An lowerbound on the bounce action.

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Lifetime of metastable vacuum.

> Lifetime of metastable

$$\Gamma \propto \exp(-S_E[\phi])$$

[Coleman (1977), Callan Coleman (1977)]

> Bounce solution

$$\nabla^2 \phi_i - \frac{\partial V}{\partial \phi_i} = 0, \qquad S_E[\phi] = \int d^4x \left[\frac{1}{2}(\nabla\phi)^2 + V(\phi)\right]$$

In general, it is tough to solve EOM explicitly

Lowerbound on the bounce action.

[Sato, Takimoto (2017)]

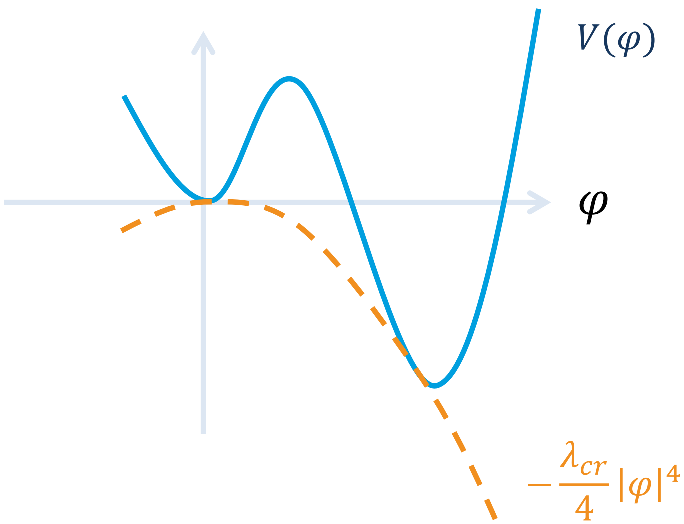
> Setup

Bounce sol. : $\nabla^2 \phi_i - \frac{\partial V}{\partial \phi_i} = 0, \qquad \lim_{|x| \to \infty} \phi_i(x) = 0$ (false vacuum)

Bounce action : $S_E[\phi] = \int d^4x \left[\frac{1}{2} (\nabla \phi)^2 + V(\phi) \right]$

> Result

$$S_E[\phi] \geq \frac{24}{\lambda_{cr}} \qquad \lambda_{cr} \equiv \max_{\varphi} \left[-\frac{4V(\varphi)}{|\varphi|^4} \right]$$



Derivation (1/3) : preparation.

Let us assume the existence of the bounce sol.

Bounce = sol. of EOM $\rightarrow$ $\delta S = 0$ for any perturbation $\delta\phi$

$$\phi_\lambda(x) \equiv \phi(\lambda x) \qquad S[\phi_\lambda] = \lambda^2 T + \lambda^4 V \qquad \begin{cases} T = \int d^4x \frac{1}{2}(\nabla\phi)^2 \\ V = \int d^4x\, V(\phi) \end{cases}$$

$$\left.\frac{dS[\phi_\lambda]}{d\lambda}\right|_{\lambda=1} = 0 \quad \Rightarrow \quad 2T = -4V, \qquad \boxed{S = \frac{1}{2}T}$$

Bounce sol. is spherical. [Callan Glaser Martin (1977)] [Blum Honda Sato Takimoto Tobioka (2016)]

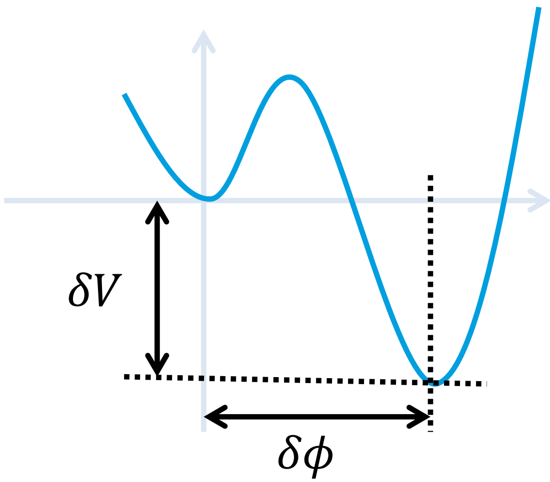
$$\boxed{S = \frac{1}{2}T = \frac{1}{2}\int_0^\infty dr\, \pi^2 r^3 \left(\frac{d\phi}{dr}\right)^2}$$

Derivation (2/3) : Lagrange multiplier.

$$S = \frac{1}{2}T = \int_0^\infty dr\, \pi^2 r^3 \left(\frac{d\phi}{dr}\right)^2$$

$$\delta\phi = \phi(0) - \phi(\infty) = -\int_0^\infty dr \frac{d\phi}{dr}$$

$$\delta V = V(\infty) - V(0) = \int_0^\infty dr \frac{3}{r}\left(\frac{d\phi}{dr}\right)^2$$



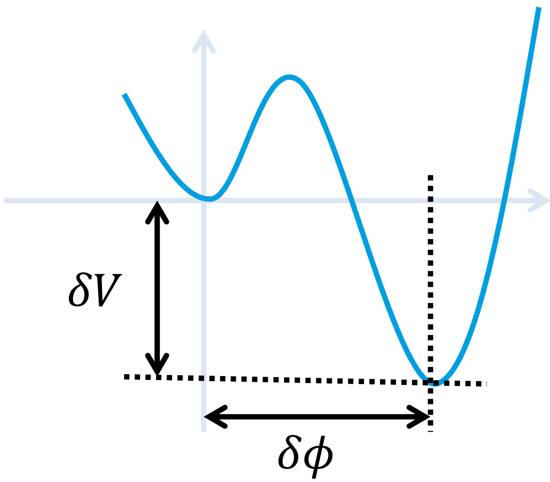
Let us fix $\delta\phi$ and $\delta V (> 0)$ by hand for the moment

$\rightarrow T$ can be minimized by Lagrange multiplier method.

$$\tilde{T} = T + 2\alpha\left(\delta\phi + \int_0^\infty dr \frac{d\phi}{dr}\right) - \beta\left(\delta V - \int_0^\infty dr \frac{3}{r}\left(\frac{d\phi}{dr}\right)^2\right) \Rightarrow T_{min}[\delta\phi, \delta V] = \frac{12(\delta\phi)^4}{\delta V}$$

Derivation (3/3) : bound on S.

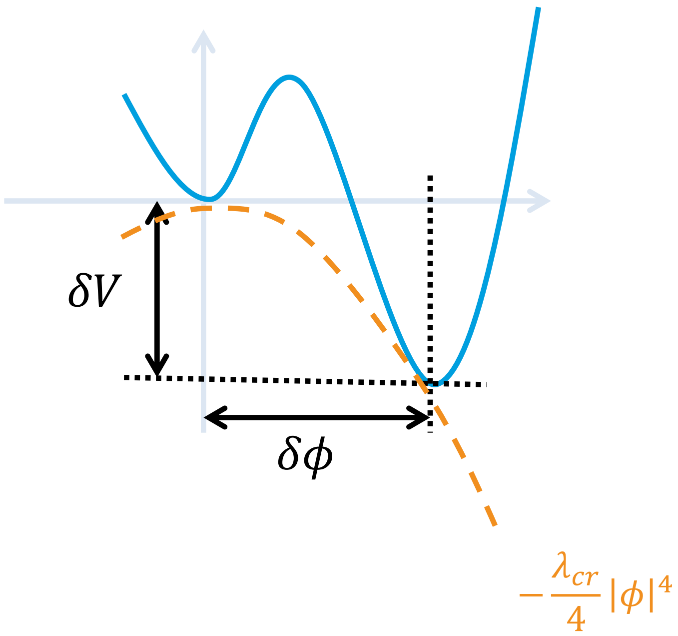
$$S = \frac{T}{2} \geq \frac{T_{min}[\delta\phi, \delta V]}{2} = \frac{6(\delta\phi)^4}{\delta V}$$



We do not know $\delta\phi$ and δV

Derivation (3/3) : bound on S.

$$S = \frac{T}{2} \geq \min_{\phi} \frac{T_{min}[\delta\phi, \delta V]}{2} = \min_{\phi} \frac{6(\delta\phi)^4}{\delta V}$$



We do not know $\delta\phi$ and δV

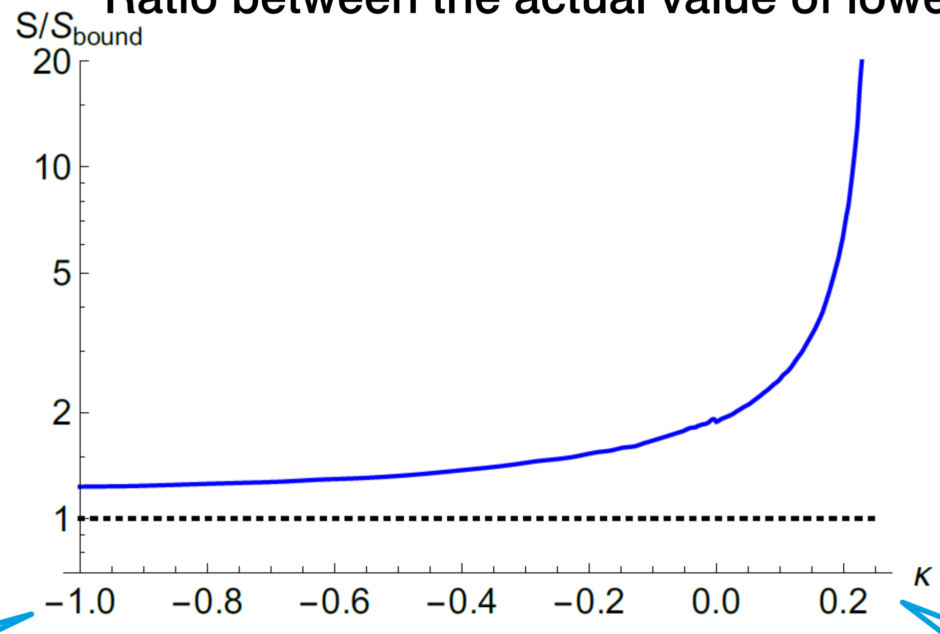
The minimum of lowerbound is smaller than S

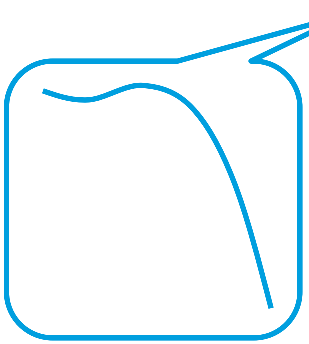
$$S \geq \frac{24}{\lambda_{cr}}, \qquad \lambda_{cr} \equiv \max_{\phi}\left[-\frac{4V(\phi)}{|\phi|^4}\right]$$

Example 1 : single scalar field.

$$V = \frac{1}{2}M^2\phi^2 - \frac{1}{3}A\phi^3 + \frac{1}{4}\lambda_4\phi^4, \qquad \kappa \equiv \frac{9\lambda_4 M^2}{8A^2}$$

Ratio between the actual value of lowerbound

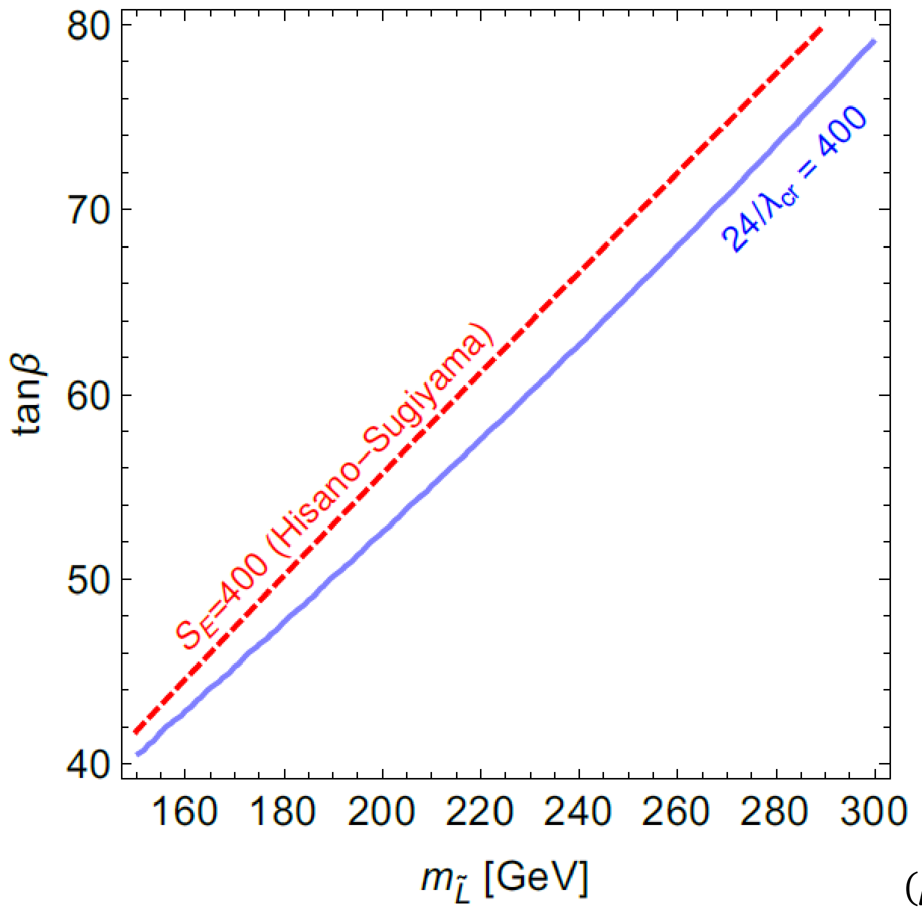




~ negative quartic
Fubini instanton like

~ Double-well
Thin-wall

Example 2 : MSSM with large tanbeta.



$$L \ni \frac{m_\tau}{v}\mu\tan\beta\ h\tilde{\tau}_L\tilde{\tau}_R$$

$(\mu = 700\ \text{GeV},\ m_{\tau R} = m_L + 200\ \text{GeV})$

Red line is given by Hisano-Sugiyama (2011)

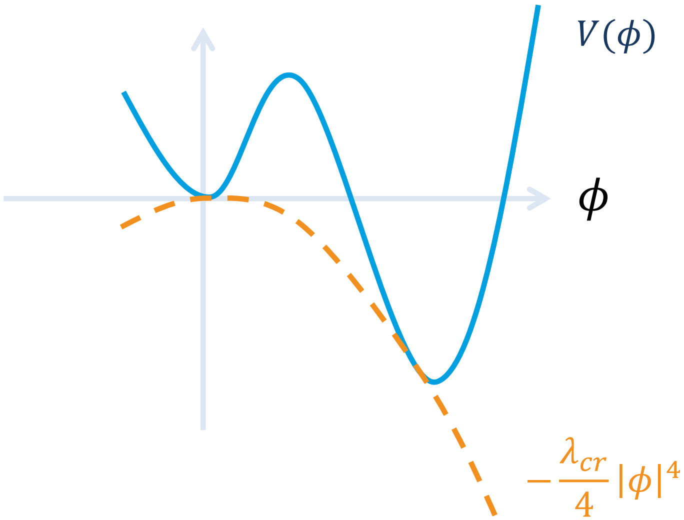
Summary.

> **In general, it is technically tough to calculate lifetime of metastable vacuum.**

> **We derived a lowerbound without solving EOM**

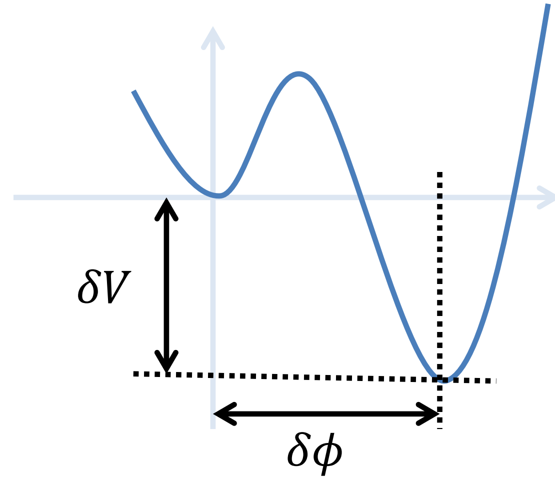
> **It is useful to analyze multiscalar models.**

$$S_E[\phi] \geq \frac{24}{\lambda_{cr}} \qquad \lambda_{cr} \equiv \max_{\phi}\left[-\frac{4V(\phi)}{|\phi|^4}\right]$$



Backup

Lagrange multiplier



$$T = \int_0^\infty dr\, \pi^2 r^3 \left(\frac{d\phi}{dr}\right)^2$$

Let us determine lowerbound of T for fixed $\delta\phi$ and δU

$$\delta\phi = \phi(0) - \phi(\infty) = -\int_0^\infty dr \frac{d\phi}{dr}, \qquad \delta V = V(\infty) - V(0) = \int_0^\infty dr \frac{3}{r}\left(\frac{d\phi}{dr}\right)^2$$

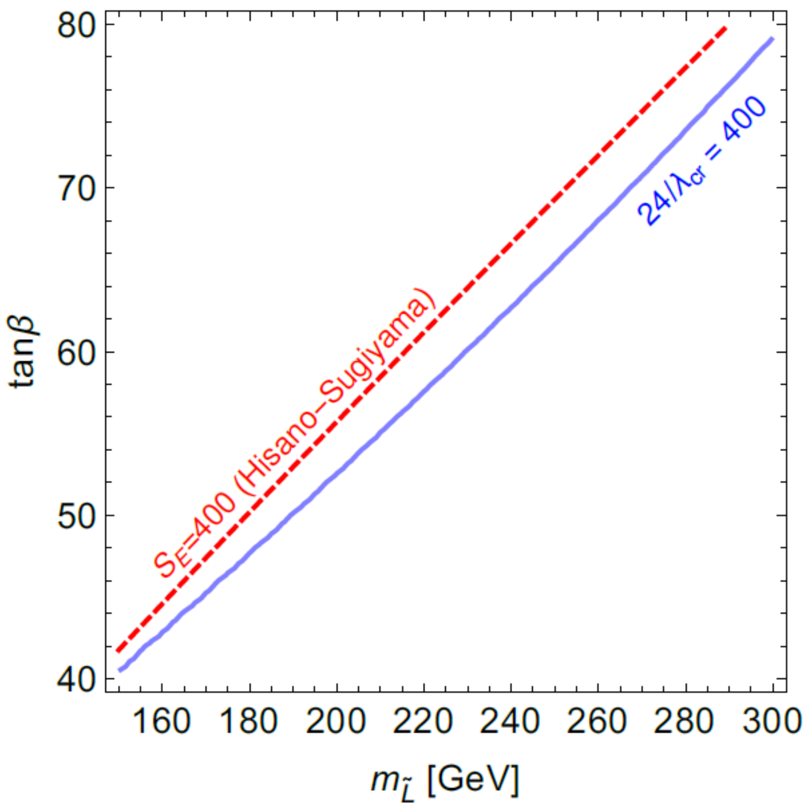
Minimize $\tilde{T}$ (Lagrange multiplier method)

$$\tilde{T} = T + 2\alpha\left(\delta\phi + \int_0^\infty dr \frac{d\phi}{dr}\right) - \beta\left(\delta V - \int_0^\infty dr \frac{3}{r}\left(\frac{d\phi}{dr}\right)^2\right)$$

極値の条件は、 $\frac{\delta\tilde{T}}{\delta\dot{\phi}} = 0, \quad \frac{\partial\tilde{T}}{\partial\alpha} = \frac{\partial\tilde{T}}{\partial\beta} = 0 \;\Rightarrow\; \dot{\phi} = -\frac{\alpha r}{\pi^2 r^4 + 3\beta}, \quad \alpha = \frac{24(\delta\phi)^3}{\delta V}, \quad \beta = \frac{12(\delta\phi)^4}{(\delta V)^2}$

さらに、 $\tilde{T}\left[\alpha = \frac{24(\delta\phi)^3}{\delta V}, \beta = \frac{12(\delta\phi)^4}{(\delta V)^2}\right] = \frac{12(\delta\phi)^4}{\delta U} + \int_0^\infty dr \left(\frac{\pi^2 r^4 + 3\beta}{r}\right)\left(\dot{\phi} + \frac{\alpha r}{\pi^2 r^4 + 3\beta}\right)^2$

For fixed $\delta\phi$ and δV, $T \geq \frac{12(\delta\phi)^4}{\delta U}$



$$
\begin{aligned}
V =&(m_{H_u}^2 + \mu^2)|H_u|^2 + m_{\tilde{L}}^2|\tilde{L}|^2 + m_{\tilde{\tau}_R}^2|\tilde{\tau}_R|^2 \\
&+ \frac{g_2^2}{8}(|\tilde{L}|^2 + |H_u|^2)^2 + \frac{g_Y^2}{8}(|\tilde{L}|^2 - 2|\tilde{\tau}_R|^2 - |H_u|^2)^2 \\
&+ \frac{g_2^2 + g_Y^2}{8}\delta_H|H_u|^4 + y_\tau^2|\tilde{L}\tilde{\tau}_R|^2 \\
&- (y_\tau \mu H_u^* \tilde{L}\tilde{\tau}_R + h.c.).
\end{aligned}
$$