$B \to D^{(*)}\ell\nu$ Semileptonic Decays using Oktay-Kronfeld Heavy Quarks on $2+1+1$-flavor Lattice QCD

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(LANL/SWME Collaboration)

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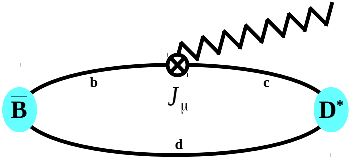
Form Factors from Lattice QCD: $\mathcal{F}(w)\,,\mathcal{G}(w)$



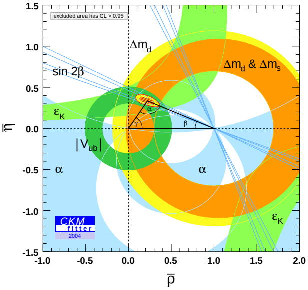
- $B \to D^*\ell\nu$

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_{D^*}^3}{4\pi^3}(M_B - M_{D^*})^2(w^2-1)^{1/2}\chi(w)|\eta_{\mathrm{EW}}|^2|V_{cb}|^2|\mathcal{F}(w)|^2$$

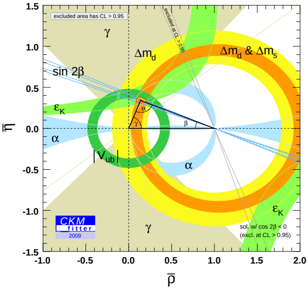
- $B \to D\ell\nu$

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_B^5}{48\pi^3}(w^2-1)^{3/2}r^3(1+r)^2|\eta_{\mathrm{EW}}|^2|V_{cb}|^2|\mathcal{G}(w)|^2$$

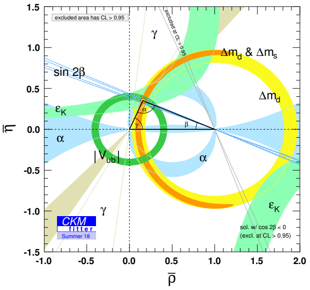
Impact on Unitarity Triangle Analysis



2004



2009



2018

- As B experiments improved, the measurements of angles (α, β, γ) and the mass differences $(\Delta m_{d,s})$ became more precise.
- The parabolic band ε_K still has a largest uncertainty in the global fit.
- The leading dependence of ε_K is: $\varepsilon_K \propto c_\varepsilon \hat{B}_K |V_{cb}|^4$

P. Gambino [THU], M. Ciuchini [FRI], S. Gottlieb [SAT]

V_{cb} from Exclusive Decays $B \to D^* \ell \nu$

- $\sim 3\sigma$ tension between inclusive and excluive V_{cb} determinations
- FF normalizations $\mathcal{F}(1)$ from experiments depend on a parameterization:
  CLN (Caprini, Lellouch, Neubert) [Nucl. Phys. B **530**, 153 (1998)]
  BGL (Boyd, Grinstein, Lebed) [Phys. Rev. Lett. **74**, 4603 (1995)]

- $10^3 \times |V_{cb}||\eta_{\text{EW}}|\mathcal{F}(1) = 35.90(45) \to 35.61(43)$ [HFLAV(2016)]

$$\mathcal{F}(1) = h_{A_1}(1) = 0.906(4)(12) \quad \text{[FNAL/MILC(2014)]}$$
$$\mathcal{F}(1) = h_{A_1}(1) = 0.895(10)(24) \quad \text{[HPQCD(2018)]}$$

$$\begin{aligned} 10^3|V_{cb}| &= 39.04(49)_{\text{exp}}(53)_{\text{QCD}}(19)_{\text{QED}} \quad \text{[FNAL/MILC]} \\ &\to 38.72(46)_{\text{exp}}(53)_{\text{QCD}}(19)_{\text{QED}} \\ &c.f.\ 38.9(7) \quad \text{[FNAL/MILC + HPQCD + HFLAV(2016)]} \end{aligned}$$

$\to$ lattice form factors / exclusive V_{cb} with $\sim 1\%$ error

Lattice Setup

Ensemble	$N_S^3 \times N_T$	$M_\pi L$	M_π	$10/g^2$	am_l	am_s	am_c	u_0	r_1/a_{r_1}	a_{r_1}(fm)	$N_{\rm conf} \times N_{\rm src}$
$a12m310$	$24^3 \times 64$	4.54	305.3(4)	6.00	0.0102	0.0509	0.635	0.86372	2.575(17)	0.1207(11)	1053×3
$a12m220$	$32^3 \times 64$	4.29	216.9(2)	6.00	0.00507	0.0507	0.628	0.86372	2.626(13)	0.1184(10)	1000×3
$a09m310$	$32^3 \times 96$	4.50	312.7(6)	6.30	0.0074	0.037	0.440	0.874164	3.499(24)	0.0888(8)	1001×3
$a09m220$	$48^3 \times 96$	4.71	220.3(2)	6.30	0.00363	0.0363	0.430	0.874164	3.566(14)	0.0872(7)	1001×3

[arXiv:1812.07675]

- The highly improved heavy quark (c, b) action (Oktay-Kronfeld action) will reduce the heavy quark discretization error (dominant error)

- Using MILC HISQ ensemble [Phys. Rev. D87 (2013) 054505] of $N_f = 2(u,d) + 1(s) + 1(c)$ dynamical quarks will improve continuum/chiral extrapolation (second largest error)

- Coherent sequential sources: 1 sequential inversion for $N_{\rm src}$
- Gaussian smeared quark source and sink operators improve statistics and systematics, e.g., excited-state contamination in $B, D^{(*)}$

$\rightarrow$ lattice form factors / exclusive V_{cb} with $\sim 1\%$ error

Reduce the Heavy Quark Discretization Error

- Symanzik improved action / operator:
  - discretization error enters as $\mathcal{O}((am)^n) \to am_Q < 1$
  - $am_b \sim 1$ for $a \sim 0.045\,\mathrm{fm}$
  - MILC $2+1+1$-flavor HISQ ensemble includes $a \sim 0.03\,\mathrm{fm}$ [arXiv:1712.09262]

- Fermilab method: tune (or *interprete*) the Wilson clover action in *nonrelativistic* way
  [A. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, PRD **55**, 3933 (1997)]

$$E = m_1 + \frac{\boldsymbol{p}^2}{2m_2} - \frac{(\boldsymbol{p}^2)^2}{8m_4^3} - \frac{a^3 w_4}{6} \sum_i p_i^4 + \cdots$$

$$(m_1 = m_2)\,,\ m_Q = m_2 \to \mathcal{O}(\lambda)$$

  - discretization error enters with $\sim (\Lambda/m_Q)^n < 1$ for heavy quark
  - and smoothly connects to the light quark scaling $\sim (am_0)^n$
  - $m_1 = m_2$ by lifting the axis interchange symmetry
- OK action: $m_Q = m_2 = m_4\,,\ w_4 = 0 \to \mathcal{O}(\lambda^3)$

Oktay-Kronfeld (OK) Action

[M. B. Oktay and A. S. Kronfeld, PRD 78, 014504 (2008)]

- OK action includes dim-6 and -7 operators ($S_{\rm new}$) necessary for tree-level matching to QCD through order $\mathcal{O}(\Lambda^3/m_Q^3)$:

$$S_{\rm OK} = S_{\rm Fermilab} + S_{\rm new}$$

$$\begin{aligned} S_{\rm new} &= c_1 a^2 \sum_x \bar{\psi}_x \sum_i \gamma_i D_i \triangle_i \psi_x \\ &+ c_2 a^2 \sum_x \bar{\psi}_x \{\vec{\gamma} \cdot \vec{D}, \triangle^{(3)}\} \psi_x \\ &+ c_3 a^2 \sum_x \bar{\psi}_x \{\vec{\gamma} \cdot \vec{D}, i\vec{\Sigma} \cdot \vec{B}\} \psi_x \\ &+ c_{EE} a^2 \sum_x \bar{\psi}_x \{\gamma_4 D_4, \vec{\alpha} \cdot \vec{E}\} \psi_x \\ &+ c_4 a^3 \sum_x \bar{\psi}_x \sum_i {\triangle_i}^2 \psi_x \\ &+ c_5 a^3 \sum_x \bar{\psi}_x \sum_i \sum_{j \neq i} \{\mathrm{i}\Sigma_i B_i, \triangle_j\} \psi_x \\ &: \mathcal{O}(\lambda^3) \end{aligned}$$

- Tree-level on-shell matching:
  Energy (quark dispersion relation)
  Current (quark-gluon vertex)
  Quark-quark scattering
  Compton scattering

Improvement Test: *I*nconsistency

[S. Collins *et al.*, NPB 47, 455 (1996) , A. S. Kronfeld, NPB 53, 401 (1997)]

$$I \equiv \frac{2\delta M_{\overline{Q}q} - (\delta M_{\overline{Q}Q} + \delta M_{\overline{q}q})}{2M_{2\overline{Q}q}} = \frac{2\delta B_{\overline{Q}q} - (\delta B_{\overline{Q}Q} + \delta B_{\overline{q}q})}{2M_{2\overline{Q}q}}$$

$$\rightarrow 0 \quad \text{in the continuum limit } (B_1 = B2)$$

$$M_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q} \qquad \delta M_{\overline{Q}q} = M_{2\overline{Q}q} - M_{1\overline{Q}q}$$

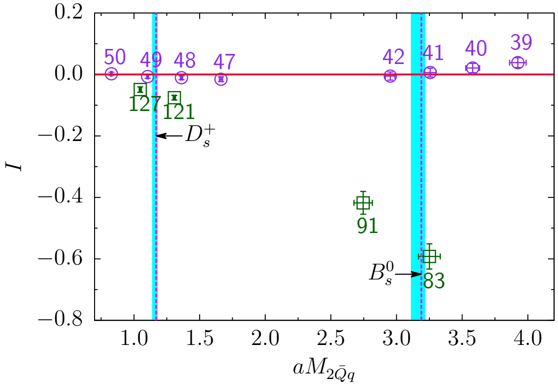
$$M_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q} \qquad \delta B_{\overline{Q}q} = B_{2\overline{Q}q} - B_{1\overline{Q}q}$$

- By design, the inconsistency parameter I isolates the δB of $\mathcal{O}(\boldsymbol{p}^2)$ effect.
- The leading $\mathcal{O}(\boldsymbol{p}^2)$ contributions in I are cancelled for the OK action. (tree-level)
- I can examine the size of improvements by $\mathcal{O}(\boldsymbol{p}^4)$ terms of spin independent.

Improvement Test: *I*nconsistency

[S. Collins *et al.*, NPB 47, 455 (1996) , A. S. Kronfeld, NPB 53, 401 (1997)]

- Near B_s^0 mass, the coarse ($a = 0.12$fm) ensemble data shows that the OK action achieves a significant improvement compared to the Fermilab action.



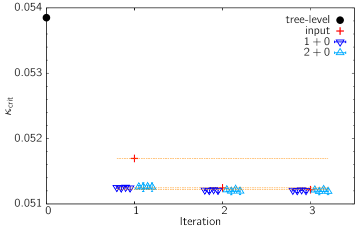
The action closer to $I = 0$ is closer to the renormalized trajectory S_{RT}.

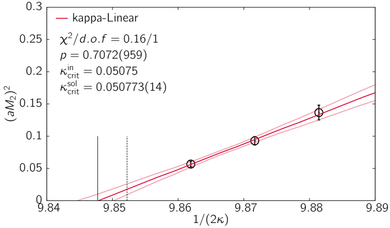
[Y.-C. Jang *et al.*, Eur. Phys. J. C77 (2017) 768]

Critical Hopping Parameter

Ensemble	κ_{crit}
$a12m310$	0.051211
$a12m220$	0.051218
$a09m310$	0.05075
$a09m220$	0.05077

$$am_0 = \frac{1}{u_0}\left(\frac{1}{2\kappa} - \frac{1}{2\kappa_{\text{crit}}}\right)$$



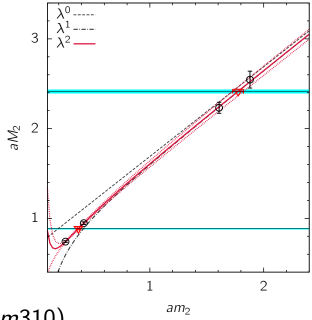


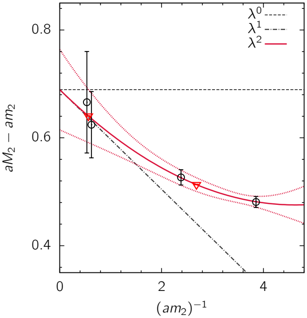
- action coefficients $c_i(m_0 a)$ depends on κ and κ_{crit}.
- use heavy pion (OK-OK) $450\text{MeV} \le M_\pi \le 900\text{MeV}$
- fit to $(aM_\pi)^2 = c_0(am_2) + \ldots$ or $a^2 M_\pi^2 = c_0'/(2\kappa) + \ldots$
- a very fast convergence of κ_{crit} (left: $a12m310$)
- Practically 1 iteration step is enough. (right: $a09m220$)

Heavy Quark Hopping Parameters

Ensemble	κ_c	κ_b
$a12m310$	0.048524	0.04102
$a12m220$	0.048613	0.04070
$a09m310$	0.04894	0.0429
$a09m220$	0.04902	0.0431

$$M_2(\kappa) = m_2(\kappa) + c_0 + \frac{c_1}{m_2(\kappa)} + \frac{c_2}{m_2^2(\kappa)}$$



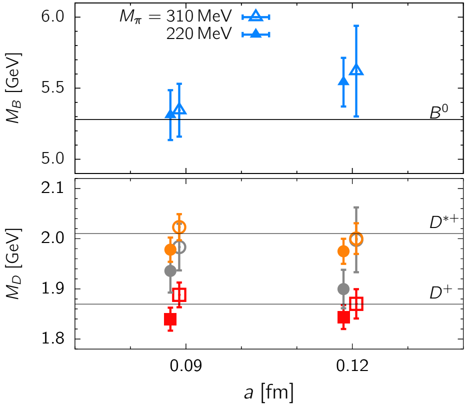


($a09m310$)

- Charm and bottom quarks are tuned to reproduce physical D_s and B_s masses.
- The errorbands of the physical mass points come mostly from lattice spacing error.

Meson Masses

- $M(X) = M_2$, kinetic mass M_2 from fit to dispersion relation
- Vector meson correlator is noisier than the pseudoscalar meson
- $M(D^*)$ from M_2 [gray]
- $M(D^*) = M(D) + M_1(D^*) - M_1(D)$ [orange]
- Consistent with the physical masses



Current Improvement

improved field $\Psi = \mathcal{R}\psi$: $\mathcal{J}_\Gamma^{fg} = \overline{\psi}^g \Gamma \psi^f \to \mathcal{J}^{\mathrm{Imp}} = \overline{\Psi}^g \Gamma \Psi^f$ improved current

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$$\begin{aligned}\mathcal{R} = {} & d_0 + d_1 a\boldsymbol{\gamma}\cdot\boldsymbol{D} + d_2 a^2 \triangle^{(3)} + d_B a^2 i\boldsymbol{\Sigma}\cdot\boldsymbol{B} + d_E a^2 \boldsymbol{\alpha}\cdot\boldsymbol{E} + d_{r_E} a^3 \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} \\ & + d_3 a^3 \sum_i \gamma_i D_i \triangle_i + d_4 a^3 \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \triangle^{(3)}\} + d_5 a^3 \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\} \\ & + d_{EE} a^3 \{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} + d_6 a^4 \sum_i {\triangle_i}^2 + d_7 a^4 \sum_i \sum_{j\neq i} \{\mathrm{i}\Sigma_i B_i, \triangle_j\} \\ & + d_{r4} a^3 [\gamma_4 D_4, \triangle^{(3)}] + d_{r5} a^3 [\gamma_4 D_4, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}] \\ & = \sum_i d_i(m_0 a)\mathcal{R}_i\end{aligned}$$

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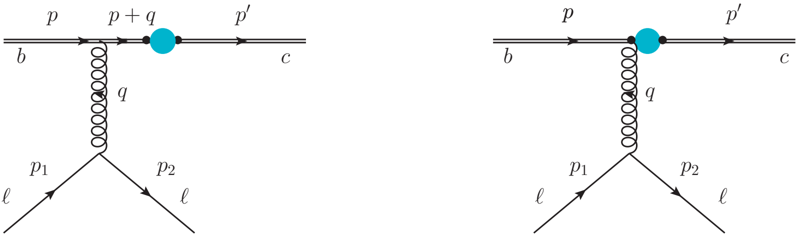
\end{aligned}$$



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improved field $\Psi = \mathcal{R}\psi$: $\mathcal{J}_\Gamma^{fg} = \overline{\psi}^g \Gamma \psi^f \to \mathcal{J}^{\mathrm{Imp}} = \overline{\Psi}^g \Gamma \Psi^f$ improved current

$$
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& = \sum_i d_i (m_0 a) \mathcal{R}_i
\end{aligned}
$$

- $d_0 \equiv 1$
- tree-level, λ^3 matching: 11 terms $d_{i(\neq 0)} \neq 0$, $d_6 = d_7 = 0$ [arXiv:1711.01777]
- 12^2 terms $\to$ 44 terms by truncating the series at $\lambda_c^n \lambda_b^m$, $m + n = 3$.

Current Improvement

$$\text{improved field} \quad \Psi = \mathcal{R}\psi : \quad \mathcal{J}^{fg}_{\Gamma} = \overline{\psi}^{g}\Gamma\psi^{f} \to \mathcal{J}^{\mathrm{Imp}} = \overline{\Psi}^{g}\Gamma\Psi^{f} \quad \text{improved current}$$

$$\begin{aligned}
\mathcal{R} = {} & d_0 + d_1 a\boldsymbol{\gamma}\cdot\boldsymbol{D} + d_2 a^2\triangle^{(3)} + d_B a^2 i\boldsymbol{\Sigma}\cdot\boldsymbol{B} + d_E a^2\boldsymbol{\alpha}\cdot\boldsymbol{E} + d_{r_E} a^3\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} \\
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& + d_{EE} a^3\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} + d_6 a^4 \sum_i \triangle_i{}^2 + d_7 a^4 \sum_i \sum_{j\neq i}\{\mathrm{i}\Sigma_i B_i, \triangle_j\} \\
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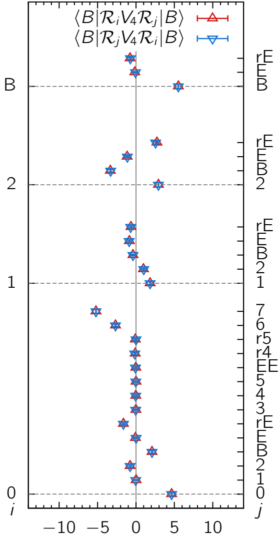
- 12^2 terms $\to$ 44 terms by truncating the series at $\lambda_c^n \lambda_b^m$, $m + n = 3$.

$$C^{X(f)\to Y(g)}(t,\tau) = \sum_{i,j} d_i(m_0^g a) d_j(m_0^f a)\langle 0|\mathcal{O}_Y^\dagger(0) R_i \mathcal{J}^{fg}_{\Gamma}(t) R_j \mathcal{O}_X(\tau)|0\rangle$$

Current Improvement: $a09m310$, $\langle B|\mathcal{R}_i V_4 \mathcal{R}_j|B\rangle \equiv \langle B|V_4^{(i,j)}|B\rangle$



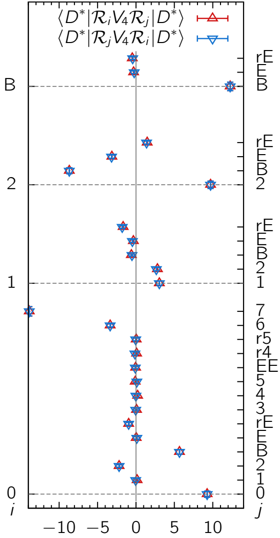
- For each i, improvement terms by $V^{(i,j)}$ are smaller in most cases than the diagonal term $V^{(i,i)}$.
- $\langle B|V_4^{(j,k)}|B\rangle = \langle B|V_4^{(k,j)}|B\rangle$ provides a sanity check.
- Up to $\mathcal{O}(\lambda^2)$ in HQET,

  $\langle B|V_4^{(0,1)}|B\rangle = 0,$

  $\langle B|V_4^{(0,E)}|B\rangle = 0,$

  and lattice results agree with the expectation.
- Comparison with HQET up to $\mathcal{O}(\lambda^3)$ is underway.

Current Improvement: $a09m310$, $\langle D^*|\mathcal{R}_i V_4 \mathcal{R}_j|D^*\rangle \equiv \langle D^*|V_4^{(i,j)}|D^*\rangle$



- Data pattern is similar to the $\langle B|V_4^{(i,j)}|B\rangle$.
- $\langle D^*|V_4^{(j,k)}|D^*\rangle = \langle D^*|V_4^{(k,j)}|D^*\rangle$ provides a sanity check.
- Up to $\mathcal{O}(\lambda^2)$ in HQET,

  $\langle D^*|V_4^{(0,1)}|D^*\rangle = 0,$

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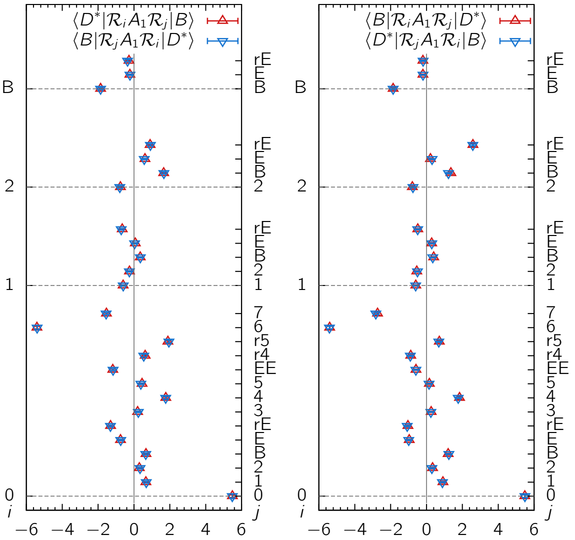
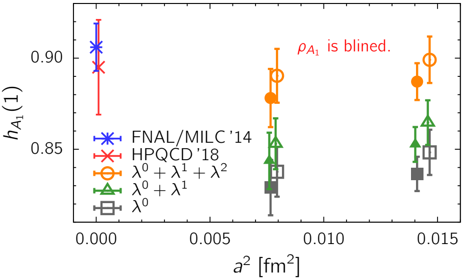
Current Improvement: $a09m310$ $\langle D^*|\mathcal{R}_i A_1 \mathcal{R}_j|B\rangle$, $\langle B|\mathcal{R}_i A_1 \mathcal{R}_j|D^*\rangle$



- Leading contribution $V^{(0,0)}$ is dominant.
- More correction terms are nonzero.
- Comparison with HQET up to $\mathcal{O}(\lambda^3)$ is underway.
- The improvement coefficients have to be incorporated.

Zero-recoil Form Factor $h_{A_1}(1)$ **Preliminary**

$$R_{A_1} = \frac{\langle B|A_1|D^*\rangle\langle D^*|A_1|B\rangle}{\langle B|V_4|B\rangle\langle D^*|V_4|D^*\rangle} \to \left|\frac{h_{A_1}(1)}{\rho_{A_1}}\right|^2$$



- The matching factor ρ_{A_1} is expected to be close to 1.
- Current improvement is crucial.
- Improved action and current operator are expected to give a mild dependence on a and M_π. However, the shown dependence in a and M_π needs to be understood.

Summary & Outlook

- Renormalization (matching) factor calculation is underway.
- Current improvement is crucial, and the preliminary results are close to the continuum limit.
- Improved action and current operator are expected to give a mild (flat) dependence on a and M_π. However, the shown dependence in a and M_π needs to be understood.
- Physical pion mass and finer lattice spacing ensembles will be included.
- Statistics will be increased.
- $\mathcal{O}(\lambda^3)$ current improvement is being implemented.
- $B \to D^* \ell\nu$ and $B \to D\ell\nu$ nonzero recoil data will be analyzed.

Thank you for your attention.