On the new resonance d*(2380)

---calculations in a chiral quark model

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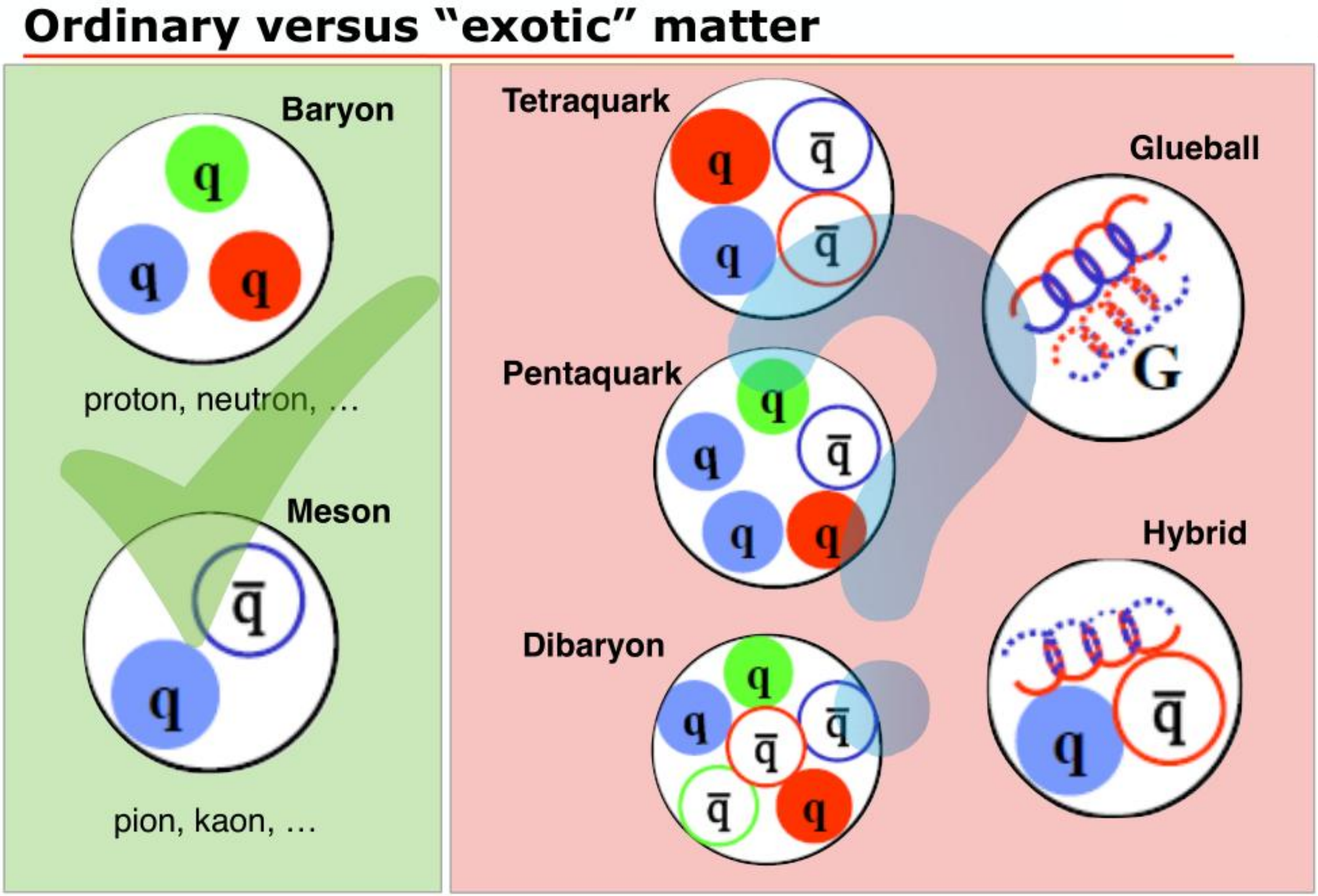
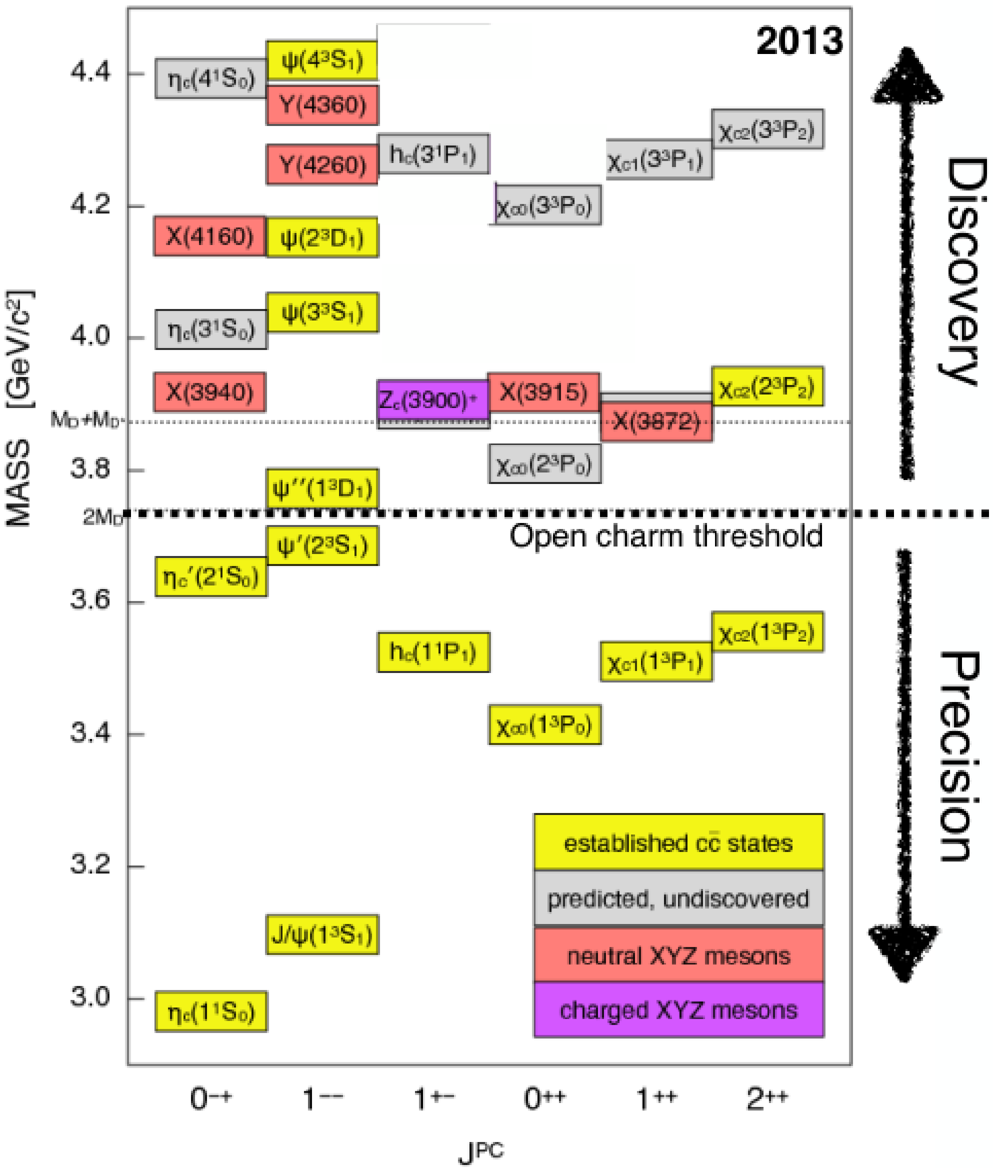
1 Observations of exotic



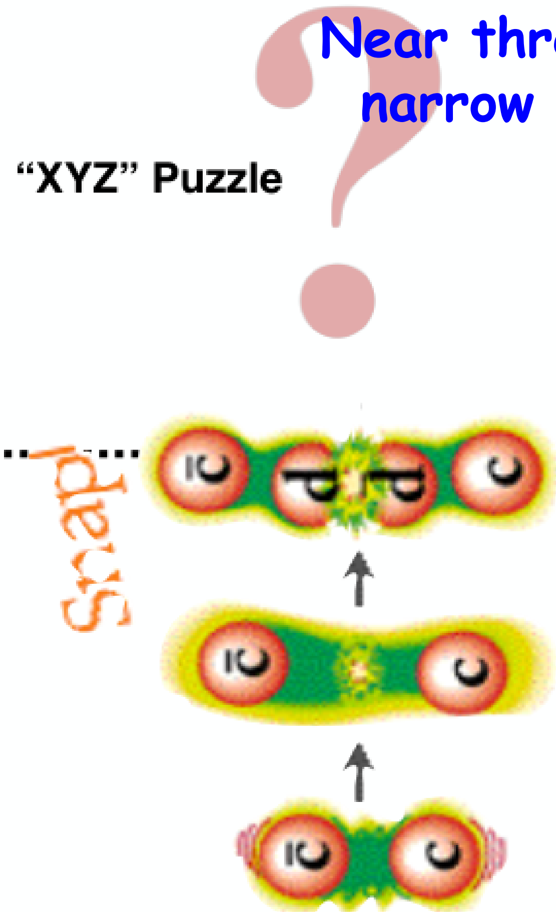
Courtesy of J. Messchendorp

Charmonium-like particles - XYZ states



"XYZ" Puzzle

Near threshold, narrow width



(New resonances, Five-Quark)

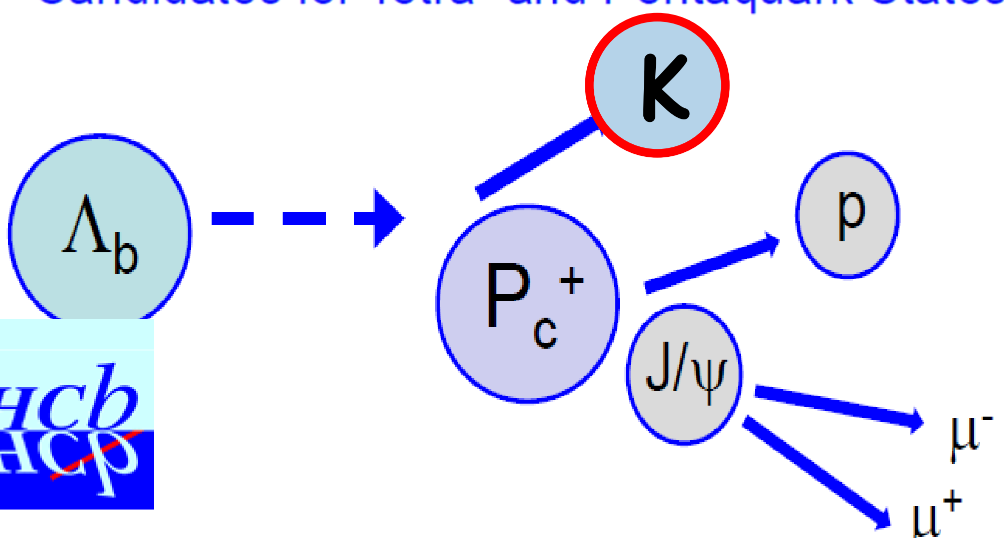
Pentaquark states Pc(4380)+, & Pc(4450)+

Observation of J/ψp resonances consistent with pentaquark states

PRL 115, 07201, arXiv:150

Exotic Hadron Spectroscopy at LHCb:

Candidates for Tetra- and Pentaquark States



$$b \to c + \overline{c}s$$

Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \to J/\psi K^- p$ decays

The LHCb collaboration†

Abstract

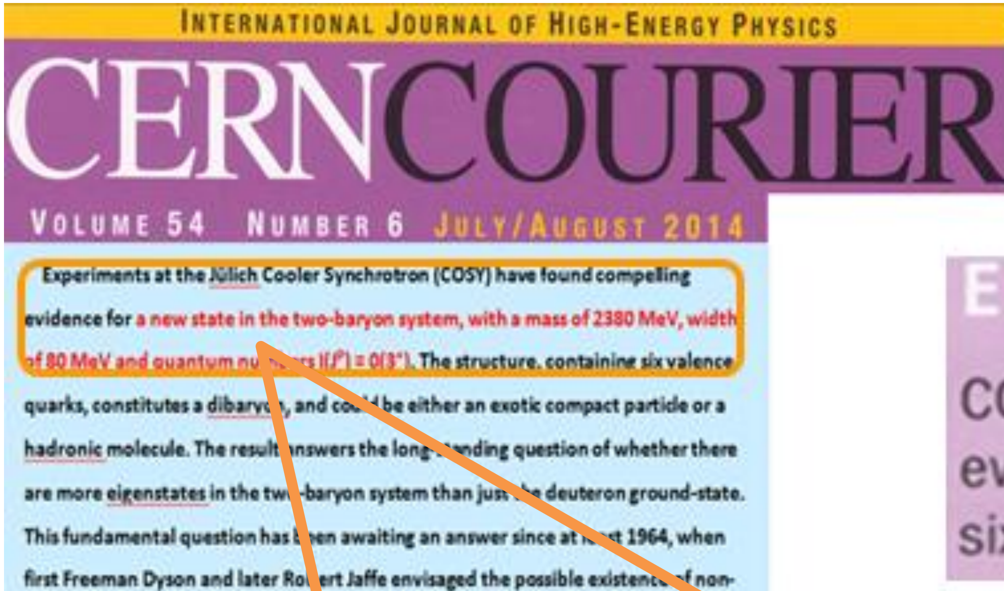
Observations of exotic structures in the $J/\psi p$ channel, that we refer to as pentaquark-charmonium states, in $\Lambda_b^0 \to J/\psi K^- p$ decays are presented. The data sample corresponds to an integrated luminosity of 3 fb^{-1} acquired with the LHCb detector from 7 and 8 TeV pp collisions. An amplitude analysis is performed on the three-body final-state that reproduces the two-body mass and angular distributions. To obtain a satisfactory fit of the structures seen in the $J/\psi p$ mass spectrum, it is necessary to include two Breit-Wigner amplitudes that each describe a resonant state. The significance of each of these resonances is more than 9 standard deviations. One has a mass of $4380 \pm 8 \pm 29$ MeV and a width of $205 \pm 18 \pm 86$ MeV, while the second is narrower, with a mass of $4449.8 \pm 1.7 \pm 2.5$ MeV and a width of $39 \pm 5 \pm 19$ MeV. The preferred J^P assignments are of opposite parity, with one state having spin 3/2 and the other 5/2.

Five-quark

$\Sigma_c\overline{D}$, $\Sigma_c^*\overline{D}$, $\Sigma_c\overline{D}^*$, $\Sigma_c^*\overline{D}^*$, $p\,\chi_{c1}$, $\psi(2S)p$

$3^-/2$, $P_c(4380)$, $5^+/2(J^P\,?)$ $P_c'(4449)$

Observation of d*(2380)


INTERNATIONAL JOURNAL OF HIGH-ENERGY PHYSICS

CERN COURIER

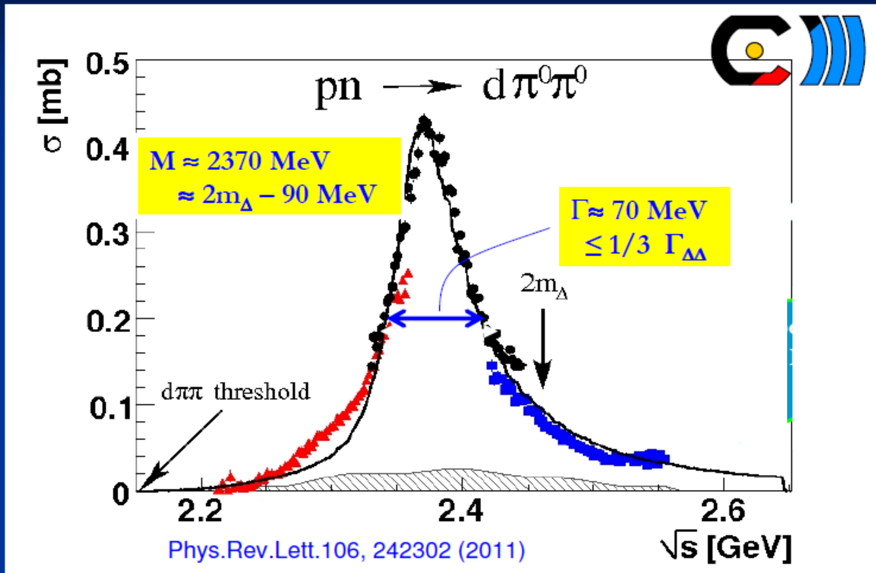
VOLUME 54 NUMBER 6 JULY/AUGUST 2014

Experiments at the Jülich Cooler Synchrotron (COSY) have found compelling evidence for a new state in the two-baryon system, with a mass of 2380 MeV, width of 80 MeV and quantum nu... I(J^P) = 0(3^+). The structure, containing six valence quarks, constitutes a dibary..., and co... be either an exotic compact particle or a hadronic molecule. The result ...nswers the long-...nding question of whether there are more eigenstates in the tw...-baryon system than jus... deuteron ground-state. This fundamental question has b...en awaiting an answer since at ...t 1964, when first Freeman Dyson and later Ro...ert Jaffe envisaged the possible existenc... of non-

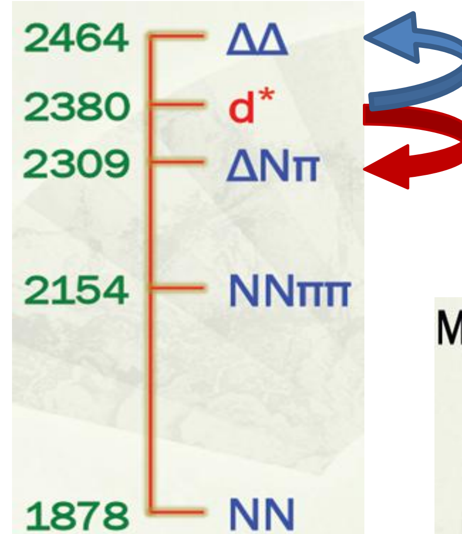
cerncourier.com/cws/article/cern/57836 (2014)

Experiments at the Jülich Cooler Synchrotron (COSY) have found compelling evidence for a new state in the two-baryon system, with a mass of 2380 MeV, width of 80 MeV and quantum numbers $I(J^P) = 0(3^+)$…Since 2009

The d* Resonance $I(J^P) = 0(3^+)$



Baryon number=2



84MeV

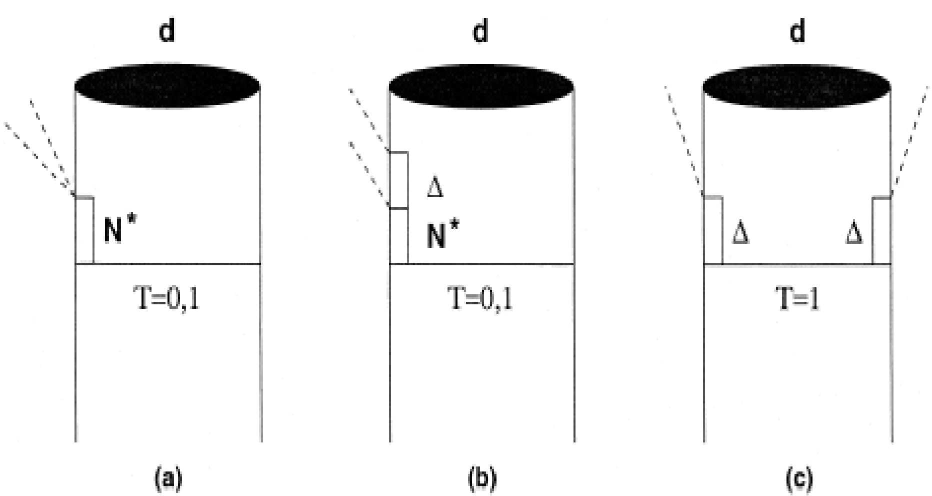
71MeV

$M_{d*} \approx 2380$ MeV
$\approx 2M_\Delta - 84$ MeV
$> M_{\Delta N\pi}$
$> M_{NN\pi\pi}$
$> M_{NN}$

$2\Gamma_\Delta \approx 230$ MeV

Unusual narrow width

$\Gamma_{d*} \approx 70$ MeV
$< 1/3 \times 2\Gamma_\Delta$?



Neither NN (Roper),
nor ΔΔ
Intermediate state

d*(2380)

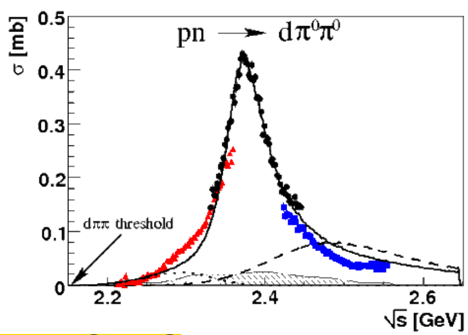
Signals in np procese @ COSY

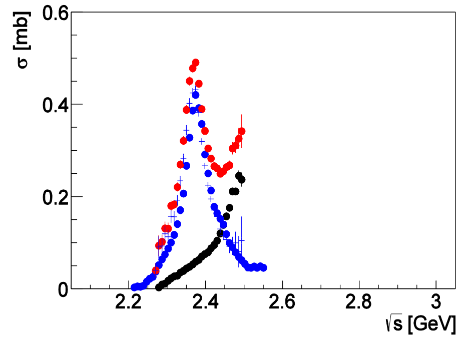
2π production processes

PRL 106 (2011) 242302

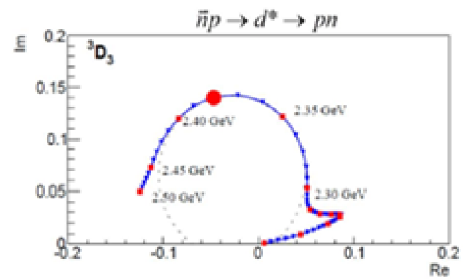
PLB 721 (2013) 229

● ● ● WASA data





np scattering process



Fusion

$d\pi^0\pi^0$

$d\pi^+\pi^-$

pn → d*(2380)

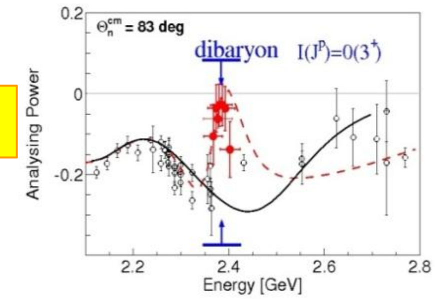
pn

Non-fusion

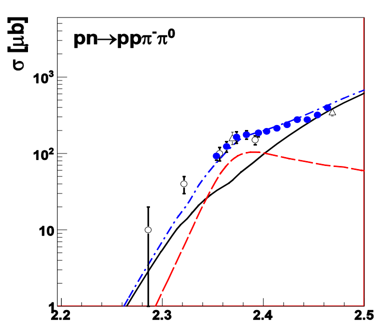
$pp\pi^-\pi^0$

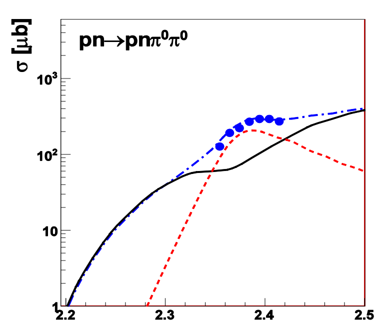
$pn\pi^0\pi^0$

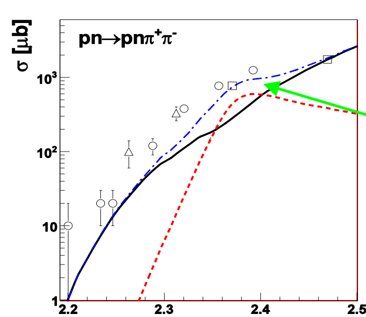
$pn\pi^+\pi^-$



PRL 112 (2014) 202301







HADES

PRC 88 (2013) 055208

PLB 743 (2015) 325

Proc. STORI 2015

Signals in other reactions @ COSY

fusion 2π processes

Measured also in fusion reactions to helium isotopes:

$p + d \rightarrow {}^3He + \pi^0 + \pi^0$

$p + d \rightarrow {}^3He + \pi^+ + \pi^-$

$d + d \rightarrow {}^4He + \pi^0 + \pi^0$

$d + d \rightarrow {}^4He + \pi^+ + \pi^-$

Characters of d*(2380)

- **d* mass locates between ΔΔ and ΔNπ thresholds**

  Effect from threshold is expected small

$\Gamma_{d^*} \approx 70\mathrm{MeV}$

$M_{\Delta N\pi} = 2310\mathrm{MeV}$ $M_{d^*} \approx 2380\mathrm{MeV}$ $M_{\Delta\Delta} = 2464\mathrm{MeV}$

71MeV 84MeV

- **d* narrow width** → **Possible 6q structure**

  might be different from normal hadrons

Review article: by Heinz Clement, Progress in Particle and Nuclear Physics, 93 (2017), 195-142

2、Possible interpretations

d*(2380)

$I(J^P) = 0(3^+)$

▲ Before COSY's observation

- Consists with COSY's measurement

Dyson(64) ------- symmetry analysis

Thomas(83) ----- bag model

Yuan(99) -------- ΔΔ+CC quark cluster model

Jaffe(77)

Swart(78)

Oka(80)

Maltman(85)

Goldman(89)

Wang(95)……

▲ After COSY's observation

- **Quark model**

J.Ping (09/14)-10 coupled channels QM

F.Huang,Y.B.Dong et al. (14-18)--ΔΔ+CC QM

Bashkanov, Brodsky, Clement (13) -- ΔΔ+CC

A. Compact 6q dominated exotic state

- **Hadronic model**

Gal (14) ---ΔNπ

Kukulin(15,16) - $D_{12}\pi$

B. ΔNπ (or $D_{12}\pi$) resonant state

3、Compact 6q dominated d* (2380) in a chiral constituent quark model

(A),Mass and wave function

PRC 60 (1999) 045203
CPC 39 (2015) 071001

SU(3) chiral QM + RGM approach

▲ **Interaction:** $$\mathbf{V_{ij} = V_{ij}^{conf} + V_{ij}^{OGE} + V_{ij}^{ch} + V_{ij}^{chv}}$$

q-q potential $$\mathbf{V_{ij}^{ch} = \sum_{a} (V_{ij}^{s(a)} + V_{ij}^{ps(a)})}$$

Interactive Lagrangian

$$\mathcal{L}_I = -g_{ch}\bar{\Psi}(\sum_{a=0}^{8}\sigma_a\lambda_a + i\sum_{a=0}^{8}\pi_a\lambda_a\gamma_5)\Psi$$

Model parameters: reproduce experimental data for NN systems---NN phase shifts, $\mathrm{BE}_d^{exp't} = 2.22\,\mathrm{MeV}$

▲ Trial wavefunction:

$\mathbf{I(J^P) = 0(3^+)}$

$$\Psi_{6q} = \mathcal{A}\left[\phi_\Delta(\xi_1,\xi_2)\,\phi_\Delta(\xi_4,\xi_5)\,\eta_{\Delta\Delta}(\boldsymbol{r}) + \phi_C(\xi_1,\xi_2)\,\phi_C(\xi_4,\xi_5)\,\eta_{CC}(\boldsymbol{r})\right]_{S=3,I=0,C=(00)} .$$

$$\Delta: \quad (0s)^3\,[3]_{\rm orb}, S=3/2, I=3/2, C=(00),$$
$$C: \quad (0s)^3\,[3]_{\rm orb}, S=3/2, I=1/2, C=(11),$$

$\eta_{\Delta\Delta}$ (r) and η_{CC} (r) are not orthogonal

▲ Hadronization----Channel wave function:

Using the projection method to integrate out the internal coordinates inside the clusters (or Hadronization approach)

$$\Psi_{d^*} = |\Delta\Delta\rangle\,\chi_{\Delta\Delta}(\boldsymbol{r}) + |CC\rangle\,\chi_{CC}(\boldsymbol{r})$$

$$\chi_{\Delta\Delta}(\boldsymbol{r}) \equiv \langle\phi_\Delta(\xi_1,\xi_2)\,\phi_\Delta(\xi_4,\xi_5)\,|\,\Psi_{6q}\rangle\,,$$
$$\chi_{CC}(\boldsymbol{r}) \equiv \langle\phi_C(\xi_1,\xi_2)\,\phi_C(\xi_4,\xi_5)\,|\,\Psi_{6q}\rangle\,,$$

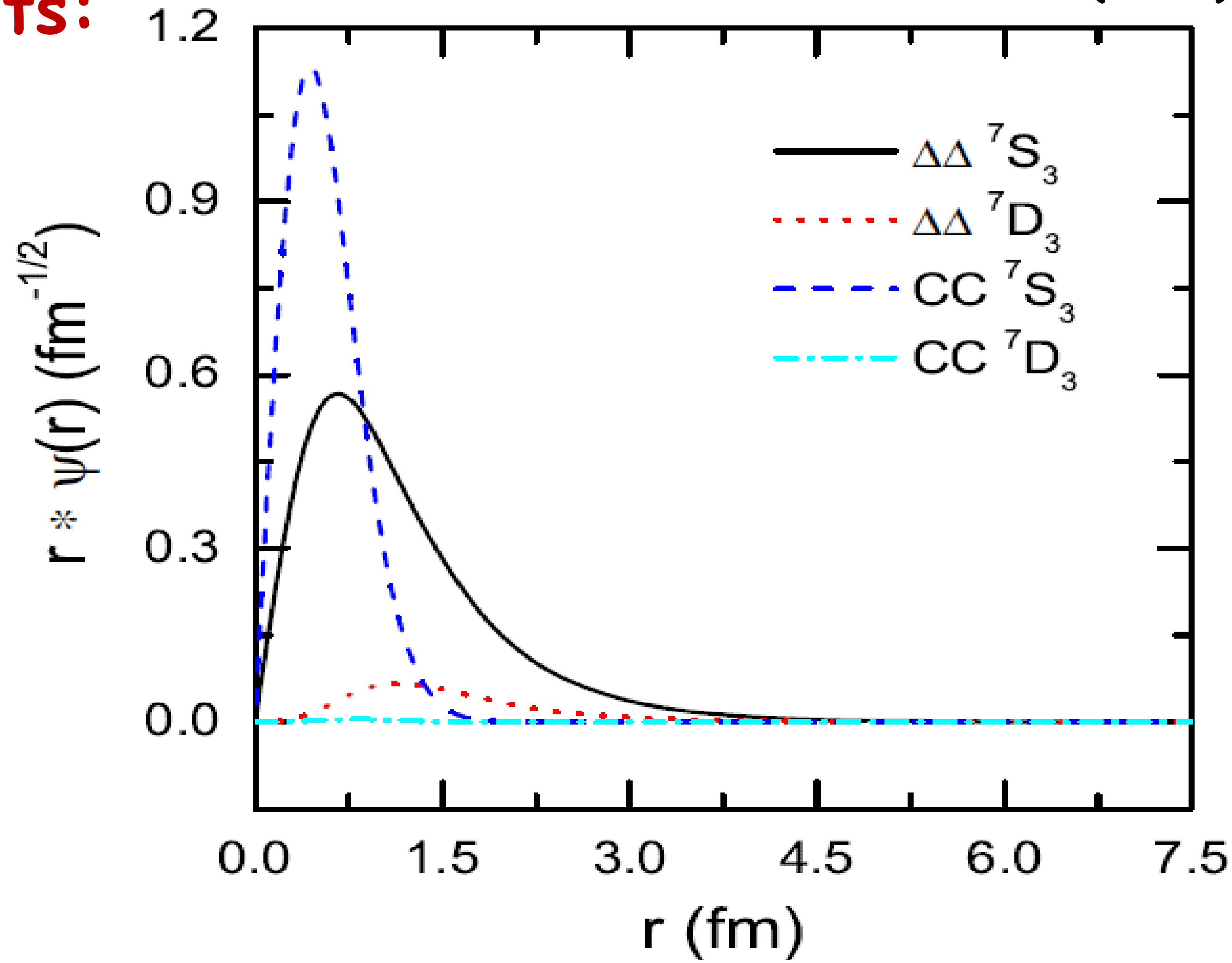
The two components are orthogonal due to the quark exchange effect

(A), Mass and wave function

CPC 39 (2015) 071001

▲ Results:

d* WFs



- **Binding energy**

$\mathrm{BE}_{d^*}^{th} = 84\mathrm{MeV}$ $\mathrm{BE}_{d^*}^{exp't} = 84\mathrm{MeV}$

		Ext. SU(3) (f/g=0)	
		ΔΔ (L=0,2)	ΔΔ-CC (L=0,2)
d^* Binding Energy(MeV)		62.3	83.9
Fraction of Wave Function (%)	ΔΔ (L=0)	98.01	31.22
	ΔΔ (L=2)	1.99	0.45
	CC (L=0)	0	68.33
	CC (L=2)	0	0.00

Reason for the large component of CC (68%)

$P_{36} = P^{r}_{36} \; P^{sfc}_{36}$

$I(J^P) = 0(3^+)$

1). Intrinsic character of d* ----- $<P^{sfc}_{36}>$

quark exchange effect of sfc large (negative:-4/9)

2). Dynamical effect-----

(SI=30) , OGE and vector meson exchange induced Δ-Δ short range interaction is attractive

Two cluster closer → large CC component

→ d* deep bound and narrow width

d* might be a 6q dominant state!

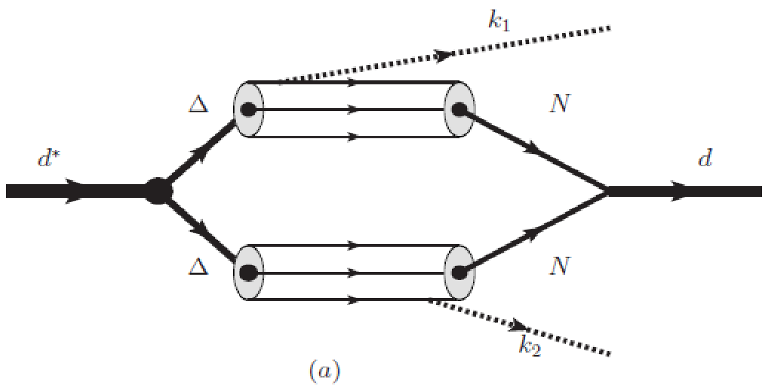
(B), Strong decays

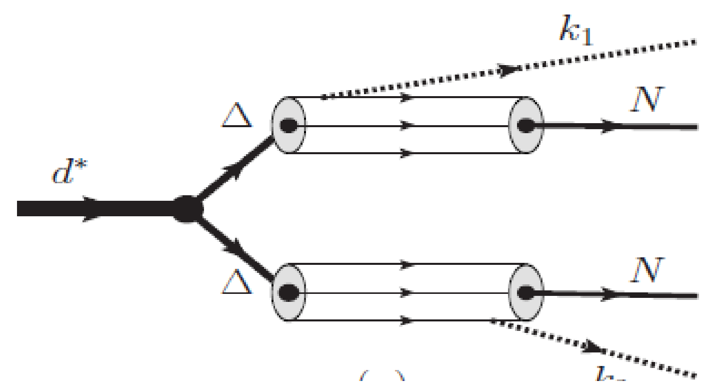
PRC91, (2015) 064002

PRC94, (2016) 014003

▲ 2π decay widths

Three-body decay		Four-body decay
$d^* \to d\pi^0\pi^0 \ (d\pi^+\pi^-)$	Typical diagrams	$d^* \to np\pi^0\pi^0 \ (np\pi^+\pi^-)$
$d^* \to pp\pi^-\pi^0$		$d^* \to nn\pi^0\pi^+$





Parameter: $qq\pi$ Interaction

$$\mathcal{H}_{qq\pi} = g_{qq\pi}\vec{\sigma}\cdot\vec{k}_\pi\tau\cdot\phi\frac{1}{(2\pi)^{3/2}\sqrt{2\omega_\pi}},$$

$\Delta \to N\pi$ Coupling & form factor

$$\Gamma_{\Delta\to\pi N} = \frac{4}{3\pi}k_\pi^3(g_{qq\pi}I_o)^2\frac{\omega_N}{M_\Delta},$$

	Theor.(MeV)	Expt.(MeV)
$d^* \to d\pi^+\pi^-$	16.8	16.7
$d^* \to d\pi^0\pi^0$	9.2	10.2
$d^* \to pn\pi^+\pi^-$	20.6	21.8
$d^* \to pn\pi^0\pi^0$	9.6	8.7
$d^* \to pp\pi^0\pi^-$	3.5	4.4
$d^* \to nn\pi^0\pi^+$	3.5	4.4
$d^* \to pn$	8.7	8.7
Total	71.9	74.9

Discussions:

* FSI is about 26~30%
* Isospin breaking factor

$$\frac{\Gamma(d^* \to d\pi^+\pi^-)}{\Gamma(d^* \to d\pi^0\pi^0)} \sim 1.8 \quad (1.6, \quad 2.0)$$

$$\frac{\Gamma(d^* \to pn\pi^+\pi^-)}{\Gamma(d^* \to pn\pi^0\pi^0)} \sim 2.2 \quad (2.5, \quad 2.5)$$

Ideal

* Too large width for (ΔΔ) component only

M_{d^*}(MeV)	(100%)ΔΔ 2374	Expt 2375
Decay channel	Γ(MeV)	Γ(MeV)
$d^* \to d\pi^0\pi^0$	17.0	10.2
$d^* \to d\pi^+\pi^-$	30.8	16.7
Total	132.8	74.9

* All partial and total widths agree with data

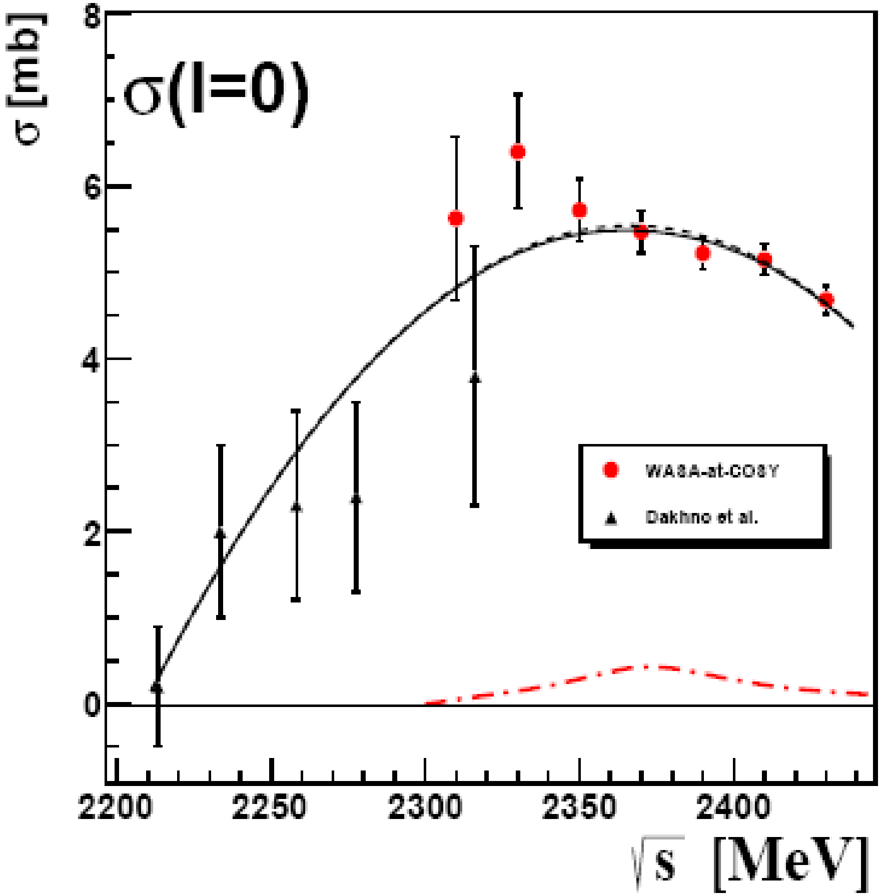
$$\Gamma^{exp't} = 70 \sim 75\,MeV$$

$$\Gamma^{th} \approx 72\,MeV$$

The narrow width is due to large CC component

▲ Single-π decay

$\sigma_{NN\to NN\pi}(I=0) = 3(2\sigma_{np\to pp\pi^-} - \sigma_{pp\to pp\pi^0})$



- **Experimental status**

The WASA-@-COSY Collaborations, arXiv:1702.07212v1 [nucl-ex]

PLB774 (2017), 599-607

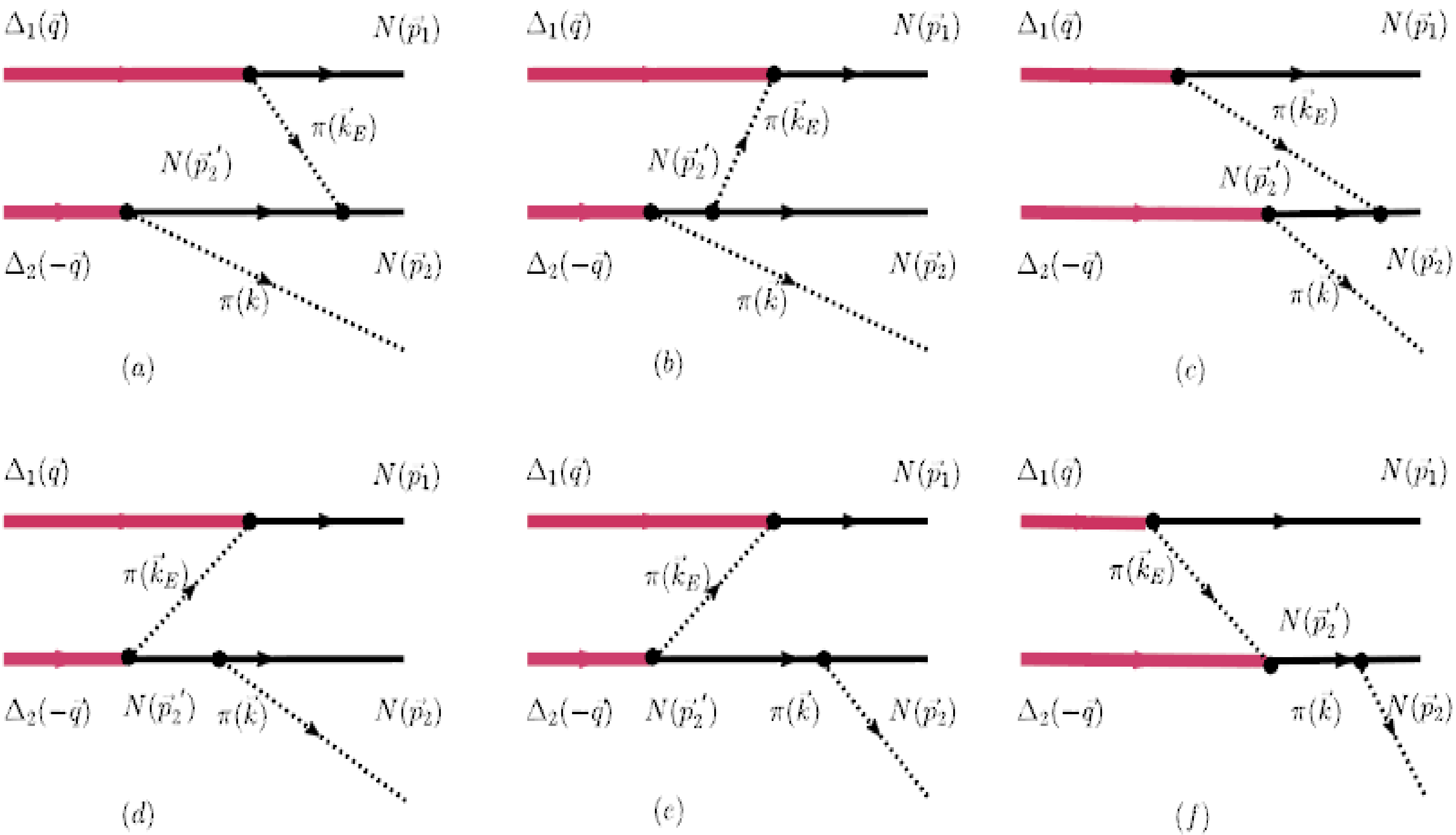
Dash-dotted line illustrates a 10% d* resonance contribution

Upper limit of branching ratio for $d^*(2380) \to NN\pi$ is 9%.

This channel might serve as a test

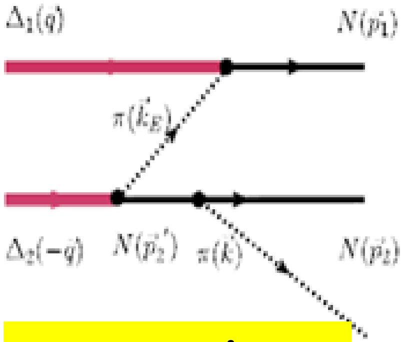
- **compact 6q dominated case:**

PLB769 (2017) 223-226



Typical diagrams: pion emitted from cluster II

Fig. 1. Six possible ways to emit pion only from the $\Delta\Delta$ component of d^* in the $d^* \to NN\pi$ decay process. The outgoing pion with momenta $\vec{k}$ is emitted from Δ_2. The other six sub-diagrams with pion emitted from Δ_1 are similar, and then are not shown here for reducing the size of the figure.



Sample

$$\Psi_{d^*} = |\Delta\Delta\rangle\,\chi_{\Delta\Delta}(r) + |CC\rangle\,\chi_{CC}(r)$$

$\Delta:$	$(0s)^3\,[3]_{\rm orb}, S=3/2, I=3/2, C=(00),$
$C:$	$(0s)^3\,[3]_{\rm orb}, S=3/2, I=1/2, C=(11),$

$$L = L_{\pi NN} + L_{\Delta N\pi}$$

Intermediate states: (N,N*,Δ,Δ*)

Low-lying resonances need to be considered

From quark model

$$\frac{g^2_{\pi\Delta\Delta}}{4\pi} = \frac{1}{25}\frac{M^2_\Delta}{M^2_N}\frac{g^2_{\pi NN}}{4\pi}, \quad g_{\pi\Delta\Delta}\ \ small$$

1, C→Δ, interaction should be color and isospin-dependent

2, CC(SI=3,0)→NN*(1400), D-wave of OGE is required

The suppressions enable to ignore the contribution from the CC component in d*

Our prediction, 1% is compatible with the Exp't upper-limits

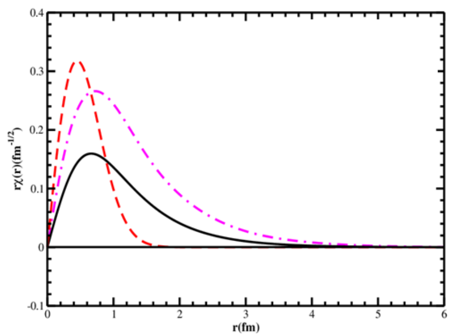
(C), Charge distribution of d*(2380)

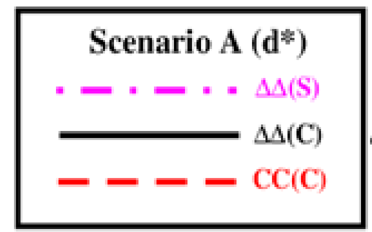
PRD96 094001 (2017)

For a spin=3 system:

2S+1=7 form factors (related to the size of system)

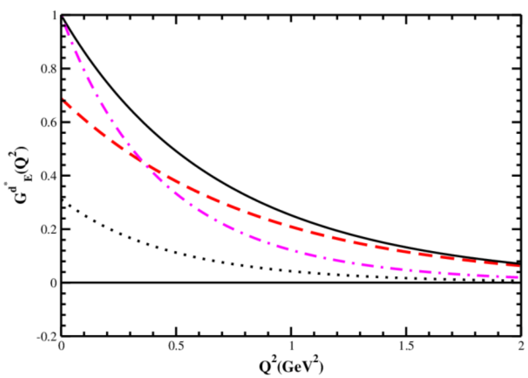
Wave Function

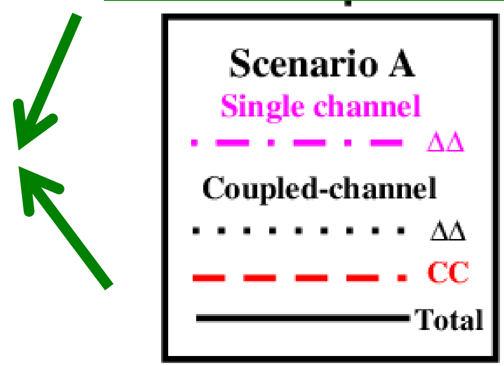




Compact ΔΔ+CC

Charge Distribution

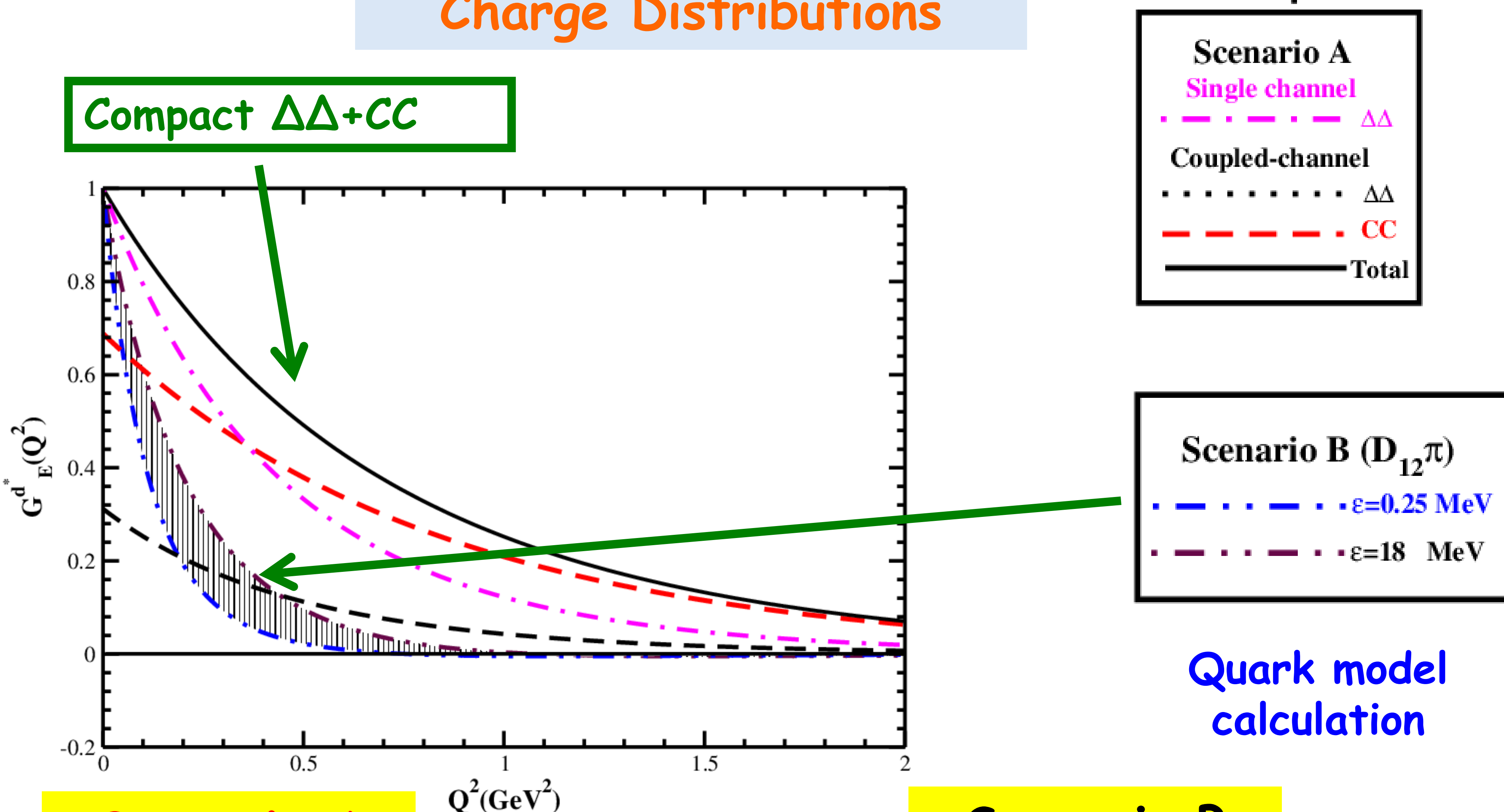




rms

$d^*(2380)$		
Cases	A1	A2
rms (fm)	1.09	0.72

Charge Distributions



Quark model calculation

Scenario A

rms

$d^*(2380)$		
Cases	A1	A2
rms (fm)	1.09	0.72

Scenario B

D_{12}		
Cases	B1	B2
rms (fm)	2.64	1.87

4、Summary, Remarks, and outlook

d*: Hexaquark dominated state :

(CC component ~ 66-68% in ΔΔ+CC)

Compact 6q dominated

ΔNπ (or $D_{12}\pi$) system

A.Gal, PLB769 (2017) 436

Good ← **Mass** / **Double-pion strong decays** → Good

d*(2380) single-π decay   Exp't BR ≤ 9% $d^*(2380) \to NN\pi$

↙ our predicted BR of 1%

↘ the BR for ΔNπ (or $D_{12}\pi$) is large

in the mixing case

Suggest other experimental searches

- $\gamma + d$ Process (Mainz, Jlab.)
- $\Upsilon \to \bar{d}^* + X$ Process (Belle)

  [BR($\Upsilon \to \bar{d} + X$) ~ 2.86 X 10^{-5}]
- $e^+ + e^- \to \bar{d}^* + p + n$ Processes (Belle?)

If the d* is further confirmed by experiments, Our interpretation looks reasonable. Thus, it might be a state with 6q structure dominant and moreover, the more information about the short range interaction is expected.

Thanks !

BACKUP SLICES

Analysis: Large component of CC (67%) in d* ?

(1) (2)

$$\Psi_{6q} = (1 - 9P_{36})[\phi_\Delta \phi_\Delta \eta_{\Delta\Delta}(\mathbf{r})]_{SIC=30(00)}$$
$$+ (1 - 9P_{36})[\phi_C \phi_C \eta_{CC}(\mathbf{r})]_{SIC=30(00)}$$

(3) (4)

$$\chi_{\Delta\Delta}(r) \equiv \langle \phi_\Delta(\xi_1, \xi_2)\, \phi_\Delta(\xi_4, \xi_5) \,|\, \Psi_{6q} \rangle \,,$$ (1) (2) (4) terms

$$\chi_{CC}(r) \equiv \langle \phi_C(\xi_1, \xi_2)\, \phi_C(\xi_4, \xi_5) \,|\, \Psi_{6q} \rangle \,,$$ (3) (4) (2) terms

$$\Psi_{d^*} = |\Delta\Delta\rangle\, \chi_{\Delta\Delta}(r) + |CC\rangle\, \chi_{CC}(r)$$

χ_{CC} contains the contri. From (2), from ΔΔ exchanged terms。

Thus P_{36} Exchange is important!

$P_{36} = P^{r}_{36}\ P^{sfc}_{36}$ d* has ΔΔ and CC components

Before the discovery of d*

- *A* pioneer discussion from symmetry: J.Dyson, PRL 13, 815 (1964)

Two baryon systems
SU(6) classification :

Anti-symmetric representations:
Non-strange states

$(I, J) = (3,0)(2,1)(1,0)(1,2)(0,1)(0,3)$ 6 states

Casmir operator reduced
a mass formula

$M = A + B'(T(T+1)) + B''(J(J+1))$

If $B' = B'' = B$, the obtained deuteron mass 1876MeV, and then, obtain A,

Choose B = 50MeV, Then, $M_{d*} = 2376$MeV

$<P^{sfc}_{36}>$ exchange effect in spin-flavor-color spaces

Reason for the large component of CC (67%)

intrinsic	$(\Delta\Delta)_{SI=30}$	$(\Delta\Delta)_{SI=30}$	$(CC)_{SI=30}$
	$(\Delta\Delta)_{SI=30}$	$(CC)_{SI=30}$	$(CC)_{SI=30}$
$<P^{sfc}_{36}>$	$-\frac{1}{9}$	$-\frac{4}{9}$	$-\frac{7}{9}$

$P_{36} = P^{r}_{36}\ P^{sfc}_{36}$ if it large

$<P^{r}_{36}> \sim 1$ should also large

$<P^{r}_{36}>$ is determined by the dynamical wave function

For d* The effective Δ-Δ interaction induced by OGE and vector meson exchange enables the short range interaction attractive.

$\longrightarrow$ Two clusters ΔΔ closer, $<P^{r}_{36}>$ is not small

1). d* special characters

spin-flavor-color spaces exchange effect :model independent

2). ΔΔ (SI=30) , Δ-Δ short range interaction is attractive

Dynamical effect —— Model dependent

P_{36} Effect large, large CC componet

d* might be a 6q dominant state

$\longrightarrow$ **d* deep bounded and narrow width**

A. Compact 6q dominated exotic state

(a) In 1999, proposed d* with ΔΔ+CC structure

X.Q.Yuan, Z.Y.Zhang, Y.W.Yu, P.N.Shen, PRC 60 (1999) 045203

- d* binding energy: 40-80 MeV
- CC enhances binding energy by 20 MeV

(b) In 2013, proposed narrow d* width due to Harvey formula

$$|\Psi_{d^*}\rangle = \sqrt{\frac{1}{5}}|\Delta\Delta\rangle + \sqrt{\frac{4}{5}}|6Q\rangle$$

Bashkanov, Brodsky, H.Clement, Phys.Lett.B727 (2013) 438

(c) In 2014, gave CC fraction of 68% in d*(ΔΔ+CC)

F.Huang, Z.Y.Zhang, P.N.Shen, W.L.Wang, CPC 39 (2015) 071001

Decay widths

Three-body decay

$$\Gamma_{d^*\to d\pi^0\pi^0} = \frac{1}{2!}\int d^3k_1 d^3k_2 d^3p_d (2\pi)\delta^3(\vec{k}_1+\vec{k}_2+\vec{p}_d) \times \delta\left(\omega_{k_1}+\omega_{k_2}+E_{p_d}-M_{d^*}\right)\left|\overline{\mathcal{M}_{if}^{\pi^0\pi^0}}\right|^2$$

$$\mathcal{M}_{if}^{\pi^0\pi^0} = \frac{1}{\sqrt{3}}\sum F_1F_2k_{1,\mu}k_{2,\nu}I_S^0I_I^0C_{1\nu,1\mu}^{jm_j}C_{3m_{d^*},jm_j}^{1m_d} \times \int d^3q\Bigg[\frac{\chi_d^*(\vec{q}-\frac{1}{2}\vec{k}_{12})}{E_\Delta(q)-E_N(q-k_1)-\omega_1} + \frac{\chi_d^*(\vec{q}+\frac{1}{2}\vec{k}_{12})}{E_\Delta(q)-E_N(q-k_2)-\omega_2} + \frac{\chi_d^*(\vec{q}+\frac{1}{2}\vec{k}_{12})}{E_\Delta(-q)-E_N(-q-k_1)-\omega_1} + \frac{\chi_d^*(\vec{q}-\frac{1}{2}\vec{k}_{12})}{E_\Delta(-q)-E_N(-q-k_2)-\omega_2}\Bigg]\chi_{d^*}(\vec{q})$$

Four-body decay

$$\Gamma_{d^*\to pn\pi^0\pi^0} = \frac{1}{2!2!}\int d^3k_1 d^3k_2 d^3p_1 (2\pi)\delta(\Delta E) \times |\overline{\mathcal{M}(k_1,k_2;p_1)}|^2$$

$$\mathcal{M}(k_1,k_2;p_1) = \mathcal{M}^{\text{bare}}(k_1,k_2;p_1)\times\mathcal{I}$$

⟺ FSI

$$\mathcal{I} = \mathcal{J}^{-1}(k) = C(k^2)\frac{\sin\delta e^{i\delta}}{k}$$

$$\mathcal{M}^a(k_1,k_2;p_1) = \int d^3p_2d^3q[\mathcal{H}\mathcal{S}_f\mathcal{H}]\Psi_{d^*}(q) \times \delta^3(\vec{p}_1+\vec{k}_1-\vec{q})\delta(\vec{p}_2+\vec{k}_2+\vec{q}) = \int d^3p_2\delta^3(\vec{p}_1+\vec{p}_2+\vec{k}_1+\vec{k}_2)[\mathcal{H}\mathcal{S}_f\mathcal{H}] \times \Psi_{d^*}(-\vec{p}_2-\vec{k}_2)$$

$$d^*\to np\pi^0\pi^0\ \ (np\pi^+\pi^-)$$

Magnetic Moment

Naïve quark model

Nucleon $$\frac{\mu_p}{\mu_n} = -\frac{3}{2} \rightarrow -\frac{2.79}{1.91}_{EXPT.}$$

d*(2380) ΔΔ+CC $$\mu_{d*} = \frac{M_{d*}}{m_q} \approx 7.6$$

d*(2380) $D_{12}\pi$ $$\mu_{d*} = \frac{2M_{d*}}{3m_q} \approx 5.1$$

(C), Form factors

Form factors: 2S+1 relative to size arXiv:1704.01253

Nucleon(1/2): $$< N(p') \mid J_N^\mu \mid N(p) > = \bar{U}_N(p')\Big[F_1(Q^2)\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{2M_N}F_2(Q^2)\Big]U(p),$$

$$G_E(Q^2) = F_1(Q^2) - \eta F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$

Breit frame

$$\begin{aligned} < N(\vec{q}/2) \mid J_N^0 \mid N(-\vec{q}/2) > &= (1+\eta)^{-1/2}\chi_{s'}^+\chi_s G_E(Q^2) \\ < N(\vec{q}/2) \mid \vec{J}_N \mid N(-\vec{q}/2) > &= (1+\eta)^{-1/2}\chi_{s'}^+\frac{\vec{\sigma}\times\vec{q}}{2M_N}\chi_s G_M(Q^2). \end{aligned}$$

Deuteron(1): $$J_{jk}^\mu(p',p) = \epsilon_j^{'*\alpha}(p')S_{\alpha\beta}^\mu\epsilon_k^\beta(p)$$

$$S_{\alpha\beta}^\mu = -\Big[G_1(Q^2)g_{\alpha\beta} - G_3(Q^2)\frac{Q_\alpha Q_\beta}{2m_D^2}\Big]P^\mu - G_2(Q^2)(Q_\alpha g_\beta^\mu - Q_\beta g_\alpha^\mu)\ ,$$

$$\begin{aligned} G_C(Q^2) &= G_1(Q^2) + \frac{2}{3}\eta_D G_2(Q^2)\ , \qquad G_M(Q^2) = G_2(Q^2)\ , \\ G_Q(Q^2) &= G_1(Q^2) - G_2(Q^2) + (1+\eta_D)G_3(Q^2)\ , \end{aligned}$$

Breit frame $$G_C(Q^2) \longrightarrow \frac{1}{3}\sum_\lambda < p', \lambda \mid J^0 \mid p, \breve{\lambda} >.$$