Hadronic matrix elements for Dark Matter and other searches

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Budapest-Marseille-Wuppertal collaboration (BMWc)

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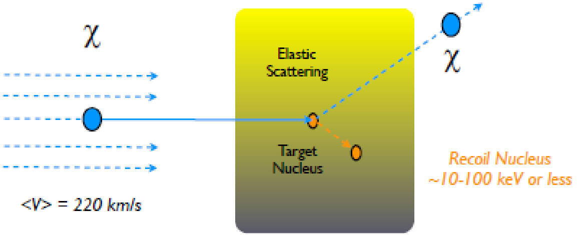
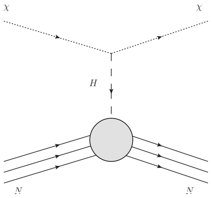
Direct WIMP dark matter detection



$$\downarrow$$



$$\mathcal{L}_{q\chi} = \sum_q \lambda_q^\Gamma [\bar{q}\Gamma q][\bar{\chi}\Gamma\chi] \longrightarrow \mathcal{L}_{N\chi} = \lambda_N^\Gamma [\bar{N}\Gamma N][\bar{\chi}\Gamma\chi]$$

Quark are confined within nucleons
$\rightarrow$ nonperturbative QCD tool

WIMP-nucleus spin-independent cross section . . .

- In low-E limit

$$\frac{d\sigma^{\rm SI}_{\chi\,{}^A_Z X}}{dq^2} = \frac{1}{\pi v^2}\,[Zf_p + (A-Z)f_n]^2\;|F_X(q^2)|^2$$

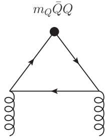
w/ $F_X(\vec{q}=0)=1$ nuclear FF and χN couplings $(N=p,n)$

$$\frac{f_N}{M_N} = \sum_{q=u,d,s} f_q^N \frac{\lambda_q}{m_q} + \sum_{Q=c,b,t} f_Q^N \frac{\lambda_Q}{m_Q}$$

such that $(f=u,\ldots,t$ and $\langle N(\vec{p}\,')|N(\vec{p})\rangle = (2\pi)^3\delta^{(3)}(\vec{p}\,'-\vec{p}))$

$$f_{ud}^N M_N = \sigma_{\pi N} = m_{ud}\langle N|\bar{u}u+\bar{d}d|N\rangle, \qquad f_f^N M_N = \sigma_{fN} = m_f\langle N|\bar{f}f|N\rangle$$

- For heavy $Q=c,b,t$ (Shifman et al '78)



$$\longrightarrow \qquad m_Q\,\bar{Q}Q = -\frac{1}{3}\frac{\alpha_s}{4\pi}G^2 + O\!\left(\frac{\alpha_s^2\mathcal{O}_6}{4m_Q^2}\right)$$

. . . and relevant hadronic matrix elements

- Then obtain f_Q^N in terms of f_q^N through to $M_N = \langle N|\theta^\mu{}_\mu|N\rangle$, w/

$$\theta^\mu{}_\mu = (1-\gamma_m(\alpha_s))\left[\sum_{q=u,d,s} m_q\bar{q}q + \sum_{Q=c,b,t} m_Q\bar{Q}Q\right] + \frac{\beta(\alpha_s)}{2\alpha_s}G^2$$

- Integrate out $Q=t,b,c$ and obtain f_c^N from f_q^N, $q=u,d,s$, etc.

- Will be done to $O(\alpha_s^3)$ (Hill et al '15), but at LO find

$$f_Q^N \equiv \frac{\langle N|m_Q\bar{Q}Q|N\rangle}{M_N} = \frac{2}{27}\left[1-\sum_{q=u,d,s} f_q^N\right] + O\left(\alpha_s, \alpha_s^2\frac{\Lambda_{\rm QCD}^2}{4m_Q^2}\right)$$

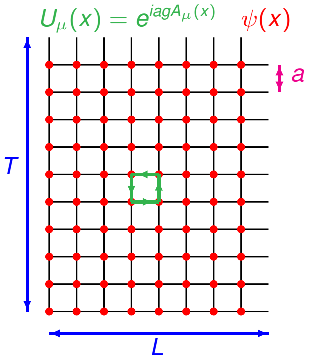
since $4\pi\beta(\alpha_s) = -\beta_0\alpha_s^2 + O(\alpha_s^3)$ and $\beta_0 = 11 - \frac{2}{3}N_q - \frac{2}{3}N_Q$

- For f_q^N, $q=u,d,s$, use lattice QCD and Feynman-Hellman theorem

$$f_q^N M_N = \langle N|m_q\bar{q}q|N\rangle = m_q \left.\frac{\partial M_N}{\partial m_q}\right|_{m_q^\Phi}$$

What is lattice QCD (LQCD)?

To describe ordinary matter, QCD requires $\geq$ 104 numbers at every point of spacetime
$\rightarrow \infty$ number of numbers in our continuous spacetime
$\rightarrow$ must temporarily "simplify" the theory to be able to calculate (regularization)
$\Rightarrow$ Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:



- UV (& IR) cutoff $\rightarrow$ well defined path integral in Euclidean spacetime:

$$\langle O \rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-S_G - \int \bar{\psi}D[M]\psi}\, O[U,\psi,\bar{\psi}]$$
$$= \int \mathcal{D}U\, e^{-S_G} \det(D[M])\, O[U]_{\text{Wick}}$$

- $\mathcal{D}Ue^{-S_G}\det(D[M]) \geq 0$ & finite # of dofs
  $\rightarrow$ evaluate numerically using stochastic methods

LQCD is QCD but only when $N_f \geq 2+1$, $m_q \rightarrow m_q^{\text{phys}}$, $a \rightarrow 0$, $L \rightarrow \infty$

HUGE conceptual and numerical challenge (integrate over $\sim 10^9$ real variables)

$\Rightarrow$ very few calculations control all necessary limits

Strategy of calculation

Objective:

- Determine slope of M_N wrt m_q, $q = u, d, s$, at physical point

Method:

- Perform many high-statistics simulations with various m_q around physical values, various $a \lesssim 0.1\,\mathrm{fm}$ and various $L \gtrsim 6\, fm$
- For each compute M_π $(\to m_{ud})$, $M_{\eta_s} = \sqrt{2M_K^2 - M_\pi^2}$ $(\to m_s)$, M_{D_s} $(\to m_c)$ and M_N $(\to \Lambda_{\mathrm{QCD}})$
- Study dependence of m_q, $q = ud, s, c$ and M_N on M_π, M_{η_s}, M_{D_s}, a and L
- For each simulation determine a, m_q^Φ's such that M_π, ... take their physical value in $a \to 0$ and $L \to \infty$ limit
- Compute, at physical point

$$f_q^N = \sum_{P=\pi,\eta_s} \frac{\partial \ln M_P^2}{\partial \ln m_q} \frac{\partial \ln M_N}{\partial \ln M_P^2}$$

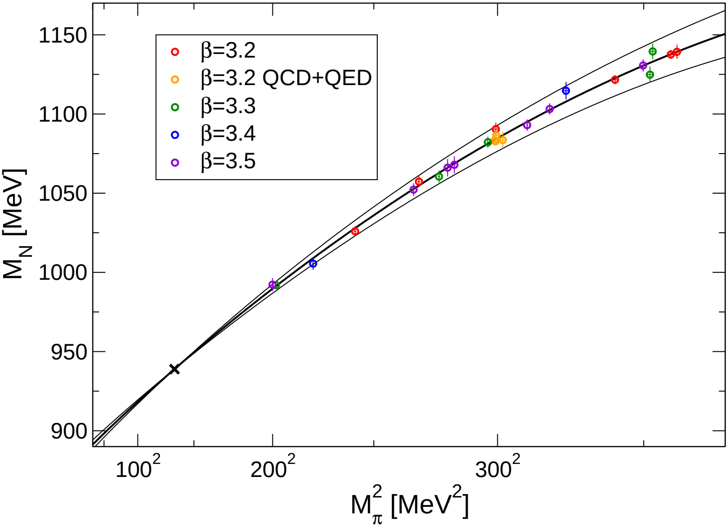
Lattice details

- $N_f = 1+1+1+1$
- 3HEX clover-improved Wilson fermions on tree-level improved Symanzik gluons
- 33 ensembles w/ total $\sim$ 169000 trajectories
- $\sim$ 500 measurements per configuration
- 4 $a \in [0.064, 0.102]\,\mathrm{fm}$;
- $M_\pi \in [195, 450]\,\mathrm{MeV}$ w/ $LM_\pi > 4$

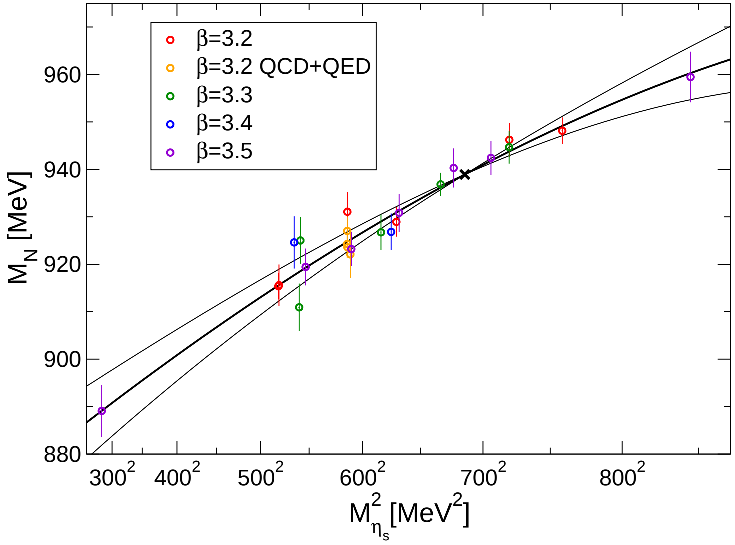
Improvements over BMWc, PRL '16

✓ Charm in sea
✓ $\gtrsim \times 100$ in statistics
✓ $\gtrsim \times 2$ lever arm in m_s
✓ Like PRL '16 FH in terms of quark and not meson masses
✗ No physical m_{ud}, but small enough and know M_N from experiment

$M_\pi^2 \sim m_{ud}$ dependence of M_N (preliminary)



$M_{\eta_s} \sim m_s$ dependence of M_N (preliminary)



Chain rule conversion matrix (preliminary)

- Have:

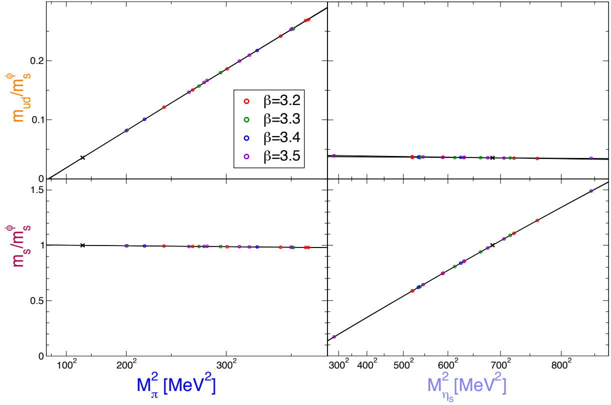
$$\frac{\partial \ln M_N}{\partial \ln M_P^2}$$

w/ $P = \pi, \eta_s$

- Want:

$$\frac{\partial \ln M_N}{\partial \ln m_q}$$

w/ $q = u, d, s$



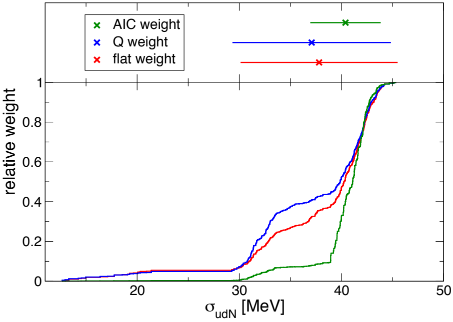
$$\begin{pmatrix} \frac{\partial \ln M_\pi^2}{\partial \ln m_{ud}} & \frac{\partial \ln M_{\eta_s}^2}{\partial \ln m_{ud}} \\ \frac{\partial \ln M_\pi^2}{\partial \ln m_s} & \frac{\partial \ln M_{\eta_s}^2}{\partial \ln m_s} \end{pmatrix} = \begin{pmatrix} 0.94(1)(1) & 0.002(0)(1) \\ 0.06(2)(3) & 1.02(0)(2) \end{pmatrix}$$

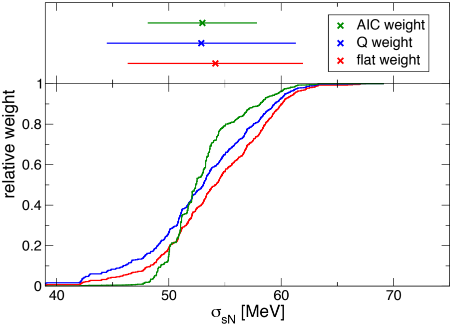
Systematic error assessment (preliminary)

Estimated using extended frequentist approach (BMWc, Science '08, Science '15)

- Excited state contamination: 4 time intervals for correlations functions
- Mass interpolation/extrapolation errors
  - $M_\pi \leq 330/360/420\,\mathrm{MeV}$
  - different M_{π/η_s} dependences (polynomials, Padés, χPT)
- continuum extrapolation: $O(\alpha_s a)$ vs $O(a^2)$

$\Rightarrow$ 672 analyses which differ by higher order effects





Preliminary results

- Direct results

$$f_{ud}^N = 0.0430(19)(32)\ [8.6\%] \qquad f_s^N = 0.0564(38)(35)\ [9.2\%]$$

- Using SU(2) isospin (BMWc, PRL116) w/ $\Delta_{\rm QCD} M_N = 2.52(17)(24)\,\mathrm{MeV}$ (BMWc, Science 347) & $m_u/m_d = 0.485(11)(16)$ (BMWc, PRL117)

$$f_u^p = 0.0153(6)(11)\ [8.1\%] \qquad f_d^p = 0.0264(13)(21)\ [9.4\%]$$

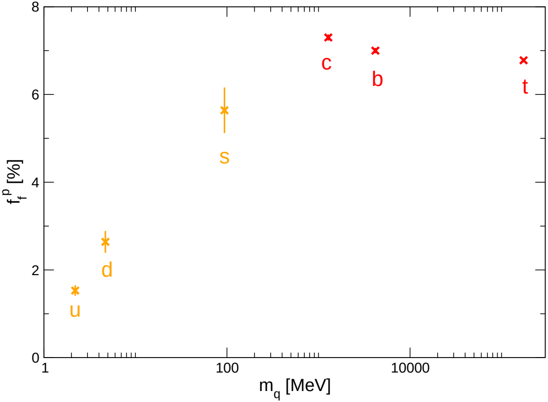
$$f_u^n = 0.0128(6)(11)\ [9.7\%] \qquad f_d^n = 0.0316(13)(21)\ [7.9\%]$$

- Using $f_{ud,s}^N$ & HQ expansion up to $O(\alpha_s^4, \Lambda_{\rm QCD}^2/m_c^2)$ corrections (Hill et al '15)

$$f_c^N = 0.0730(5)(5)(??)\ [1.0 + ??\%] \qquad f_b^N = 0.0700(4)(4)(??)\ [0.9 + ??\%]$$

$$f_t^N = 0.0678(3)(3)(??)\ [0.7 + ??\%]$$

Low-energy effective *h*-*N* coupling (preliminary)



- f_{fN} is q contribution to effective coupling of Higgs to nucleon in units of M_N
- $\Rightarrow$ fraction of M_N coming from q contribution to coupling of N to Higgs vev
- HQ expansion $\Rightarrow$ $Q = c, b, t$ contributions mainly through their impact on the running of α_s

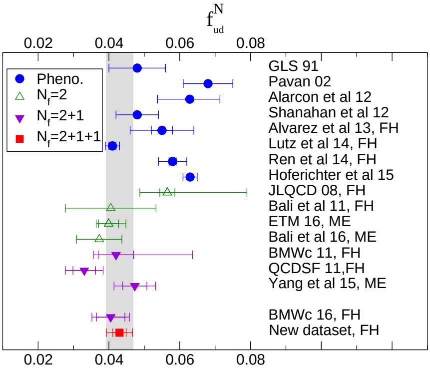
$$f_N = 0.310(3)(3)(??)\ [1.4 + ??\%]$$

Conclusion

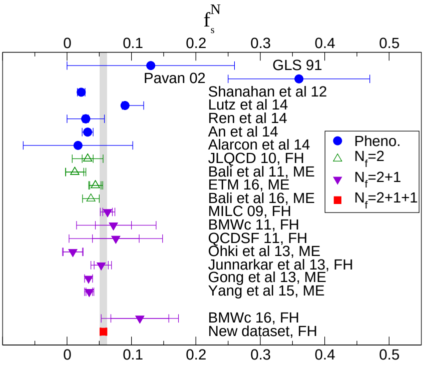
- Scalar quark contents of p & n have been computed with full control over all sources of uncertainties
- Important for: DM searches; coherent LFV $\mu \to e$ conversion in nuclei; describing low-energy coupling of N to the Higgs; understanding M_N; πN and KN scattering, etc.
- f_q^N, $q = u, d, s$ & $N = n, p$ are now known to better than 10%
- f_Q^N, $Q = c, b, t$ and f_N to even better precision
- Correlations between the various quantities will be given
- Hadronic ME are no longer the dominant source of uncertainty in DM direct detection rate predictions ...
- ...or in the determination of WIMP couplings from possible DM signals

BACKUP

Comparison

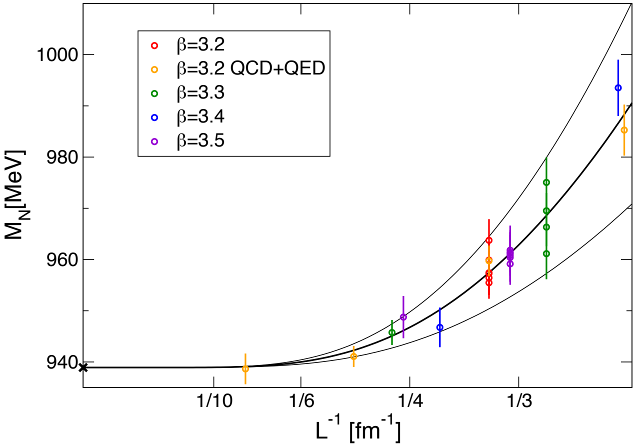


f^N_{ud}
0.02
0.04
0.06
0.08
Pheno.
N_f=2
N_f=2+1
N_f=2+1+1
GLS 91
Pavan 02
Alarcon et al 12
Shanahan et al 12
Alvarez et al 13, FH
Lutz et al 14, FH
Ren et al 14, FH
Hoferichter et al 15
JLQCD 08, FH
Bali et al 11, FH
ETM 16, ME
Bali et al 16, ME
BMWc 11, FH
QCDSF 11,FH
Yang et al 15, ME
BMWc 16, FH
New dataset, FH



f^N_s
0
0.1
0.2
0.3
0.4
0.5
GLS 91
Pavan 02
Shanahan et al 12
Lutz et al 14
Ren et al 14
An et al 14
Alarcon et al 14
JLQCD 10, FH
Bali et al 11, ME
ETM 16, ME
Bali et al 16, ME
MILC 09, FH
BMWc 11, FH
QCDSF 11, FH
Ohki et al 13, ME
Junnarkar et al 13, FH
Gong et al 13, ME
Yang et al 15, ME
BMWc 16, FH
New dataset, FH
Pheno.
N_f=2
N_f=2+1
N_f=2+1+1

Finite-volume effects

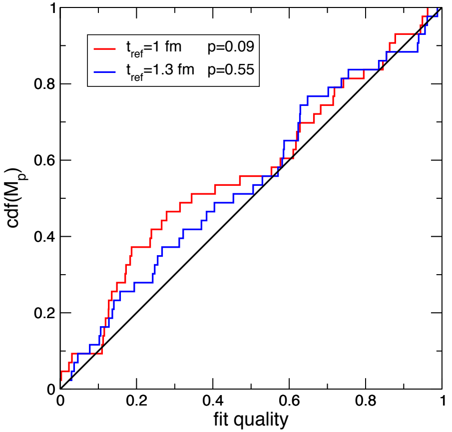


- Fit away leading effects $\frac{M_X(L)-M_X}{M_X} = cM_\pi^{1/2}L^{-3/2}e^{-LM_\pi}$
- Compabtible w/ χPT expectation (Colangelo et al '10)

Kolmogorov-Smirnov test and ground state extraction

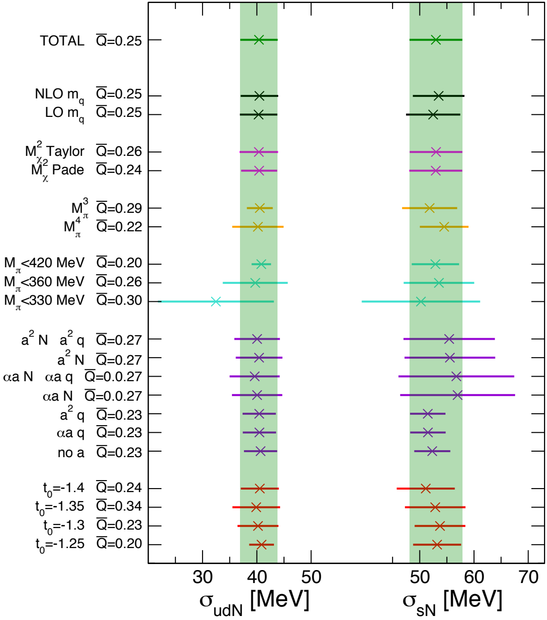
Selection of fit-time range is crucial and delicate to isolate *N* ground state in correlation functions

- Consider cumulative distributions of the fit qualities over 33 ensembles for different $t_{\min}$ and hadrons
- Fit quality should be uniformly distributed
- Apply Kolmogorov-Smirnov analysis to test measured distributions
- Keep $t_{\min} \ni$ distribution compatible w/ uniform distribution w/ prob. $> 30\%$

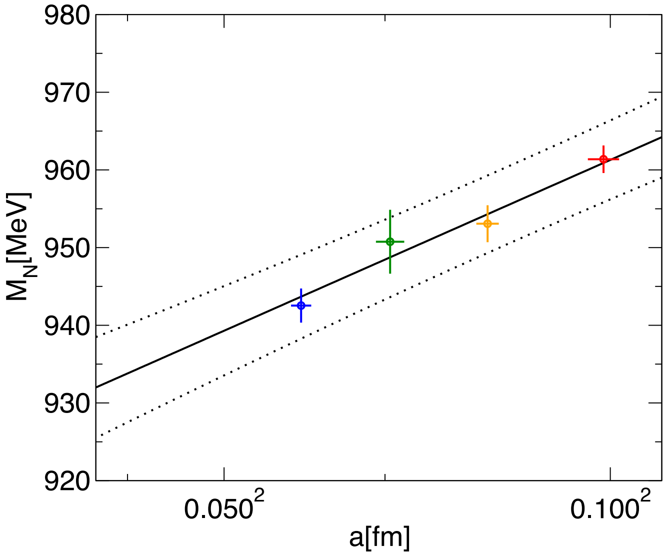


Systematic error decomposition

- ✓ All analyses in good agreement
- ✓ Reasonable fit quality
  $0.03 < Q < 0.65$



Example continuum extrapolation of M_N (preliminary)



New method for obtaining $f_{u/d}^{p/n}$

[BMWc, PRL 116 (2016)]

- Input: f_{ud}^N and $\Delta_{\rm QCD}M_N = M_n - M_p$ (from BMWc, Science '15)
- SU(2) relations w/ $\delta m = m_d - m_u$

$$H = H_{\rm iso} + H_{\delta m}\,, \quad H_{\delta m} = \frac{\delta m}{2}\int d^3x\,(\bar{d}d - \bar{u}u)$$

$$\Delta_{\rm QCD}M_N = \delta m\langle p|\bar{u}u - \bar{d}d|p\rangle$$

lead to, w/ $r = m_u/m_d$,

$$f_u^{p/n} = \left(\frac{r}{1+r}\right) f_{ud}^N \pm \frac{1}{2}\left(\frac{r}{1-r}\right)\frac{\Delta_{\rm QCD}M_N}{M_N}$$

$$f_d^{p/n} = \left(\frac{1}{1+r}\right) f_{ud}^N \mp \frac{1}{2}\left(\frac{1}{1-r}\right)\frac{\Delta_{\rm QCD}M_N}{M_N}$$

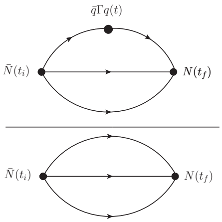
- Huge improvement on usual SU(3)-flavor approach

systematic: $\left(\frac{m_s - m_{ud}}{\Lambda_{QCD}}\right)^2 \approx 10\% \quad \longrightarrow \quad \left(\frac{m_d - m_u}{\Lambda_{QCD}}\right)^2 \approx 0.01\%\,.$

σ-terms from LQCD: matrix element (ME) method

$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle \qquad \sigma_{sN} = m_s\langle N|\bar{s}s|N\rangle$$

Extract directly from time-dependence of 3-pt fns:



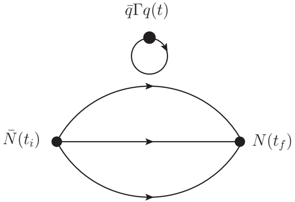
$$\xrightarrow{(t-t_i),(t_f-t)\to\infty} \quad \langle N(\vec{0})|\bar{q}\Gamma q|N(\vec{0})\rangle$$

✓ Desired matrix element appears at leading order

✗ Must compute more noisy 3-pt fn

✗ Quark-disconnected contribution difficult, though $1/N_c$ suppressed

✗ $m_q\bar{q}q$ renormalization challenging (Wilson fermions)

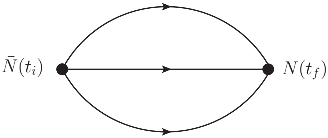


σ-terms from LQCD: Feynman-Hellmann (FH) method

Feynman-Hellmann theorem yields:

$$\langle N|m_q\bar{q}q|N\rangle = m_q \left.\frac{\partial M_N}{\partial m_q}\right|_{m_q^\Phi}$$

On lattice get M_N from time-dependence of 2pt-fn, e.g.:



$$\xrightarrow{(t-t_i)\to\infty} \quad \langle 0|N|N\rangle\langle N|\bar{N}|0\rangle \exp\{-M_N(t_f - t_i)\}$$

- ✓ Only simpler and less noisy 2pt-fn is needed
- ✓ No difficult quark-disconnected contributions
- ✓ No difficult renormalization
- ✗ $\partial M_N/\partial m_q$ small for $q = [ud]$ and even smaller for $q = s, c, \ldots$