$1 \to 2$ transition amplitudes from lattice QCD

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Introduction

Many important processes in flavor physics have two (or more) hadrons in the final state. This includes the B decays

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and

$$B \to \rho(\to \pi\pi)\ell^-\bar{\nu},$$

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In our past lattice QCD calculation of the $B \to K^*$ form factors [R. R. Horgan, Z. Liu, S. Meinel, M. Wingate, arXiv:1310.3722/PRD 2014], we performed the analysis as if the K^* was stable, which introduces uncontrolled systematic errors.

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To properly determine the $B \to K^*$ and $B \to \rho$ resonance form factors, and to obtain information beyond the resonant contribution, lattice QCD calculations of $B \to K\pi$ and $B \to \pi\pi$ form factors are needed.

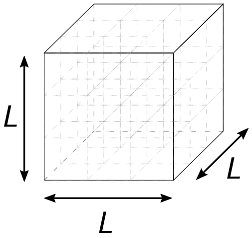
Lattice QCD calculations involving multi-hadron states are substantially more complicated than for single-hadron states, but the finite-volume formalism needed to compute $1 \to 2$ (as well as $0 \to 2$ and $2 \to 2$) transition matrix elements has been fully developed. I will discuss our progress toward applying this formalism to $B \to K\pi$ and $B \to \pi\pi$.

1. Hadron-hadron scattering on the lattice
2. $1 \to 2$ transition matrix elements on the lattice
3. $\pi\gamma \to \pi\pi$
4. Prospects for $B \to \pi\pi\ell^-\bar{\nu}$, $B \to K\pi\ell^+\ell^-$, ...

Lattice QCD

Lattice QCD allows us to nonperturbatively compute Euclidean correlation functions in a finite volume:

$$\langle O_1 ... O_n \rangle_L = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}, U] \; O_1 ... O_n \; e^{-S_E[\psi, \overline{\psi}, U]}.$$



With periodic b.c., the total spatial momentum of a finite-volume state can take on the values $\mathbf{P} = \frac{2\pi}{L}(n_x, n_y, n_z)$, where n_x, n_y, n_z are integers.

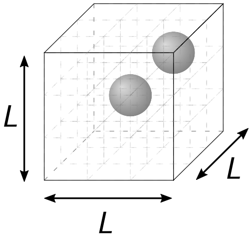
The finite-volume energy spectrum can be extracted from two-point correlation functions of operators with the desired quantum numbers (irreps):

$$\langle O_1(\mathbf{P}, t_1) O_2^\dagger(\mathbf{P}, t_2) \rangle_L = \sum_n \frac{1}{2E_n} \langle 0 | O_1 | n, \mathbf{P}, L \rangle \langle n, \mathbf{P}, L | O_2^\dagger | 0 \rangle e^{-E_n |t_1 - t_2|}.$$

Hadron-hadron scattering on the lattice

In 1991, Martin Lüscher showed that infinite-volume elastic hadron-hadron scattering amplitudes can be extracted from the finite-volume energy levels.

[M. Lüscher, Nucl. Phys. B **354**, 531 (1991)]



A recent review of this very active field can be found in:

R. A. Briceño, J. J. Dudek, R. D. Young, `arXiv:1706.06223`/RMP 2018

Hadron-hadron scattering on the lattice

Simple case: single channel, partial-wave mixing neglected

Noninteracting energies: $\sqrt{m_1^2 + (\frac{2\pi}{L}\mathbf{n}_1)^2} + \sqrt{m_2^2 + (\frac{2\pi}{L}\mathbf{n}_2)^2}$

Hadron-hadron scattering on the lattice

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The energy levels E_n correspond to the solutions k_n of the Lüscher quantization condition

$$\cot \underbrace{\delta(k)}_{\text{infinite-volume phase shift}} = \cot \underbrace{\phi(k, L, \mathbf{P}, \Lambda)}_{\text{known finite-volume function}},$$

where $\mathbf{P}$ is the total momentum, and the scattering momentum k is related to the center-of-mass energy $E_{\mathrm{CM}} = \sqrt{s}$ via

$$\sqrt{m_1^2 + k^2} + \sqrt{m_2^2 + k^2} = E_{\mathrm{CM}} = \sqrt{s}.$$

The finite-volume geometric function is given by

$$\cot \phi(k, L, \mathbf{P}, \Lambda) = \sum_{l,m} c_{lm}^{\mathbf{P},\Lambda} \frac{Z_{lm}^{\mathbf{P}}\left(1; (kL/(2\pi))^2\right)}{\pi^{3/2}\sqrt{2l+1}\gamma\left(\frac{kL}{2\pi}\right)^{l+1}}, \qquad Z_{lm}^{\mathbf{P}}(s; x) = \sum_{\mathbf{r}\in P_{\mathbf{P}}} \frac{r^l Y_{lm}(\mathbf{r})}{(\mathbf{r}^2 - x)^s}.$$

The coefficients $c_{lm}^{\mathbf{P},\Lambda}$ depend on the irrep Λ of the lattice symmetry group.
[See, e.g., L. Leskovec, S. Prelovsek, arXiv:1202.2145/PRD 2012]

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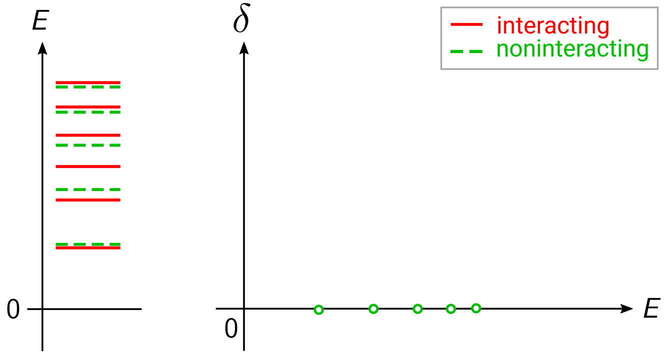

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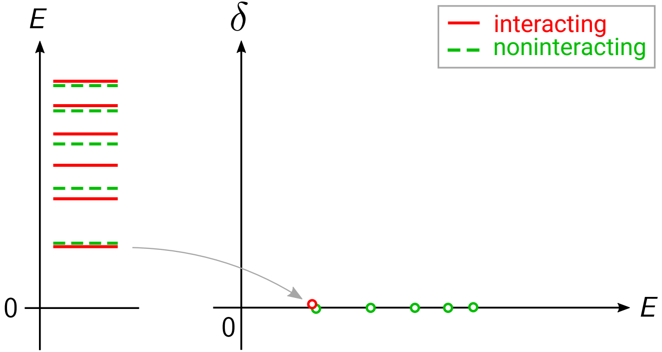

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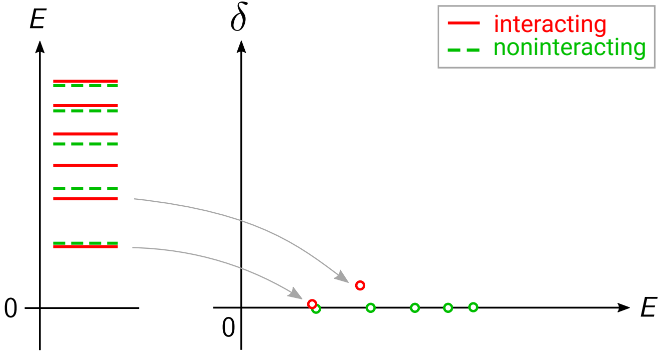

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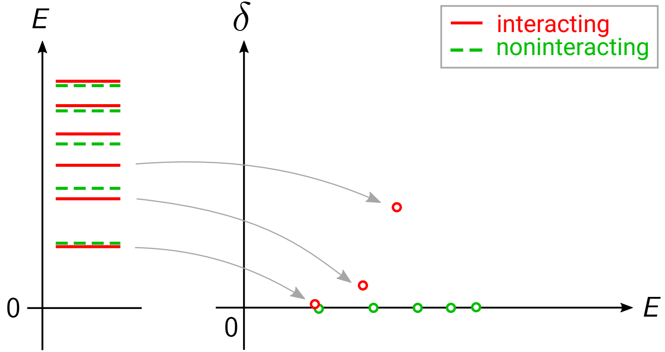

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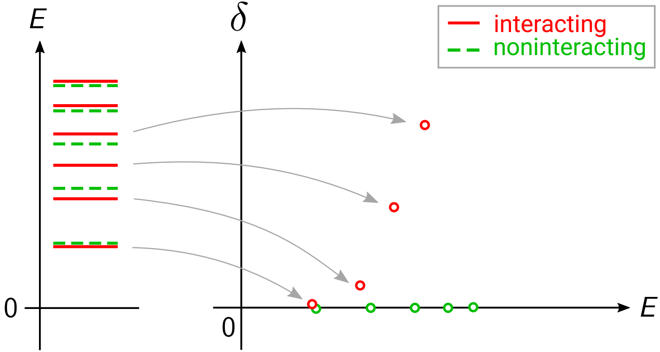

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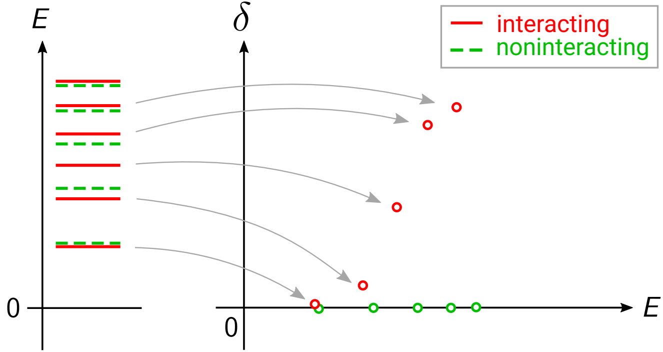

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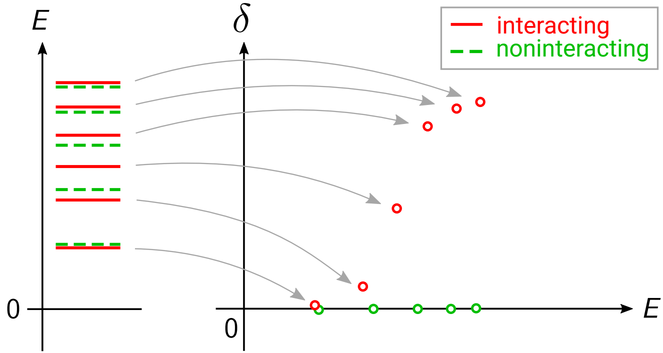

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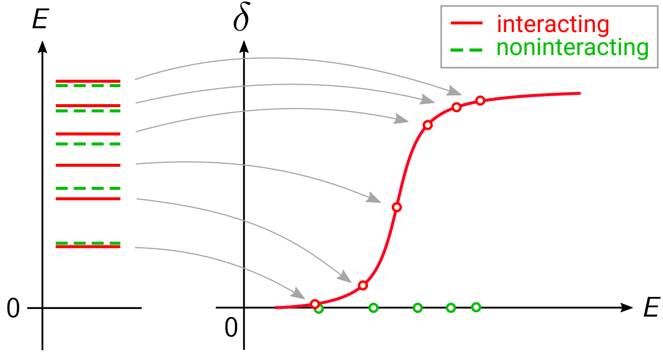

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$1 \to 2$ transition matrix elements on the lattice

The goal is to determine matrix elements with infinite-volume two-hadron "out" states, such as

$$\langle \pi^0\pi^+, s, \mathbf{P}, l, m | \, J^\mu \, | B, \mathbf{p}_B \rangle \qquad \text{(infinite volume)},$$

where $J^\mu = \bar{u}\gamma^\mu b, \;\; \bar{u}\gamma^\mu\gamma_5 b$.

On the lattice, the single-meson initial state is not significantly affected by the finite volume. However, instead of the continuum of noninteracting $\pi^0\pi^+$ "out" states, we have the interacting finite-volume states, and we only get

$$\langle n, L, \mathbf{P}, \Lambda, r | \, J^\mu \, | B, \mathbf{p}_B \rangle \qquad \text{(finite volume)}.$$

Here, Λ is the irrep of the (little group of the) cubic group, and r is the row of the irrep.

$1 \to 2$ transition matrix elements on the lattice

In the year 2000, L. Lellouch and M. Lüscher showed how the finite-volume and infinite-volume matrix elements are related for the case of the $K \to \pi\pi$ nonleptonic weak decay.

[L. Lellouch, M. Lüscher, arXiv:hep-lat/0003023/CMP 2001].

The formalism has since been generalized to arbitrary $1 \to 2$ transition matrix elements with nonzero four-momentum transfer, and including the effects of coupled-channel interactions.

[C. J. D. Lin, G. Martinelli, C. T. Sachrajda, M. Testa, arXiv:hep-lat/0104006/NPB 2001;
N. H. Christ, C. Kim, T. Yamazaki, arXiv:hep-lat/0507009/PRD 2005;
M. T. Hansen and S. R. Sharpe, arXiv:1204.0826/PRD 2012;
R. A. Briceño and Z. Davoudi, arXiv:1204.1110/PRD 2013;
R. A. Briceño, M. T. Hansen, A. Walker-Loud, arXiv:1406.5965/PRD 2015;
R. A. Briceño, M. T. Hansen, arXiv:1502.04314/PRD 2015].

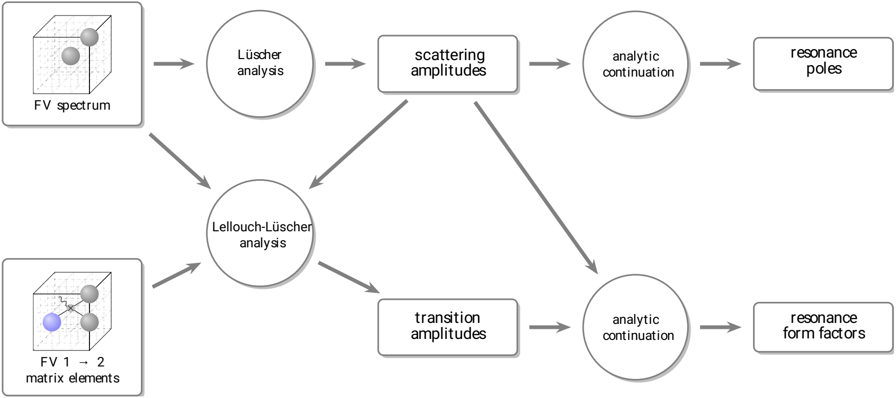
$1 \to 2$ transition matrix elements on the lattice

Considering again the simple case without coupled channels and neglecting partial-wave mixing, the relation is given by

$$\frac{|\langle \pi^0\pi^+, s_n, \mathbf{P}, \Lambda, r|\, J^\mu \,|B, \mathbf{p}_B\rangle|^2}{|\langle n, L, \mathbf{P}, \Lambda, r|\, J^\mu(\mathbf{q}) \,|B, \mathbf{p}_B\rangle|^2} = \frac{1}{2E_n}\frac{16\pi\sqrt{s_n}}{k_n}\left[\frac{\partial\delta}{\partial E} + \frac{\partial\phi}{\partial E}\right]_{E=E_n},$$

where $\delta = \delta_l$ is the scattering phase shift for the partial wave l considered here, and ϕ is the finite-volume function that also appears in the Lüscher quantization condition.

$1 \to 2$ transition matrix elements on the lattice



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$\pi\gamma \to \pi\pi$

The electromagnetic process $\pi\gamma \to \pi\pi$ is a good starting point to test the lattice methods for $1 \to 2$ transitions.

We allow the photon to be virtual. We take the $\pi\pi$ system to have angular momentum 1 and isospin 1, so that we expect the ρ resonance to appear.

The process is described by the hadronic matrix element

$$\langle \pi\pi, s, \mathbf{P}, 1, m | \, J^\mu \, | \pi, \mathbf{p}_\pi \rangle = \frac{2i \, V(q^2, s)}{m_\pi} \epsilon^{\nu\mu\alpha\beta} \underbrace{\varepsilon_\nu(P, m)}_{\pi\pi \text{ polarization vector}} (p_\pi)_\alpha P_\beta,$$

where J^μ is the quark electromagnetic current. The form factor $V(q^2, s)$ is a function of the photon virtuality

$$q^2 = (p_\pi - P)^2$$

and the $\pi\pi$ invariant mass

$$s = P^2.$$

$\pi\gamma \to \pi\pi$

There is one previous calculation of $\pi\gamma \to \pi\pi$, by the Hadron Spectrum Collaboration, with $m_\pi \approx 400$ MeV:

R. A. Briceno, J. J. Dudek, R. G. Edwards, C. J. Shultz, C. E. Thomas, D. J. Wilson, arXiv:1507.06622/PRL 2015; arXiv:1604.03530/PRD 2016.

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Our calculation of $\pi\gamma \to \pi\pi$, with $m_\pi \approx 320$ MeV, is published in

C. Alexandrou, L. Leskovec, S. Meinel, J. Negele, S. Paul, M. Petschlies, A. Pochinsky, G. Rendon, S. Syritsyn, arXiv:1807.08357/PRD 2018.

We use gauge configurations with $2+1$ flavors of clover fermions, generated by the JLab and William & Mary lattice QCD groups.

$\pi\gamma \to \pi\pi$

The first step is to determine the $\pi\pi$ finite-volume energy spectra for various total momenta $\mathbf{P}$ and irreps Λ that contain the P wave. We compute matrices of two-point correlation functions

$$C_{ij}^{\mathbf{P},\Lambda,r}(t) = \left\langle O_i^{\Lambda,r}(\mathbf{P},t)\, O_j^{\Lambda,r\,\dagger}(\mathbf{P},0) \right\rangle,$$

using operators with both quark-antiquark and two-pion structure:

$$O_{1,2}(\mathbf{P},t) \sim \sum_{\mathbf{x}} \bar{d}(\mathbf{x},t)\Gamma u(\mathbf{x},t) e^{i\mathbf{P}\cdot\mathbf{x}},$$

$$O_{3,4}(\mathbf{P},t) \sim \frac{1}{\sqrt{2}}\left(\pi^+(\mathbf{p}_1,t)\pi^0(\mathbf{p}_2,t) - \pi^0(\mathbf{p}_1,t)\pi^+(\mathbf{p}_2,t)\right),$$

where $\pi^+(\mathbf{p}_1,t) = \sum_{\mathbf{x}} \bar{d}(\mathbf{x},t)\Gamma u(\mathbf{x},t) e^{i\mathbf{p}_1\cdot\mathbf{x}}$ etc.

$\pi\gamma \to \pi\pi$

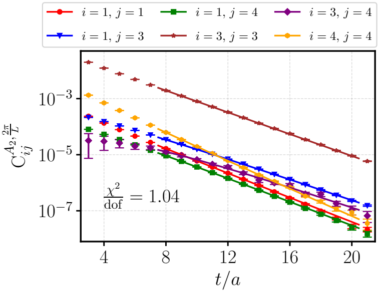
We then solve the generalized eigenvalue problem (GEVP)

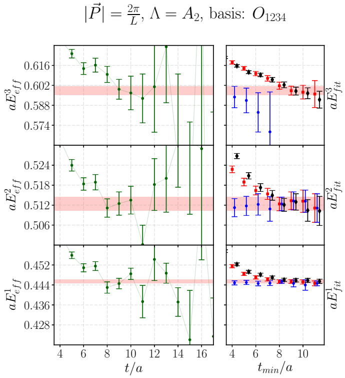
$$\sum_j C_{ij}^{\mathbf{P},\Lambda,r}(t) v_j^{n,\mathbf{P},\Lambda}(t_0) = \lambda_n^{\mathbf{P},\Lambda}(t,t_0) \sum_j C_{ij}^{\mathbf{P},\Lambda,r}(t_0) v_j^{n,\mathbf{P},\Lambda}(t_0).$$

For large t_0 and $t - t_0$, the eigenvalues satisfy

$$\lambda_n^{\mathbf{P},\Lambda}(t,t_0) = e^{-E_n^{\mathbf{P},\Lambda}(t-t_0)}.$$

Example:

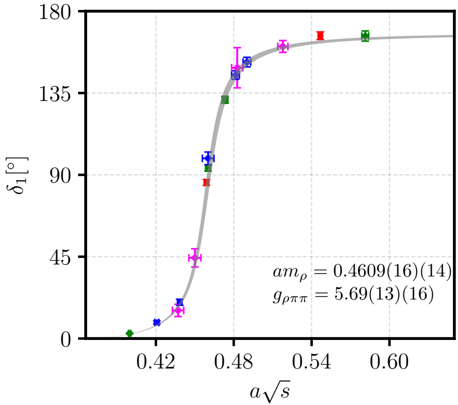




Correlation matrix GEVP E_{eff} Fitted energies

$\pi\gamma \to \pi\pi$

In step 2, we use Lüscher's method to extract the P-wave $\pi\pi$ scattering phase shifts, and we perform a Breit-Wigner fit:



Fit:

$$\cot\delta(s) = \frac{m_R^2 - s}{\sqrt{s}\,\Gamma(s)},$$

$$\Gamma(s) = \frac{g_{\rho\pi\pi}^2}{6\pi}\,\frac{k^3}{s}$$

The colors indicate the total momenta used on the lattice, $|\frac{L}{2\pi}\mathbf{P}|^2 = 0, 1, 2, 3$.

$\pi\gamma \to \pi\pi$

Step 3 is to determine the finite-volume transition matrix elements of the electromagnetic current from three-point functions.

To compute the matrix element for the nth excited state for a given momentum and irrep, we use the optimized operator

$$\mathcal{O}^{n,\,\Lambda,\,r}(\mathbf{P},t,t_0) = \sum_i v_i^{n,\,\mathbf{P},\,\Lambda\,\dagger}(t_0)\, O_i^{\Lambda,\,r}(\mathbf{P},t),$$

where $v_j^{n,\,\mathbf{P},\,\Lambda}(t_0)$ is the nth generalized eigenvector obtained previously from the two-point-function analysis.

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where $v_j^{n,\,\mathbf{P},\,\Lambda}(t_0)$ is the nth generalized eigenvector obtained previously from the two-point-function analysis.

The optimized three-point function is then defined as

$$\Omega_{3,\,\mu,\,n}^{\mathbf{p}_\pi,\,\mathbf{P},\,\Lambda,\,r}(t_\pi,t_J,t_{\pi\pi},t_0) = \langle O_\pi(\mathbf{p}_\pi,t_\pi)\, J_\mu(t_J,\mathbf{q})\, \mathcal{O}^{n,\,\Lambda,\,r}(\mathbf{P},t_{\pi\pi},t_0)\rangle.$$

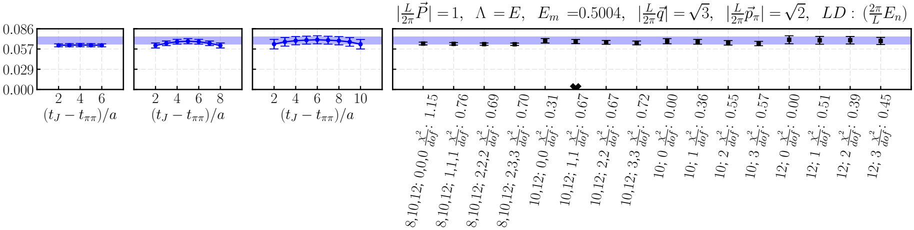
$\pi\gamma \to \pi\pi$

The finite-volume matrix elements can then be obtained from the following ratios:

$$R^{\mathbf{p}_\pi,\,\mathbf{P},\,\Lambda,\,r}_{\mu,\,n}(t_\pi, t_J, t_{\pi\pi}) = \frac{\Omega^{\mathbf{p}_\pi,\,\mathbf{P},\,\Lambda,\,r}_{3,\,\mu,\,n}(t_\pi, t_J, t_{\pi\pi}, t_0)\,\Omega^{\mathbf{p}_\pi,\,\mathbf{P},\,\Lambda,\,r\,\dagger}_{3,\,\mu,\,n}(t_\pi, t', t_{\pi\pi}, t_0)}{C^{\mathbf{p}_\pi}_\pi(\Delta t)\,\lambda^{\mathbf{P},\Lambda}_n(\Delta t, t_0)}$$

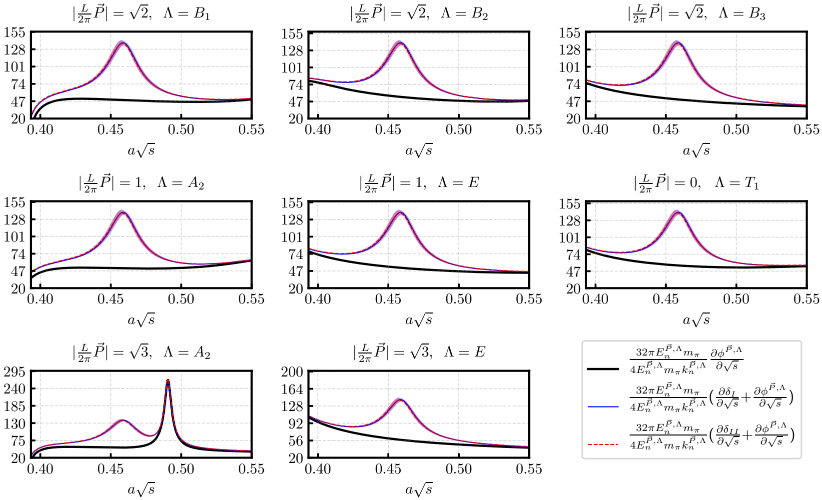
$$\xrightarrow[\text{large times}]{} \frac{|\langle n, L, \mathbf{P}, \Lambda, r|\, J^\mu(\mathbf{q})\, |\pi, \mathbf{p}_\pi\rangle|^2}{4E^{\mathbf{P},\Lambda}_n E^{\mathbf{p}_\pi}_\pi}.$$

Example:



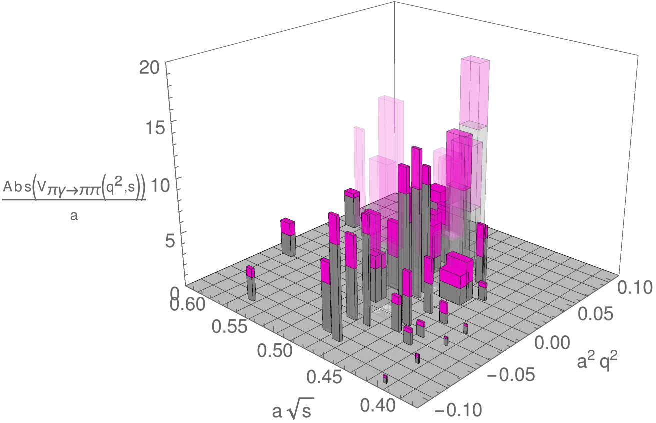
$\pi\gamma \to \pi\pi$

In step 4, we map the finite-volume matrix elements to the infinite-volume matrix elements using the Lellouch-Lüscher (LL) factors. The LL factors look like this:



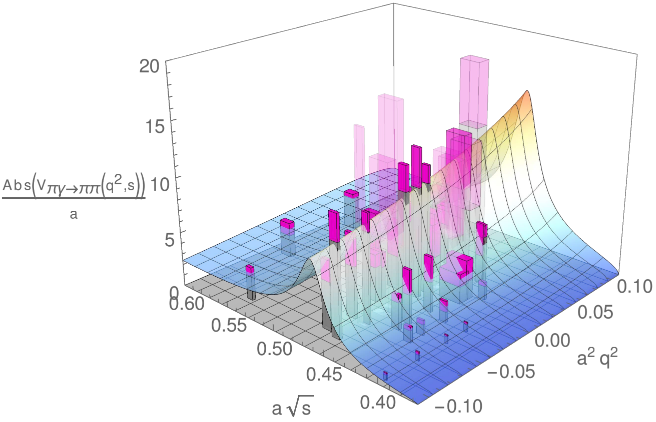
$\pi\gamma \to \pi\pi$

Here are the data points we obtain for the infinite-volume $\pi\gamma \to \pi\pi$ transition form factor:



$\pi\gamma \to \pi\pi$

We perform fits of the q^2 and s dependence of the form factor using a two-dimensional power series:

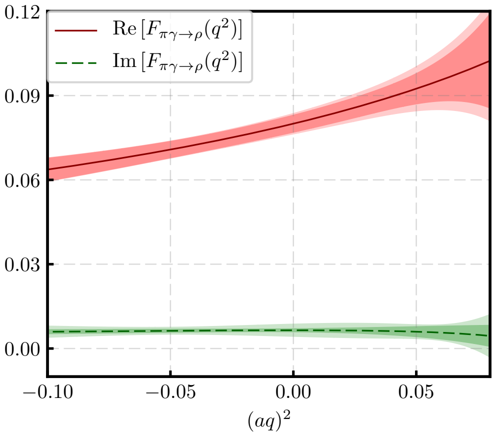


$$V(q^2,s) = \frac{F(q^2,s)}{m_R^2 - s - i\sqrt{s}\,\Gamma(s)}\sqrt{\frac{16\pi s\Gamma(s)}{k}}, \qquad F(q^2,s) = \frac{1}{1-\frac{q^2}{m_P^2}}\sum_{n,m} A_{nm} z^n \mathcal{S}^m, \qquad \mathcal{S} = \frac{s-m_R^2}{m_R^2}$$

$\pi\gamma \to \pi\pi$

Finally, we can also obtain the $\pi\gamma \to \rho$ resonant form factor by analytically continuing to the resonance pole at complex s:

$$F_{\pi\gamma\to\rho}(q^2) = F(q^2,\, m_R^2 + i m_R \Gamma_R).$$



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The $\pi\gamma \to \pi\pi$ calculation presented so far is part of a larger program to determine the $B \to \pi\pi\ell\bar{\nu}$, $D \to \pi\pi\ell\nu$, $\pi\gamma \to \pi\pi$, $B \to K\pi\ell^+\ell^-$, $D \to K\pi\ell\nu$, and $K\gamma \to K\pi$ form factors.

The charm decays are ideal to test the methods, because detailed experimental data for the decay distributions are available.

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Our code computes the correlation functions for all of these processes simultaneously. The production status is as follows:

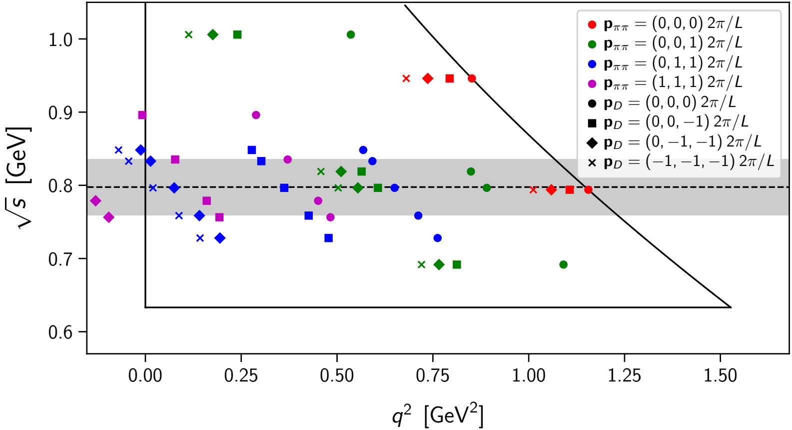
Label	$N_s^3 \times N_t$	a (fm)	m_π (MeV)	Status
C13	$32^3 \times 96$	≈ 0.114	≈ 320	done
D5	$32^3 \times 64$	≈ 0.088	≈ 280	running
D6	$48^3 \times 96$	≈ 0.088	≈ 170	running
D7	$64^3 \times 128$	≈ 0.088	≈ 170	planned
D8	$72^3 \times 196$	≈ 0.088	≈ 140	planned

Thanks to the JLab and W&M LQCD groups for generating the gauge configurations!

The analyses of the other processes are underway.

Prospects for $B \to \pi\pi\ell^-\bar{\nu}$, $B \to K\pi\ell^+\ell^-$, ...

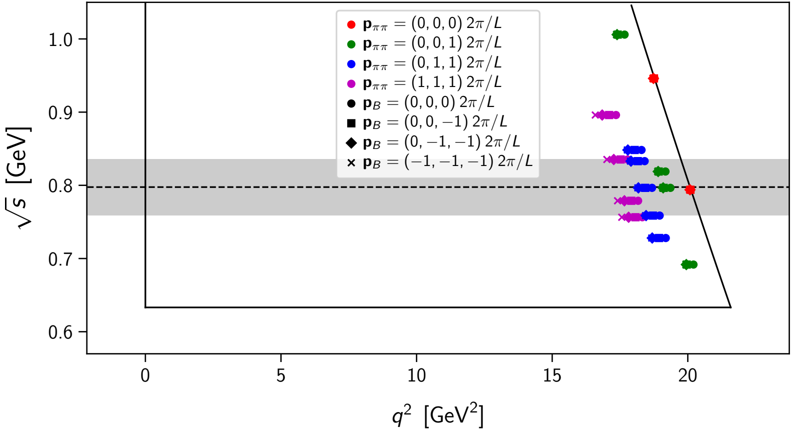
This plot shows the kinematic points we expect to obtain for the P-wave $D \to \pi\pi\ell\nu$ form factors on the `C13` ensemble:



($D \to K\pi\ell\nu$ will be similar.)

Prospects for $B \to \pi\pi\ell^-\bar{\nu}$, $B \to K\pi\ell^+\ell^-$, ...

This plot shows the kinematic points we expect to obtain for the P-wave $B \to \pi\pi\ell\bar{\nu}$ form factors on the C13 ensemble:



($B \to K\pi\ell^+\ell^-$ will be similar.)

Prospects for $B \to \pi\pi\ell^-\bar{\nu}$, $B \to K\pi\ell^+\ell^-$, ...

The formalism requires $\sqrt{s}$ to be below any $\geq$ 3-body thresholds, such as $\pi\pi\pi\pi$ and $K\eta\eta$. This becomes more restrictive at lower pion mass.

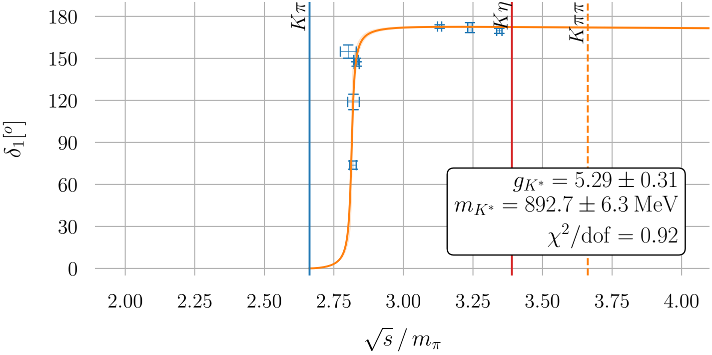
The coupling of the $\pi\pi$-$\bar{K}K$ and $K\pi$-$K\eta$ channels can be included. Again, this becomes more relevant at high $\sqrt{s}$ and at lower pion mass.

Partial-wave mixing can also be included in the analysis. This is particularly important for the unequal-mass ($K\pi$) case, where even and odd partial waves can mix (some irreps contain both the S wave and the P wave).

There is no limitation on the q^2 range from the finite-volume formalism, but statistical and discretization errors grow as the final-state total momentum is increased.

Prospects for $B \to \pi\pi\ell^-\bar{\nu}$, $B \to K\pi\ell^+\ell^-$, ...

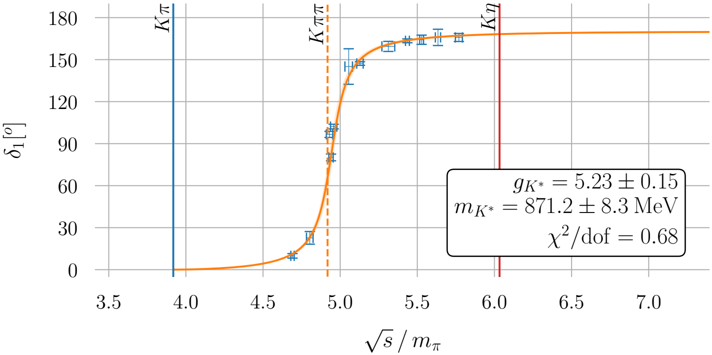
Finally, here are our preliminary results for the $K\pi$, P-wave, $I = 1/2$ scattering amplitude at $m_\pi \approx 320$ MeV:



[G. Rendon *et al.*, arXiv:1811.10750]

Prospects for $B \to \pi\pi\ell^-\bar{\nu}$, $B \to K\pi\ell^+\ell^-$, ...

... and at $m_\pi \approx 180$ MeV:



[G. Rendon *et al.*, arXiv:1811.10750]

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Lattice QCD computations are underway for $\pi\gamma \to \pi\pi$, $D \to \pi\pi\ell\nu$, $B \to \pi\pi\ell\bar{\nu}$, $K\gamma \to K\pi$, $D \to K\pi\ell\nu$, and $B \to K\pi\ell^+\ell^-$. There is still a lot of work to do.