetry breaking

$$\mathcal{L}_m = \bar{q}\hat{m}q$$

¹ $\Sigma = (s_a - ip_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a i\gamma_5 q$, and $a = 0, 1, 3, 4, 5, 6, 7, 8$

Multi-quark interactions (u , d and s)¹

$$\mathcal{L}_{\text{eff}} = \bar{q}i\not{\partial}q + \mathcal{L}_m + \mathcal{L}_{NJL}$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$

- **Nambu–Jona-Lasinio** (4 q)

$$\mathcal{L}_{NJL} = G \text{tr} [\Sigma^\dagger \Sigma]$$

¹ $\Sigma = (s_a - ip_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a i\gamma_5 q$, and $a = 0, 1, 3, 8$

Multi-quark interactions (u , d and s)¹

$$\mathcal{L}_{\text{eff}} = \bar{q}i\not{\partial}q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$

- $\mathcal{L}_{NJL} = G \text{tr} [\Sigma^\dagger \Sigma]$

- **'t Hooft determinant** (6 q)

$$\mathcal{L}_H = \kappa (\det [\Sigma] + \det [\Sigma^\dagger])$$

¹ $\Sigma = (s_a - ip_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a i\gamma_5 q$, and $a = 0, 1, \dots, 8$

Multi-quark interactions (u , d and s)¹

$$\mathcal{L}_{\text{eff}} = \bar{q}i\not{\partial}q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q}$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$
- $\mathcal{L}_{NJL} = G \text{tr} [\Sigma^\dagger \Sigma]$
- $\mathcal{L}_H = \kappa (\det [\Sigma] + \det [\Sigma^\dagger])$
- **Eight quark interaction term**

$$\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}, \quad \mathcal{L}_{8q}^{(1)} = g_1 (\text{tr} [\Sigma^\dagger \Sigma])^2, \quad \mathcal{L}_{8q}^{(2)} = g_2 \text{tr} [\Sigma^\dagger \Sigma \Sigma^\dagger \Sigma]$$

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OZI violation in \mathcal{L}_H and $\mathcal{L}_{8q}^{(1)}$.

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Multi-quark interactions (u , d and s)¹

$$\mathcal{L}_{\text{eff}} = \bar{q}i\not{\partial}q + \mathcal{L}_m + \mathcal{L}_{NJL} + \mathcal{L}_H + \mathcal{L}_{8q} + \mathcal{L}_\chi$$

- $\mathcal{L}_m = \bar{q}\hat{m}q$
- **Extended Explicit Chiral symmetry breaking** \mathcal{L}_χ
- $\mathcal{L}_{NJL} = G \text{tr} [\Sigma^\dagger \Sigma]$
- $\mathcal{L}_H = \kappa (\det [\Sigma] + \det [\Sigma^\dagger])$
- $\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}$, $\mathcal{L}_{8q}^{(1)} = g_1 (\text{tr} [\Sigma^\dagger \Sigma])^2$, $\mathcal{L}_{8q}^{(2)} = g_2 \text{tr} [\Sigma^\dagger \Sigma \Sigma^\dagger \Sigma]$

Non canonical explicit chiral symmetry breaking terms

¹ $\Sigma = (s_a - ip_a)\frac{1}{2}\lambda_a$, $s_a = \bar{q}\lambda_a q$, $p_a = \bar{q}\lambda_a i\gamma_5 q$, and $a = 0, 1, \dots, 8$

Inclusion of explicit chiral symmetry breaking terms

$$\mathcal{L}_\chi = \sum_{i=1}^{10} \mathcal{L}_\chi^i,$$

$$\mathcal{L}_\chi^1 = -\kappa_1 \mathbf{e}_{ijk} \mathbf{e}_{mnl} \Sigma_{im} \chi_{jn} \chi_{kl} + h.c.,$$

$$\mathcal{L}_\chi^3 = g_3 \text{tr} [\Sigma^\dagger \Sigma \Sigma^\dagger \chi] + h.c.,$$

$$\mathcal{L}_\chi^5 = g_5 \text{tr} [\Sigma^\dagger \chi \Sigma^\dagger \chi] + h.c.$$

$$\mathcal{L}_\chi^7 = g_7 (\text{tr} [\Sigma^\dagger \chi] + h.c.)^2$$

$$\mathcal{L}_\chi^9 = -g_9 \text{tr} [\Sigma^\dagger \chi \chi^\dagger \chi] + h.c.$$

$$\mathcal{L}_\chi^2 = \kappa_2 \mathbf{e}_{ijk} \mathbf{e}_{mnl} \chi_{im} \Sigma_{jn} \Sigma_{kl} + h.c.,$$

$$\mathcal{L}_\chi^4 = g_4 \text{tr} [\Sigma^\dagger \Sigma] \text{tr} [\Sigma^\dagger \chi] + h.c.,$$

$$\mathcal{L}_\chi^6 = g_6 \text{tr} [\Sigma \Sigma^\dagger \chi \chi^\dagger + \Sigma^\dagger \Sigma \chi^\dagger \chi],$$

$$\mathcal{L}_\chi^8 = g_8 (\text{tr} [\Sigma^\dagger \chi] - h.c.)^2,$$

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- $\kappa_1, g_9, g_{10} \rightarrow 0$ without loss of generality

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- $\chi \rightarrow \frac{1}{2} \hat{m}$

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- $\kappa_1, g_9, g_{10} \rightarrow 0$ without loss of generality
- $\chi \rightarrow \frac{1}{2} \hat{m}$
- $\kappa_1, \kappa_2, g_4, g_7, g_8, g_{10}$ OZI violating

Thermodynamic potential

$$\Omega = \mathcal{V}_{st} [h_i] + \sum_i \frac{N_c}{8\pi^2} (J_{-1} [M_i [h_i], T, \mu_i] + C [T, \mu_i])$$

$$\begin{aligned} \mathcal{V}_{st} [h_i] = & \frac{1}{16} \left(4G (h_i^2) + 3g_1 (h_i^2)^2 + 3g_2 (h_i^4) + 4g_3 (h_i^3 m_i) \right. \\ & + 4g_4 (h_i^2) (h_j m_j) + 2g_5 (h_i^2 m_i^2) + 2g_6 (h_i^2 m_i^2) + 4g_7 (h_i m_i)^2 \\ & \left. + 8\kappa h_u h_d h_s + 8\kappa_2 (m_u h_d h_s + h_u m_d h_s + h_u h_d m_s) \right) \Big|_0^{M_i} \end{aligned}$$

$$\Delta_f = M_f - m_f$$

$$\begin{aligned} = & -G h_f - \frac{g_1}{2} h_f (h_f^2) - \frac{g_2}{2} (h_f^3) - \frac{3g_3}{4} h_f^2 m_f - \frac{g_4}{4} \left(m_f (h_f^2) + 2h_f (m_i h_i) \right) \\ & - \frac{g_5 + g_6}{2} h_f m_f^2 - g_7 m_f (h_i m_i) - \frac{\kappa}{4} t_{ij} h_i h_j - \kappa_2 t_{ij} h_i m_j \end{aligned}$$

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Thermodynamic potential: the fermionic integrals

$$\Omega = \mathcal{V}_{st} [h_i] + \sum_i \frac{N_c}{8\pi^2} (J_{-1} [M_i [h_i], T, \mu_i] + C [T, \mu_i])$$

$$J_{-1}^{vac} [M, \Lambda] = -16\pi^2 \int \frac{d^4 p}{(2\pi)^4} \int_{E_0^2}^{E_M^2} dE^2 \hat{\rho} \frac{1}{E^2 + p_4^2}$$

$$J_{-1} [M, \Lambda, \mu, T] = -16\pi^2 \int \frac{d^3 \vec{p}}{(2\pi)^3} \int_{E_0^2}^{E_M^2} dE^2 T \sum_{n=-\infty}^{+\infty} \hat{\rho} \frac{1}{E^2 + (\pi(2n+1)T - i\mu)^2}$$

$$= - \int \frac{d^3 \vec{p}}{(2\pi)^3} \int_{E_0^2}^{E_M^2} dE^2 \hat{\rho} \frac{8\pi^2}{E} (1 - n_q [E, \mu, T] - n_{\bar{q}} [E, \mu, T])$$

$$\hat{\rho}_{PV}^E = 1 - (1 - \Lambda^2 \frac{\partial}{\partial E^2}) e^{-\Lambda^2 \frac{\partial}{\partial E^2}}$$

$$C(T, \mu) = \int \frac{d^3 p}{(2\pi)^3} 16\pi^2 T \log \left(\left(1 + e^{-\frac{|p|-\mu}{T}} \right) \left(1 + e^{-\frac{|p|+\mu}{T}} \right) \right)$$

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Inclusion of the Polyakov loop.

Introduce homogeneous background A_4 gluonic field

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + iA^\mu, \quad A^\mu = \delta_0^\mu g A_a^0 \frac{\lambda^a}{2}, \quad L = \mathcal{P} e^{\int_0^\beta dx_4 i A_4}, \quad \phi = \frac{1}{N_c} \text{Tr} L, \quad \bar{\phi} = \frac{1}{N_c} \text{Tr} L^\dagger$$

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- \sim order parameter (exact in the quenched limit) for (de)confinement ($\phi = 0 \leftrightarrow$ confined)

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Introduce homogeneous background A_4 gluonic field

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + \imath A^\mu, \quad A^\mu = \delta_0^\mu g A_a^0 \frac{\lambda^a}{2}, \quad L = \mathcal{P} e^{\int_0^\beta dx_4 \imath A_4}, \quad \phi = \frac{1}{N_c} \text{Tr} L, \quad \bar{\phi} = \frac{1}{N_c} \text{Tr} L^\dagger$$

Polyakov loop:

- \sim order parameter (exact in the quenched limit) for (de)confinement ($\phi = 0 \leftrightarrow$ confined)
- enters the action as an imaginary μ

$$n_q(M, p, \mu, T) = \left(1 + e^{(\sqrt{M^2 + p^2} - \mu)/T} \right)^{-1}$$

$$n_{\bar{q}}(M, p, \mu, T) = \left(1 + e^{(\sqrt{M^2 + p^2} + \mu)/T} \right)^{-1}$$

$$\tilde{n}_q(M, p, \mu, T, \phi, \bar{\phi}) \equiv \frac{1}{N_c} \sum_{i=1}^{N_c} n_q(M, p, \mu + \imath (A_4)_{ii}, T)$$

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$$n_{\bar{q}}(M, p, \mu, T) = \left(1 + e^{(\sqrt{M^2 + p^2} + \mu)/T} \right)^{-1}$$

$$\tilde{n}_q(M, p, \mu, T, \phi, \bar{\phi}) \equiv \frac{1}{N_c} \sum_{i=1}^{N_c} n_q(M, p, \mu + \imath (A_4)_{ii}, T)$$

$$\tilde{n}_{\bar{q}}(M, p, \mu, T, \phi, \bar{\phi}) \equiv \frac{1}{N_c} \sum_{i=1}^{N_c} n_{\bar{q}}(M, p, \mu + \imath (A_4)_{ii}, T)$$

- $\Omega \left[M_i, T, \mu, \phi, \bar{\phi} \right] = \mathcal{V}st[h_i] + \frac{N_c}{8\pi^2} \sum_{f=u,d,s} \left(J_{-1} \left[M_f, T, \mu, \phi, \bar{\phi} \right] + C(T, \mu) \right) + \mathcal{U} \left[\phi, \bar{\phi}, T \right]$



Polyakov potentials

■ Logarithmic form²

($a_0 = 3.51$, $a_1 = -2.47$, $a_2 = 15.2$, $b_3 = -1.75$, $T_0 = 200$ MeV) :

$$\frac{\mathcal{U}_I[\phi, \bar{\phi}, T]}{T^4} = -\frac{1}{2}a[T]\bar{\phi}\phi + b[T]\ln\left[1 - 6\bar{\phi}\phi + 4(\bar{\phi}^3 + \phi^3) - 3(\bar{\phi}\phi)^2\right]$$

$$a[T] = a_0 + a_1\frac{T_0}{T} + a_2\left(\frac{T_0}{T}\right)^2; \quad b[T] = b_3\left(\frac{T_0}{T}\right)^3$$

■ Exponential K-Log form³



($a_0 = 6.75$, $a_1 = -9.8$, $a_2 = 0.26$, $b_3 = 0.805$, $b_4 = 7.555$, $K = 0.1$, $T_0 = 175$ MeV):

$$\frac{\mathcal{U}'_{II}[\phi, \bar{\phi}, T]}{T^4} = -\frac{1}{2}a[T]\bar{\phi}\phi - \frac{b_3}{6}(\bar{\phi}^3 + \phi^3) + \frac{b_4}{4}(\bar{\phi}\phi)^2 + K\ln\left[\frac{27}{24\pi^2}\left(1 - 6\bar{\phi}\phi + 4(\bar{\phi}^3 + \phi^3) - 3(\bar{\phi}\phi)^2\right)\right]$$

$$a[T] = a_0 + a_1\left(\frac{T_0}{T}\right)e^{-a_2\frac{T_0}{T}}$$

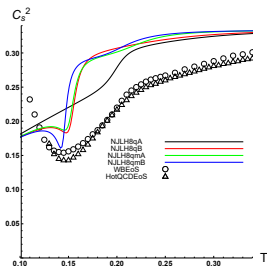
$$\mathcal{U}_{II}[\phi, \bar{\phi}, T] = \mathcal{U}'_{II}[\phi, \bar{\phi}, T] - \mathcal{U}'_{II}[0, 0, T] \quad (1)$$

²S. Rössner, C. Ratti, and W. Weise, Phys. Rev. D 75, 034007 (2007)

³A. Bhattacharyya, S. K. Ghosh, S. Maity, S. Raha, R. Ray, K. Saha, and S. Upadhyaya, Phys. Rev. D 95, 054005 (2017)  

NJL: C_S^2 and $\Theta_{\mu}^{\mu} |_{\mu=0}$ vs IQCD ⁴

$$\blacksquare C_S^2 \equiv \frac{\partial P}{\partial \epsilon} = \frac{-\frac{\partial \Omega}{\partial T}}{T \frac{\partial^2 \Omega}{\partial T^2}}$$

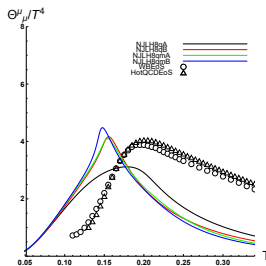
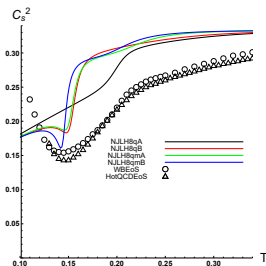


⁴WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
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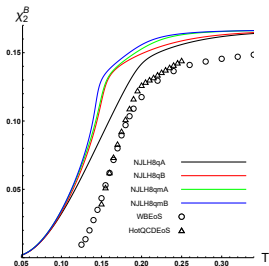
$$\blacksquare \Theta^\mu_\mu = \epsilon - 3P$$



⁴WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
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NJL: $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ⁵

$$\blacksquare \chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$



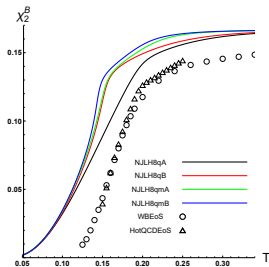
$$\chi_2^B = \frac{1}{9} (\chi_2^u + \chi_2^d + \chi_2^s + 2\chi_{11}^{us} + 2\chi_{11}^{ds} + 2\chi_{11}^{ud})$$

⁵WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat]

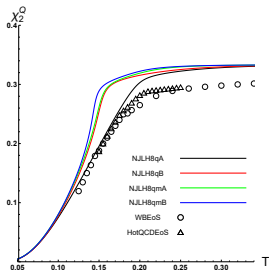
NJL: $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ⁵

$$\blacksquare \chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

$$\blacksquare \chi_2^Q = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_Q}{T})^2}$$



$$\chi_2^B = \frac{1}{9} (\chi_2^u + \chi_2^d + \chi_2^s + 2\chi_{11}^{us} + 2\chi_{11}^{ds} + 2\chi_{11}^{ud})$$



$$\chi_2^Q = \frac{1}{9} (4\chi_2^u + \chi_2^d + \chi_2^s - 4\chi_{11}^{us} + 2\chi_{11}^{ds} - 4\chi_{11}^{ud})$$

⁵WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012),

arXiv:1112.4416 [hep-lat].

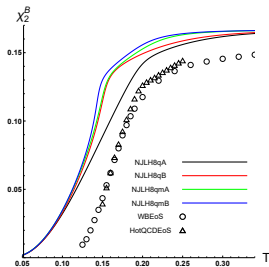
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

NJL: $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ⁵

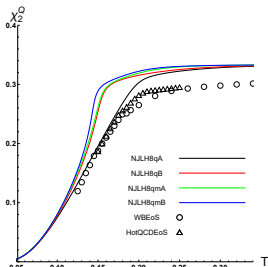
$$\blacksquare \chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

$$\blacksquare \chi_2^Q = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_Q}{T})^2}$$

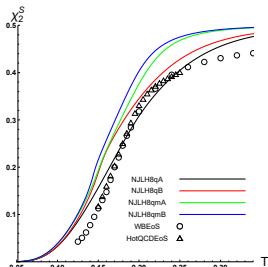
$$\blacksquare \chi_2^S = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_S}{T})^2}$$



$$\chi_2^B = \frac{1}{9} (\chi_2^u + \chi_2^d + \chi_2^s + 2\chi_{11}^{us} + 2\chi_{11}^{ds} + 2\chi_{11}^{ud})$$



$$\chi_2^Q = \frac{1}{9} (4\chi_2^u + \chi_2^d + \chi_2^s - 4\chi_{11}^{us} + 2\chi_{11}^{ds} - 4\chi_{11}^{ud})$$

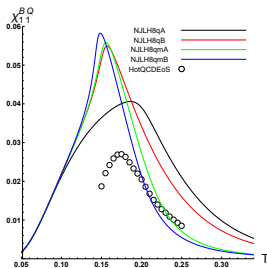


$$\chi_2^S = \chi_2^s$$

⁵WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

NJL: $\chi_{11}^{BQ}, \chi_{11}^{QS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ⁶

$$\blacksquare \chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$



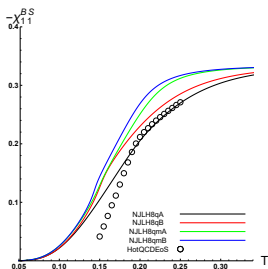
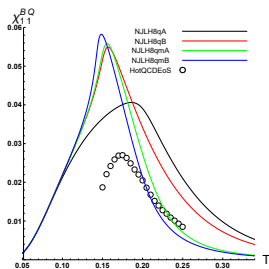
$$\chi_{11}^{BQ} = \frac{1}{9} (2\chi_2^u - \chi_2^d - \chi_2^s + \chi_{11}^{ud} + \chi_{11}^{us} - 2\chi_{11}^{ds})$$

⁶ HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

NJL: $\chi_{11}^{BQ}, \chi_{11}^{QS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ⁶

$$\blacksquare \chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$

$$\blacksquare \chi_{11}^{BS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_S}{T})}$$



$$\chi_{11}^{BQ} = \frac{1}{9} (2\chi_2^u - \chi_2^d - \chi_2^s + \chi_{11}^{ud} + \chi_{11}^{us} - 2\chi_{11}^{ds})$$

$$\chi_{11}^{BS} = -\frac{1}{3} (\chi_2^s + \chi_{11}^{us} + \chi_{11}^{ds})$$

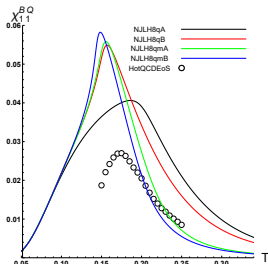
⁶ HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

NJL: $\chi_{11}^{BQ}, \chi_{11}^{QS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ⁶

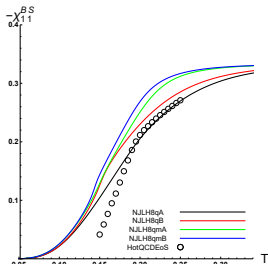
$$\blacksquare \chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$

$$\blacksquare \chi_{11}^{BS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_S}{T})}$$

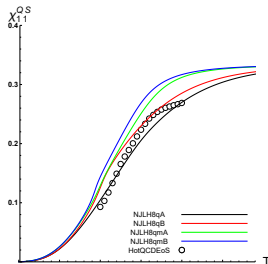
$$\blacksquare \chi_{11}^{QS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_Q}{T}) \partial(\frac{\mu_S}{T})}$$



$$\chi_{11}^{BQ} = \frac{1}{9} (2\chi_2^u - \chi_2^d - \chi_2^s + \chi_{11}^{ud} + \chi_{11}^{us} - 2\chi_{11}^{ds})$$



$$\chi_{11}^{BS} = -\frac{1}{3} (\chi_2^s + \chi_{11}^{us} + \chi_{11}^{ds})$$

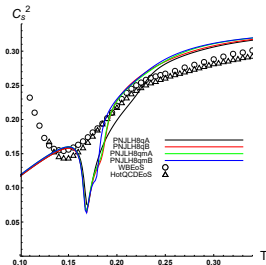


$$\chi_{11}^{QS} = \frac{1}{3} (\chi_2^s - 2\chi_{11}^{us} + \chi_{11}^{ds})$$

⁶ HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Log): C_S^2 and $\Theta^\mu{}_\mu |_{\mu=0}$ vs IQCD ⁷

$$\blacksquare C_S^2 \equiv \frac{\partial P}{\partial \epsilon} = \frac{-\frac{\partial \Omega}{\partial T}}{T \frac{\partial^2 \Omega}{\partial T^2}}$$

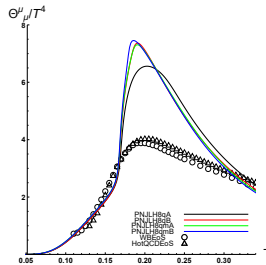
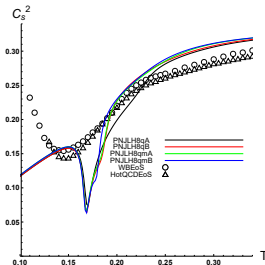


⁷WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D90, 094503 (2014), arXiv:1407.6387 [hep-lat]

PNJL (Log): C_s^2 and $\Theta^\mu{}_\mu \big|_{\mu=0}$ vs IQCD ⁷

$$\blacksquare C_s^2 \equiv \frac{\partial P}{\partial \epsilon} = \frac{-\frac{\partial \Omega}{\partial T}}{T \frac{\partial^2 \Omega}{\partial T^2}}$$

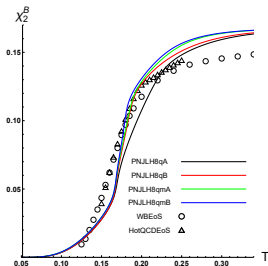
$$\blacksquare \Theta^\mu{}_\mu = \epsilon - 3P$$



⁷WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D90, 094503 (2014), arXiv:1407.6387 [hep-lat]

PNJL (Log): $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ⁸

$$\chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

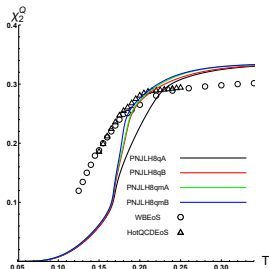
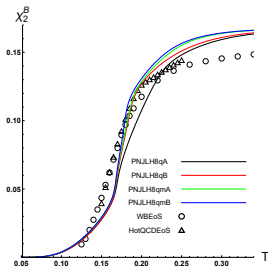


⁸ WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
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PNJL (Log): $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ⁸

$$\chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

$$\chi_2^Q = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_Q}{T})^2}$$



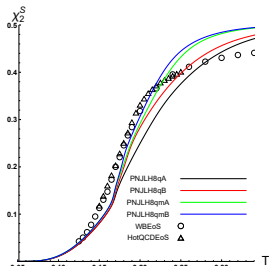
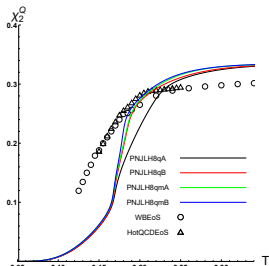
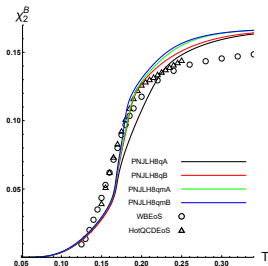
⁸WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
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PNJL (Log): $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ⁸

$$\chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

$$\chi_2^Q = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_Q}{T})^2}$$

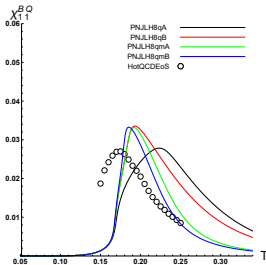
$$\chi_2^S = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_S}{T})^2}$$



⁸WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Log): $\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ⁹

$$\chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$

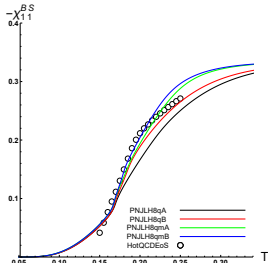
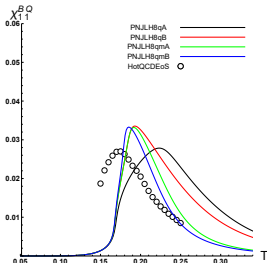


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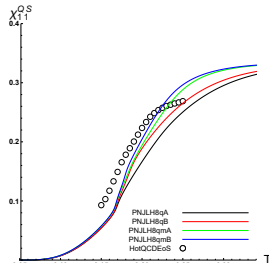
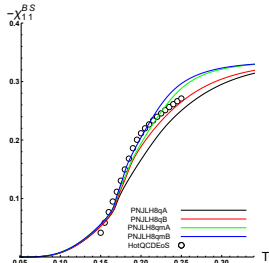
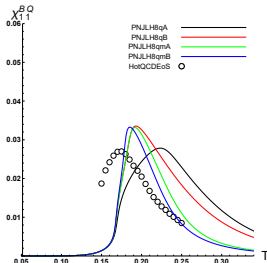
⁹ HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Log): $\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ⁹

$$\chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$

$$\chi_{11}^{BS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_S}{T})}$$

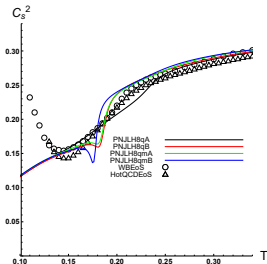
$$\chi_{11}^{QS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_Q}{T}) \partial(\frac{\mu_S}{T})}$$



⁹ HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): C_S^2 and $\Theta_\mu^\mu |_{\mu=0}$ vs IQCD ¹⁰

$$\blacksquare C_S^2 \equiv \frac{\partial P}{\partial \epsilon} = \frac{-\frac{\partial \Omega}{\partial T}}{T \frac{\partial^2 \Omega}{\partial T^2}}$$

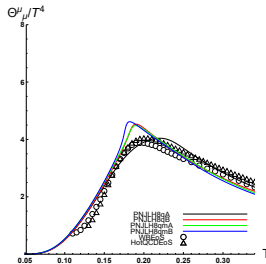
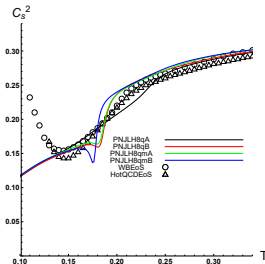


¹⁰ WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
 HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D90, 094503 (2014), arXiv:1407.6387 [hep-lat]

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$$\blacksquare C_s^2 \equiv \frac{\partial P}{\partial \epsilon} = \frac{-\frac{\partial \Omega}{\partial T}}{T \frac{\partial^2 \Omega}{\partial T^2}}$$

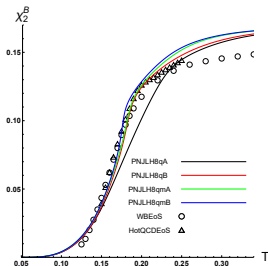
$$\blacksquare \Theta^\mu{}_\mu = \epsilon - 3P$$



¹⁰WB: S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg, and K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258 [hep-lat]
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D90, 094503 (2014), arXiv:1407.6387 [hep-lat]

PNJL (Exp K-Log): $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ¹¹

$$\chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

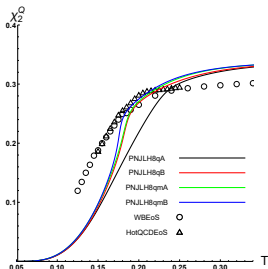
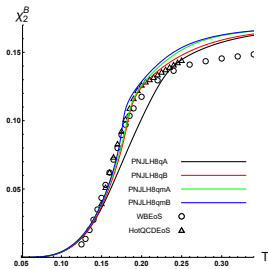


¹¹ WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
 HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ¹¹

$$\chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

$$\chi_2^Q = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_Q}{T})^2}$$



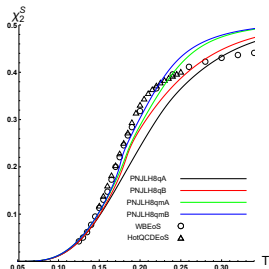
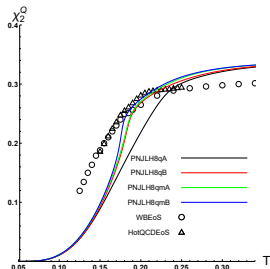
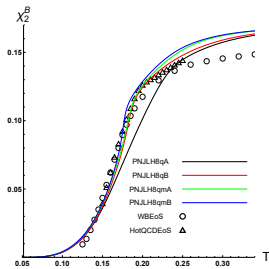
¹¹ WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): $\chi_2^B, \chi_2^Q, \chi_2^S |_{\mu=0}$ vs IQCD ¹¹

$$\blacksquare \chi_2^B = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_B}{T})^2}$$

$$\blacksquare \chi_2^Q = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_Q}{T})^2}$$

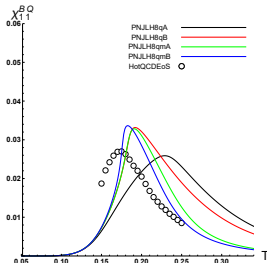
$$\blacksquare \chi_2^S = \frac{1}{2} \frac{\partial^2 \Omega / T^4}{\partial (\frac{\mu_S}{T})^2}$$



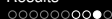
¹¹ WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].
 HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): $\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ¹²

$$\chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$



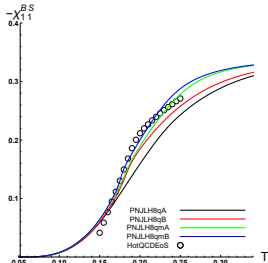
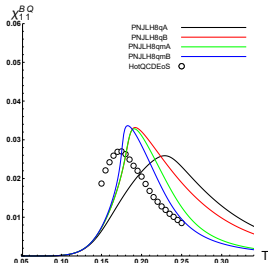
¹²HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].



PNJL (Exp K-Log): $\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ¹²

$$\blacksquare \chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$

$$\blacksquare \chi_{11}^{BS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_S}{T})}$$



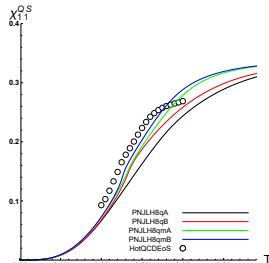
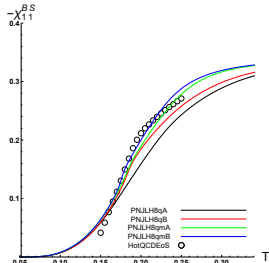
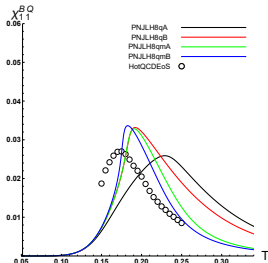
¹²HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL (Exp K-Log): $\chi_{11}^{BQ}, \chi_{11}^{BS}, \chi_{11}^{QS} |_{\mu=0}$ vs IQCD ¹²

$$\blacksquare \chi_{11}^{BQ} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_Q}{T})}$$

$$\blacksquare \chi_{11}^{BS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_B}{T}) \partial(\frac{\mu_S}{T})}$$

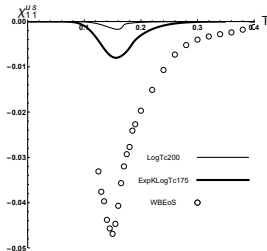
$$\blacksquare \chi_{11}^{QS} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_Q}{T}) \partial(\frac{\mu_S}{T})}$$



¹²HotQCD: A. Bazavov et al. (HotQCD), Phys. Rev. D86, 034509 (2012), arXiv:1203.0784 [hep-lat].

PNJL: $\chi_{11}^{us} |_{\mu=0}$ vs IQCD¹³ : gluonic signature?

$$\chi_{11}^{us} = \frac{\partial^2 \Omega / T^4}{\partial(\frac{\mu_u}{T}) \partial(\frac{\mu_s}{T})}$$



¹³WB: S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti, and K. Szabo, JHEP 01, 138 (2012), arXiv:1112.4416 [hep-lat].

Conclusions

- Multiquark interaction and full pattern of explicit chiral symmetry breaking play a key role in the reproduction of several key IQCD results
- Perfect fit across the board is not achieved with this Polykov potential but very promising results
- PNJL can however shift several results in temperature towards IQCD data
- Correlations in the uds base disappear without Polyakov loop

Additional information:

- “Thermodynamical properties of strongly interacting matter in a model with explicit chiral symmetry breaking interactions” J. Moreira, J. Morais, B. Hiller, A.A. Osipov, A.H. Blin, e-Print: arXiv:1806.00327, Phys. Rev. D 98, 074010 (2018)
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