Lattice QCD and current tensions in the Standard Model: $|V_{cb}|$ from $\bar{B} \to D^* \ell \bar{\nu}$ at non-zero recoil

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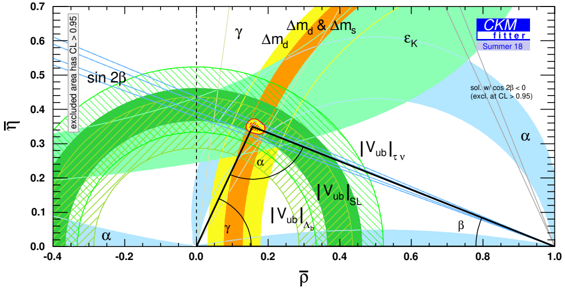
The V_{cb} matrix element: Tensions

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Determination	$\lvert V_{cb}\rvert\,(\cdot 10^{-3})$
Exclusive	39.2 ± 0.7
Inclusive	42.2 ± 0.8

PDG 2016

- Matrix must be unitary (preserve the norm)
- Based on CLN, motivated this work



The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B} \to D^* \ell \bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{\frac{G_F^2 m_B^5}{48\pi^2}(w^2-1)^{\frac{1}{2}} P(w)\left|\eta_{ew}\right|^2}_{\text{Known factors}} \underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}} \left|V_{cb}\right|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q \to \infty} \mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know how $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $\left|V_{cb}\right|^2 \left|\eta_{ew}\mathcal{F}(w)\right|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations

The V_{cb} matrix element: The parametrization issue

All the parametrizations perform an expansion on the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Boyd-Grinstein-Lebed (BGL)

  Phys. Rev. Lett. 74 (1995) 4603-4606

  Phys.Rev. D56 (1997) 6895-6911

  Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Unitarity constrains $\sum_n |a_n|^2 \leq 1$

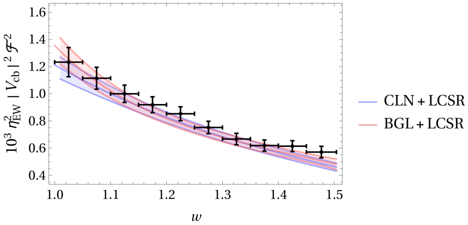
- Caprini-Lellouch-Neubert (CLN)

  Nucl. Phys. B530 (1998) 153-181

$$\mathcal{F}(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data at non-zero recoil and lattice data at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

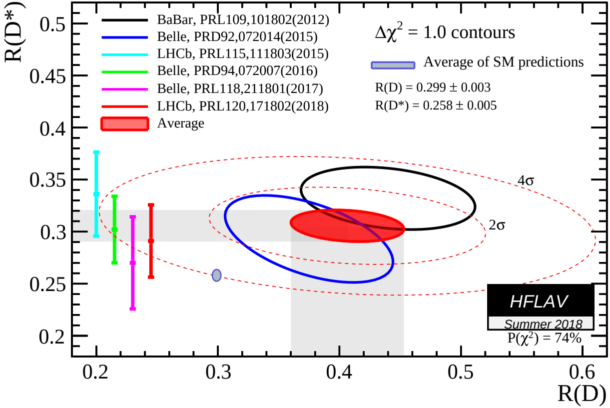
$$|V_{cb}| = 41.7 \pm 2.0 (\times 10^{-3})$$

- Current discrepancy might be an artifact
- Data at $w \gtrsim 1$ is urgently needed to settle the issue
- Experimental measurements perform badly at low recoil

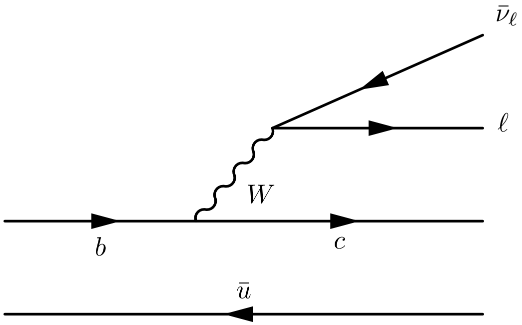
We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

The V_{cb} matrix element: Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current 4σ tension with the SM

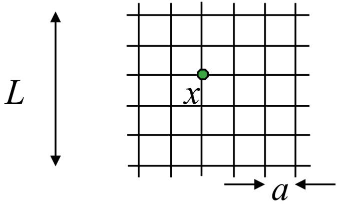


$$\text{(Experiment)} = \text{(Known)}\times\text{(CKM)}\times\text{(Had. Matrix El.)}$$

- The lattice allows us to compute hadronic matrix elements from first principles
- Requires experimental data to make full predictions about nature

Introduction to Lattice QCD

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f \left(\gamma^\mu D_\mu + m_f\right) \psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu} F^{\mu\nu}$$



- Discretize space-time in a computer
- Perform simulations approaching the physical limit
  - Finite lattice spacing $a \to 0$
  - Finite volume $L \to \infty$
  - $m_\pi \to m_\pi^{\mathrm{Phys}}$, $m_Q \to m_Q^{\mathrm{Phys}}$

- Use the path integral formulation and montecarlo simulations

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, e^{-S}, \qquad S = \int d^4x \, \mathcal{L}_{QCD}(\bar{\psi}, \psi, A)$$

Introduction to Lattice QCD

Steps in a lattice calculation

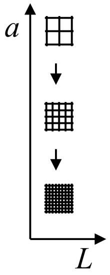
1. Generate gluon field configurations with the right distribution
2. Calculate quark propagators for each valence quark at source points
3. Tie together the propagators to construct correlators (normally 2- or 3-point functions)
4. Extract hadron masses, energies, hadronic matrix elements... from correlators
5. Extra/Inter-polate results to the continuum limit ($a \to 0$) and to the right quark masses
6. Perform a systematic error analysis

The last point determines how reliable the lattice result is

Introduction to Lattice QCD

The systematic error analysis is based on **EFT** descriptions of QCD
The EFT description:

- provides functional form for different extrapolations (or interpolations)
- can be used to construct improved actions
- can estimate the size of the systematic errors



In order to keep the systematic errors under control we must repeat the calculation for several lattice spacings, volumes, light quark masses... and use the EFT to extrapolate to the physical theory

Calculating V_{cb} on the lattice

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\rho\sigma}v_B^\rho v_{D^*}^\sigma h_V(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\left(1+w\right)h_{A_1}(w) - v_B^\nu\left(v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w)\right)\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Introduction: The weak decay $\bar{B} \to D^* \ell \bar{\nu}$

- Helicity amplitudes

$$H_\pm = \sqrt{m_B\, m_{D^*}}(w+1)\left(h_{A_1}(w) \mp \sqrt{\frac{w-1}{w+1}} h_V(w)\right)$$

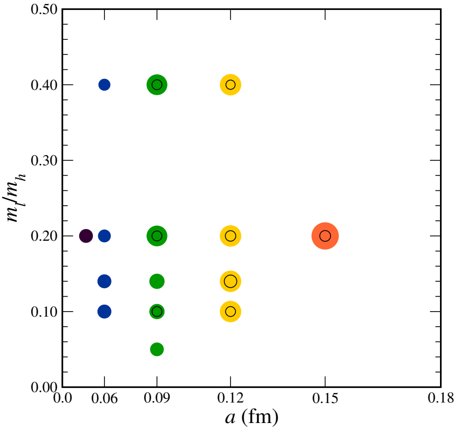
$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)h_{A_1}(w) - (w-1)\left(r\, h_{A_2}(w) + h_{A_3}(w)\right)\right]/\sqrt{q^2}$$

$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)h_{A_1}(w) + (wr-1)h_{A_2}(w) + (r-w)h_{A_3}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

Calculating V_{cb} on the lattice: Available ensembles



- $N_f = 2 + 1$ staggered asqtad sea quarks
- Heavy quarks use the fermilab action
- Size of the point proportional to the statistics (min 2372, max 15072)

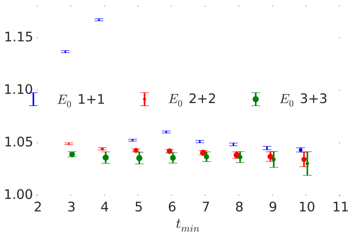
Calculating V_{cb} on the lattice: Two-point functions

$$\langle X(t)|\, X(0)\rangle = \sum_i (-1)^{it}\, Z_i^2\, \left(e^{-E_i t} + e^{-E_i (T-t)}\right)$$

- Any state that shares quantum numbers with X can be created
  - Use smearing to increase overlap with the desired state
- The staggered formulation adds new (unphysical) particles to the mix
  - It respects a subgroup of chiral symmetry that allows for parity partners
  - The parity partners oscillate with $(-1)^t$ factor and must be fitted

- Consider $N+N$ states (N oscillating, N non-oscillating)

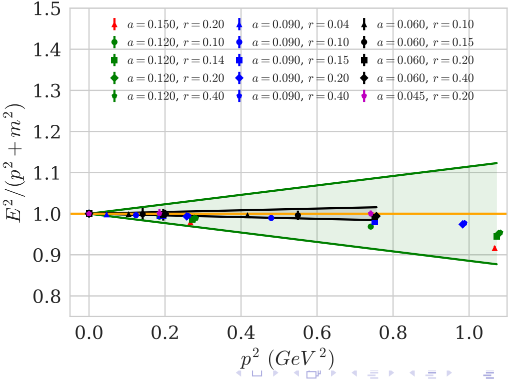
$$\sum_i^{2N-1} (-1)^{i(t+1)}\, Z_i^2\, \left(e^{-E_i t} + e^{-E_i (T-t)}\right)$$



- Discretization effects coming from the heavy quark break the dispersion relation
- The Fermilab action uses tree-level matching, discretization errors $O(\alpha\, m)$

$$a^2E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1 w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

- As long as the discretization errors are under control, this is all right
- In the Fermilab action we interpret the kinetic mass am_2 as the particle mass



Calculating V_{cb} on the lattice: Three-point functions

$$\langle Y(0)|\, \mathcal{W}^{\mu}(t)\, |X(T_s)\rangle = \sum_i \sum_j (-1)^{it}(-1)^{j(T_s-t)}\, W_{X_j \to Y_i} \times$$

$$Z_{Y,i} Z_{X,j}\, e^{-E_{Y,i}t}\, e^{-E_{X,j}(T_s-t)}$$

- As before, any state that shares quantum numbers with X, Y can be created
- We construct ratios to cancel leading order exponentials, overlap and renormalization factors
- Oscillating states with weight $(-1)^t$ also appear in the three-point functions
  - A clever average removes them up to a negligible error

$$\bar{R}(t, T_s) = \frac{1}{2}R(t, T_s) + \frac{1}{4}R(t, T_s+1) + \frac{1}{4}R(t+1, T_s+1)$$

- Fit function

$$r\left(1 + Ae^{-\Delta E_{D^*}t} + Be^{-\Delta E_B(T_s-t)}\right)$$

Calculating V_{cb} on the lattice: The recoil parameter w

- The recoil parameter is measured dynamically
- In the lab frame (B meson at rest)

$$w^2 = 1 + v_{D^*}^2$$

- Ratio of three point functions

$$X_f(p) = \frac{\langle D^*(p)| \mathbf{V} |D^*(0)\rangle}{\langle D^*(p)| V_4 |D^*(0)\rangle} = \frac{\mathbf{v}_{D^*}}{w+1}$$

- From here

$$w(p) = \frac{1+\mathbf{x}_f^2}{1-\mathbf{x}_f^2}$$

- Alternatively one can use the dispersion relation

Calculating V_{cb} on the lattice: form factors

$$\left|R_{A_1}(w)\right|^2 = \frac{\langle D^*(p_\perp)|\, \mathbf{A} \,|\bar{B}(0)\rangle \langle \bar{B}(0)|\, \mathbf{A} \,|D^*(p_\perp)\rangle}{\langle D^*(0)|\, V_4 \,|D^*(0)\rangle \langle \bar{B}(0)|\, V_4 \,|\bar{B}(0)\rangle} = \left(\frac{1+w}{2}\right)^2 \left|h_{A_1}(w)\right|^2$$

- The double ratio cancels leading overlap factors and energy exponentials

Phys. Rev. D83 (2011) 036004

$$X_V(p) = \frac{\langle D^*(p_\perp)|\, \mathbf{V} \,|\bar{B}(0)\rangle}{\langle D^*(p_\perp)|\, \mathbf{A} \,|\bar{B}(0)\rangle} = \sqrt{\frac{w-1}{w+1}}\frac{h_V(w)}{h_{A_1}(w)}$$

$$R_0(p) = \frac{\langle D^*(p)|\, A_4 \,|\bar{B}(0)\rangle}{\langle D^*(p_\perp)|\, \mathbf{A} \,|\bar{B}(0)\rangle} = \sqrt{w^2-1}\left(1 - \frac{h_{A_2}(w) + w h_{A_3}(w)}{(1+w)\, h_{A_1}(w)}\right)$$

$$R_1(p) = \frac{\langle D^*(p_\parallel)|\, \mathbf{A} \,|\bar{B}(0)\rangle}{\langle D^*(p_\perp)|\, \mathbf{A} \,|\bar{B}(0)\rangle} = w - \frac{\left(w^2-1\right) h_{A_3}(w)}{(1+w)\, h_{A_1}(w)}$$

Calculating V_{cb} on the lattice: Current renormalization

- In the coefficients of the terms of our effective theory a dependence arises with the scale (i.e. a)
- The renormalization tries to account for the right dependence
- The scheme we employ is called *Mostly non-perturbative renormalization* of results

$$Z_{V^{1,4},A^{1,4}} = \underbrace{\rho_{V^{1,4},A^{1,4}}}_{\text{Perturbative factor}} \times \underbrace{\sqrt{Z_{V_{bb}} Z_{V_{cc}}}}_{\text{Non-perturbative piece}}$$

- The (relatively large) non-perturbative piece cancels in our ratios
- The (close to one) perturbative piece (matching factor ρ) is calculated at one-loop level for $w = 1$
- The error for $w \neq 1$ is estimated and added to the factor

This analysis is **blinded** and the blinding happens at the level of the matching factor

Calculating V_{cb} on the lattice: The chiral-continuum extrapolation

- Our data represents the form factors at non-zero a and unphysical m_π
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the a and the m_π dependence
- Functional form explicitly known

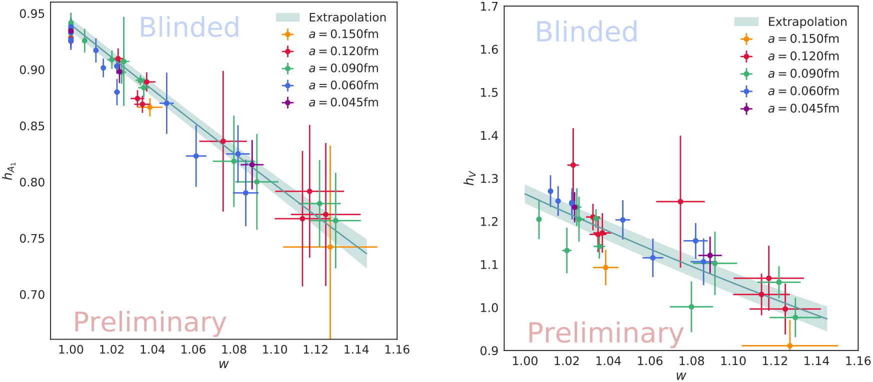
$$h_{A_1}(w) = \underbrace{1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*D\pi}}{48\pi^2 f_\pi^2 r_1^2} \mathrm{logs}_{\mathrm{SU3}}(a, m_l, m_s, \Lambda_{QCD})}_{\mathrm{NLO}\ \chi\mathrm{PT} + \mathrm{HQET}} -$$

$$\underbrace{\rho^2(w-1) + k(w-1)^2}_{w\ \text{dependence}} \underbrace{+ c_1 x_l + c_2 x_l^2 + c_{a1} x_{a^2} + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}}_{\mathrm{NNLO}\ \chi\mathrm{PT}}$$

with

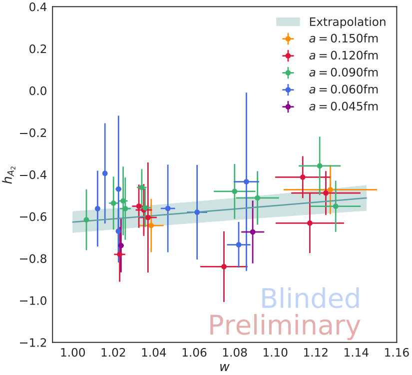
$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

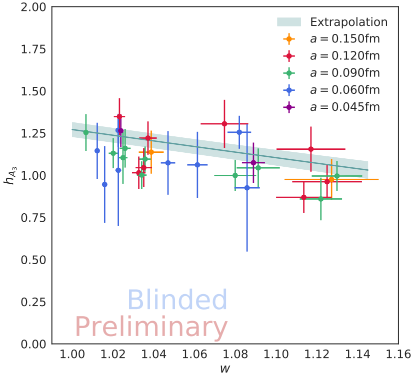
Results: Chiral-continuum fits



- Preliminary **blinded** results

Results: Chiral-continuum fits





- Preliminary **blinded** results

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z)B_g(z)}\sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w)h_{A_1}(w) = \frac{1}{\phi_f(z)B_f(z)}\sum_j b_j z^j$$

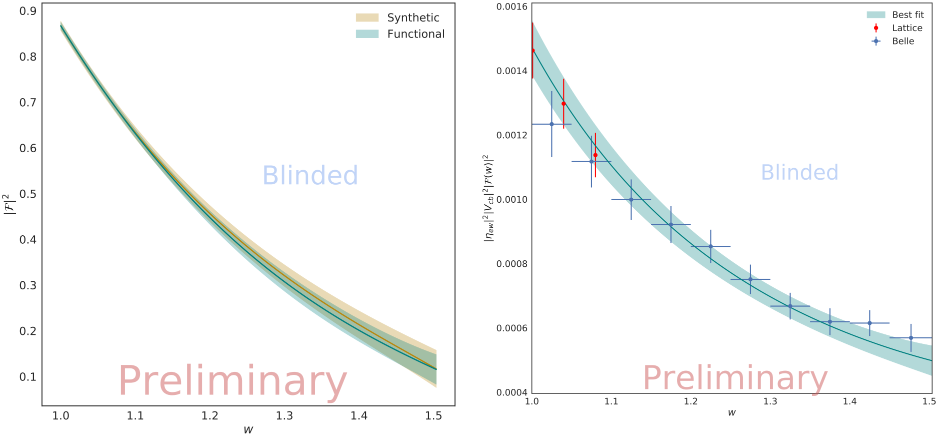
$$\mathcal{F}_1 = \sqrt{q^2}H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z)B_{\mathcal{F}_1}(z)}\sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}}H_S = \frac{1}{\phi_{\mathcal{F}_2}(z)B_{\mathcal{F}_2}(z)}\sum_j d_j z^j$$

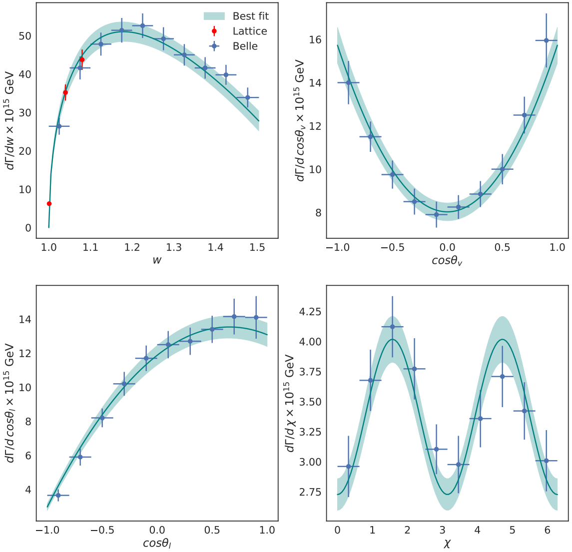
- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*})f(z=0)$
- Constraint $\mathcal{F}_1(z=z_{\mathrm{Max}}) = \frac{1+r}{(1+w)m_B^2(1-r)}\mathcal{F}_2(z=z_{\mathrm{Max}})$
- BGL (weak) unitarity constraints

$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

Results: Pure-lattice prediction and joint fit



Results: Angular bins



Analysis: What happens if I use CLN?

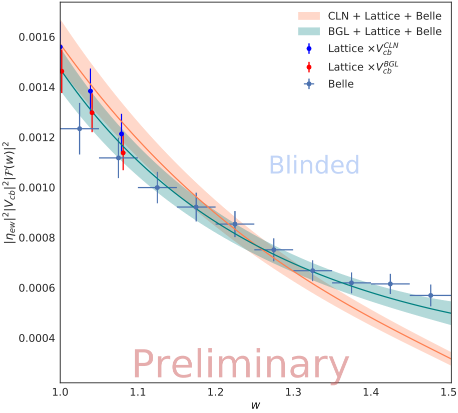
- CLN is much more constraining than BGL, using only 4 fit parameters

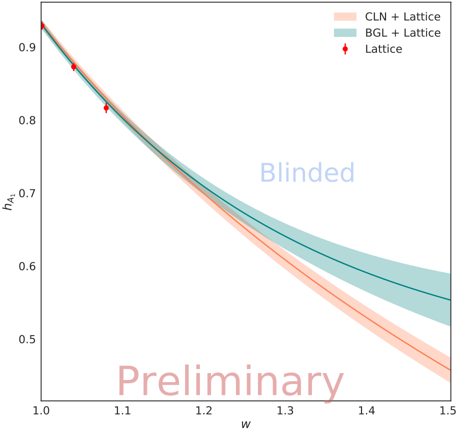
$$
\begin{aligned}
h_{A_1}(w) &= h_{A_1}(1)\left[1 - 8\rho^2 z + \left(53\rho^2 - 15\right) z^2 - \left(231\rho^2 - 91\right) z^3\right] \\
R_1(w) &= R_1(1) - 0.12\,(w-1) + 0.05\,(w-1)^2 \\
R_2(w) &= R_2(1) + 0.11\,(w-1) - 0.06\,(w-1)^2
\end{aligned}
$$

with

$$
\begin{aligned}
R_1(w) &= \frac{h_{A_1}(w)}{h_V(w)} \\
R_2(w) &= \frac{\frac{m_{D^*}}{m_B} h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}
\end{aligned}
$$

Analysis: What happens if I use CLN?

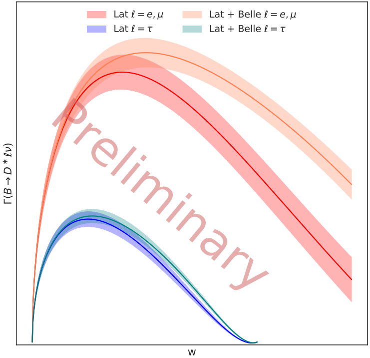




- When lattice data is added, CLN breaks down
- Prediction for h_{A_1} very constrained in the CLN parametrization

Results: $R(D^*)$

- Pure lattice QCD prediction of $R(D^*)$
- Includes constraint $\mathcal{F}_1(w_{\mathrm{Max}}) = \frac{1+r}{(1+w)m_B^2(1-r)}\mathcal{F}_2(w_{\mathrm{Max}})$



Conclusions

What to expect

- Errors might not be improved compared to previous lattice estimations
- The main new information of this analysis won't come from the zero-recoil value, but from the slope
- Main source of errors of our form factor seems to be discretization errors (to be confirmed in error budget)
- Missing only discretization errors + last checks

The future

- Well established roadmap to reduce errors in our calculation
  - Light HISQ quarks + heavy Fermilab quarks aim to reduce mainly chiral fit errors
  - Light HISQ quarks + heavy HISQ quarks aim to reduce discretization and renormalization errors
  - New runs measure other interesting quantities (i.e. the tensor form factor)
- This roadmap is to be followed in other processes involving other CKM matrix elements
- Would be interesting to add existing $B \to D$ data to the mix, to further constrain the form factors