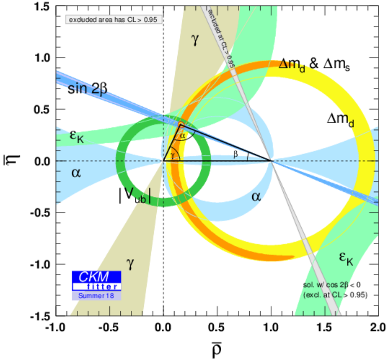
# $B \to \pi\ell\nu$ Form Factors with Fully Relativistic Lattice QCD

Brian Colquhoun
York University
w/ Shoji Hashimoto, Takashi Kaneko
& Jonna Koponen
for the JLQCD Collaboration

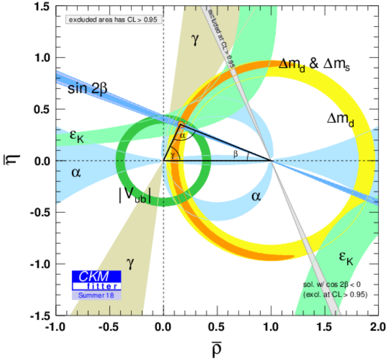
KEK-FF Feb 26 2019

# Motivation




$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\bar{B}_s\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

# Motivation



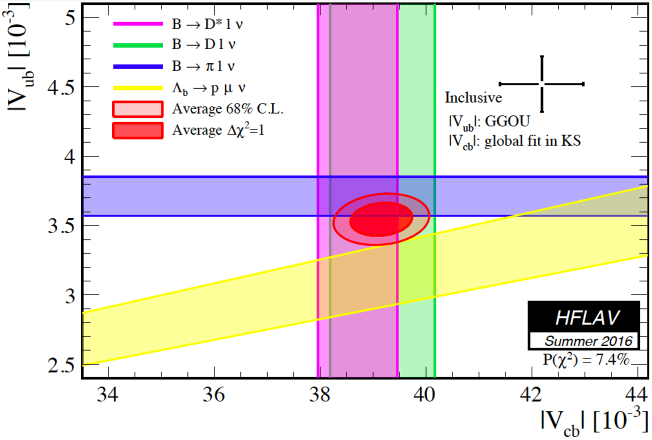

This talk

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\bar{B}_s\rangle & \langle B_s|\bar{B}_s\rangle & \end{pmatrix}$$

T. Kaneko
Fri @ 17:30

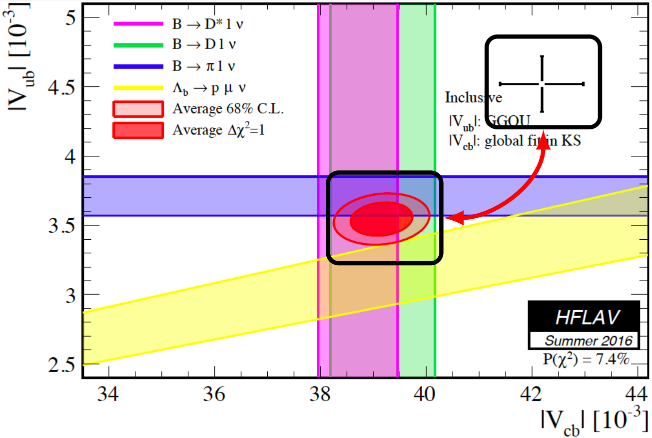
Inclusive: S. Hashimoto Poster

# Tension



$|V_{ub}|$ [$10^{-3}$]
5
4.5
4
3.5
3
2.5
34
36
38
40
42
44
$|V_{cb}|$ [$10^{-3}$]
$B \to D^* l \nu$
$B \to D l \nu$
$B \to \pi l \nu$
$\Lambda_b \to p \mu \nu$
Average 68% C.L.
Average $\Delta\chi^2$=1
Inclusive
$|V_{ub}|$: GGOU
$|V_{cb}|$: global fit in KS
HFLAV
Summer 2016
$P(\chi^2)$ = 7.4%

# Tension



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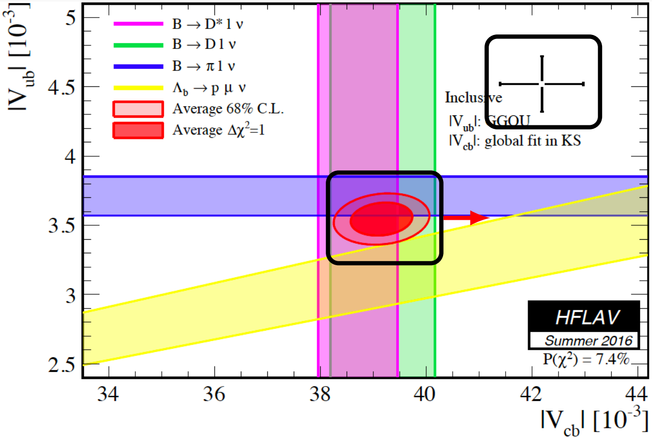

B → D* l ν
B → D l ν
B → π l ν
Λb → p μ ν
Average 68% C.L.
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|Vub| [10^-3]
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5
4.5
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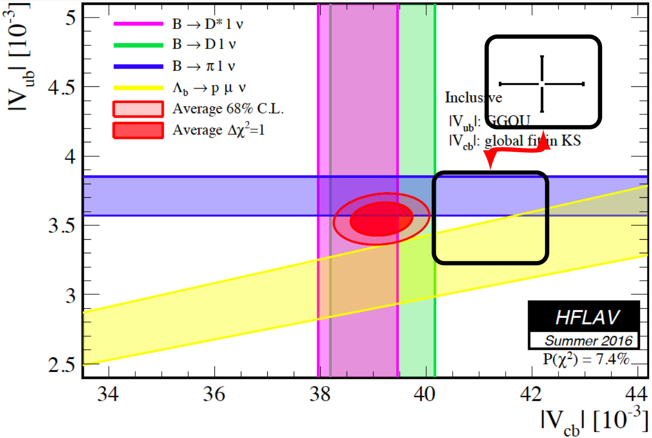
36
38
40
42
44

# Tension



B → D* l ν
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|Vub|: GGOU
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HFLAV
Summer 2016
P(χ²) = 7.4%
|Vub| [10⁻³]
|Vcb| [10⁻³]

# Background

CKM element $V_{ub}$ relates to the differential decay rate:

$$\frac{\mathrm{d}\Gamma\left(B \to \pi \ell \nu\right)}{\mathrm{d}q^2} = \frac{G_F^2 \left|V_{ub}\right|^2}{24\pi^3} \left|k_\pi\right|^3 \left|f_+(q^2)\right|^2$$

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$$\frac{\mathrm{d}\Gamma\left(B \to \pi \ell \nu\right)}{\mathrm{d}q^2} = \frac{G_F^2 \left|V_{ub}\right|^2}{24\pi^3} \left|k_\pi\right|^3 \left|f_+(q^2)\right|^2$$

Experiment

CKM

Lattice

# Semileptonic Decays

For pseudoscalar to pseudoscalar decays: :

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[(p_B+k_\pi)^\mu - \frac{m_B^2-m_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2-m_\pi^2}{q^2}q^\mu,$$

- $p_B$ and $k_\pi$: $B$ and $\pi$ four-momenta
- $q^\mu = p_B^\mu - k_\pi^\mu$: four-momentum transfer
- Constraint: $f_0(0) = f_+(0)$

# Semileptonic Decays

In the context of HQET, a useful parametrisation is:

[Burdman et al. (1994)]

$$\langle \pi(k_\pi)|V^\mu|B(p_B)\rangle = 2\sqrt{m_B}\left[f_1\left(v\cdot k_\pi\right)v^\mu + f_2\left(v\cdot k_\pi\right)\frac{k^\mu}{v\cdot k_\pi}\right]$$

- $v^\mu = \dfrac{p_B}{m_B}$: heavy quark velocity
- $E_\pi = v\cdot k_\pi = \dfrac{m_B^2 + m_\pi^2 - q^2}{2m_B}$

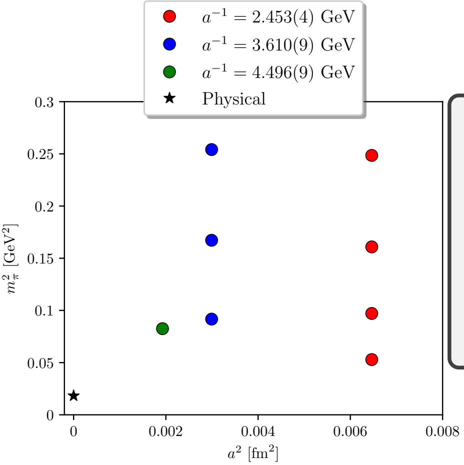
These two form factor definitions are related by:

$$f_+\left(q^2\right) = \sqrt{m_B}\left[\frac{f_2\left(v\cdot k\right)}{v\cdot k_\pi} + \frac{f_1\left(v\cdot k\right)}{m_B}\right];$$

$$f_0\left(q^2\right) = \frac{2}{\sqrt{m_B}}\frac{m_B^2}{m_B^2 - m_\pi^2}\Bigg[f_1\left(v\cdot k\right) + f_2\left(v\cdot k\right) - \frac{v\cdot k_\pi}{m_B}\left(f_1\left(v\cdot k\right) + \frac{m_\pi^2}{\left(v\cdot k_\pi\right)^2} f_2\left(v\cdot k\right)\right)\Bigg]$$

# Lattice Calculation

# Gauge Configurations




Möbius Domain Wall Fermions

$N_f = 2+1$

$m_\pi \approx 225, 300, 400, 500$ MeV

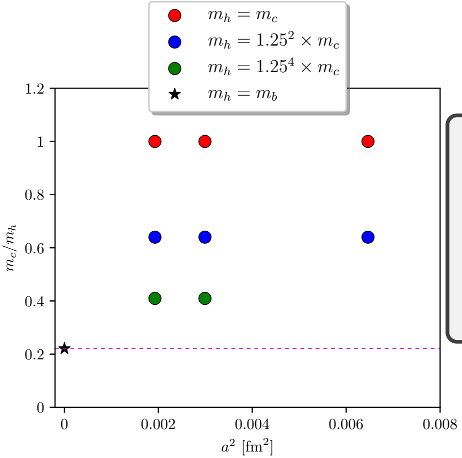
$\beta = 4.17, 4.35, 4.47$

$a \approx 0.080, 0.055, 0.044$ fm

$a^{-1} \approx 2.453, 3.610, 4.496$ GeV

$L^3 \times T = 32^3 \times 64, 48^3 \times 96, 64^3 \times 128$

# Valence Quarks



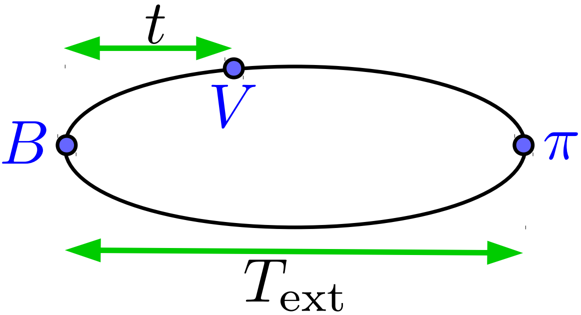

Möbius Domain Wall Fermions

$m_l$ same values as sea quarks

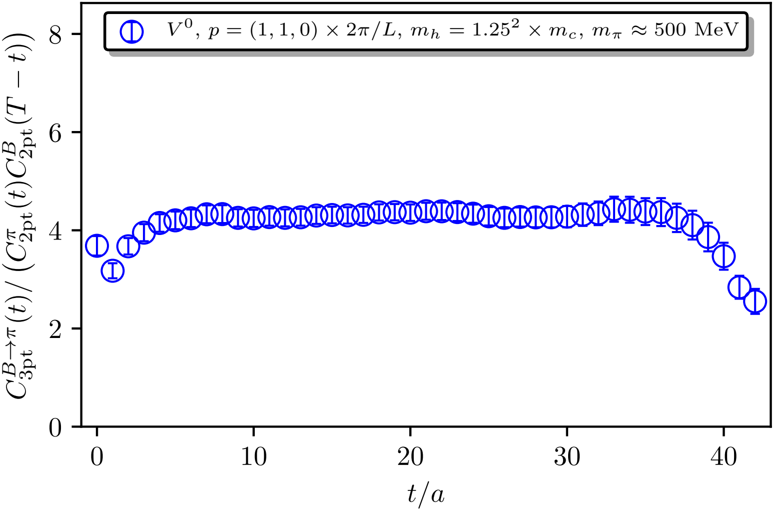
$m_h \in \{m_c,\ 1.25^2 m_c,\ 1.25^4 m_c\}$

- $m_{H_l} \approx 1.95,\ 2.55,\ 3.40$ GeV
- Fully relativistic formalism

- $B$ and $\pi$ mesons are separated by time $T_{\text{ext}}$
- Operators $V^{\mu}$ are inserted at $0 \leq t \leq T_{\text{ext}}$



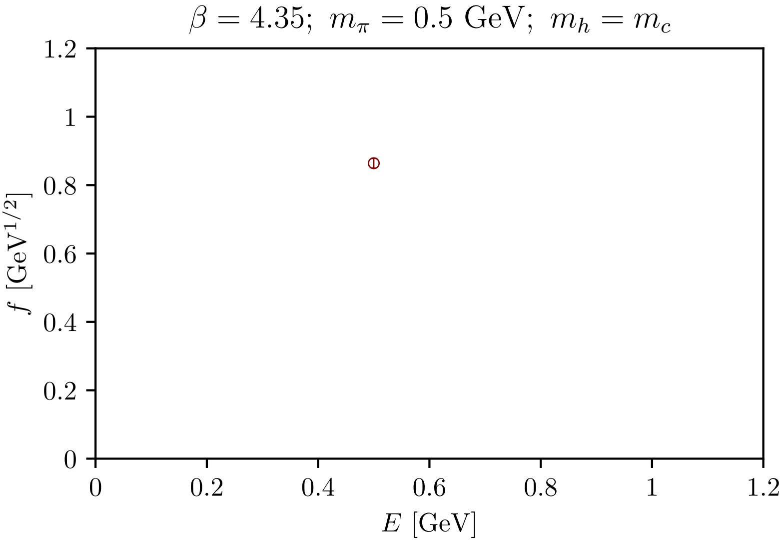
# Correlators: 3-point




q

# Results

# $E_\pi$ Dependence



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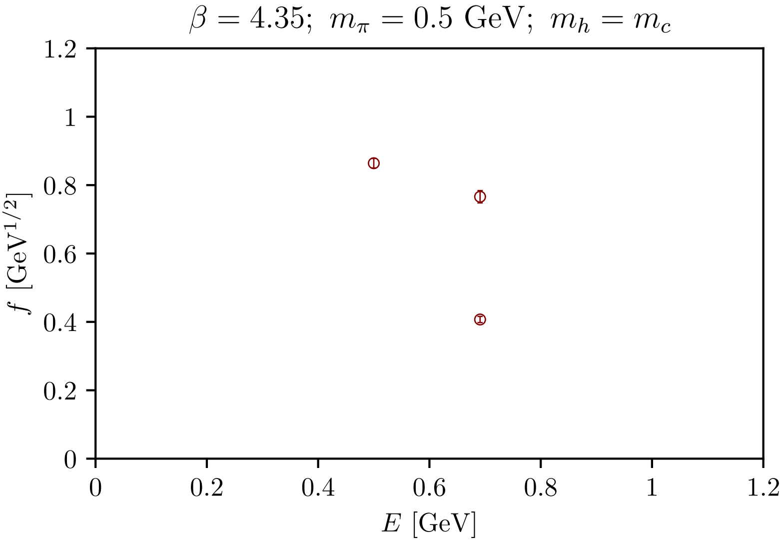

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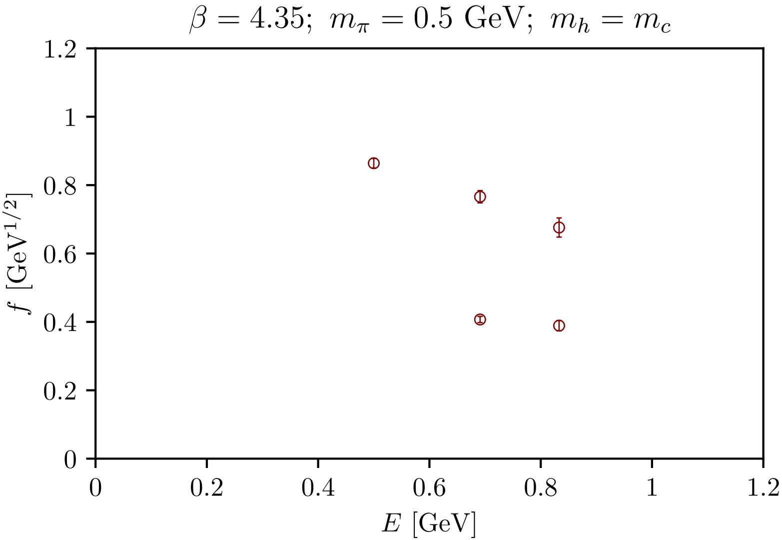

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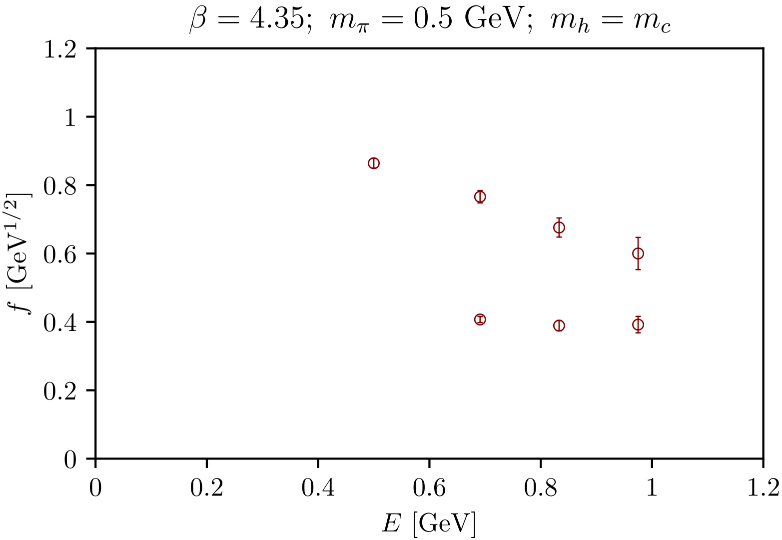
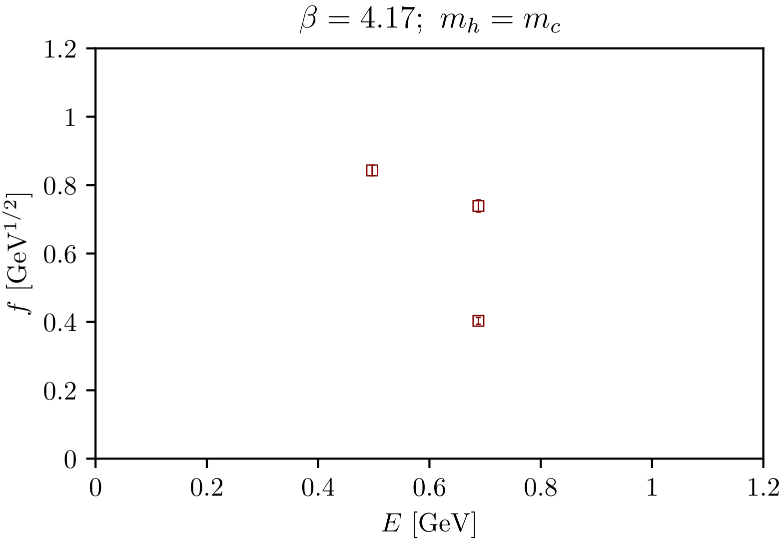


# $m_\pi$ Dependence



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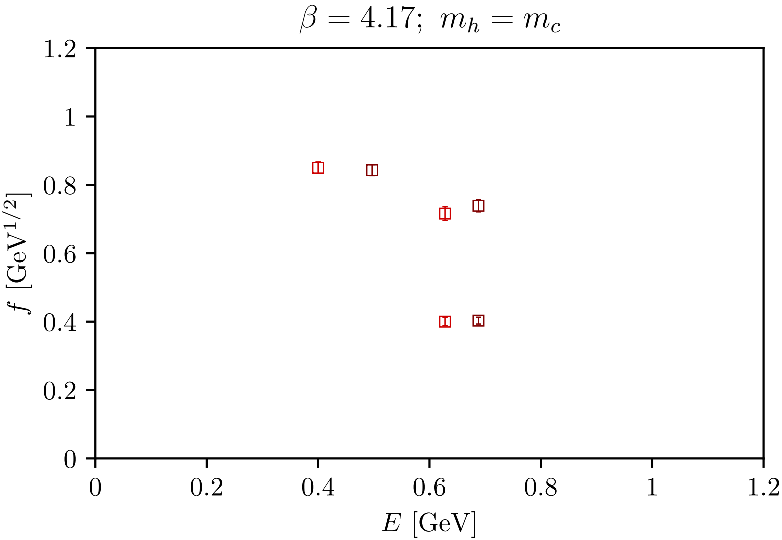

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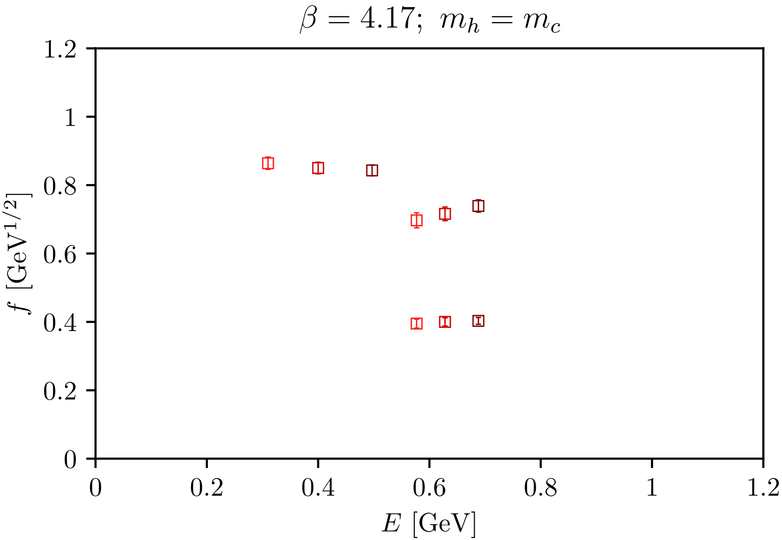

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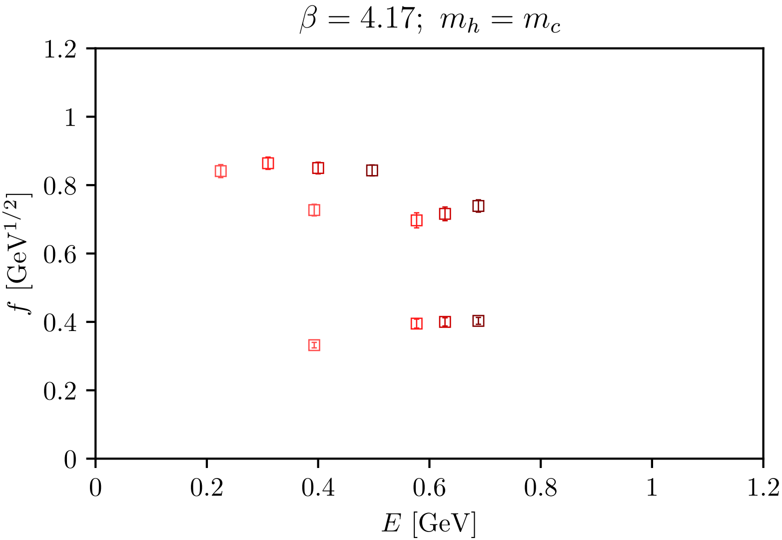
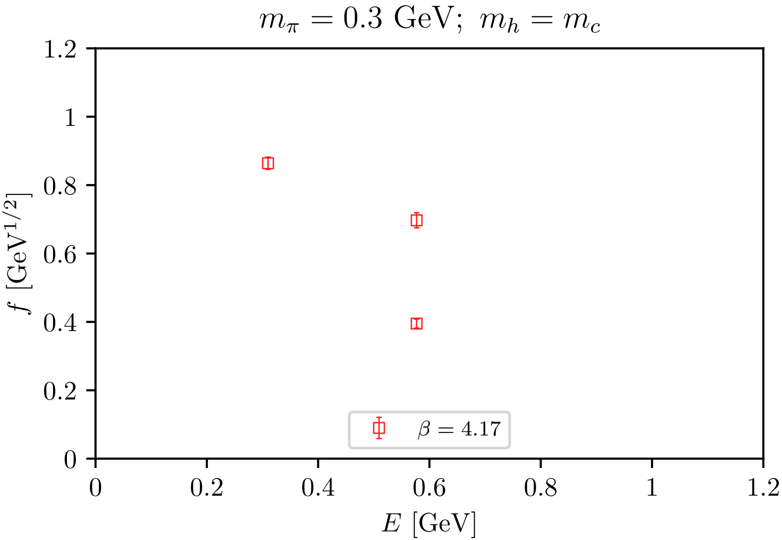


# Lattice Spacing Dependence



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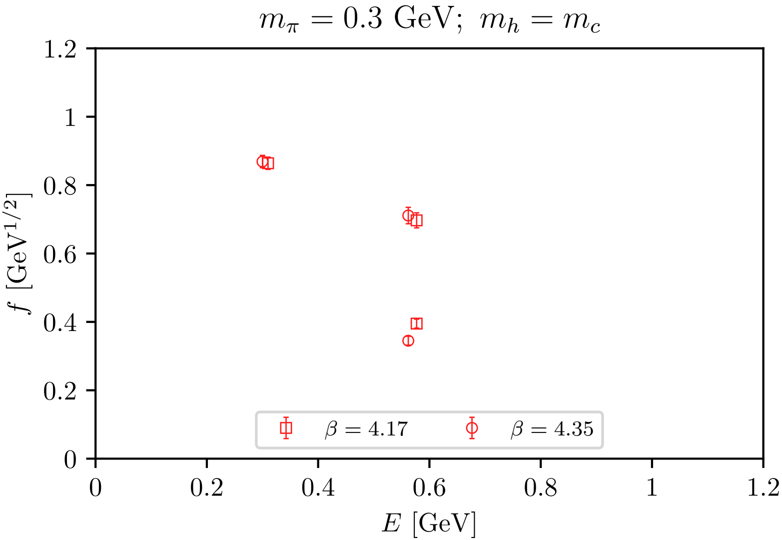

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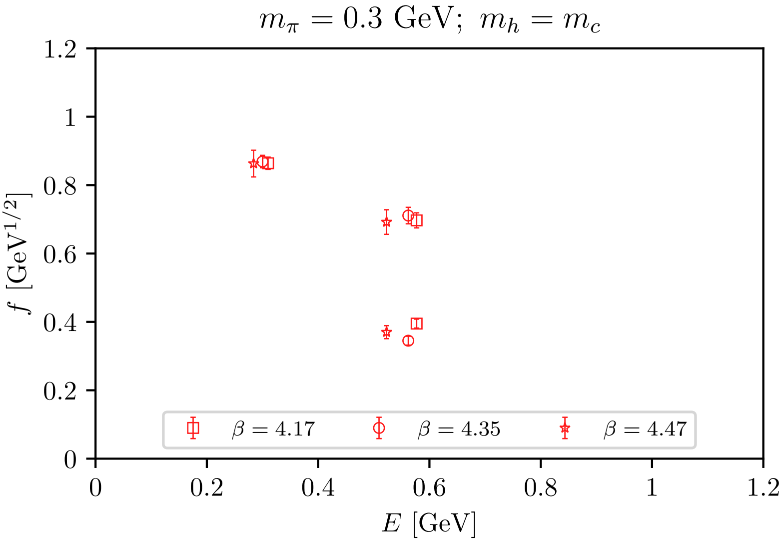
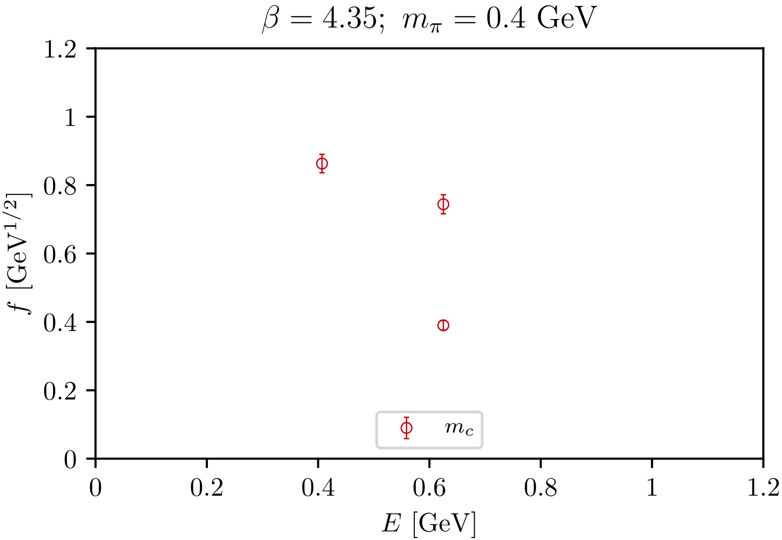
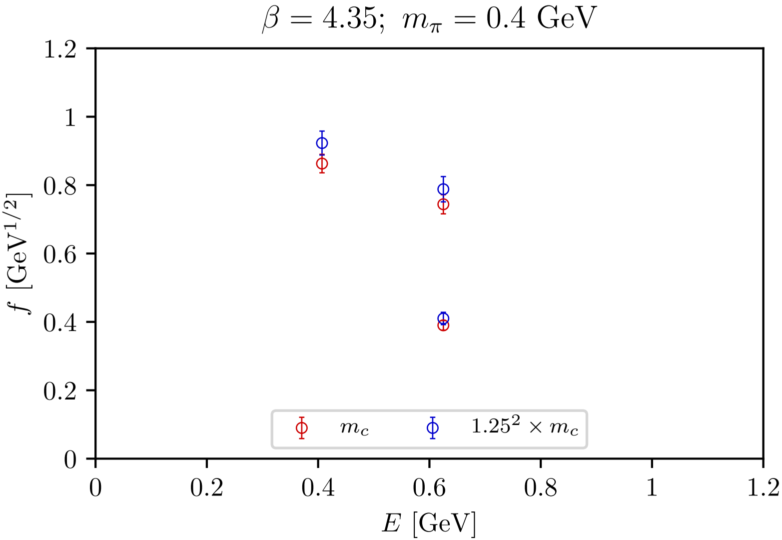


# $m_h$ Dependence



# $m_h$ Dependence



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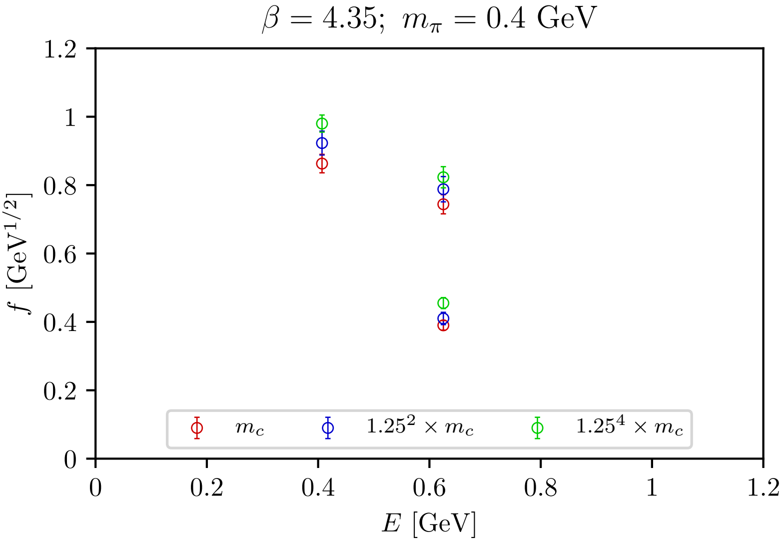
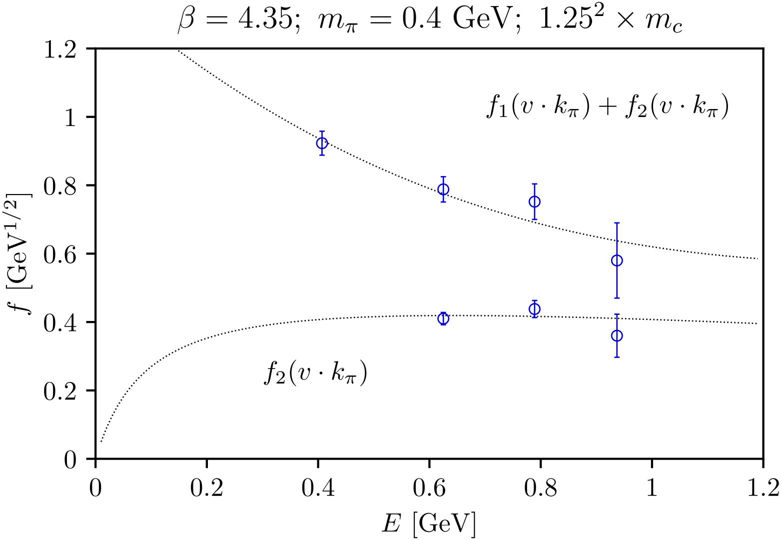
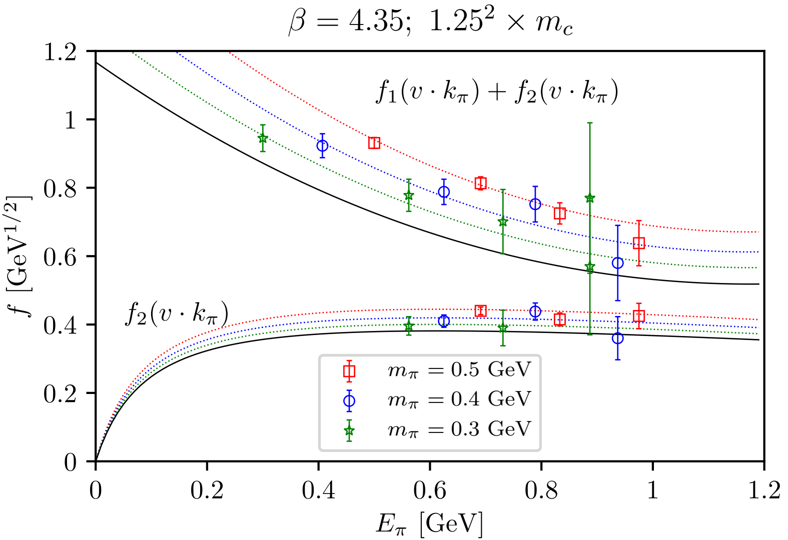


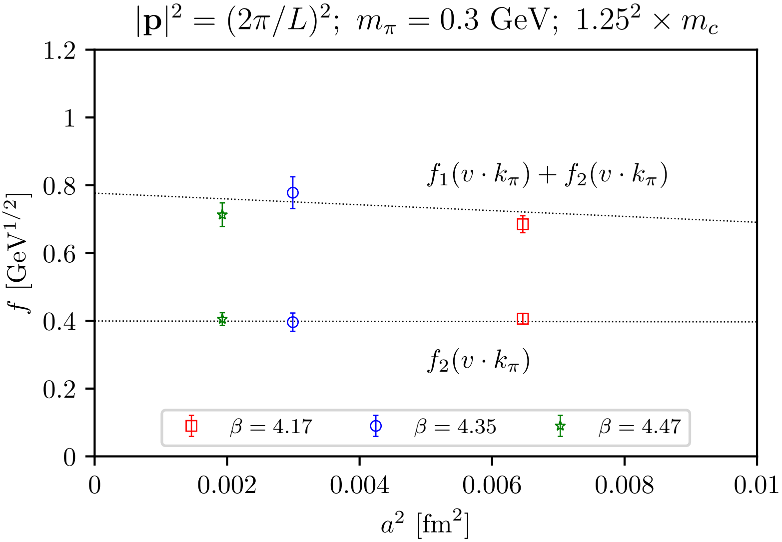
# $E_\pi$ Dependence





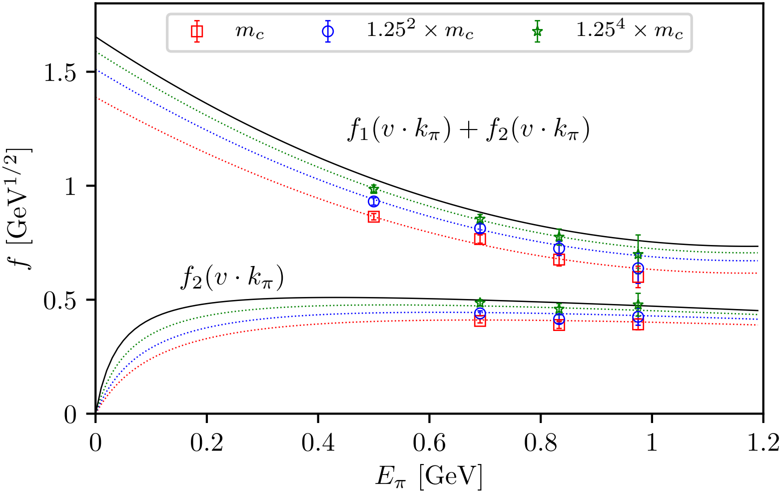
$\beta = 4.35;\ 1.25^2 \times m_c$
$f_1(v \cdot k_\pi) + f_2(v \cdot k_\pi)$
$f_2(v \cdot k_\pi)$
$m_\pi = 0.5$ GeV
$m_\pi = 0.4$ GeV
$m_\pi = 0.3$ GeV
$f$ [GeV$^{1/2}$]
$E_\pi$ [GeV]
0
0.2
0.4
0.6
0.8
1
1.2

# $a^2$ dependence

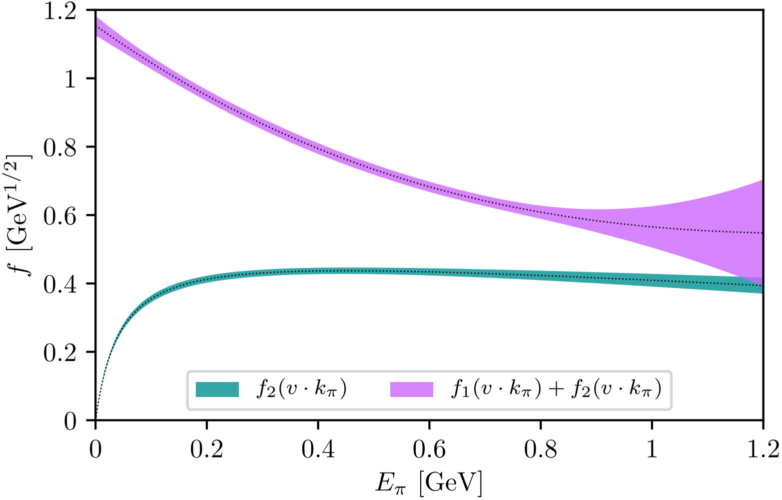


# $m_h$ dependence

$\beta = 4.35;\ m_\pi = 0.5$ GeV



# $f_1(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$



$q^2$ range for $B \to \pi$ is large. Reparametrise:

$$z(t, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where

$$t_\pm = (M_B \pm M_\pi)^2.$$

$t_0$ is a choice. So choosing:

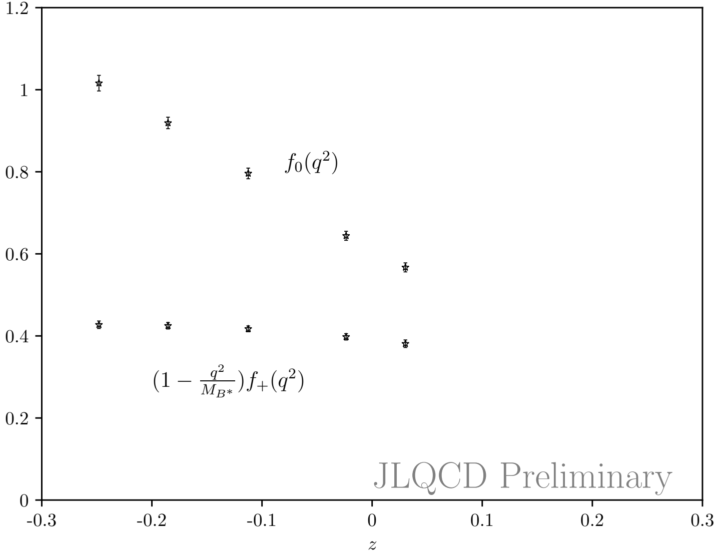
$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

will center full kinematic range around $z = 0$, with $-0.3 < z < 0.3$. Fit form factors as:

$$f_+(z) = \frac{1}{1 - q^2(z)/M_{B^*}^2} \sum_{n=0}^{N_z-1} b^{(n)} \left[ z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right]$$

$$f_0 = \sum_{n=0}^{N_z-1} a^{(n)} z^n$$

# $z$-parameter expansion: Unconstrained



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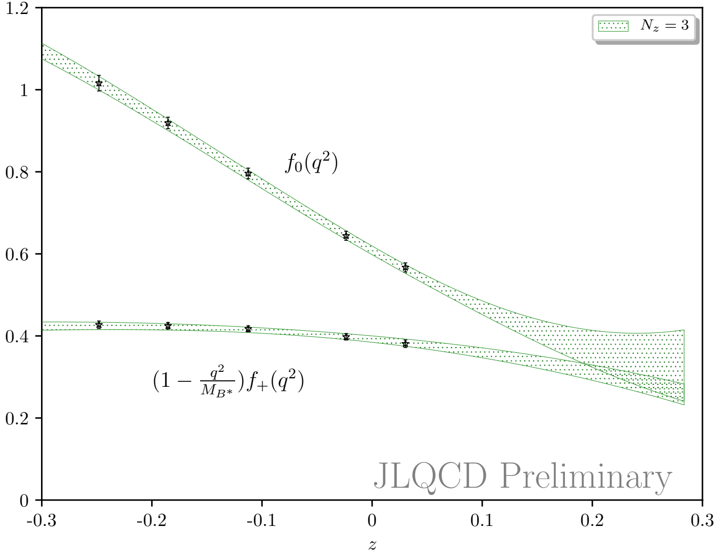

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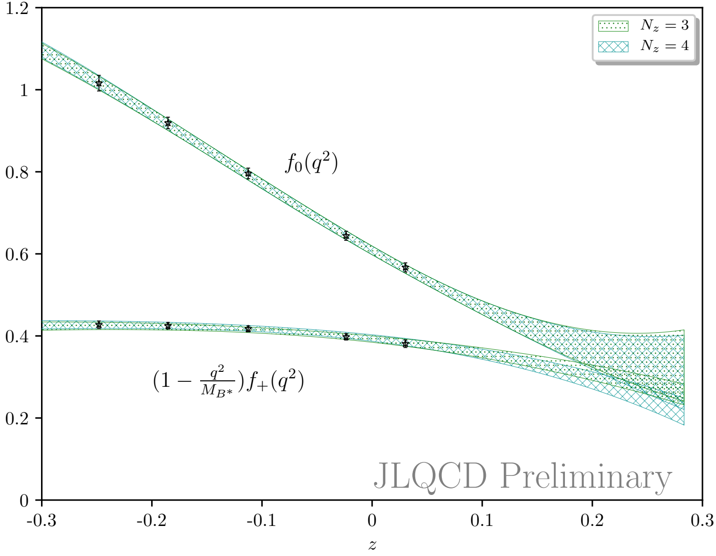

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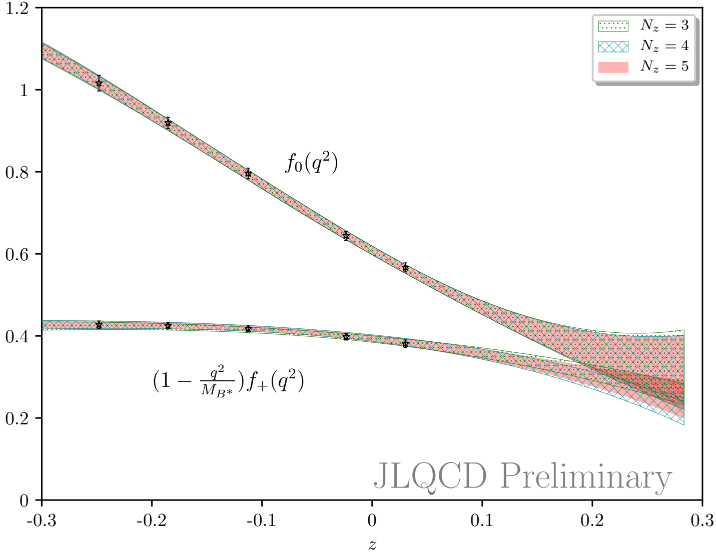

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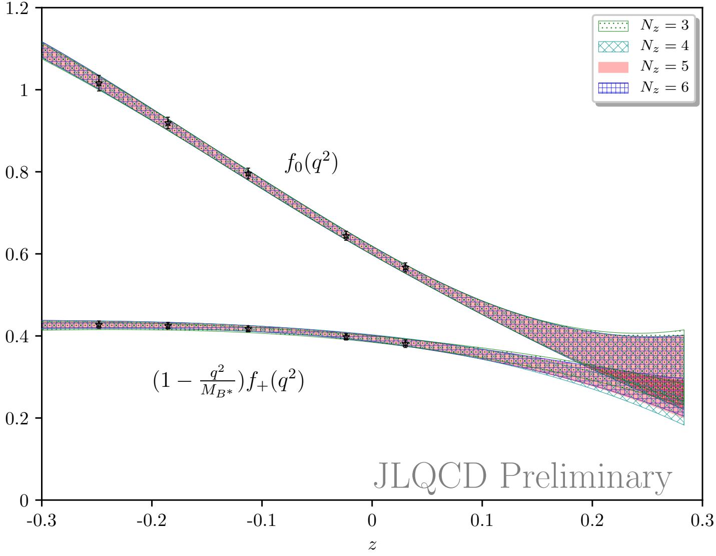
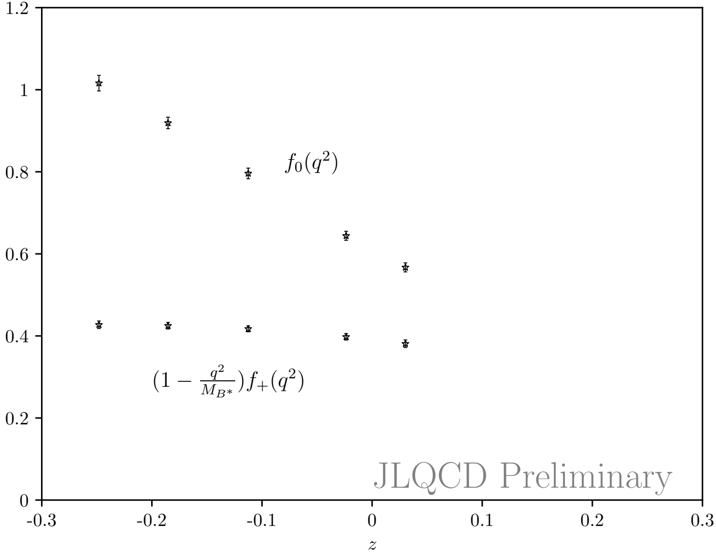


# $z$-parameter expansion: Constrained



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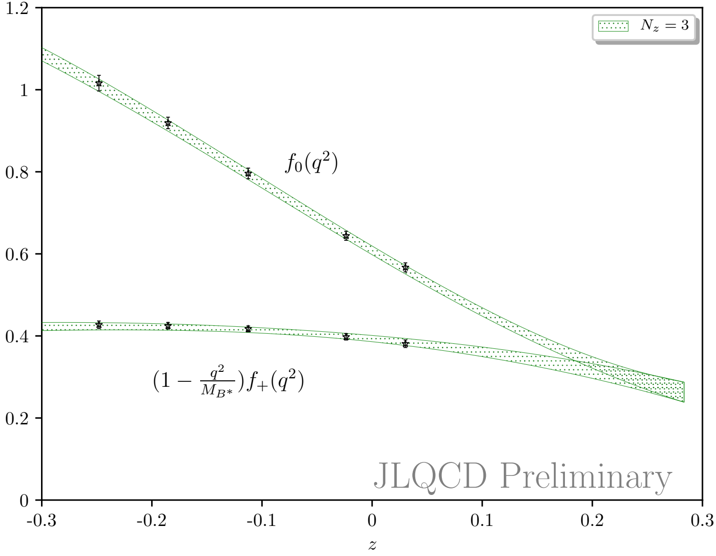

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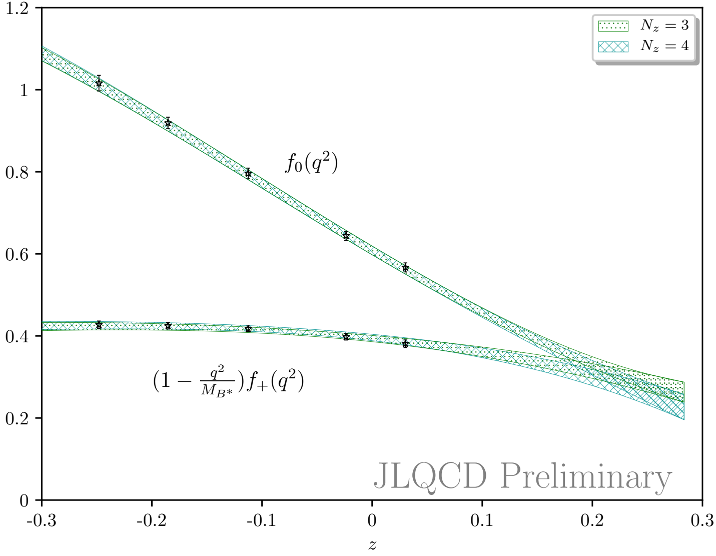
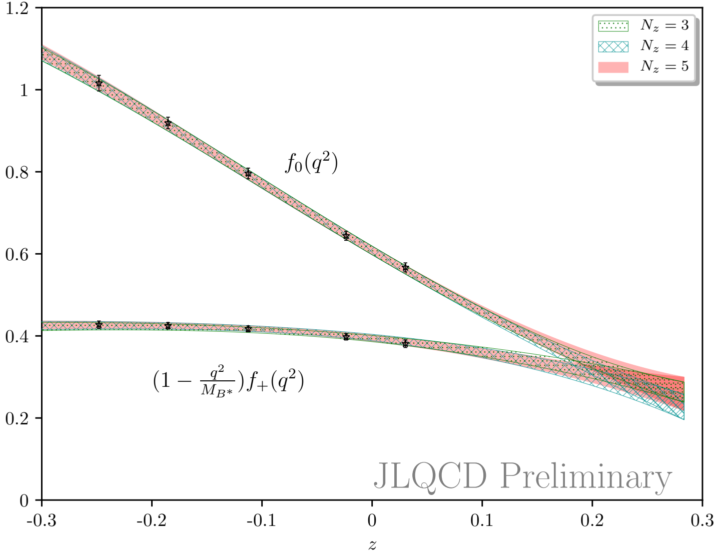
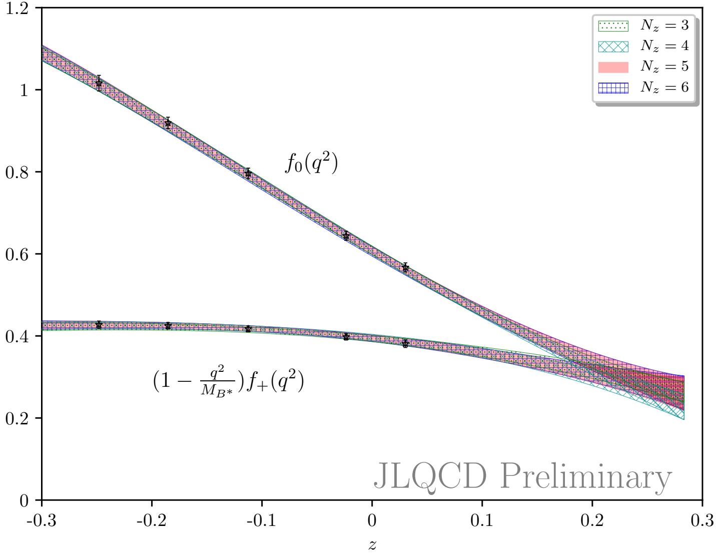
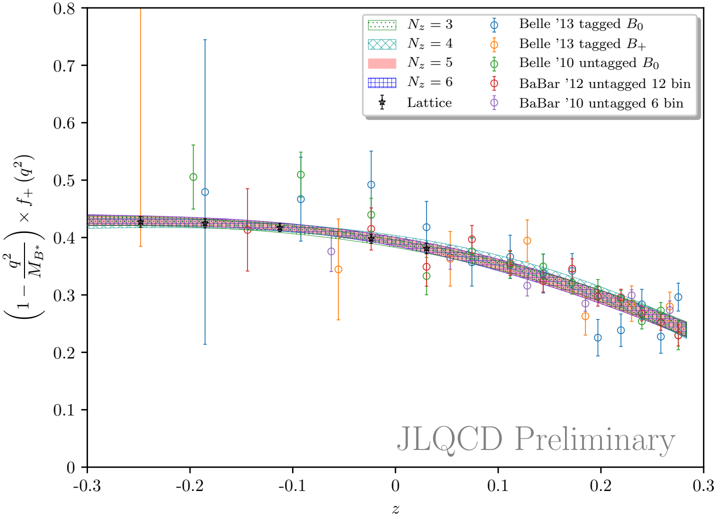



1.2
1
0.8
0.6
0.4
0.2
0
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
$z$
$N_z = 3$
$N_z = 4$
$N_z = 5$
$f_0(q^2)$
$(1 - \frac{q^2}{M_{B^*}})f_+(q^2)$
JLQCD Preliminary

# $z$-parameter expansion: Constrained



# Combined Fit



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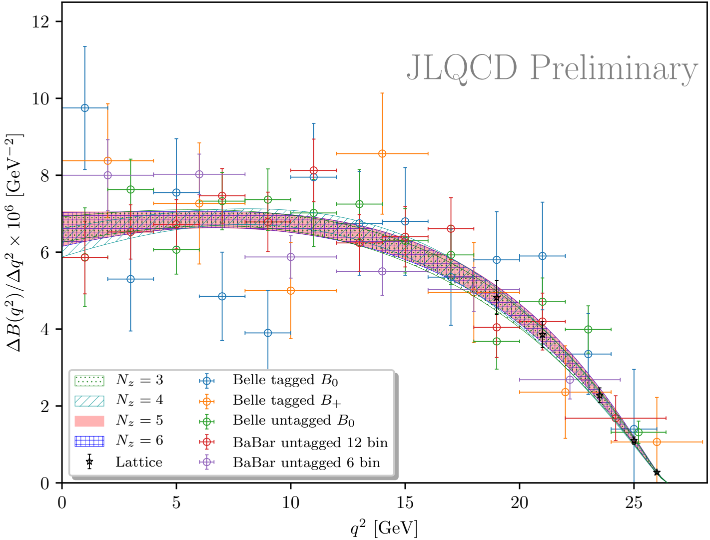

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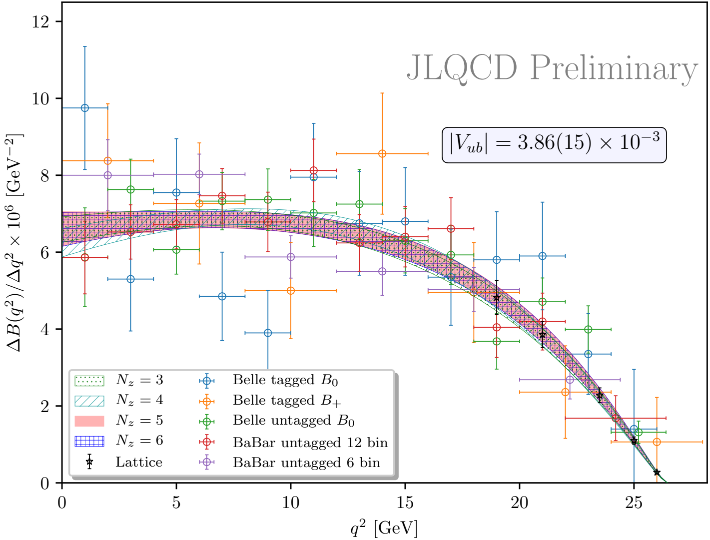
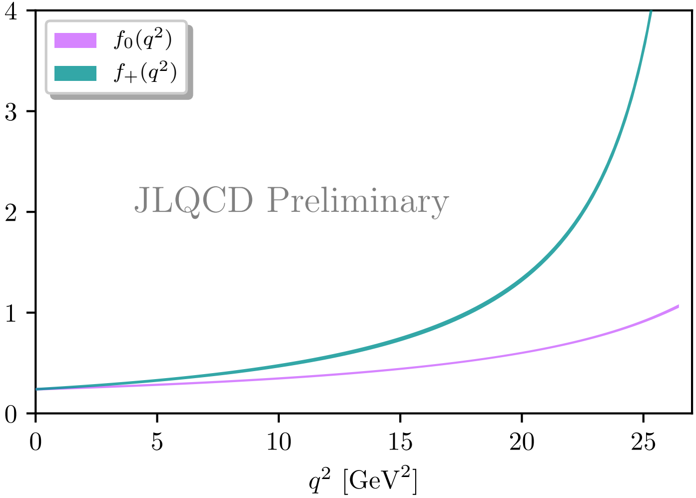
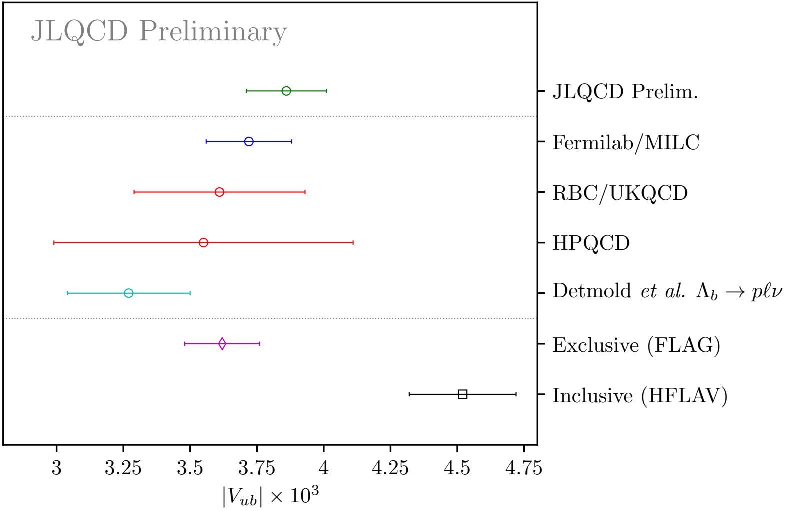


# Fitted Form Factors



# $|V_{ub}|$



JLQCD Preliminary
JLQCD Prelim.
Fermilab/MILC
RBC/UKQCD
HPQCD
Detmold *et al.* $\Lambda_b \to p\ell\nu$
Exclusive (FLAG)
Inclusive (HFLAV)
3
3.25
3.5
3.75
4
4.25
4.5
4.75
$|V_{ub}| \times 10^3$

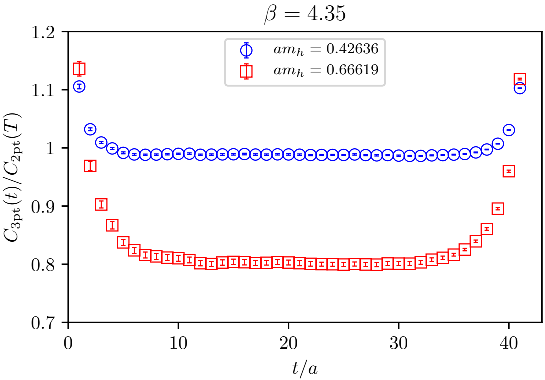
# Summary

- We can use MDWF for semileptonic $B$ decays w/ fine lattices
  - Effectively an extension of JLQCD $D \to \pi \ell \nu$ calculation
  - Also in $B \to D^{(*)} \ell \nu$ [T. Kaneko, Fri @ 17:30]
  - Inclusive decays [S. Hashimoto, Poster]
- HQET behaviour to reach $m_b$
  - $1/m_b$ term explains data
  - (Possible: Push to higher $m_h$ on finest lattice: $am_h \lesssim 0.8$)
  - Should anticipate fully relativistic results from other groups
- Preliminary results of $|V_{ub}|$ consistent w/ other lattice results

# Backup

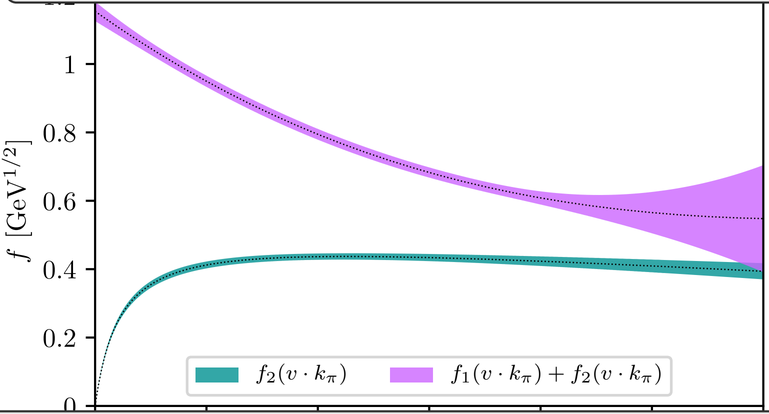
# Renormalisation

- Renormalise with $Z_{V,bl} = \sqrt{Z_{V,bb} Z_{V,ll}}$.
- Require $C_{3\mathrm{pt}}(t)/C_{2\mathrm{pt}}(T_{\mathrm{ext}}) = Z^{-1}_{V,bb}$.



# $f_1(v \cdot k_\pi)$ and $f_2(v \cdot k_\pi)$

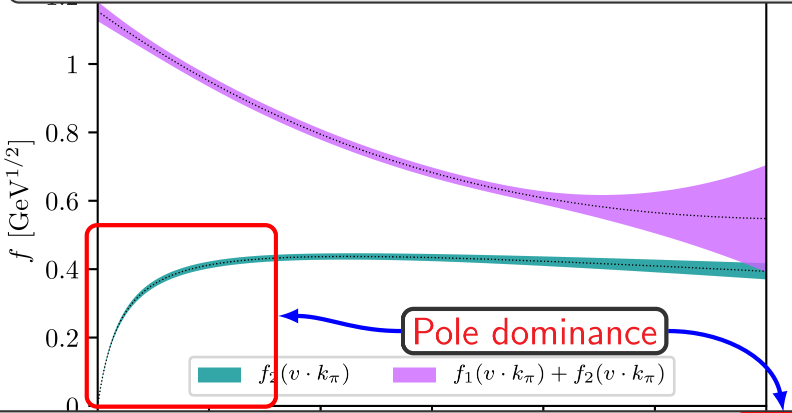
$$D_0 \left(1 + \sum_{n=1}^{3} D_{E_\pi^n} E_\pi^n\right) \left(1 + D_{m_\pi} m_\pi^2\right) \left(1 + D_{a^2} a^2\right) \left(1 + \frac{D_{m_b}}{m_b}\right) (1 + \chi_{\log})$$




$$\left[C_0 \left(1 + C_a a^2\right) \left(1 + \frac{C_{m_b}}{m_b}\right) \left(1 + C_\pi m_\pi^2\right) \left(1 + C_{E_\pi} E_\pi\right) (1 + \chi_{\log})\right] \frac{E_\pi}{E_\pi + \Delta_B}$$

# Pole Dominance

$$D_0 \left(1 + \sum_{n=1}^{3} D_{E_\pi^n} E_\pi^n\right) \left(1 + D_{m_\pi} m_\pi^2\right) \left(1 + D_{a^2} a^2\right) \left(1 + \frac{D_{m_b}}{m_b}\right) \left(1 + \chi_{\log}\right)$$




$$\left[C_0 \left(1 + C_a a^2\right) \left(1 + \frac{C_{m_b}}{m_b}\right) \left(1 + C_\pi m_\pi^2\right) \left(1 + C_{E_\pi} E_\pi\right) \left(1 + \chi_{\log}\right)\right] \frac{E_\pi}{E_\pi + \Delta_B}$$