A test of gauge invariant canonical angular momentum in Landau level problem

M.Wakamatsu, Y.K, P.M.Zhang,
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Nucleon Spin

- Nucleon (p,n) has spin ½ (known since 1927 for p).

Can we understand nucleon spin by quark and gluon ?

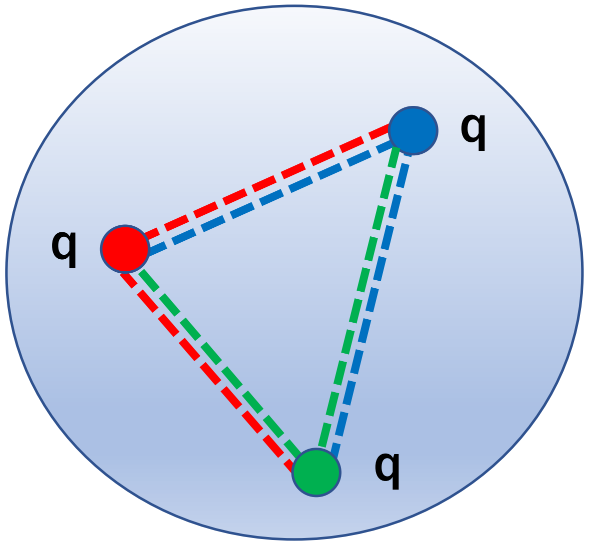
Nucleon (1927)

- If true,

Quark spin + OAM   **Gluon spin + OAM**

Nucleon spin: $$\frac{1}{2} = \underbrace{S_q + L_q}_{J_q} + \underbrace{S_g + L_g}_{J_g}$$

$$S_q = \frac{1}{2}\Delta\Sigma = \frac{1}{2}\sum_{i=u,d,s}\int_0^1 dx \Delta q_i(x) \qquad S_g = \Delta G = \int_0^1 dx \Delta G(x)$$



Nucleon (2018)

Duer, Brodsky,Teramond,
arXiv:1807.05250 (review)

- Expectation from Constituent Quark Model

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q$$

Quark model

$\Delta\Sigma = 1$ $L_q = 0$ (non-relativistic)

$\Delta\Sigma \approx 0.65$ $L_q \approx 0.18$ (relativistic)

- EMC experiment (1987)

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.14 \pm 0.18$$

EMC Collaboration
PLB 206,364(1988);
NPB 328,1(1989)

Very small amount of Nucleon spin is carried by quarks !

(Nucleon Spin Crisis)

- After EMC shock (1st hot debate:1987-)

QCD correction **Gluon spin**

Sea quark **etc.**

- New decompositions (2nd hot debate:2008-)

Gauge invariant canonical (g.i.c.) decompositions

Gauge invariant canonical decomposition

Chen et al.:
PRL,100,232002 (2008)

- Chen et al.:

q-spin / q-OAM / g-spin / g-OAM

$$\mathbf{J}_{\mathrm{Chen}} = \int d^3x \bar{\psi} \frac{1}{2} \mathbf{\Sigma} \psi + \int d^3x \bar{\psi} \mathbf{x} \times (-i\mathbf{D}_{\mathrm{pure}}) \psi + \int d^3x (\mathbf{E}^a \times \mathbf{A}^a_{\mathrm{phys}}) + \int d^3x E^{ai} (\mathbf{x} \times \mathbf{D}_{\mathrm{pure}}) A^{ai}_{\mathrm{phys}}$$

$$-i\mathbf{D}_{\mathrm{pure}} = -i\mathbf{\nabla} - g\mathbf{A}_{\mathrm{pure}}$$

- Decompositions of gauge field: $\mathbf{A} = \mathbf{A}_{\mathrm{phys}} + \mathbf{A}_{\mathrm{pure}}$

Conditions

$$\mathbf{\nabla} \cdot \mathbf{A}_{\mathrm{phys}} = 0$$
$$\mathbf{\nabla} \times \mathbf{A}_{\mathrm{pure}} = 0$$

Gauge trans law

$$\mathbf{A}'_{\mathrm{phys}} = \mathbf{A}_{\mathrm{phys}}$$
$$\mathbf{A}'_{\mathrm{pure}} = \mathbf{A}_{\mathrm{pure}} + \mathbf{\nabla}\chi$$

*) Helmholtz theorem

$$\mathbf{A} = \mathbf{A}_{\mathrm{trans}} + \mathbf{A}_{\mathrm{long}}$$

uniqueness
(if field vanishes at spatial infinity)

- We can make **gauge-invariant-canonical (g.i.c.) momentum/OAM**

$$\mathbf{p}^{\mathrm{gic}} \equiv \mathbf{p}^{\mathrm{can}} - g\mathbf{A}_{\mathrm{pure}} \qquad \mathbf{L}^{\mathrm{gic}} \equiv \mathbf{r} \times [\mathbf{p}^{\mathrm{can}} - g\mathbf{A}_{\mathrm{pure}}]$$

Several decompositions (theory)

Leader, Lorce (2014), Phys.Rep.541,163 (2014)

Wakamatsu(2014), Int.J.Mod.Phys.A29,1430012(2014)

- **Jaffe-Manohar:**

$$\mathbf{J}_{\mathrm{JM}} = \int d^3x \bar{\psi}\frac{1}{2}\mathbf{\Sigma}\psi + \boxed{\int d^3x \bar{\psi}\mathbf{x}\times(-i\mathbf{\nabla})\,\psi} + \int d^3x (\mathbf{E}^a \times \mathbf{A}^a) + \int d^3x E^{ai}(\mathbf{x}\times\mathbf{\nabla})A^{ai}$$

NPB,337,509 (1990)

q-spin, q-OAM, g-spin, g-OAM

- **Ji:**

$$\mathbf{J}_{\mathrm{Ji}} = \int d^3x \bar{\psi}\frac{1}{2}\mathbf{\Sigma}\psi + \boxed{\int d^3x \bar{\psi}\mathbf{x}\times(-i\mathbf{D})\,\psi} + \int d^3x\, \mathbf{x}\times(\mathbf{E}^a \times \mathbf{B}^a)$$

$$\mathbf{D} = \mathbf{\nabla} - ig\mathbf{A}$$

PRL,78,610 (1997)

q-spin, q-OAM, g-(spin+OAM)

- **Chen et al.:**

PRL,100,232002 (2008)

$$\mathbf{J}_{\mathrm{Chen}} = \int d^3x \bar{\psi}\frac{1}{2}\mathbf{\Sigma}\psi + \boxed{\int d^3x \bar{\psi}\mathbf{x}\times(-i\mathbf{D}_{\mathrm{pure}})\,\psi} + \int d^3x (\mathbf{E}^a \times \mathbf{A}^a_{\mathrm{phys}}) + \int d^3x E^{ai}(\mathbf{x}\times\mathbf{D}_{\mathrm{pure}})A^{ai}_{\mathrm{phys}}$$

q-spin, q-OAM, g-spin, g-OAM

$$\mathbf{D}_{\mathrm{pure}} = \mathbf{\nabla} - ig\mathbf{A}_{\mathrm{pure}}$$

- **Wakamatsu:**

$$\mathbf{J}_{\mathrm{Waka}} = \int d^3x \bar{\psi}\frac{1}{2}\mathbf{\Sigma}\psi + \boxed{\int d^3x \bar{\psi}\mathbf{x}\times(-i\mathbf{D})\,\psi} + \int d^3x (\mathbf{E}^a \times \mathbf{A}^a_{\mathrm{phys}}) + \int d^3x E^{ai}(\mathbf{x}\times\mathbf{D}_{\mathrm{pure}})A^{ai}_{\mathrm{phys}} + \int d^3x \rho^a(\mathbf{x}\times\mathbf{A}^a_{\mathrm{phys}})$$

PRD,81,114010 (2010)

q-spin, q-OAM, g-spin, g-OAM

- **Question:**

  If there are two gauge invariant OAMs,
  which one corresponds to the observable ? Both ?

Mechanical OAM

$$\mathbf{r} \times [-i\boldsymbol{\nabla} - g\mathbf{A}]$$

g.i.c. OAM

$$\mathbf{r}\,[-i\boldsymbol{\nabla} - g\mathbf{A}_{\text{pure}}]$$

- **Idea:**

  Try to answer the question in a simple gauge theory,
  → Test of g.i.c. OAM in QED (**Landau problem**) !

*) Relation between OAM in nucleon spin and OAM in Landau problem:

$$\hat{L}^{\text{gic}} \to \hat{L}^{\text{can}}$$

(take a gauge)

Nucleon spin problem

⟷

PS: pseudo

$$\hat{L}^{\text{ps}} \to \hat{L}^{\text{can}}$$

(take a gauge)

Landau problem

Landau level and OAM

- Schroedinger eq.

$$\hat{H}\psi(x,y) = E\psi(x,y)$$

$$\hat{H} = \frac{(\hat{\mathbf{p}}^{\mathrm{mech}})^2}{2m_e}$$

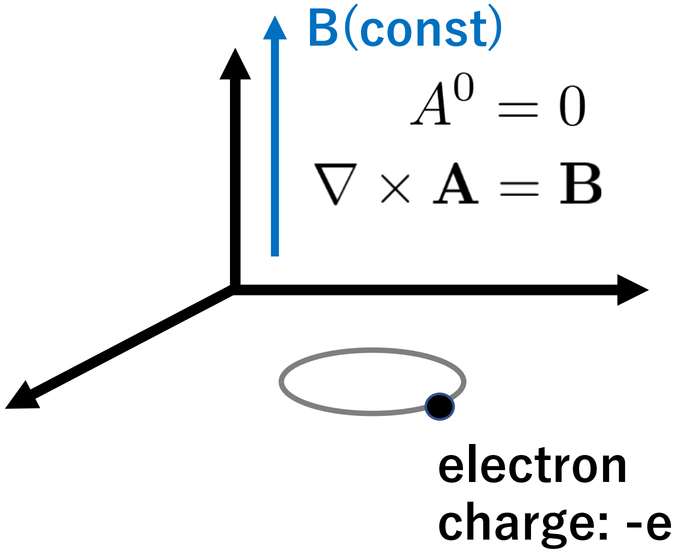
$$\hat{\mathbf{p}}^{\mathrm{can}} = -i\boldsymbol{\nabla}$$

Canonical momentum

$m_e \mathbf{v}$ ↕

$$\hat{\mathbf{p}}^{\mathrm{mech}} = \hat{\mathbf{p}}^{\mathrm{can}} + e\mathbf{A}$$

Mechanical (kinetic) momentum



- Advantages of this system:

-**Simpler** than general QCD for nucleon spin physics

-**Analytical solutions** are known (energy, wave functions, ⋯)

-**Gauge dependence** appears in Hamiltonian (not in energy)

Standard method (well-known)

Landau gauge

$$\mathbf{A}_{\mathrm{L}_1} = (-By, 0, 0)$$
$$\mathbf{A}_{\mathrm{L}_2} = (0, +Bx, 0)$$

Symmetric gauge

$$\mathbf{A}_{\mathrm{S}} = \left(-\frac{By}{2}, +\frac{Bx}{2}, 0\right)$$

……

Other gauges (I have never tried)

- Solutions

Landau, Lifshitz, *Quantum Mechanics, non-relativistic theory, vol.3, chap15*

Landau gauge(L)

$$\psi^{(\mathrm{L})}_{n,k_y}(x,y) = N_n \frac{e^{ik_y y}}{\sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2l_B^2}} H_n\left(\frac{x-x_0}{l_B}\right)$$

$$x_0 = -l_B^2 k_y \qquad l_B = \frac{1}{\sqrt{eB}}$$

Symmetric gauge(S)

$$\psi^{(\mathrm{S})}_{n,m}(r,\phi) = N_{n,m} \frac{e^{im\phi}}{\sqrt{2\pi}} \left(\frac{r^2}{2l_B^2}\right)^{\frac{|m|}{2}} e^{-\frac{r^2}{4l_B^2}} L^{|m|}_{n-\frac{m+|m|}{2}}\left(\frac{r^2}{2l_B^2}\right)$$

$$x = r\cos\phi \qquad y = r\sin\phi$$

- Energy level (Landau level)

$$E_n = \omega\left(n + \frac{1}{2}\right) \quad (n = 0, 1, 2, \dots) \qquad \omega = \frac{eB}{m_e}$$

- Two solutions are not related to gauge transformation

→ **Problem in comparison**

Our method (gauge invariant method)

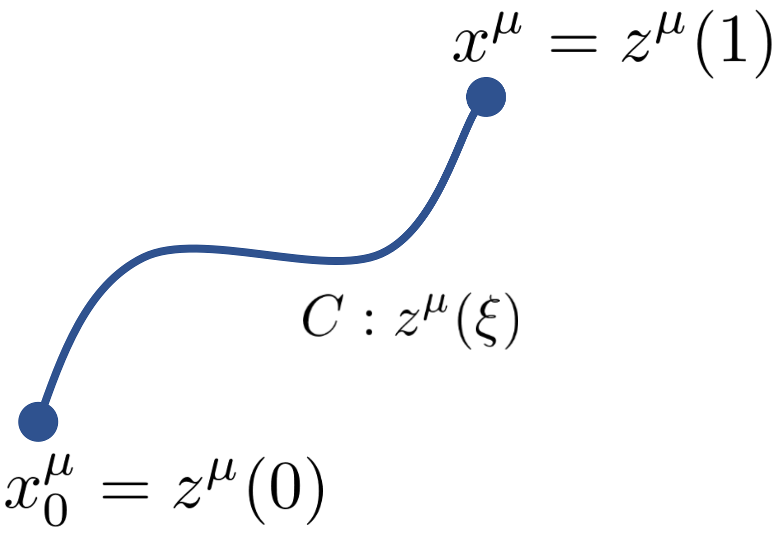
- Fixing gauge is needed in solving Schroedinger eq in standard method.
- Our method, **Gauge invariant method**, does not need gauge fixing.
- DeWitt's gauge invariant fields: B.S.DeWitt, Phys.Rev.125,2189(1962)

$$\tilde{\psi}(x) \equiv e^{ie\Lambda_C(x)}\psi(x)$$

$$\tilde{A}_\mu(x) \equiv A_\mu(x) - \partial_\mu \Lambda_C(x)$$

DeWitt's gauge invariant potential

$$\Lambda_C(x) \equiv \int_C A_\alpha(z) dz^\alpha$$



- No gauge-fixing → But path-fixing is needed.

Solutions in DeWitt's method

$$U^{(\mathrm{C})} = e^{-ie\int_C \mathbf{A}(x)\cdot d\mathbf{x}}$$
$$\tilde{H} = U^{(\mathrm{C})\ -1} H U^{(\mathrm{C})}$$
$$\psi(x,y) = U^{(\mathrm{C})}\tilde{\psi}^{(\mathrm{C})}(x,y)$$

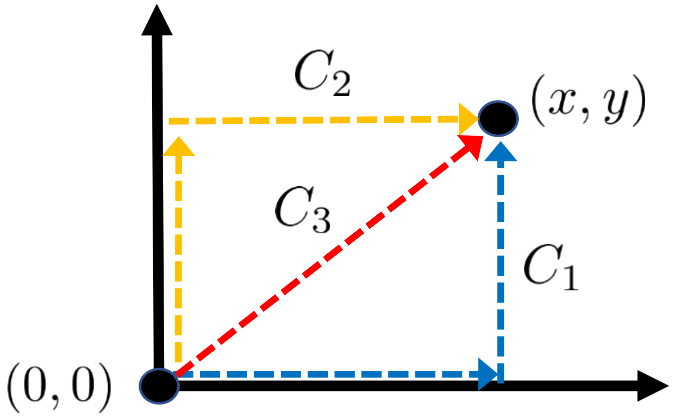
- Unitary transformed eigen eq:

$$H\psi(x,y) = E\psi(x,y)$$
$$H = \frac{(\mathbf{p}^{\mathrm{can}} + e\mathbf{A})^2}{2m_e}$$

Unitary trans →

$$\tilde{H}\tilde{\psi}^{(\mathrm{C})}(x,y) = E\tilde{\psi}^{(\mathrm{C})}(x,y)$$
$$\tilde{H} = \frac{(\mathbf{p}^{\mathrm{can}} + e\tilde{\mathbf{A}})^2}{2m_e}$$

- Eigen eq. can be solved by specifying a path:



Fix a path →

DeWitt's gauge invariant potential

$C_1 : \tilde{\mathbf{A}}^{(\mathrm{C}_1)} = (-By, 0, 0)$ — 1st Landau gauge

$C_2 : \tilde{\mathbf{A}}^{(\mathrm{C}_2)} = (0, +Bx, 0)$ — 2nd Landau gauge

$C_3 : \tilde{\mathbf{A}}^{(\mathrm{C}_3)} = (-\frac{By}{2}, \frac{Bx}{2}, 0)$ — Symmetric gauge

ex) solution for C_3 →

gauge d.o.f.

$$\psi^{(\mathrm{C}_3)}_{n,m}(r,\phi) = \boxed{U^{(\mathrm{C}_3)}} N_{n,m}\frac{e^{im\phi}}{\sqrt{2\pi}}\left(\frac{r^2}{2l_B^2}\right)^{\frac{|m|}{2}} e^{-\frac{r^2}{4l_B^2}} L^{|m|}_{n-\frac{m+|m|}{2}}\left(\frac{r^2}{2l_B^2}\right)$$

Johnson, Lippmann, Phys.Rev. 76 (1949)

Konstantinou, Moulopoulos, Int.J.Theo.Phys. 1484 (2017)

- Pseudo OAM from **Classical EOM**:

$$m_e \frac{d\mathbf{v}(t)}{dt} = -e(\mathbf{v}(\mathbf{t}) \times \mathbf{B}) \Rightarrow \begin{aligned} &\frac{d}{dt}[m_e \mathbf{v}(t) + e\mathbf{x}(t) \times \mathbf{B}] \equiv \frac{d}{dt}\mathbf{p}^{\mathrm{ps}} = 0 \\ &\frac{d}{dt}\left[m_e (\mathbf{x}(t) \times \mathbf{v}(t))_z - \frac{e}{2}(x^2(t) + y^2(t))B_z\right] \equiv \frac{d}{dt}L_z^{\mathrm{ps}} = 0 \end{aligned}$$

pseudo(forgotten) Momentum/OAM

- Two gauge invariant OAM in Landau level system (**Quantised theory**):

1) Pseudo OAM

$$\hat{L}_z^{\mathrm{ps}} = (\mathbf{x} \times \hat{\mathbf{p}}_{\mathrm{mech}}) - \frac{eB}{2}(x^2 + y^2)$$

$$\rightarrow (\mathbf{x} \times \hat{\mathbf{p}}_{\mathrm{can}})$$ **(in symmetric gauge)**

$$\left[\hat{L}_z^{\mathrm{ps}}, \hat{H}\right] = 0$$ Commutable in arbitrary gauge (conserved)

2) Mechanical OAM

$$\hat{L}_z^{\mathrm{mech}} = (\mathbf{x} \times \hat{\mathbf{p}}_{\mathrm{mech}})_z$$

$$\hat{\mathbf{p}}^{\mathrm{mech}} = -i\boldsymbol{\nabla} + e\mathbf{A}$$

$$\left[\hat{L}_z^{\mathrm{mech}}, \hat{H}\right] \neq 0$$ Not commutable

Which OAM describes an observable ?

- Results of $\langle \hat{L}_z^{\mathrm{can}} \rangle$ $\langle \hat{L}_z^{\mathrm{mech}} \rangle$ $\langle \hat{L}_z^{\mathrm{ps}} \rangle$

note: $H\psi_{n,m}^{(\mathrm{C}_3)} = \omega \left(n + \frac{1}{2} \right) \psi_{n,m}^{(\mathrm{C}_3)}$

$$L_z^{\mathrm{ps}} \psi_{n,m}^{(\mathrm{C}_3)} = m \psi_{n,m}^{(\mathrm{C}_3)}$$

$$L_z^{\mathrm{can}} \psi_{n,m}^{(\mathrm{C}_3)} = m \psi_{n,m}^{(\mathrm{C}_3)}$$

Multi-valued gauge

	1st Landau $\mathbf{A}_{\mathrm{L}_1}$	2nd Landau $\mathbf{A}_{\mathrm{L}_2}$	Symmetric $\mathbf{A}_{\mathrm{S}}$	Bawin-Burnel $\mathbf{A}_{\mathrm{BB}}$	
$\langle \hat{L}_z^{\mathrm{can}} \rangle$	m	m	m	$2n+1$	gauge dep.
$\langle \hat{L}_z^{\mathrm{mech}} \rangle$	$2n+1$	$2n+1$	$2n+1$	$2n+1$	gauge inv.
$\langle \hat{L}_z^{\mathrm{ps}} \rangle$	m	m	m	m	gauge inv.

- Both mechanical/ps OAM are gauge invariant. However,

1) $L^{\mathrm{mech}}_z = 2n+1$ is related to energy

→ physical OAM (observable)

2) measurement of $L^{\mathrm{ps}}_z = m$,

→ measurement of $L^{\mathrm{can}}_z = m$.

Not observable

Summary

- We studied **pseudo OAM** in Landau level problem.
- The pseudo OAM is an analogy with **g.i.c. OAM** in nucleon spin problem.
- Expectation value of pseudo OAM is same with canonical OAM,

  → Measurement of pseud OAM = Measurement of canonical OAM.

  gauge dependent
- **The pseudo OAM does not describe the observable.**

  (at least in Landau level)
- Hence g.i.c. OAM in nucleon spin will not describe an observable.

Extra Slides

Gauge transformation from one to another gauge

- Integrate degeneracy to compensate different degeneracy !

$$\mathbf{A}^{(\mathrm{L}_2)} = \mathbf{A}^{(\mathrm{S})} + \nabla \frac{eB}{2} xy$$

$$\psi^{(\mathrm{L}_2)}_{n,m} = e^{-i\frac{eB}{2}xy} \psi^{(\mathrm{S})}_{n,m}$$

$$\psi^{(\mathrm{L}_2)}_{n,m} \equiv \int dk_x U_{n,m}(k_y) \psi^{(\mathrm{L}_2)}_{n,k_y}$$

Haugset et al (1993)

Wakamatsu, Y.K.P.M. Zhang(2018)

Symmetric gauge

$$\mathbf{A}_{\mathrm{S}} = \left(-\frac{By}{2}, +\frac{Bx}{2}, 0\right)$$

$$\psi^{(\mathrm{S})}_{n,m}(x,y)$$

Swenson (1989)

Konstantinou, Moulopous (2017)

2nd Landau gauge

$$\mathbf{A}_{\mathrm{L}_2} = (0, +Bx, 0)$$

$$\psi^{(\mathrm{L}_1)}_{n,k_x}(x,y)$$

$$\mathbf{A}^{(\mathrm{L}_2)} = \mathbf{A}^{(\mathrm{L}_1)} + \nabla eBxy$$

$$\psi^{(\mathrm{L}_2)}_{n,k_x} = e^{-ieBxy} \psi^{(\mathrm{L}_1)}_{n,k_x}$$

$$\psi^{(\mathrm{L}_2)}_{n,k_x} \equiv \int dk_y U_{n,k_y}(k_y) \psi^{(\mathrm{L}_2)}_{n,k_y}$$

1st Landau gauge

$$\mathbf{A}_{\mathrm{L}_1} = (-By, 0, 0)$$

$$\psi^{(\mathrm{L}_2)}_{n,k_y}(x,y)$$

Recent data for polarized quark and gluon

NNPDF, Nucl. Phys. B 887, 276 (2014)

$$\Delta\Sigma = 0.23 \pm 0.15$$ (NNPDFpol 1.0)

$$\Delta\Sigma = 0.25 \pm 0.10$$ (NNPDFpol 1.1)

$$\int_0^1 dx \Delta q(Q^2, x) \to \int_{10^{-3}}^1 dx \Delta q(Q^2, x)$$

$$\Delta\Sigma = 0.366^{+0.042}_{-0.062}$$ (DSSV08)

$$\Delta G = 0.013^{+0.702}_{-0.314}$$ (DSSV08)

$$Q^2 = 10\mathrm{GeV}^2$$

COMPASS, Phys. Lett. B 753, 18 (2016)

$$\Delta\Sigma = [0.26, 0.36] \qquad Q^2 = 3\mathrm{GeV}^2$$

Lattice calculation

- Results for $\Delta\Sigma$

Huey-Wen Lin et al. (PNDME Collaboration), arXiv:1806.10604 (2018)

2+1+1 flavor, Highly Improved Staggered Quark

Chiral continuum extrapolation

MS scheme at 2 GeV

$$\frac{1}{2}\Delta\Sigma = 0.143(31)(29)$$

$$\mathrm{at} \quad 2\mathrm{GeV}$$

$$m_\pi = 135 \ \mathrm{GeV}$$

Agree with COMPASS Exp data

- First Lattice QCD result for ΔG

Yi-Bo Yang et al. (χQCD Collaboration), PRL118,102001(2017)

$$\Delta G = 2\int d^3x \mathrm{Tr}\left[\mathbf{E}_c(x) \times \mathbf{A}_c(x)\right]$$

JM-type

Valence overlap fermions on 2+1 flavor

MS with one-loop perturbative matching

Coulomb gauge

$$\Delta G = 0.251(47)(16)$$

$$\mathrm{at} \quad 10\mathrm{GeV}^2$$

$$0 < |\vec{p}| < 1.5 \ \mathrm{GeV}$$

50% of proton

Example of gauge invariant gauge field

- DeWitt's gauge invariant fields:

$$\tilde{\psi}(x) \quad \equiv \quad e^{ie\Lambda_C(x)}\psi(x) \qquad \tilde{A}_\mu(x) \equiv A_\mu(x) - \partial_\mu \Lambda_C(x) \qquad \Lambda_C(x) \equiv \int_C A_\alpha(z) dz^\alpha$$

- For path C_1:

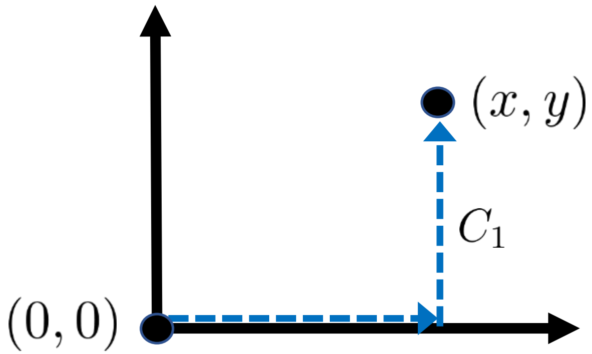
$$\Lambda_{C_1} = \int_0^x dx' A_x(x', 0) + \int_0^y dy' A_y(x, r') \qquad A_0 = 0$$

$$\tilde{A}_i^{(\mathrm{C}_1)}(x, y) = A_i(x, y) - \partial_i \Lambda_{C_1}$$

$$\partial_y \Lambda_{C_1} = A_y(x, y)$$

$$\partial_x \Lambda_{C_1} \quad = \quad A_x(x, 0) + \int_0^y dy' \frac{\partial A_y(x, y')}{\partial x} = A_x(x, 0) + \int_0^y dy' \left[B + \frac{A_x(x, y')}{\partial y'} \right]$$

$$= \quad By + A_x(x, y)$$



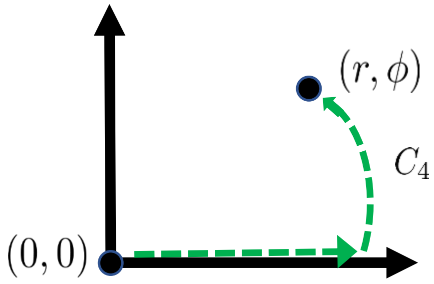
def. of magnetic field

$$\tilde{\mathbf{A}}^{(\mathrm{C}_1)}(x, y) = (-By, 0, 0)$$

1st Landau gauge

Bawin-Burnel (gauge) path

$$C_4 \,:\, \psi^{(C_4)}_{n,m}(r,\phi)$$



- Wave function for C_4,

$$\psi^{(C_4)}_{n,m}(r,\phi) = U^{(C_4)}\tilde{\psi}^{(C_4)}_{n,m}(r,\phi)$$

$$U^{(C)_4} = e^{-ie\left[\int_0^r A_r(r',0)dr' + \int_0^\phi A_\phi(r,\phi')r d\phi'\right]}$$

$$\tilde{\psi}^{(C_4)}_{n,m}(r,\phi) = N_{n,m}\frac{e^{i\left(m+\frac{eB}{2}r^2\right)\phi}}{\sqrt{2\pi}}\left(\frac{r^2}{2l_B^2}\right)^{\frac{|m|}{2}} e^{-\frac{r^2}{4l_B^2}} L^{|m|}_{n-\frac{m+|m|}{2}}\left(\frac{r^2}{2l_B^2}\right)$$

- Periodicity,

$$m = \text{integer}$$

$$e^{im(\phi+2\pi)} = e^{im\phi} \qquad e^{i\frac{eBr^2}{2}(\phi+2\pi)} \neq e^{i\frac{eBr^2}{2}\phi} \qquad \frac{eBr^2}{2} \neq \text{integer}$$

$$\Rightarrow \quad \tilde{\psi}^{(C_4)}_{n,m}(r,\phi+2\pi) = e^{i\pi eBr^2}\tilde{\psi}^{(C_4)}_{n,m}(r,\phi)$$

Not single-valued

(Multi-valued)