Lattice calculation of $B \to D^* \ell \bar{\nu}$ semileptonic decay form factors and determination of $|V_{cb}|$

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imagine **the impossible**

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CKM matrix

- In the Standard Model, the weak interaction with charged-currents are

$$-\frac{g}{\sqrt{2}}\begin{pmatrix}\bar{u}_L & \bar{c}_L & \bar{t}_L\end{pmatrix}_L \gamma^\mu W_\mu^+ V_{\mathrm{CKM}} \begin{pmatrix} d_L \\ u_L \\ b_L \end{pmatrix} + \mathrm{h.c.}, \tag{1}$$

- The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3 by 3 unitary matrix parametrized by three mixing angles and the CP-violating phase.

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 1-\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \tag{2}$$

- Consistency of SM : check unitarity of V_{CKM}.

Why V_{cb}?

- The indirect CP violation parameter in the neutral kaon system ε_K depends on ρ and η : it gives constraint on the apex of the unitarity triangle.

$$\varepsilon_K \equiv \frac{\mathcal{A}[K_L \to \pi\pi(I=0)]}{\mathcal{A}[K_S \to \pi\pi(I=0)]} \tag{3}$$

- Standard model evaluation of $|\varepsilon_K^{\mathrm{SM}}|$ using inputs determined from lattice QCD ($\hat{B}_K$, exclusive $|V_{cb}|$, and etc.) has a strong tension ($\sim 4\sigma$) with the experimental value [Weonjong Lee, et al., PRD 98, 094505 (2018)].

$$|\varepsilon_K^{\mathrm{Exp}}| = (2.228 \pm 0.011) \times 10^{-3}, \qquad |\varepsilon_K^{\mathrm{SM}}| = (1.57 \pm 0.16) \times 10^{-3}, \tag{4}$$

- The largest error of $|\varepsilon_K^{\mathrm{SM}}|$ comes from the uncertainty of V_{cb}.

$$\varepsilon_K^{\mathrm{SM}} = \sqrt{2}\exp(i\theta)\sin(\theta)\Big(C_\varepsilon X_{\mathrm{SD}} \hat{B}_K + \cdots\Big) + \cdots \tag{5}$$

$$X_{\mathrm{SD}} = \bar{\eta}\lambda^2 |V_{cb}|^4 (1-\bar{\rho}) + \cdots \tag{6}$$

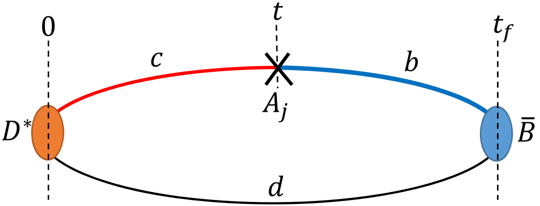
Around 30 % of total errors of $|\varepsilon_K^{\mathrm{SM}}|$ is from the uncertainty of $|V_{cb}|$.

Current status of V_{cb}

- The determination of V_{cb} : Inclusive vs Exclusive

Determination	$\|V_{cb}\| \times 10^{-3}$	memo
Exclusive($\bar{B} \to D^* \ell \bar{\nu}$)	39.3(7)	FNAL/MILC + HFLAG
Inclusive($\bar{B} \to X_c \ell \bar{\nu}$)	42.5(9)	HFLAG

[HFLAG, Eur.Phys.J.C77, 895 (2017)] [Fermilab-MILC, PRD89,114504 (2014)]



- Determination of CKM matrix elements with lattice QCD

Experiment = known factors $\times V_{CKM} \times$ Hadronic matrix element

$|V_{cb}|$ from the exclusive decay $\bar{B} \to D^* \ell \bar{\nu}$

1. **Experiment**:

$$\frac{d\Gamma}{dw}(\bar{B} \to D^* \ell \bar{\nu}) = \frac{G_F^2 m_{D^*}^3}{48\pi^3}(m_B - m_{D^*})^2 (w^2 - 1)^{1/2} |\eta_{\mathrm{EW}}|^2 |V_{cb}|^2 \chi(w) |\mathcal{F}(w)|^2$$

where $w \equiv v_B \cdot v_{D^*}$, where $v_M = p_M / m_M$ $(M = B, D^*)$ are meson velocities.

2. **Lattice QCD**: Calculate form factors from the matrix element.

$$\frac{1}{\sqrt{m_B m_{D^*}}} \langle D^*(p_{D^*}, \epsilon) | A_\mu | \bar{B}(p_B) \rangle = -i h_{A_1}(w)(w+1)\epsilon^*_\mu + i h_{A_2}(w)(\epsilon^* \cdot v_B) v_{B\mu} + i h_{A_3}(w)(\epsilon^* \cdot v_B) v_{D\mu},$$

$$\frac{1}{\sqrt{m_B m_{D^*}}} \langle D^*(p_D, \epsilon) | V_\mu | \bar{B}(p_B) \rangle = h_V(w) \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} v_D^\alpha v_B^\beta.$$

At zero recoil, $\mathcal{F}(1) = h_{A_1}(1)$.

Green's function

- 2-point (Euclidean) Green's function : meson propagator

$$C_B^{2\text{pt}}(\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle O_B(t,\boldsymbol{x}) O_B^\dagger(0)\rangle = \sum_n |\langle 0|O_B^\dagger|B_n\rangle|^2 e^{-E_n(\boldsymbol{p})t}$$
$$= |\langle 0|O_B^\dagger|\bar{B}\rangle|^2 e^{-E_0(\boldsymbol{p})t} + |\langle 0|O_B^\dagger|B_1\rangle|^2 e^{-E_1(\boldsymbol{p})t} + \cdots \quad (7)$$

$O_B^\dagger$ is a meson interpolating operator which creates B-meson as acting on the vacuum.

- 3-point (Euclidean) Green's function : extract hadronic matrix elements

$$C_{A_j}^{B\to D^*}(t_s,t_f) = \sum_{\boldsymbol{x},\boldsymbol{y}} \langle O_{D^*}(\boldsymbol{x},t_f) A_j^{cb}(\boldsymbol{y},t_s) O_B^\dagger(\boldsymbol{0},0)\rangle$$
$$= \langle 0|O_{D^*}|D^*\rangle\langle\bar{B}|O_B^\dagger|0\rangle\langle D^*|A_j|\bar{B}\rangle e^{-M_B t_s} e^{-M_{D^*}(t_f-t_s)} + \cdots \quad (8)$$

where A_j^{cb} is flavor-changing axial current.

- We calculate Green's function on Euclidean space-time lattice and do numerical analysis to extract the ground-state contribution out of excited-state contamination.

Path integral on lattice

- Calculate Green's functions by performing path integral over discretized Euclidean space-time. For example,

$$\langle O_2(x)O_1(0)\rangle = \frac{1}{Z}\int \prod dU d\bar{q} dq e^{-S_{\rm QCD}^{\rm lat}} O_2(x)O_1(0) \qquad (9)$$

  The field variable $U_\mu(x)$ is **gauge link**.

- The integral over fermionic Grassmann variables gives fermionic determinant. For example, if $O_2 \equiv \bar{b}\gamma_5 d$ and $O_1 \equiv \bar{d}\gamma_5 b$

$$\begin{aligned}\langle O_2(x)O_1(0)\rangle &= \int \frac{1}{Z}\prod dU e^{-S_g^{\rm lat}} \bar{b}(x)\gamma_5 d(x)\bar{d}(0)\gamma_5 b(0)\prod_q e^{-\sum_q \bar{q}D_q q} \\ &= -\frac{1}{Z}\int \prod dU e^{-S_g^{\rm lat}} \prod_q \det[D_q] \mathrm{tr}[\gamma_5 D_d^{-1}(x,0)\gamma_5 D_b^{-1}(0,x)] \\ &\to \sum_{i=1}^{N_{\rm MC}} w(U_i)(-\mathrm{tr}[\gamma_5 D_d^{-1}(x,0)\gamma_5 D_b^{-1}(0,x)]) \qquad (10)\end{aligned}$$

  if $\prod_q \det[D_q] \geq 0$, one can use **Monte Carlo simulation** with importance sampling with weight factor $w(U) \propto dU e^{-S_g^{\rm lat}} \prod_q \det[D_q](U)$.

Lattice QCD

- Lattice QCD simulations in three steps.
  - Generate gauge configuration : make gauge configurations to follow probability density $P(U) \propto w(U)$.
  - Compute quark propagators : need inversion of the matrix with 4(spin) $\times$3(color) $\times V$(lattice vol) rows and columns.
    For $V = 48^3 \times 96$ hypercubic lattice, the dimension of matrix is over 100 millions.
  - Calculate correlation functions by contracting quark indices in quark propagators.
- Fermions in Lagrangian : sea quark $\rightarrow$ fermion determinant
- Fermions in the operator : valence quark $\rightarrow$ propagator.

Current status of h_{A_1}

- The $h_{A_1}(w=1)$ results from FNAL/MILC collaboration [Fermilab-MILC PRD89,114504 (2014)] has error within 2%.

$$h_{A_1}(w=1) = 0.906(4)(12) \tag{11}$$

Simulation details

  - Valence b and c quark : Fermilab action
  - Valence light quark : AsqTad improved action
  - Sea light quark : $N_f = 2+1$ AsqTad improved action

- The dominant systematic error is from the discretization of the charm quark.

uncertainty	$h_{A_1}(w=1)$
statistics	0.4 %
matching	0.4 %
χPT	0.5 %
discretization	1.0 %
...	...
total	1.4 %

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  Simulation details
  - Valence b and c quark : Fermilab action → Oktay Kronfeld action
  - Valence light quark : AsqTad improved action → HISQ action
  - Sea light quark : $N_f = 2+1$ AsqTad improved action → $N_f = 2+1+1$ HISQ
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Fermilab action

- The Fermilab action [El-Khadra, Kronfeld, and Mackenzie, PRD55, 3933 (1997)]

$$S_{\text{Fermilab}} \equiv S_0 + S_E + S_B, \tag{12}$$

$$S_0 \equiv a^4 \sum_x \bar{\psi}(x)\left[m_0 + \gamma_4 D_{\text{lat},4} - \frac{a}{2}\Delta_4 + \zeta\left(\boldsymbol{\gamma}\cdot\boldsymbol{D}_{\text{lat}} - \frac{r_s a}{2}\Delta^{(3)}\right)\right]\psi(x) \tag{13}$$

$$S_E \equiv -\frac{1}{2}c_E\zeta a^5 \sum_x \bar{\psi}(x)\boldsymbol{\alpha}\cdot\boldsymbol{E}_{\text{lat}}\psi(x), \quad S_B \equiv -\frac{1}{2}c_B\zeta a^5 \sum_x \bar{\psi}(x)i\boldsymbol{\Sigma}\cdot\boldsymbol{B}_{\text{lat}}\psi(x),$$

$\Delta^{(3)}, \Delta_4$: discretized 2nd order covariant derivative $\boldsymbol{D}^2$, D_4^2.

- For b and c quark, $1/a \sim m_Q \gg \Lambda_{\text{QCD}}$ for typical lattice spacing ($0.04\text{fm} < a < 0.12\text{fm}$) in simulations. Describing the lattice action by the heavy-quark effective theory,

$$\mathcal{L}_{\text{Fermilab}} \doteq \bar{h}(D_4 + m_1)h + \frac{1}{2m_2}\bar{h}\boldsymbol{D}^2 h + \frac{Z_B}{2m_B}\bar{h}i\boldsymbol{\sigma}\cdot\boldsymbol{B}h + \cdots \tag{14}$$

- The discretization error can be estimated by using power counting parameter $\lambda \simeq a\Lambda_{\text{QCD}} \simeq \Lambda_{\text{QCD}}/(2m_q)$ and α_s.

Oktay-Kronfeld action

Extend the lattice action from $\mathcal{O}(\lambda)$ (Fermilab action) $\rightarrow \mathcal{O}(\lambda^3)$ (OK action).
[Oktay and Kronfeld, PRD78, 014504 (2008)]

- $S_{\text{OK}} \equiv S_0 + S_B + S_E + S_6 + S_7$.

$$\begin{aligned} S_6 \equiv\ & c_1 a^6 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_{\text{lat},i} \Delta_i \psi(x) + c_2 a^6 \sum_x \bar{\psi}(x) \{\boldsymbol{\gamma} \cdot \boldsymbol{D}_{\text{lat}}, \Delta^{(3)}\} \psi(x) \\ & + c_3 a^6 \sum_x \bar{\psi}(x) \{\boldsymbol{\gamma} \cdot \boldsymbol{D}_{\text{lat}}, i\boldsymbol{\Sigma} \cdot \boldsymbol{B}_{\text{lat}}\} \psi(x) \\ & + c_{EE} a^6 \sum_x \bar{\psi}(x) \{\gamma_4 D_{\text{lat},4}, \boldsymbol{\alpha} \cdot \boldsymbol{E}_{\text{lat}}\} \psi(x), \end{aligned} \tag{15}$$

$$S_7 \equiv a^7 \sum_x \bar{\psi}(x) \sum_i \left[c_4 \Delta_i^2 \psi(x) + c_5 \sum_{j \neq i} \{ i\Sigma_i B_{\text{lat},i}, \Delta_j \} \right] \psi(x). \tag{16}$$

- Coefficients c_i are fixed by matching dispersion relation, interaction with background field, and Compton scattering amplitude of on-shell quark through the tree level.

Current improvement

- In the simulation with the Fermilab action, $\mathcal{O}(\lambda)$ improvement for current was enough. Improved currents can be constructed by improved quark fields [El-Khadra, Kronfeld, and Mackenzie, PRD55, 3933]

$$V_\mu = \bar{\Psi}_c \gamma_\mu \Psi_b, \qquad A_\mu = \bar{\Psi}_c \gamma_\mu \gamma_5 \Psi_b, \tag{17}$$

  and the improved quark field is defined by,

$$\Psi_f = e^{m_{1fa} a/2}(1 + d_{1f} a\gamma \cdot \boldsymbol{D}_{\text{lat}})\psi_f, \quad f = b, c \tag{18}$$

  where the overall factor is due to field renormalization $Z_f = e^{-m_{1f} a/2}$. One can improve the current by adjusting the improvement parameter d_1.

- It is essential to improve the lattice flavor-changing current for A_μ and V_μ up to $\mathcal{O}(\lambda^3)$, in the same level as the Oktay-Kronfeld action.

Current improvement

- At the tree level, the relation between QCD current and the HQET current is given by Foldy-Wouthouysen-Tani transformation

$$\bar{c}\gamma_\mu b \doteq \bar{h}_c \Gamma h_b - \bar{h}_c \gamma_\mu \frac{\gamma \cdot \boldsymbol{D}}{2m_b} h_b + \bar{h}_c \frac{\gamma \cdot \overleftarrow{\boldsymbol{D}}}{2m_c} \gamma_\mu h_b + \cdots \tag{19}$$

- Imitating FWT transformation through $\mathcal{O}(1/m_q^3)$, we introduced improved quark field [Jaehoon Leem, arXiv:1711.01777],

$$\begin{aligned}\Psi(x) =& e^{m_1 a/2}\Big[1 + d_1 a\gamma \cdot \boldsymbol{D}_{\text{lat}} + \frac{1}{2} d_2 a^2 \Delta^{(3)} + \frac{1}{2} i d_B a^2 \boldsymbol{\Sigma} \cdot \boldsymbol{B}_{\text{lat}} + \frac{1}{2} d_E a^2 \boldsymbol{\alpha} \cdot \boldsymbol{E}_{\text{lat}} \\ &+ d_{EE} a^3 \{\gamma_4 D_{4\text{lat}}, \boldsymbol{\alpha} \cdot \boldsymbol{E}_{\text{lat}}\} + d_{r_E} a^3 \{\gamma \cdot \boldsymbol{D}_{\text{lat}}, \boldsymbol{\alpha} \cdot \boldsymbol{E}_{\text{lat}}\} \\ &+ \frac{1}{6} d_3 a^3 \gamma_i D_{\text{lat}i} \Delta_i + \frac{1}{2} d_4 a^3 \{\gamma \cdot \boldsymbol{D}_{\text{lat}}, \Delta^{(3)}\} + d_5 a^3 \{\gamma \cdot \boldsymbol{D}_{\text{lat}}, i\boldsymbol{\Sigma} \cdot \boldsymbol{B}_{\text{lat}}\} \\ &+ d_6 a^3 [\gamma_4 D_{4\text{lat}}, \Delta^{(3)}] + d_7 a^3 [\gamma_4 D_{4\text{lat}}, i\boldsymbol{\Sigma} \cdot \boldsymbol{B}_{\text{lat}}]\Big]\psi(x).\end{aligned} \tag{20}$$

- The term with d_3 is necessary to remedy rotational symmetry breaking of lattice quark.

Current improvement

- We calculate four-quark matrix elements for matching.

$$\langle q(\eta_2, \boldsymbol{p}_2) c(\eta_c, \boldsymbol{p}_c) | \bar{\Psi}_c \Gamma \Psi_b | b(\eta_b, \boldsymbol{p}_b) q(\eta_1, \boldsymbol{p}_1) \rangle_{\mathrm{lat}}, \tag{21}$$

  where q represents a light spectator quark.
- The matching condition implies [Jaehoon Leem, arXiv:1711.01777],

$$\begin{aligned}\Psi \doteq \Big[1 &- \frac{1}{2m_3}\gamma \cdot \boldsymbol{D} + \frac{1}{8m_{D_\perp^2}^2}\boldsymbol{D}^2 + \frac{i}{8m_{sB}^2}\boldsymbol{\Sigma}\cdot\boldsymbol{B} + \frac{1}{4m_{\alpha_E}^2}\boldsymbol{\alpha}\cdot\boldsymbol{E} \\ &- \frac{\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\}}{8m_{\alpha_{EE}}^3} - \frac{3\{\gamma\cdot\boldsymbol{D}, \boldsymbol{D}^2\}}{32m_{\gamma DD_\perp^2}^3} - \frac{3\{\gamma\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\}}{32m_5^3} - \frac{\{\gamma\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\}}{16m_{\alpha_{rE}}^3} \\ &+ \frac{[\gamma_4 D_4, \boldsymbol{D}^2]}{16m_6^3} + \frac{[\gamma_4 D_4, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}]}{16m_7^3} + w_1 \sum_i \gamma_i D_i^3 + w_2[\gamma\cdot\boldsymbol{D}, \boldsymbol{D}^2] \Big] h + \cdots, \end{aligned} \tag{22}$$

- The coefficients m_i and w_i : analytic functions of $(m_0 a, r_s, \zeta, c_i)$ and d_i.
- Determine d_i by the conditions that nonphysical terms vanish ($w_i \to 0$), and all the mass-like terms be identical to the lattice kinetic mass m_2.
- The kinetic mass $m_2(m_0 a)$ is tuned by tuning the bare mass $m_0 a$ numerically.

Numerical Simulation

- Sea quarks : HISQ action with $N_f = 2 + 1 + 1$ flavors. [MILC collab., PRD87, 054505 (2013)]

ID	a(fm)	Volume	am_l	am_s	am_c	$N_{conf} \times N_{src}$
a12m310	0.12	$24^3 \times 64$	0.0102	0.0509	0.635	1053 $\times 3$
a12m220	0.12	$32^3 \times 64$	0.00507	0.0507	0.628	
a12m130	0.12	$48^3 \times 64$	0.00184	0.0507	0.628	
a09m310	0.09	$32^3 \times 96$	0.0074	0.037	0.440	1001 $\times 3$
a09m220	0.09	$48^3 \times 96$	0.00363	0.0363	0.430	
a09m130	0.09	$64^3 \times 96$	0.0012	0.0363	0.432	

- Valence quark for (u, d, s) : HISQ action
- Valence quark for b and c : Oktay-Kronfeld action
- Tune the Oktay-Kronfeld action : tune the bare masses of b and c numerically by calculating meson propagators and analyzing the energy spectrum.
- Calculate 3-point Green's functions and determine hadronic matrix elements with the tuned action and the improved current.

Numerical Simulation

- We calculate following 3-point Green's functions

$$C_{A_j}^{B\to D^*}(t,\tau) \equiv \sum_{\boldsymbol{x},\boldsymbol{y}} \langle O_{D^*}(0) A_j^{cb}(\boldsymbol{y},t) O_B^\dagger(\boldsymbol{x},\tau)\rangle,$$
$$C_{A_j}^{D^*\to B}(t,\tau) \equiv \sum_{\boldsymbol{x},\boldsymbol{y}} \langle O_B(0) A_j^{bc}(\boldsymbol{y},t) O_{D^*}^\dagger(\boldsymbol{x},\tau)\rangle,$$
$$C_{V_4}^{B\to B}(t,\tau) \equiv \sum_{\boldsymbol{x},\boldsymbol{y}} \langle O_B(0) V_4^{bb}(\boldsymbol{y},t) O_B^\dagger(\boldsymbol{x},\tau)\rangle,$$
$$C_{V_4}^{D^*\to D^*}(t,\tau) \equiv \sum_{\boldsymbol{x},\boldsymbol{y}} \langle O_{D^*}(0) V_4^{cc}(\boldsymbol{y},t) O_{D^*}^\dagger(\boldsymbol{x},\tau)\rangle, \quad (23)$$

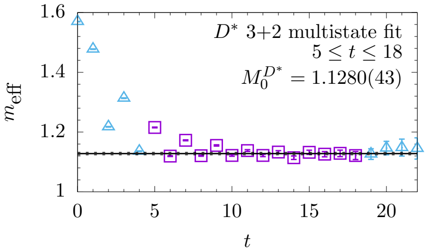
  where O_B and O_{D^*} are meson interpolating operators with a HISQ light quark and an Oktay-Kronfeld heavy quark.
- The heavy-heavy currents are given by improved quark field

$$A_j^{cb} \equiv \bar{\Psi}_c \gamma_5 \gamma_j \Psi_b,$$
$$V_4^{bb} \equiv \bar{\Psi}_b \gamma_4 \Psi_b, \quad (24)$$

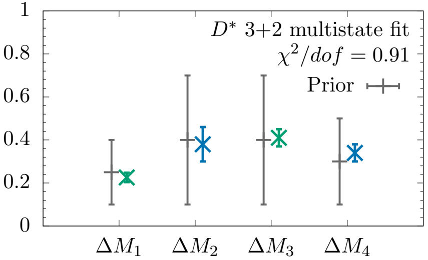
Control excited-state contamination

- Generate zero momentum meson propagators and do the multi-states fitting [Sungwoo Park, et al., Lattice 2018]

$$C^{2\text{pt}}_{B(\text{or } D^*)}(t,\mathbf{0}) = |\mathcal{A}_0|^2 e^{-M_0 t}\left(1 + \left|\frac{\mathcal{A}_2}{\mathcal{A}_0}\right|^2 e^{-\Delta M_2 t} + \left|\frac{\mathcal{A}_4}{\mathcal{A}_0}\right|^2 e^{-(\Delta M_2+\Delta M_4)t} + \cdots \right.$$
$$\left. - (-1)^t\left|\frac{\mathcal{A}_1}{\mathcal{A}_0}\right|^2 e^{-\Delta M_1 t} - (-1)^t\left|\frac{\mathcal{A}_3}{\mathcal{A}_0}\right|^2 e^{-(\Delta M_1+\Delta M_3)t} + \cdots\right) + (t \leftrightarrow T - t)$$



(a) $m_{\text{eff}}(t) \equiv \frac{1}{2}\ln|C^{2pt}(t)/C^{2pt}(t+2)|$: plot using the 3+2-state fit on the a12m310 ensemble for the D^*-meson with $m_x/m_s = 0.2$. The horizontal line is the ground state mass.



(b) The excited state masses from the 3+2-states fit using empirical Bayesian priors. (PNDME collab., PRD95, 074508 (2017))

Control excited-state contamination

- Fit 3-point function including 2+1 states for $|B_m\rangle$ and $|D_n^*\rangle$ with $n, m = 0, 1, 2$. [Sungwoo Park, et al., Lattice 2018]

$$
\begin{aligned}
C_{A_j}^{B\to D^*}(t_s,\tau) = {}& \mathcal{A}_0^{D^*}\mathcal{A}_0^{B}\langle D_0^*|A_j^{cb}|B_0\rangle e^{-M_{B_0}(\tau-t_s)}e^{-M_{D_0^*}t_s} \\
&- \mathcal{A}_0^{D^*}\mathcal{A}_1^{B}\langle D_0^*|A_j^{cb}|B_1\rangle(-1)^{(\tau-t_s)}e^{-M_{B_1}(\tau-t_s)}e^{-M_{D_0^*}t_s} \\
&- \mathcal{A}_1^{D^*}\mathcal{A}_0^{B}\langle D_1^*|A_j^{cb}|B_0\rangle(-1)^{t_s}e^{-M_{B_0}(\tau-t_s)}e^{-M_{D_1^*}t_s} \\
&+ \mathcal{A}_1^{D^*}\mathcal{A}_1^{B}\langle D_1^*|A_j^{cb}|B_1\rangle(-1)^{\tau}e^{-M_{B_1}(\tau-t_s)}e^{-M_{D_1^*}t_s} \\
&+ \mathcal{A}_2^{D^*}\mathcal{A}_0^{B}\langle D_2^*|A_j^{cb}|B_0\rangle e^{-M_{B_0}(\tau-t_s)}e^{-M_{D_2^*}t_s} \\
&+ \mathcal{A}_0^{D^*}\mathcal{A}_2^{B}\langle D_0^*|A_j^{cb}|B_2\rangle e^{-M_{B_2}(\tau-t_s)}e^{-M_{D_0^*}t_s} \\
&- \mathcal{A}_2^{D^*}\mathcal{A}_1^{B}\langle D_2^*|A_j^{cb}|B_1\rangle(-1)^{\tau-t_s}e^{-M_{B_1}(\tau-t_s)}e^{-M_{D_2^*}t_s} \\
&- \mathcal{A}_1^{D^*}\mathcal{A}_2^{B}\langle D_1^*|A_j^{cb}|B_2\rangle(-1)^{t}e^{-M_{B_2}(\tau-t_s)}e^{-M_{D_1^*}t_s} \\
&+ \mathcal{A}_0^{D^*}\mathcal{A}_2^{B}\langle D_0^*|A_j^{cb}|B_2\rangle e^{-M_{B_2}(\tau-t_s)}e^{-M_{D_0^*}t_s} + \cdots
\end{aligned}
\tag{25}
$$

- In the fitting, the 2pt amplitudes $\mathcal{A}$ and masses M are constant fixed from the 2-point function analysis.

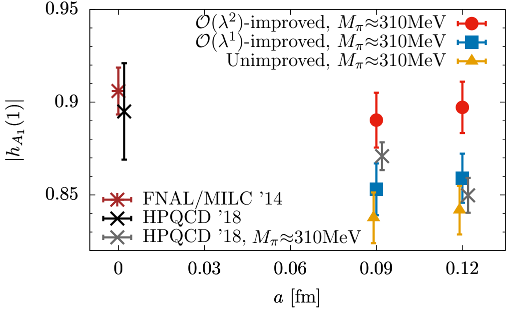
Preliminary results for $h_{A_1}(1)$

- The zero recoil form factor $h_{A_1}(1)$ can be determined by

$$\frac{\langle D^*|A^j_{cb}|\bar{B}\rangle\langle\bar{B}|A^j_{bc}|D^*\rangle}{\langle D^*|V^4_{cc}|D^*\rangle\langle\bar{B}|V^4_{bb}|\bar{B}\rangle} = \left|\frac{h_{A_1}(1)}{\rho_{A_j}}\right|^2 \tag{26}$$

where $\rho^2_{A_j} = \frac{Z_{A^j_{cb}} Z_{A^j_{bc}}}{Z^4_{cc} Z^4_{bb}} = 1 + \mathcal{O}(\alpha_s)$ is very close to 1.

- Setting $\rho^2_{A_j} = 1$ [Sungwoo Park, et al., Lattice 2018]



[Fermilab-MILC collab., PRD 89, 114504 (2014)] [HPQCD collab., PRD 97, 054502 (2018)]

Summary

- We are calculating $\bar{B} \to D^* \ell \bar{\nu}$ semileptonic decay form factor at zero recoil using the Oktay-Kronfeld action for b and c quarks.
- We improved the lattice current through $\mathcal{O}(\lambda^3)$ in the same level as the Oktay-Kronfeld action. (Analysis with the $\mathcal{O}(\lambda^3)$ terms in the current are under progress.)
- We controlled the excited-state contamination by multi-state fitting.
- The preliminary results with the improved current through $\mathcal{O}(\lambda^2)$ is consistent with the results from FNAL/MILC and HPQCD.
- Our next targets are
  - Continuum-chiral extrapolation.
  - Calculate ρ_{A_j}.
  - Calculate form factors of $\bar{B} \to D^* \ell \bar{\nu}$ and $\bar{B} \to D \ell \bar{\nu}$ at non-zero recoil.
  - Calculate leptonic decay constants f_D, f_{D_s}, f_B, and f_{B_s}.