Operator relations for gravitational form factors

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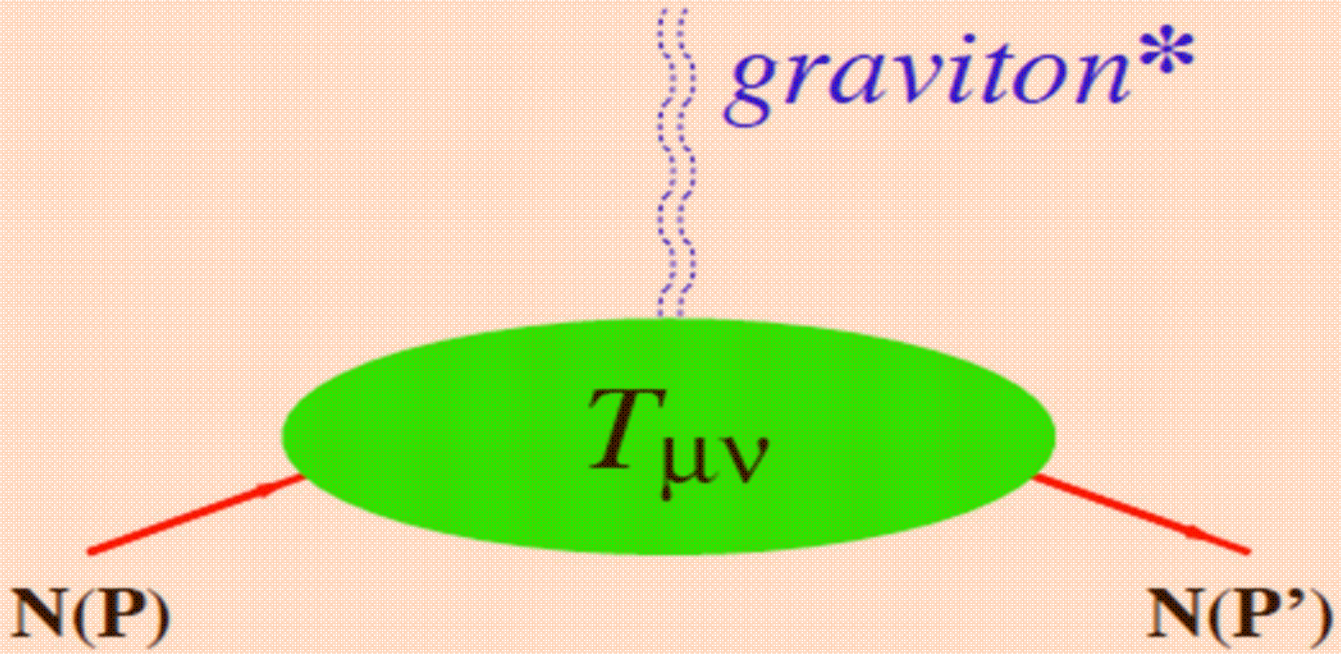
Y. Hatta, A. Rajan, KT, arXiv: 1810.05116
KT, arXiv: 1811.07879



$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\equiv \quad T_q^{\mu\nu} \quad + \quad T_g^{\mu\nu}$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\langle p' | T^{\mu\nu}_{q,g} | p \rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha}{2M}$$

$$+ D_{q,g}(t)\frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t) M g^{\mu\nu}\Big] u(p)$$

$$P = \frac{p + p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

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$$P = \frac{p+p'}{2}$$

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$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle p | T^{\mu\nu} | p \rangle = 2p^\mu p^\nu$$

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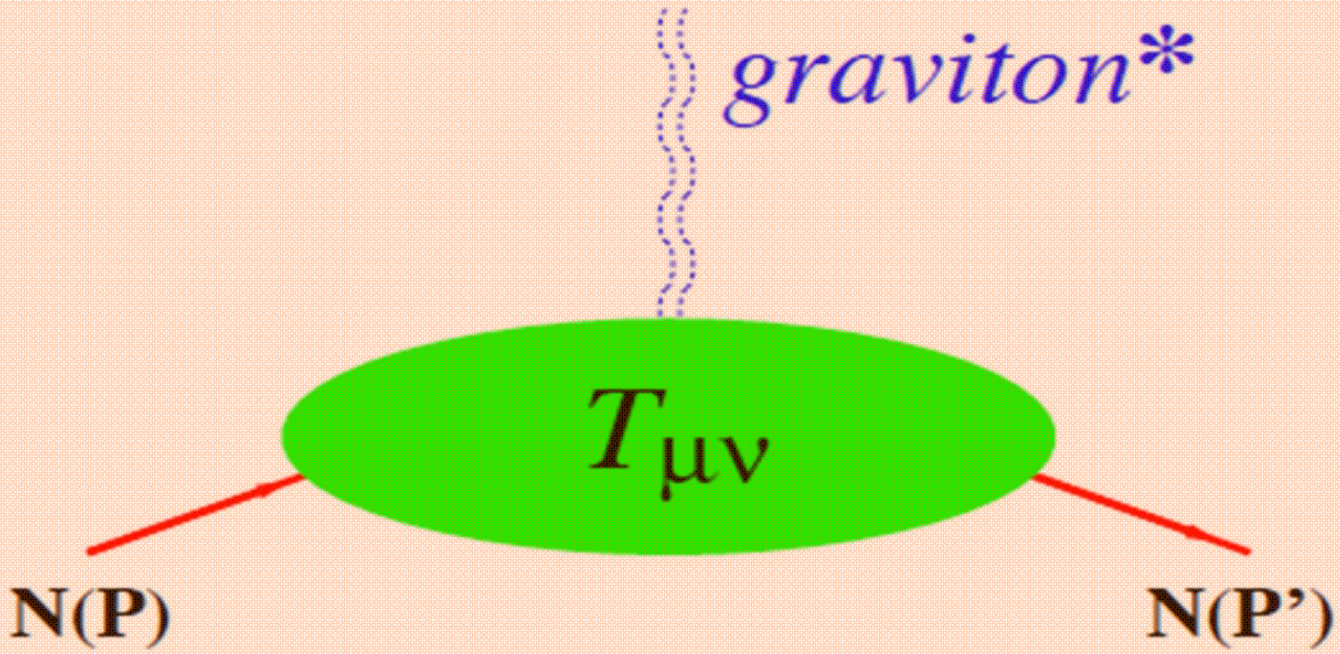
$$\frac{1}{2}\Big(A_q(0) + B_q(0) + A_g(0) + B_g(0) \Big) = \frac{1}{2}$$

$$B_q(0) + B_g(0) = 0$$

$$\frac{\langle pS | J^i | pS \rangle}{\langle pS | pS \rangle} = \frac{1}{2} S^i$$

$$J^i = \frac{1}{2} \epsilon^{ijk} \int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho}$$



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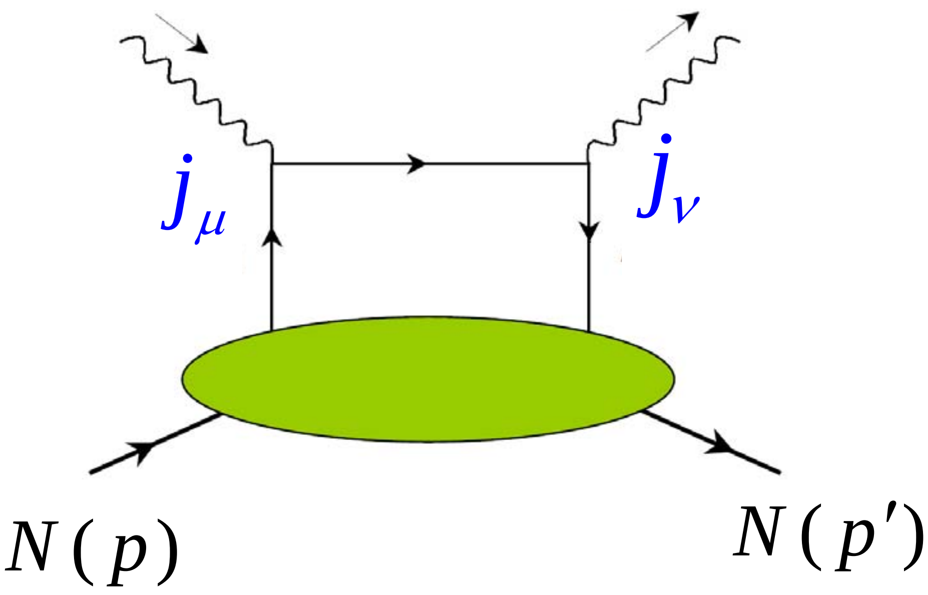


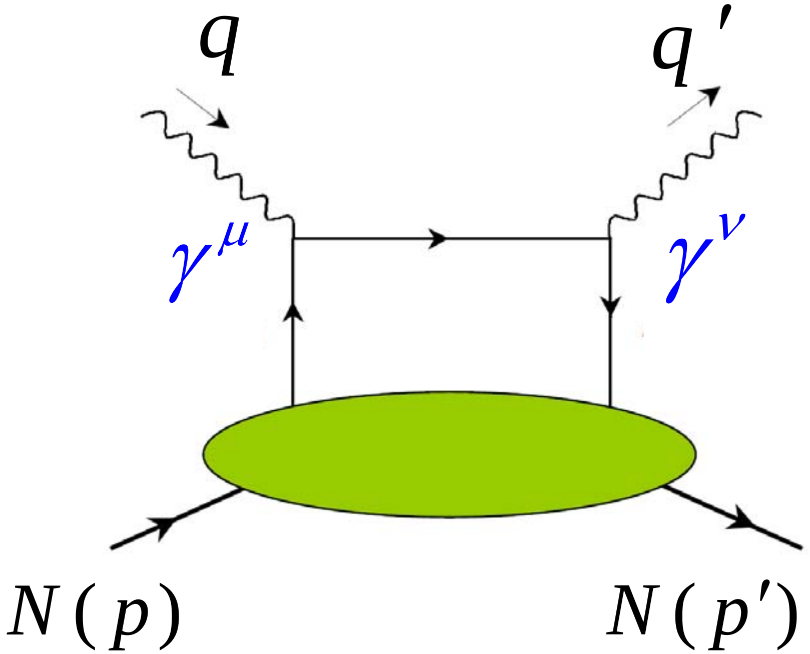
$$j_\mu(x)\, j_\nu(0) \sim \sum_i C_i(x) O_i(0)$$

$$C^q_{\mu\nu;\alpha\beta}(x) T_q^{\alpha\beta}$$

$$C^g_{\mu\nu;\alpha\beta}(x) T_g^{\alpha\beta}$$

DVCS

$$\int d^4x e^{iq'\cdot x} \langle p' | \mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x) | p \rangle$$



$$\bar{P} = \frac{p+p'}{2}, \quad \bar{q} = \frac{q+q'}{2}, \quad \Delta = p' - p = q - q',$$

$$t = \Delta^2, \quad \xi = \frac{-\bar{q}^2}{2\bar{P}\cdot\bar{q}}, \quad \eta = \frac{-\Delta\cdot n}{2\bar{P}\cdot n} = \frac{-\Delta\cdot\bar{q}}{2\bar{P}\cdot\bar{q}} + O(\text{twist-4}),$$

$$n_\mu = -\frac{2\xi}{\bar{q}^2}\left(\bar{q}_\mu + \xi\bar{P}_\mu\right), \quad \tilde{n}_\mu = \bar{P}_\mu, \qquad \left(n^2 = \tilde{n}^2 = 0, \quad n\cdot\tilde{n} = 1\right)$$

generalized Bjorken kinematics:

$$|\bar{q}^2| \to \infty, \quad |\bar{P}\cdot\bar{q}| \to \infty, \quad |\Delta\cdot\bar{q}| \to \infty, \quad \Delta^2 = \text{finite} \quad (\xi \text{ and } \eta \text{ fixed})$$

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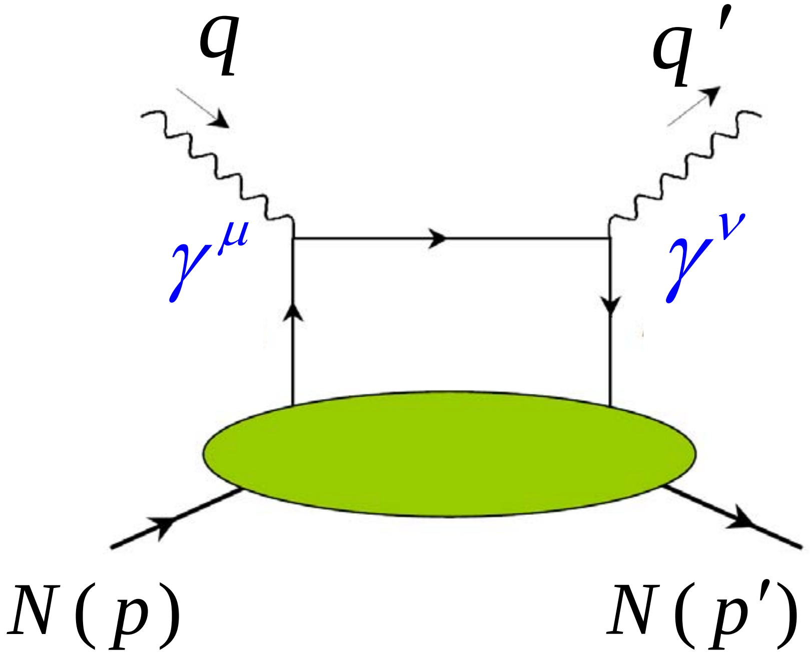
$$\int d^4x e^{iq'\cdot x} \langle p' | \mathrm{T}\, j_\mu^{\mathrm{em}}(0)\, j_\nu^{\mathrm{em}}(x) | p \rangle$$



$$\bar{P} = \frac{p+p'}{2}, \quad \bar{q} = \frac{q+q'}{2}, \quad \Delta = p' - p = q - q',$$

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generalized Bjorken kinematics: light-cone expansion

$$|\bar{q}^2| \to \infty, \quad |\bar{P}\cdot\bar{q}| \to \infty, \quad |\Delta\cdot\bar{q}| \to \infty, \quad \Delta^2 = \text{finite} \quad (\xi \text{ and } \eta \text{ fixed})$$

$$\int d^4x e^{iq'\cdot x}\langle p'|\mathrm{T}\, j_\mu^{\mathrm{em}}(0) j_\nu^{\mathrm{em}}(x)|p\rangle$$

$$= \mathcal{T}_{\mu\nu}^{(1)} \frac{ie_q^2}{\bar{q}^2}\int dx \left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2} - \frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2} \right) \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} < N(p')\left|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})\right| N(p) >\Big|_{z^+=\vec{z}_\perp=0}$$

$$+ \mathcal{T}_{\mu\nu}^{(2)} \frac{ie_q^2}{\bar{q}^2}\int dx \left(\frac{i}{1-\frac{x}{\xi}+i\epsilon\bar{q}^2} + \frac{i}{1+\frac{x}{\xi}+i\epsilon\bar{q}^2} \right) \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} < N(p')\left|\bar{q}(-\frac{z}{2})\gamma^+\gamma_5 q(\frac{z}{2})\right| N(p) >\Big|_{z^+=\vec{z}_\perp=0}$$

$$+ O(\text{twist-3})$$

$$\mathcal{T}_{\mu\nu}^{(1)} = \tilde{n}_\mu\left(\bar{q}_\nu + \xi\tilde{n}_\nu\right) + \tilde{n}_\nu\left(\bar{q}_\mu + \xi\tilde{n}_\mu\right) - g_{\mu\nu}\bar{q}\cdot\tilde{n} = \frac{\bar{q}^2}{2\xi}\left(g_{\mu\nu} - \frac{q'_\mu q_\nu}{q\cdot q'}\right) + 2\xi\left(\bar{P}_\mu - \frac{\bar{P}\cdot q}{q\cdot q'}q'_\mu\right)\left(\bar{P}_\nu - \frac{\bar{P}\cdot q'}{q\cdot q'}q_\nu\right)$$

$$\mathcal{T}_{\mu\nu}^{(2)} = i\varepsilon_{\mu\alpha\nu\rho}\bar{q}^\alpha\tilde{n}^\rho = i\varepsilon^{\lambda\beta\rho\sigma}\bar{P}_\rho\bar{q}_\sigma\left(g_{\mu\lambda} - \frac{\bar{P}_\mu q_\lambda}{\bar{P}\cdot q}\right)\left(g_{\nu\beta} - \frac{\bar{P}_\nu q'_\beta}{\bar{P}\cdot q'}\right)$$

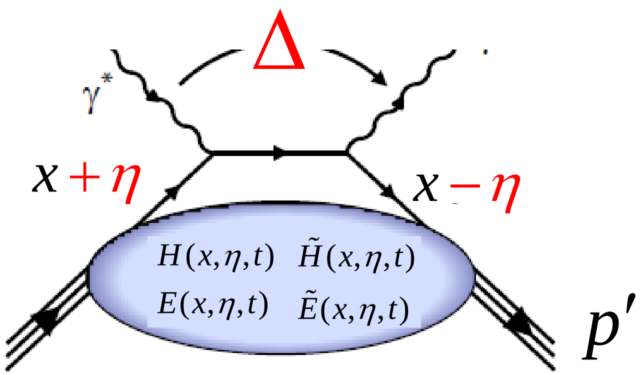
$$q^\mu\mathcal{T}_{\mu\nu}^{(1)} = q'^\nu\mathcal{T}_{\mu\nu}^{(1)} = q^\mu\mathcal{T}_{\mu\nu}^{(2)} = q'^\nu\mathcal{T}_{\mu\nu}^{(2)} = 0\,,$$

$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

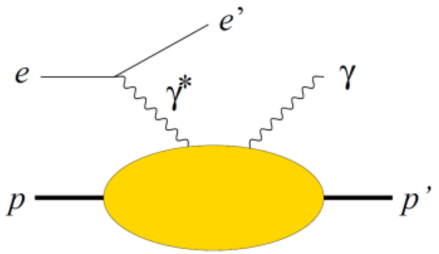
$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}^q(x,\eta,t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t) \bar{u}(p') \frac{\gamma_5 (p'-p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



$$\bar{P} = \frac{p+p'}{2}$$

JLab, HERMES, COMPASS,…

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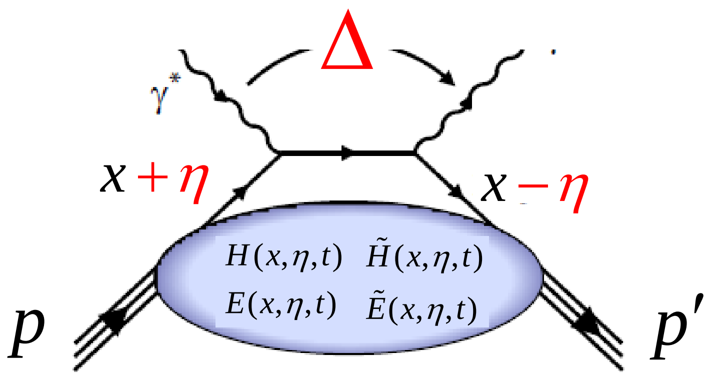

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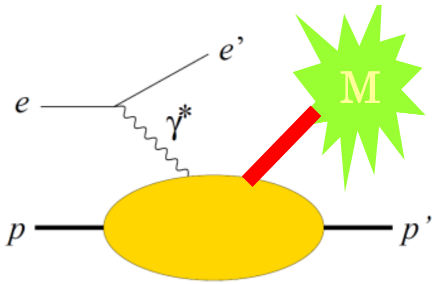

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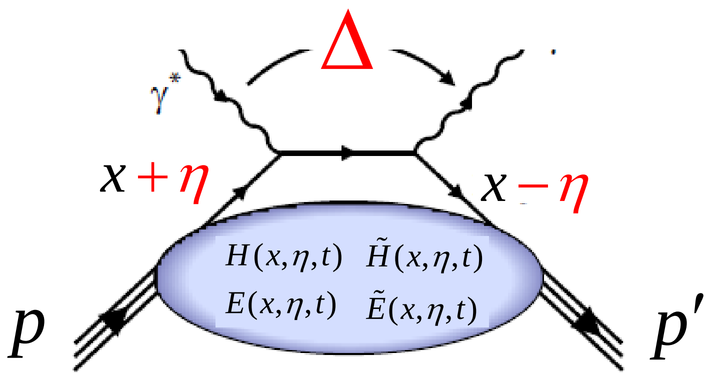

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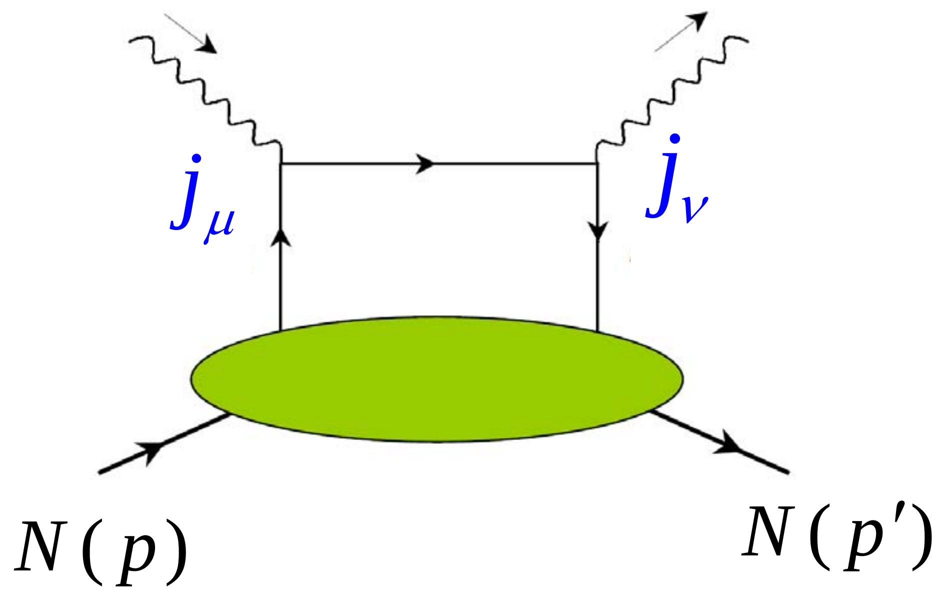

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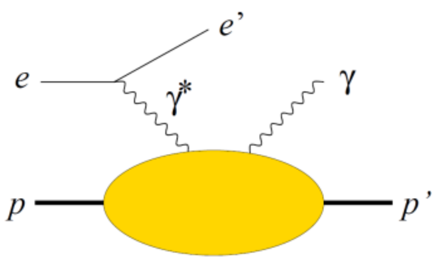

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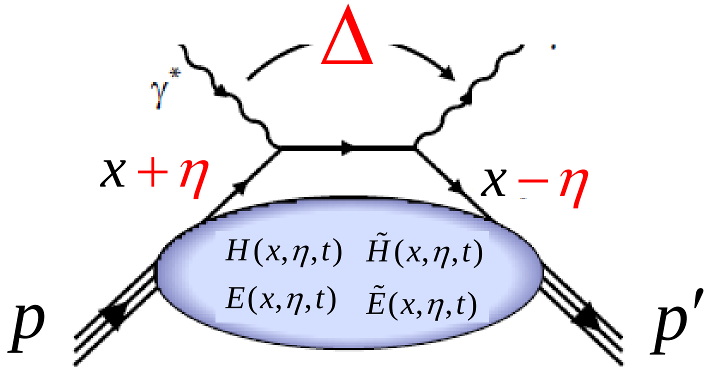

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$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

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$$H^q(x,0,0) = q(x)$$

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$$\int_{-1}^{1} dx H^q(x,\eta,t) = F_1^q(t), \qquad \int_{-1}^{1} dx E^q(x,\eta,t) = F_2^q(t)$$

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$$\int_{-1}^{1} dx x H^q(x,\eta,t) = \cdots \quad , \quad \int_{-1}^{1} dx x E^q(x,\eta,t) = \cdots$$

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$$\frac{1}{2P^+}\left\langle p' \middle| \bar{q}(0)\gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| p \right\rangle = \bar{u}(p')\gamma^+ u(p)\int_{-1}^{1} dx x H^q(x,\eta,t) + \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\int_{-1}^{1} dx x E^q(x,\eta,t)$$

$$= \frac{1}{P^+}\left\langle p' \middle| T_q^{++}(0) \middle| p \right\rangle$$

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$$\frac{1}{2P^+} \left\langle p' \middle| \bar{q}(0) \gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| p \right\rangle = \bar{u}(p') \gamma^+ u(p) \int_{-1}^{1} dx x H^q(x,\eta,t) + \bar{u}(p') \frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} u(p) \int_{-1}^{1} dx x E^q(x,\eta,t)$$

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$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \Big[A_{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} + \bar{C}_{q,g}(t) M g^{\mu\nu} \Big] u(p)$$

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$$\frac{1}{2P^+}\left\langle p' \middle| \bar{q}(0)\gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| p \right\rangle = \bar{u}(p')\gamma^+ u(p)\int_{-1}^{1} dx x H^q(x,\eta,t) + \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\int_{-1}^{1} dx x E^q(x\eta,t)$$

$$= \frac{1}{P^+}\left\langle p' \middle| T_q^{++}(0) \middle| p \right\rangle == \bar{u}(p')\gamma^+ u(p)\left(A_q(t) + 4\eta^2 D_q(t)\right) + \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\left(B_q(t) - 4\eta^2 D_q(t)\right)$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho}F_\rho{}^\nu + \frac{g^{\mu\nu}}{4}F^2$$

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$$P = \frac{p+p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2}) | p\rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M}u(p)\right]$$

$$\frac{1}{2P^+}\left\langle p' \middle| \bar{q}(0)\gamma^+ i\overleftrightarrow{D}^+ q(0) \middle| p \right\rangle = \bar{u}(p')\gamma^+ u(p)\int_{-1}^{1} dx x H^q(x,\eta,t) + \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\int_{-1}^{1} dx x E^q(x,\eta,t)$$

$$= \frac{1}{P^+}\left\langle p' \middle| T_q^{++}(0) \middle| p \right\rangle == \bar{u}(p')\gamma^+ u(p)\left(A_q(t) + 4\eta^2 D_q(t)\right) + \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(p)\left(B_q(t) - 4\eta^2 D_q(t)\right)$$

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$$\Delta^\mu = p'^\mu - p^\mu \to 0$$

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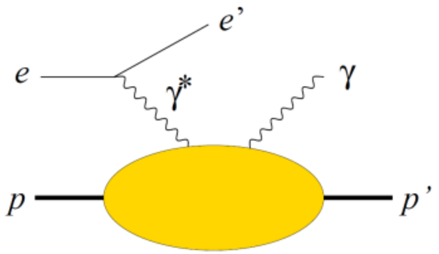


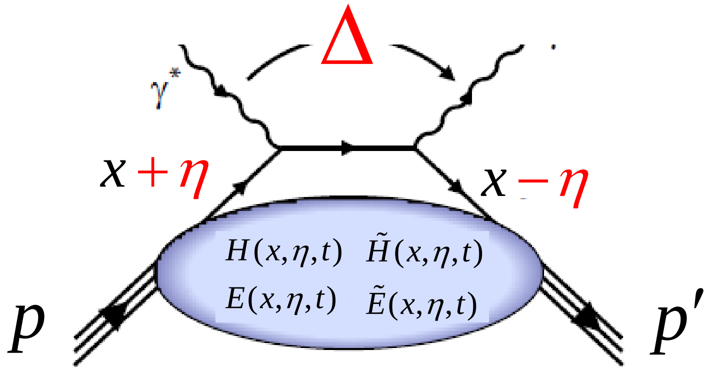
$$\bar{P} = \frac{p+p'}{2}$$

JLab, HERMES, COMPASS,...

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2})\gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+}\left[H^q(x,\eta,t)\bar{u}(p')\gamma^+ u(p) + E^q(x,\eta,t)\bar{u}(p')\frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2})\gamma^+ \gamma_5 q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+}\left[\tilde{H}^q(x,\eta,t)\bar{u}(p')\gamma^+ \gamma_5 u(p) + \tilde{E}^q(x,\eta,t)\bar{u}(p')\frac{\gamma_5(p'-p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz} \langle N(p') | \psi^\dagger(0)\psi(z^-) | N(p) \rangle$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}z} \langle p' | \bar{q}(-\frac{z^-}{2}) \gamma^+ q(\frac{z^-}{2}) | p \rangle = \frac{1}{\bar{P}^+} \left[H^q(x,\eta,t) \bar{u}(p') \gamma^+ u(p) + E^q(x,\eta,t) \bar{u}(p') \frac{i\sigma^{+\alpha}(p'-p)_\alpha}{2M} u(p) \right]$$

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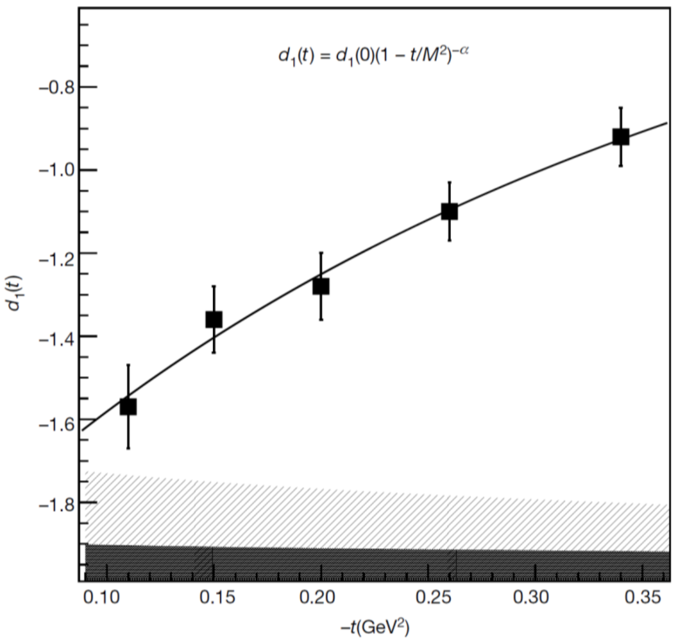

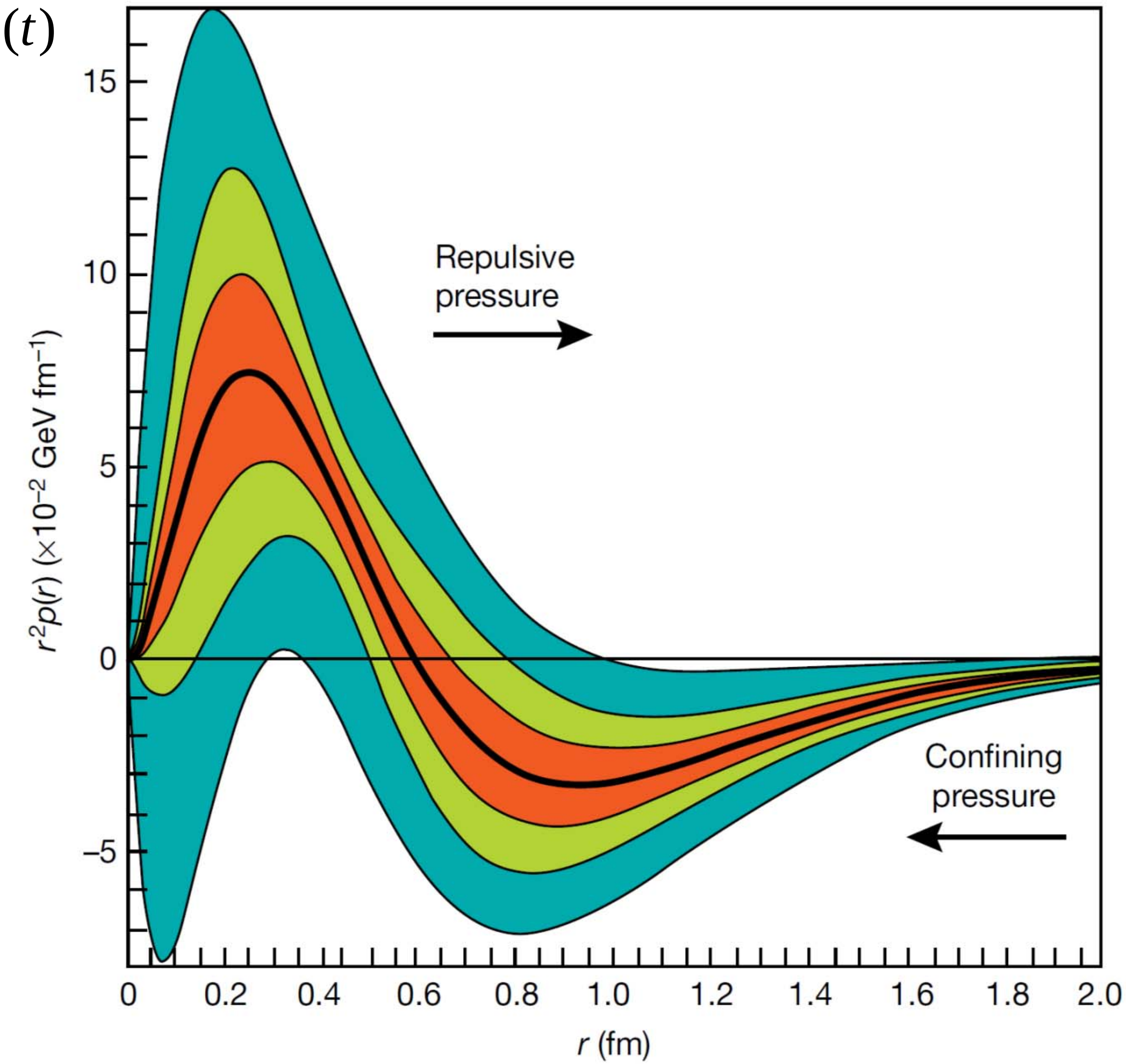

$$\langle p'|T^{ik}(0)|p\rangle \sim \left(\Delta^i\Delta^k - \delta^{ik}\boldsymbol{\Delta}^2\right)D(t), \quad T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\, p(r)$$

KT, PRD98, 034009

from QCD EOM: $\left[\hat{P}^{\mu}, q(x)\right] = -i\partial^{\mu} q(x)$

$$A_q(t) + \eta^2 D_q(t) \simeq \langle x \rangle F_1^q(t), \quad B_q(t) - \eta^2 D_q(t) \simeq \langle x \rangle F_2^q(t)$$

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KT, PRD98, 034009

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KT, PRD98, 034009

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$$-\frac{P^+\eta\Delta_\perp^\mu \bar{u}(p')u(p)}{M} D_q(t) \simeq \langle p'|\bar{q} g A_\perp^\mu \gamma^+ q|p\rangle$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} d\lambda\, \mathrm{sgn}(\lambda) n_\alpha \langle p'|g F_a^{\mu\alpha}(\lambda n)\bar{q}(0) t^a \gamma^+ q(0)|p\rangle$$

KT, PRD98, 034009

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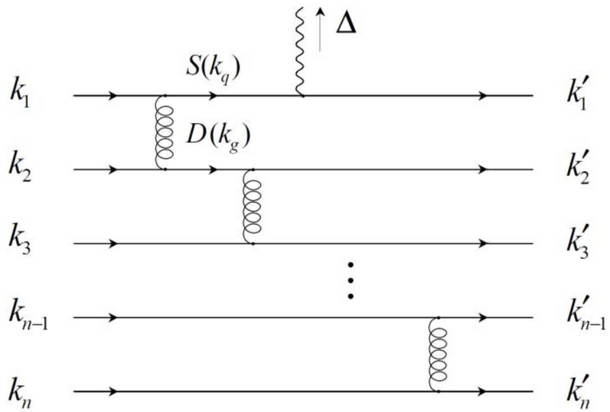
$t \to \infty$

$$A_q(t) \sim \frac{1}{t^2}, \qquad D_q(t) \sim \frac{1}{t^3}$$



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KT, PRD98, 034009

for pion

$$\langle p' | T_q^{\mu\nu} | p\rangle = \frac{1}{2}\Theta_{2q}(t)P^\mu P^\nu + \frac{1}{2}\Theta_{1q}(t)\left(tg^{\mu\nu} - \Delta^\mu\Delta^\nu\right) + \Lambda^2\bar{C}_q(t)g^{\mu\nu}$$

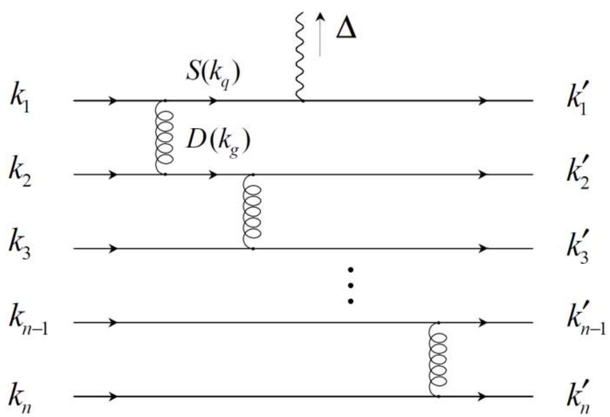
$$\frac{1}{2}\Theta_{2q}(t) - 2\eta^2\Theta_{1q}(t) \simeq 2\langle x\rangle F_v(t)$$

$$2\eta\Delta_\perp^\mu\Theta_{1q}(t) \simeq \langle p' | \bar{q}gA_\perp^\mu \not{n} q | p\rangle$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} d\lambda\, \mathrm{sgn}(\lambda) n_\alpha \langle p' | gF_a^{\mu\alpha}(\lambda n)\bar{q}(0)t^a \not{n} q(0) | p\rangle$$

$$t \to \infty$$

$$\Theta_{2q}(t) \sim \frac{1}{t}, \qquad \Theta_{1q}(t) \sim \frac{1}{t^2}$$



KT, PRD98, 034009

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$t \to \infty$

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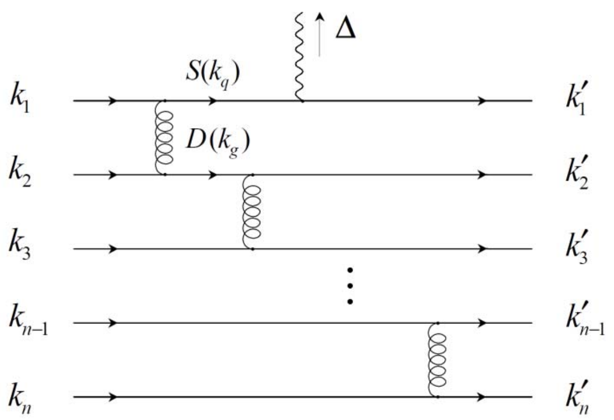


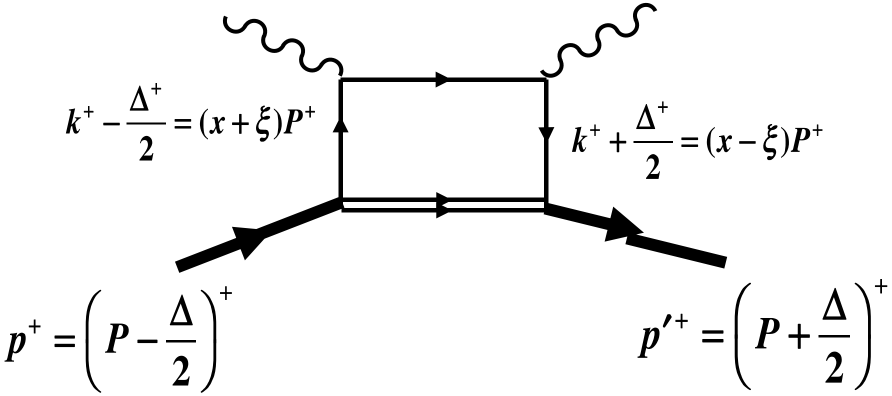
Kumano, Song, Teryaev, PRD97, 014020 ('18)

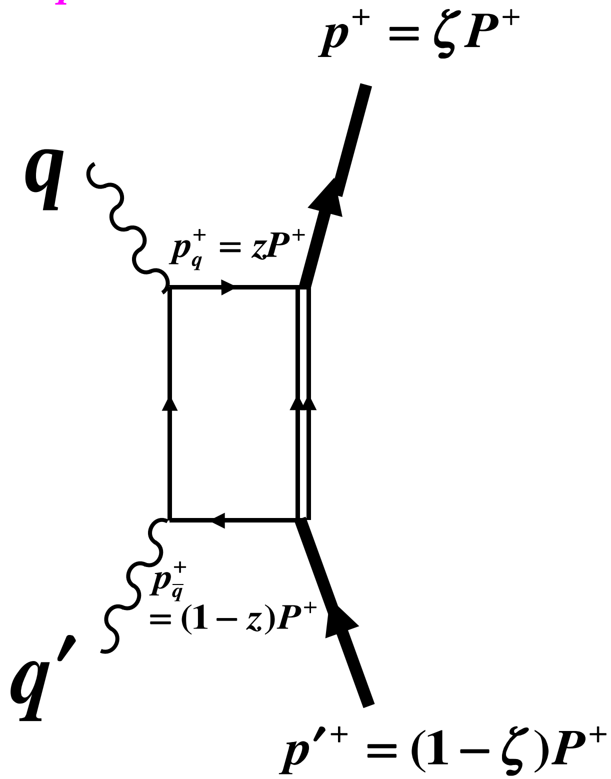
GPD: $$H_q(x,\xi,t)=\int\frac{dy^-}{4\pi}e^{ixP^+y^-}\langle\pi^0(p')|\bar{\psi}(-y/2)\gamma^+\psi(y/2)|\pi^0(p)\rangle\Big|_{y^+=0,\vec{y}_\perp=0},\qquad P^+=\frac{(p+p')^+}{2}$$

GDA: $$\Phi_q(z,\zeta,s)=\int\frac{dy^-}{2\pi}e^{izP^+y^-}\langle\pi^0(p)\pi^0(p')|\bar{\psi}(-y/2)\gamma^+\psi(y/2)|0\rangle\Big|_{y^+=0,\vec{y}_\perp=0}$$

$H_q^h(x,\xi,t)$ ⟷ $\Phi_q^{hh}(z,\zeta,W^2)$

s-t crossing





Song's talk

KT, PRD98, 034009

from QCD EOM: $\left[\hat{P}^{\mu}, q(x)\right] = -i\partial^{\mu} q(x)$

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$t \to \infty$

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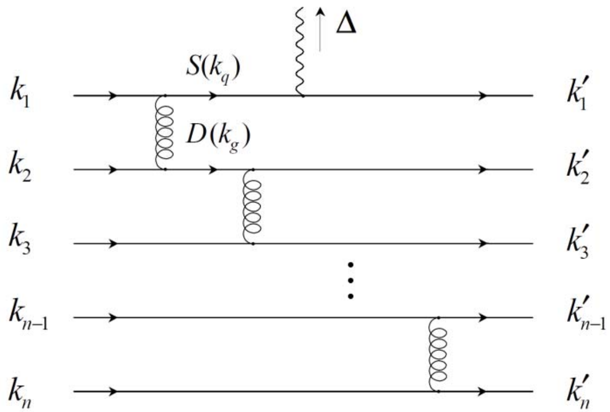
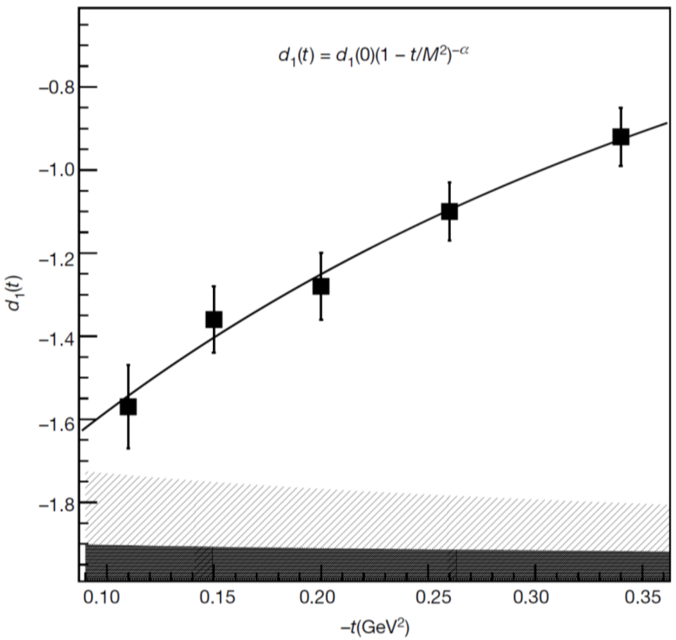
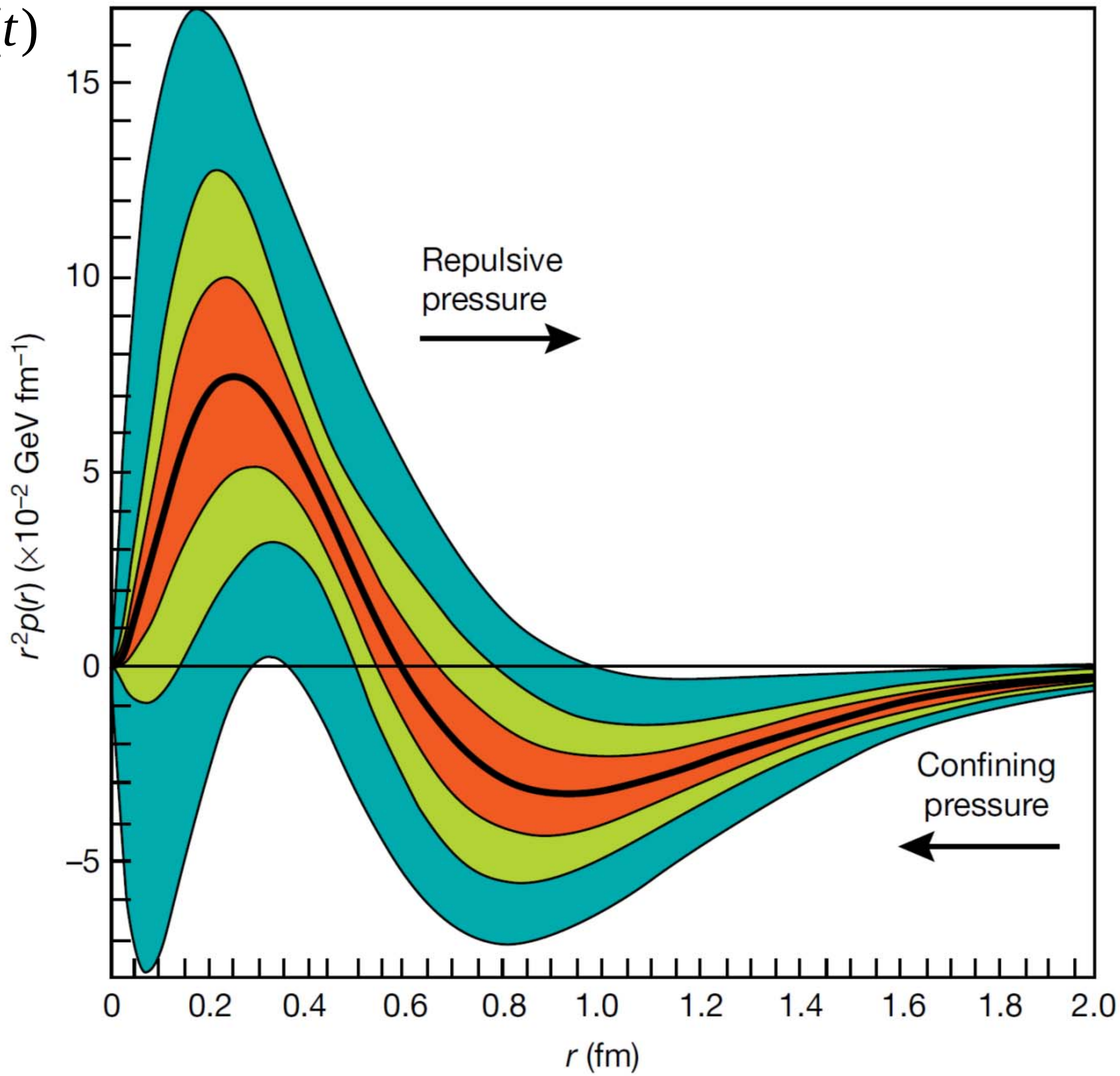


V. D. Burkert et al, Nature 557 ('18) 396

$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t)$$

$$5\sum_q e_q^2 D_q(t) \sim d_1(t)$$





$$\langle p' | T^{ik}(0) | p \rangle \sim \left(\Delta^i \Delta^k - \delta^{ik} \boldsymbol{\Delta}^2\right) D(t), \quad T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij}\, p(r)$$

V. D. Burkert et al, Nature 557 ('18) 396

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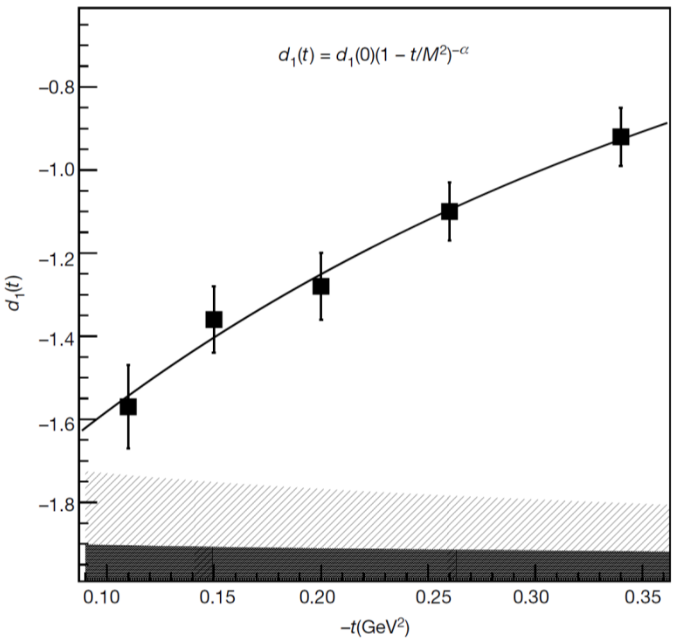

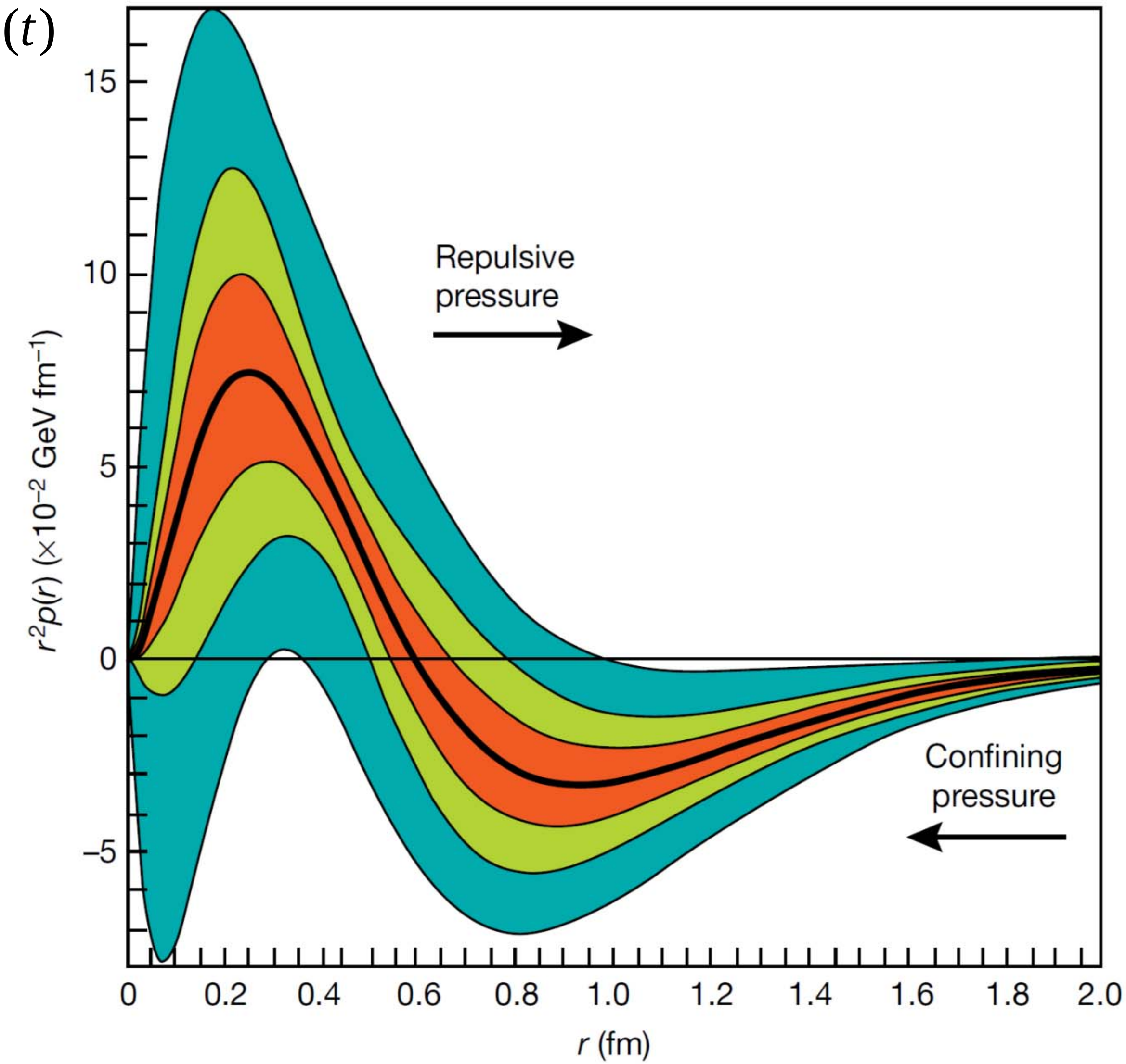

$$D(t) = D_q(t) + D_g(t)$$

$$\langle p' | T^{ik}(0) | p \rangle \sim \left(\Delta^i \Delta^k - \delta^{ik} \boldsymbol{\Delta}^2\right) D(t), \quad T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r)$$

V. D. Burkert et al, Nature 557 ('18) 396

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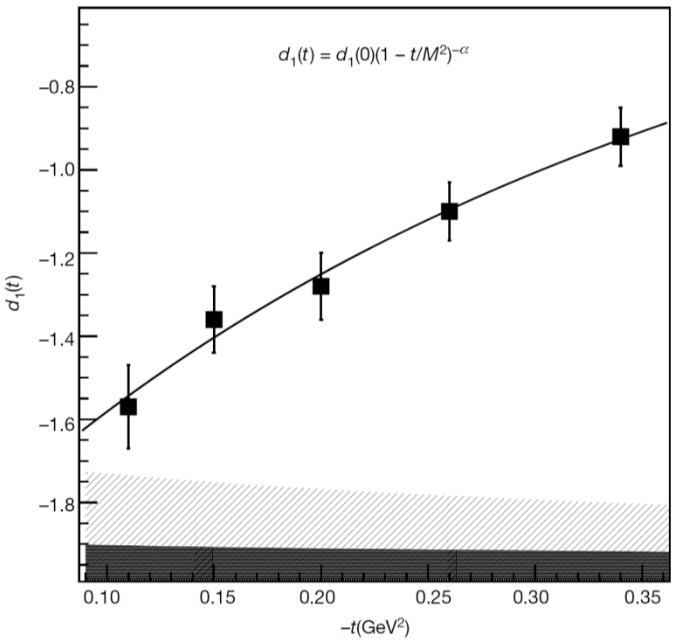

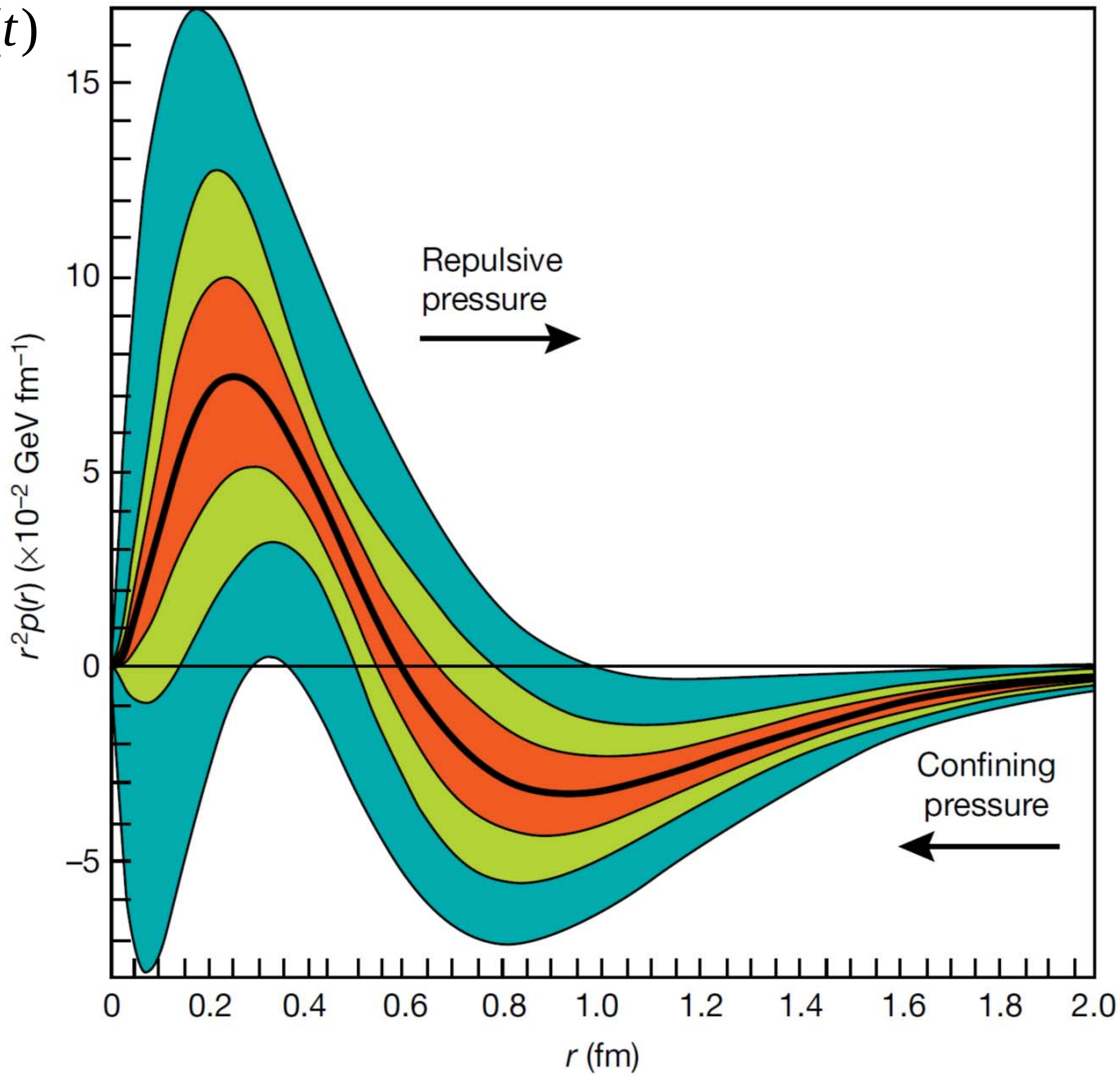

$$\langle p' | T_q^{ik}(0) | p\rangle \sim \left(\Delta^i\Delta^k - \delta^{ik}\boldsymbol{\Delta}^2\right) D_q(t) - 4M^2\delta^{ik}\bar{C}_q(t)$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \;+\; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

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$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle p | T^{\mu\nu} | p\rangle = 2p^\mu p^\nu$$

$$\frac{1}{2}\Big(A_q(0) + B_q(0) + A_g(0) + B_g(0)\Big) = \frac{1}{2}$$

$$B_q(0) + B_g(0) = 0$$

$$\frac{\langle pS | J^i | pS\rangle}{\langle pS | pS\rangle} = \frac{1}{2}S^i$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho}$$

$$\left(i\not{D}-m\right)q=0$$

KT, PRD98, 034009

$$\partial_\nu T_q^{\mu\nu}=-\bar{q}gF^{\mu\nu}\gamma_\nu q,\qquad \partial_\nu T_g^{\mu\nu}=-F_a^{\mu\nu}D_{ab}^{\rho}F_{\rho\nu}^{b}$$

$$\Delta^\mu\bar{u}(p')u(p)M\bar{C}_q(t)=\langle p'|\bar{q}igF^{\mu\nu}\gamma_\nu q|p\rangle$$

$$\Delta^\mu\bar{u}(p',S')u(p,S)M\bar{C}_g(t)=\langle p'|F_a^{\mu\nu}iD_{ab}^{\rho}F_{\rho\nu}^{b}|p\rangle$$

KT, PRD98, 034009

$$\left(i\not{D} - m\right) q = 0$$

$$\partial_\nu T_q^{\mu\nu} = -\bar{q} g F^{\mu\nu} \gamma_\nu q, \qquad \partial_\nu T_g^{\mu\nu} = -F_a^{\mu\nu} D_{ab}^{\rho} F_{\rho\nu}^{b}$$

$$\Delta^\mu \bar{u}(p') u(p) M \bar{C}_q(t) = \langle p' | \bar{q} i g F^{\mu\nu} \gamma_\nu q | p \rangle$$

$$\Delta^\mu \bar{u}(p', S') u(p, S) M \bar{C}_g(t) = \langle p' | F_a^{\mu\nu} i D_{ab}^{\rho} F_{\rho\nu}^{b} | p \rangle$$

$$A_q(t) \Longleftrightarrow \bar{C}_q(t)$$

$$A_g(t) \Longleftrightarrow \bar{C}_g(t)$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho} F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4} F^2$$

$$\langle p' | T^{\mu\nu}_{q,g} | p \rangle = \bar{u}(p') \Big[A_{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t) M g^{\mu\nu} \Big] u(p)$$

$$P = \frac{p + p'}{2}$$

$$\Delta = p' - p$$

$$t = \Delta^2$$

$$\bar{C}_q(t) + \bar{C}_g(t) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$A_q(0) + A_g(0) = 1$$

$$\langle p | T^{\mu\nu} | p \rangle = 2 p^\mu p^\nu$$

$$\frac{1}{2}\big(A_q(0) + B_q(0) + A_g(0) + B_g(0) \big) = \frac{1}{2}$$

$$B_q(0) + B_g(0) = 0$$

$$\frac{\langle pS | J^i | pS \rangle}{\langle pS | pS \rangle} = \frac{1}{2} S^i$$

$$J^i = \frac{1}{2} \epsilon^{ijk} \int d^3x M^{+jk}$$

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KT, PRD98, 034009

$$\partial_\nu T_q^{\mu\nu} = -\bar{q} g F^{\mu\nu} \gamma_\nu q, \qquad \partial_\nu T_g^{\mu\nu} = -F_a^{\mu\nu} D_{ab}^{\rho} F_{\rho\nu}^{b}$$

$$\Delta^\mu \bar{u}(p') u(p) M \bar{C}_q(t) = \langle p' | \bar{q} i g F^{\mu\nu} \gamma_\nu q | p \rangle$$

$$\Delta^\mu \bar{u}(p', S') u(p, S) M \bar{C}_g(t) = \langle p' | F_a^{\mu\nu} i D_{ab}^{\rho} F_{\rho\nu}^{b} | p \rangle$$

$$A_q(t) \Longleftrightarrow \bar{C}_q(t)$$

$$A_g(t) \Longleftrightarrow \bar{C}_g(t)$$

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KT, PRD98, 034009

$$\partial_\nu T_q^{\mu\nu} = -\bar{q} g F^{\mu\nu} \gamma_\nu q, \qquad \partial_\nu T_g^{\mu\nu} = -F_a^{\mu\nu} D_{ab}^{\rho} F_{\rho\nu}^{b}$$

$$\Delta^\mu \bar{u}(p') u(p) M \bar{C}_q(t) = \langle p' | \bar{q} i g F^{\mu\nu} \gamma_\nu q | p \rangle$$

$$\Delta^\mu \bar{u}(p', S') u(p, S) M \bar{C}_g(t) = \langle p' | F_a^{\mu\nu} i D_{ab}^{\rho} F_{\rho\nu}^{b} | p \rangle$$

$$A_q(t) \Longleftrightarrow \bar{C}_q(t)$$

$$A_g(t) \Longleftrightarrow \bar{C}_g(t)$$

Y. Hatta, A. Rajan, KT,
arXiv: 1810.05116

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho}F_{\rho}^{\;\;\nu} + \frac{g^{\mu\nu}}{4}F^2$$

$$\langle p' | T^{\mu\nu}_{q,g} | p\rangle = \bar{u}(p')\Big[A_{q,g}(t)\gamma^{(\mu}P^{\nu)} + B_{q,g}(t)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}(t)Mg^{\mu\nu}\Big]u(p)$$

$$P = \frac{p+p'}{2}$$

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$$\langle p | T^{\mu\nu} | p\rangle = 2p^\mu p^\nu$$

$$\frac{1}{2}\big(A_q(0) + B_q(0) + A_g(0) + B_g(0)\big) = \frac{1}{2}$$

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$$\frac{\langle pS | J^i | pS\rangle}{\langle pS | pS\rangle} = \frac{1}{2}S^i$$

$$J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x M^{+jk}$$

$$M^{\mu\rho\sigma} = x^\rho T^{\mu\sigma} - x^\sigma T^{\mu\rho}$$

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p | T^{\mu\nu}_{q,g} | p \rangle = \bar{u}(p)\Big[A_{q,g}(0)\gamma^{(\mu} p^{\nu)} + \bar{C}_{q,g}(0) M g^{\mu\nu}\Big]u(p)$$

$$g_{\mu\nu}\langle p | T^{\mu\nu}_{q,g} | p \rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

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Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p | T^{\mu\nu}_{q,g} | p\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu} p^{\nu)} + \bar{C}_{q,g}(0) M g^{\mu\nu}\right] u(p)$$

$$g_{\mu\nu}\langle p | T^{\mu\nu}_{q,g} | p\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu} T^{\mu\nu}_q = m\bar{q}q, \qquad g_{\mu\nu} T^{\mu\nu}_g = 0, \qquad A_{q,g}(0) + 4\bar{C}_{q,g}(0) \simeq 0$$

$$T^{\mu\nu} = \frac{1}{2}\bar{q}\gamma^{(\mu} i\overleftrightarrow{D}^{\nu)} q \; + \; F^{\mu\rho}F_{\rho}{}^{\nu} + \frac{g^{\mu\nu}}{4}F^2$$

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu} p^{\nu)} + \bar{C}_{q,g}(0) M g^{\mu\nu}\right] u(p)$$

$$g_{\mu\nu}\langle p | T_{q,g}^{\mu\nu} | p \rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu} T_q^{\mu\nu} \neq m\bar{q}q, \qquad g_{\mu\nu} T_g^{\mu\nu} \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p | T^{\mu\nu}_{q,g} | p \rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu} p^{\nu)} + \bar{C}_{q,g}(0) M g^{\mu\nu}\right]u(p)$$

$$g_{\mu\nu}\langle p | T^{\mu\nu}_{q,g} | p \rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu}T^{\mu\nu}_q \neq m\bar{q}q, \qquad g_{\mu\nu}T^{\mu\nu}_g \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

trace anomaly separately for q, g

$$g_{\mu\nu}\langle p | T^{\mu\nu}_q | p \rangle = \frac{1}{2M^2}\langle p | \left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right] | p \rangle$$

$$g_{\mu\nu}\langle p | T^{\mu\nu}_g | p \rangle = \frac{1}{2M^2}\langle p | \frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right) | p \rangle$$

$$C_A = N_c\ , \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p|T_{q,g}^{\mu\nu}|p\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

$$g_{\mu\nu}\langle p|T_{q,g}^{\mu\nu}|p\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu}T_q^{\mu\nu} \neq m\bar{q}q, \qquad g_{\mu\nu}T_g^{\mu\nu} \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

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$$g_{\mu\nu}\langle p|T_q^{\mu\nu}|p\rangle = \frac{1}{2M^2}\langle p|\left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right]|p\rangle$$

$$g_{\mu\nu}\langle p|T_g^{\mu\nu}|p\rangle = \frac{1}{2M^2}\langle p|\frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right)|p\rangle$$

$$g_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1+\gamma_m(g)\right)m\bar{q}q$$

$$(i\not{D} - m)q = 0$$

KT, PRD98, 034009

$$\partial_\nu T_q^{\mu\nu} = -\bar{q}gF^{\mu\nu}\gamma_\nu q, \qquad \partial_\nu T_g^{\mu\nu} = -F_a^{\mu\nu} D_{ab}^{\rho} F_{\rho\nu}^{b}$$

$$\Delta^\mu \bar{u}(p')u(p)M\bar{C}_q(t) = \langle p' | \bar{q}igF^{\mu\nu}\gamma_\nu q | p\rangle$$

$$\Delta^\mu \bar{u}(p',S')u(p,S)M\bar{C}_g(t) = \langle p' | F_a^{\mu\nu} iD_{ab}^{\rho} F_{\rho\nu}^{b} | p\rangle$$

$A_q(t) \Leftrightarrow \bar{C}_q(t)$

$A_g(t) \Leftrightarrow \bar{C}_g(t)$

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\bar{C}_q(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0}\right) + \frac{1}{4}\left(\frac{2n_f}{3\beta_0} + 1\right)\frac{\langle P|(m\bar{\psi}\psi)_R|P\rangle}{2M^2}$$

$$-\frac{4C_F A_q(\mu_0) + n_f\left(A_q(\mu_0) - 1\right)}{4(4C_F + n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}}$$

$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f}{4\beta_0}\left(-\frac{34C_A}{27} - \frac{49C_F}{27}\right) + \frac{\beta_1 n_f}{6\beta_0^2}\right] + \cdots$$

$$A_q(t) \Longleftrightarrow \bar{C}_q(t)$$

$$A_g(t) \Longleftrightarrow \bar{C}_g(t)$$

Y. Hatta, A. Rajan, KT,
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$$\bar{C}_q(\mu) = -\frac{1}{4}\left(\frac{n_f}{4C_F + n_f} + \frac{2n_f}{3\beta_0}\right) + \frac{1}{4}\left(\frac{2n_f}{3\beta_0} + 1\right)\frac{\langle P|\left(m\bar{\psi}\psi\right)_R|P\rangle}{2M^2}$$

$$-\frac{4C_F A_q(\mu_0) + n_f\left(A_q(\mu_0) - 1\right)}{4(4C_F + n_f)}\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{8C_F + 2n_f}{3\beta_0}}$$

$$+\frac{\alpha_s(\mu)}{4\pi}\left[\frac{n_f}{4\beta_0}\left(-\frac{34C_A}{27} - \frac{49C_F}{27}\right) + \frac{\beta_1 n_f}{6\beta_0^2}\right] + \cdots$$

$$A_q(t) \Longleftrightarrow \bar{C}_q(t)$$

$$A_g(t) \Longleftrightarrow \bar{C}_g(t)$$

Y. Hatta, A. Rajan, KT,
arXiv: 1810.05116

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p | T^{\mu\nu}_{q,g} | p \rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu} p^{\nu)} + \bar{C}_{q,g}(0) M g^{\mu\nu} \right] u(p)$$

$$g_{\mu\nu} \langle p | T^{\mu\nu}_{q,g} | p \rangle = 2M^2 \left(A_{q,g}(0) + 4\bar{C}_{q,g}(0) \right)$$

$$g_{\mu\nu} T^{\mu\nu}_{q} \neq m\bar{q}q, \qquad g_{\mu\nu} T^{\mu\nu}_{g} \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

trace anomaly separately for q, g

$$g_{\mu\nu} \langle p | T^{\mu\nu}_{q} | p \rangle = \frac{1}{2M^2} \langle p | \left[m\bar{q}q + \frac{\alpha_s}{4\pi} \left(\frac{n_f}{3} F^2 + \frac{4C_F}{3} m\bar{q}q \right) \right] | p \rangle$$

$$g_{\mu\nu} \langle p | T^{\mu\nu}_{g} | p \rangle = \frac{1}{2M^2} \langle p | \frac{\alpha_s}{4\pi} \left(-\frac{11C_A}{6} F^2 + \frac{14C_F}{3} m\bar{q}q \right) | p \rangle$$

$$g_{\mu\nu} T^{\mu\nu} = \frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q$$

Y. Hatta, A. Rajan, KT, arXiv: 1810.05116

$$\langle p | T^{\mu\nu}_{q,g} | p\rangle = \bar{u}(p)\left[A_{q,g}(0)\gamma^{(\mu}p^{\nu)} + \bar{C}_{q,g}(0)Mg^{\mu\nu}\right]u(p)$$

$$g_{\mu\nu}\langle p | T^{\mu\nu}_{q,g} | p\rangle = 2M^2\left(A_{q,g}(0) + 4\bar{C}_{q,g}(0)\right)$$

$$g_{\mu\nu}T^{\mu\nu}_q \neq m\bar{q}q, \qquad g_{\mu\nu}T^{\mu\nu}_g \neq 0, \quad A_{q,g}(0) + 4\bar{C}_{q,g}(0) = O(\alpha_s)$$

trace anomaly separately for q, g

$$g_{\mu\nu}\langle p | T^{\mu\nu}_q | p\rangle = \frac{1}{2M^2}\langle p | \left[m\bar{q}q + \frac{\alpha_s}{4\pi}\left(\frac{n_f}{3}F^2 + \frac{4C_F}{3}m\bar{q}q\right)\right] | p\rangle$$

$$g_{\mu\nu}\langle p | T^{\mu\nu}_g | p\rangle = \frac{1}{2M^2}\langle p | \frac{\alpha_s}{4\pi}\left(-\frac{11C_A}{6}F^2 + \frac{14C_F}{3}m\bar{q}q\right) | p\rangle$$

$$g_{\mu\nu}T^{\mu\nu} = \frac{\beta(g)}{2g}F^2 + \left(1 + \gamma_m(g)\right)m\bar{q}q$$

$$\langle p | T^{\mu\nu} | p\rangle = 2p^\mu p^\nu$$

$$2M^2 = \langle p | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q \right) | p \rangle \simeq \langle p | \frac{\beta(g)}{2g} F^2 | p \rangle$$

$$2M^2 = \langle p | \left(\frac{\beta(g)}{2g} F^2 + \left(1+\gamma_m(g)\right) m\bar{q}q \right) | p \rangle \simeq \langle p | \frac{\beta(g)}{2g} F^2 | p \rangle$$

$$2M^2 = g_{\mu\nu} \langle p | T_q^{\mu\nu} | p \rangle + g_{\mu\nu} \langle p | T_g^{\mu\nu} | p \rangle$$

$$2M^2 = \langle p | \left(\frac{\beta(g)}{2g} F^2 + \left(1 + \gamma_m(g)\right) m\bar{q}q \right) | p \rangle \simeq \langle p | \frac{\beta(g)}{2g} F^2 | p \rangle$$

$$2M^2 = g_{\mu\nu} \langle p | T_q^{\mu\nu} | p \rangle + g_{\mu\nu} \langle p | T_g^{\mu\nu} | p \rangle$$

$$\frac{\alpha_s}{4\pi} \frac{n_f}{3} F^2 \qquad \frac{\alpha_s}{4\pi} \left(-\frac{11C_A}{6} F^2 \right)$$

$$2M^2 = \langle p | \left(\frac{\beta(g)}{2g} F^2 + (1+\gamma_m(g)) m\bar{q}q \right) | p\rangle \simeq \langle p | \frac{\beta(g)}{2g} F^2 | p\rangle$$

$$2M^2 = g_{\mu\nu} \langle p | T_q^{\mu\nu} | p\rangle + g_{\mu\nu} \langle p | T_g^{\mu\nu} | p\rangle$$

$$\frac{\alpha_s}{4\pi} \frac{n_f}{3} F^2 \qquad \frac{\alpha_s}{4\pi} \left(-\frac{11 C_A}{6} F^2 \right)$$

Hatta, A.Rajan, KT, 1810.05116

$$\left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{17 C_A n_f}{27} + \frac{49 C_F n_f}{54} \right) F^2 \qquad \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{28 C_A n_f}{27} - \frac{17 C_A^2}{3} + \frac{5 C_F n_f}{54} \right) F^2$$

$$2M^2 = \langle p | \left(\frac{\beta(g)}{2g} F^2 + (1+\gamma_m(g)) m\bar{q}q \right) | p \rangle \simeq \langle p | \frac{\beta(g)}{2g} F^2 | p \rangle$$

$$2M^2 = g_{\mu\nu} \langle p | T_q^{\mu\nu} | p \rangle + g_{\mu\nu} \langle p | T_g^{\mu\nu} | p \rangle$$

Hatta, A.Rajan, KT, 1810.05116

$$\frac{\alpha_s}{4\pi} \frac{n_f}{3} F^2 \qquad \frac{\alpha_s}{4\pi} \left(-\frac{11C_A}{6} F^2 \right)$$

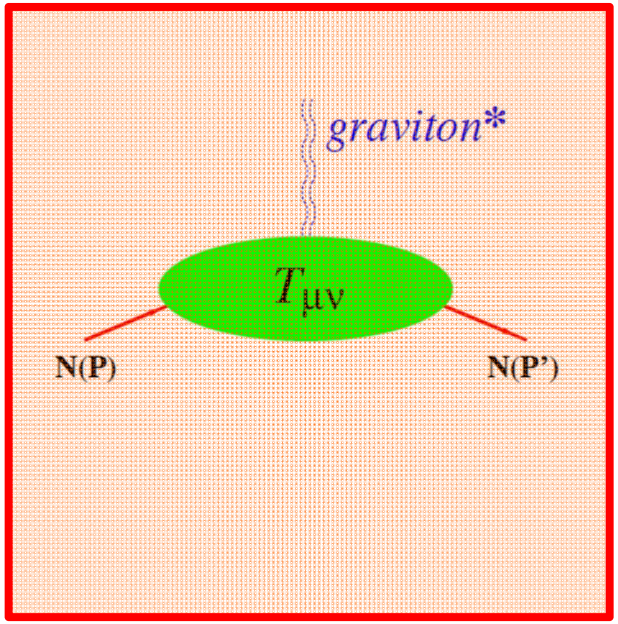
$$\left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{17C_A n_f}{27} + \frac{49C_F n_f}{54} \right) F^2 \qquad \left(\frac{\alpha_s}{4\pi}\right)^2 \left(\frac{28C_A n_f}{27} - \frac{17C_A^2}{3} + \frac{5C_F n_f}{54} \right) F^2$$

$$\left(\frac{\alpha_s}{4\pi}\right)^3 \left[\left\{ n_f \left(\frac{52\zeta(3)}{9} - \frac{401}{324} \right) C_A C_F + \left(\frac{134}{27} - 4\zeta(3) \right) C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9} \right) C_F^2 \right\} + n_f^2 \left(-\frac{697C_A}{729} - \frac{169C_F}{1458} \right) \right] F^2$$

$$\left(\frac{\alpha_s}{4\pi}\right)^3 \left[n_f \left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9} \right) C_A C_F + \left(4\zeta(3) + \frac{293}{36} \right) C_A^2 + \frac{16}{729} (81\zeta(3) - 98) C_F^2 \right) + n_f^2 \left(\frac{655C_A}{2916} - \frac{361C_F}{729} \right) - \frac{2857C_A^3}{108} \right] F^2$$

KT, 1811.07879

Summary



mass & energy
distribution

$$\int_{-1}^{1} dx\, x H^q(x,\eta,t) = A_q(t) + 4\eta^2 D_q(t)$$

$$\int_{-1}^{1} dx\, x E^q(x,\eta,t) = B_q(t) - 4\eta^2 D_q(t)$$

angular momentum distribution

force & pressure distribution

$D_q(t)$: $g\bar{q}q$ correlation

$\bar{C}_q(t)$: trace anomaly

Gravitational form factors can be accessed through GPDs