To learn about $\overset{\text{lepton}}{\mu \leftrightarrow e}$ *flavour change with EFT*

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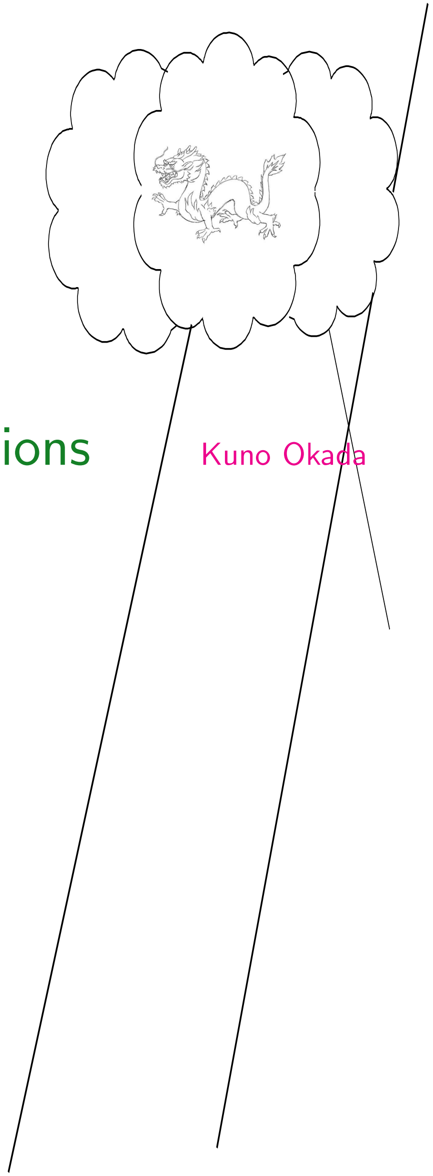
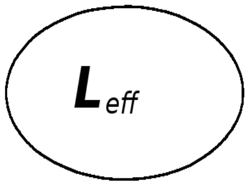


1. LFV and all that
2. EFT= parametrising LFV at low energy with contact interactions
3. EFT = path up the mountain ⇔ travels in "scale"
4. results+ questions for you

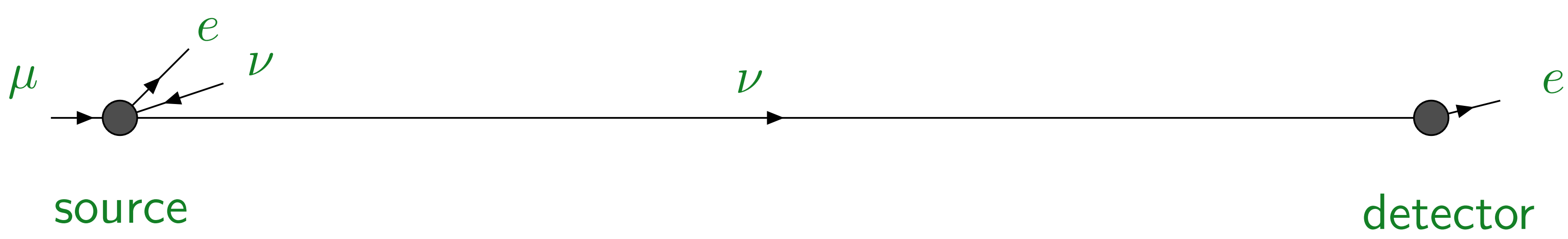


What is Lepton Flavour Violation?

- three lepton flavours in the Standard Model : $\boldsymbol{e}, \boldsymbol{\mu}, \tau$

(flavour $\equiv$ mass eigenstate) $\quad$ *different* from quarks, where 6 flavours

- LFV $\equiv$ charged lepton flavour change, at a point $= \nu$ oscillations don't count.



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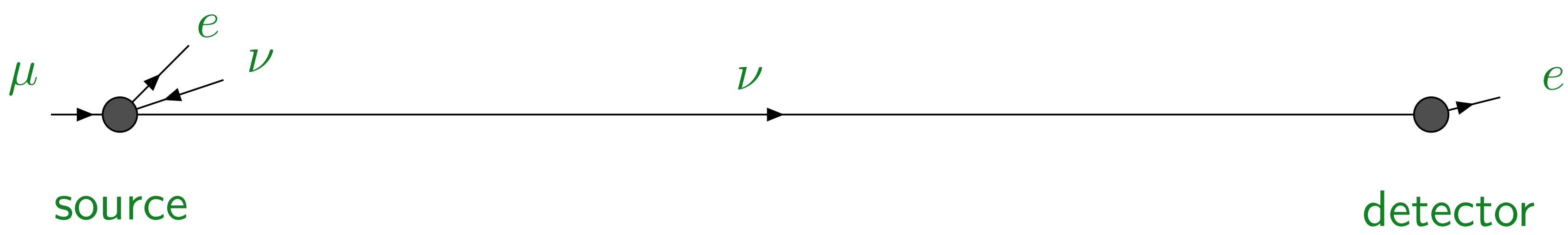


- Lepton Flavour Change is interesting:

— none in the Standard Model with $m_\nu = 0$

— occurs with m_ν and mixing matrix U

m_ν renormalisable Dirac: LFV amplitudes GIM-suppressed (like quarks)

$$\mathcal{A} \propto \frac{m_\nu^2}{m_W^2} \Rightarrow BR \lesssim 10^{-48}$$

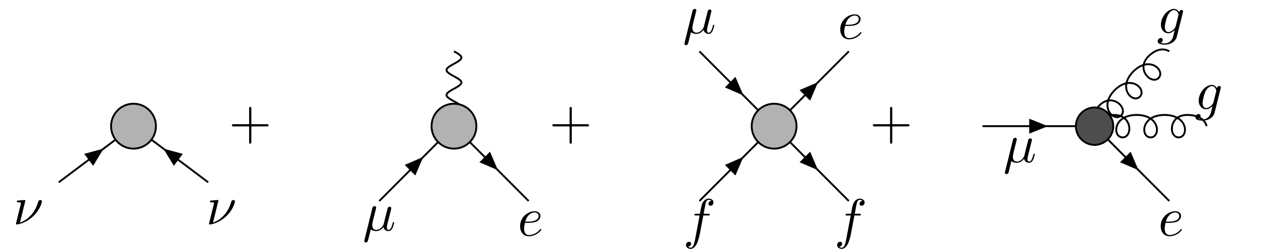
⇒ if see LFV, lepton flavour sector different from quarks!

suppose: heavy leptonic New Physics that can induce observable LFV

EFT as a parametrisation of LFV

parametrise LFV processes via contact interactions. *eg* at low E,
write down all LFV 2,3,4-point functions that respect QED and QCD:

$\sum_\zeta \sum_O \Big($



$+$ h.c.$\Big)$

$\sum_\zeta =$ sum over flavours of external legs
$\sum_O =$ sum over Lorentz structure of operators $= \{m_\nu, S, P, A, V, T\} \times$chirality .

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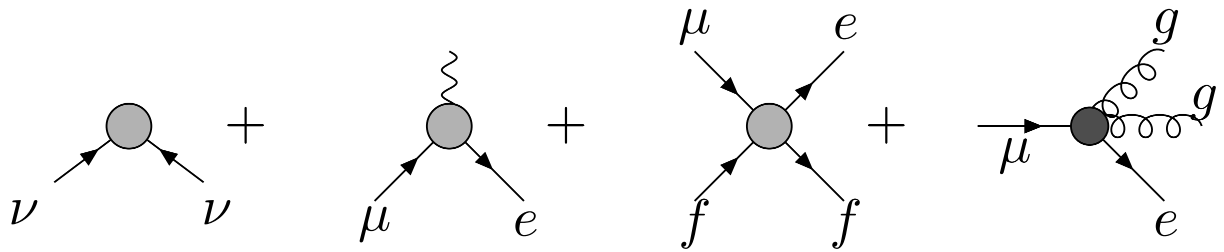

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suppose constant $\{C_O^\zeta\}$ (no form factors) $\Leftrightarrow$ New Particles are heavy

$$\delta\mathcal{L} = \sum_\zeta \sum_O \frac{C_O^\zeta}{v^n} O^\zeta + h.c. \qquad (v = 174 \text{ GeV})$$

$$\mathcal{O}_{V,LL}^{e\mu ee} = (\overline{e}\gamma^\alpha P_L \mu)(\overline{e}\gamma_\alpha P_L e)$$

$\Rightarrow$ theoretical parametrisation of the data= express LFV rates in terms of $\{C_O^\zeta\}$.

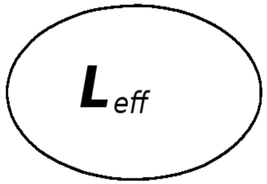
EFT as a tool to transport coefficients in scale

1. why is what dependent on which "scale"?

2. But loops are small: surely negligeable?
   ↗ data is sensitive to loop effects
   ↘ tree level is not always the best-constrained contribution

3. in practise — how does EFT allow to translate from data to models?
   ⇔ loop calculations in EFT

4. results: constraints and sensitivities +questions



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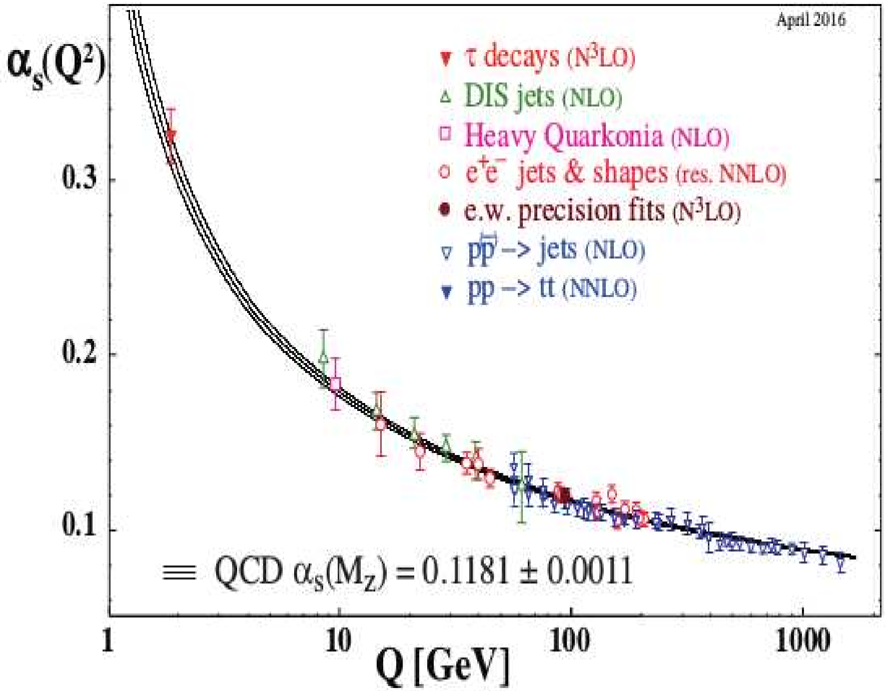

Figure 9.3: Summary of measurements of α_s as a function of the energy scale Q.

$\alpha_s(Q^2)$ larger at small Q^2
$Q^2 \equiv$ energy scale of the process
$\Leftrightarrow$ gluon loops are stickier at low energy

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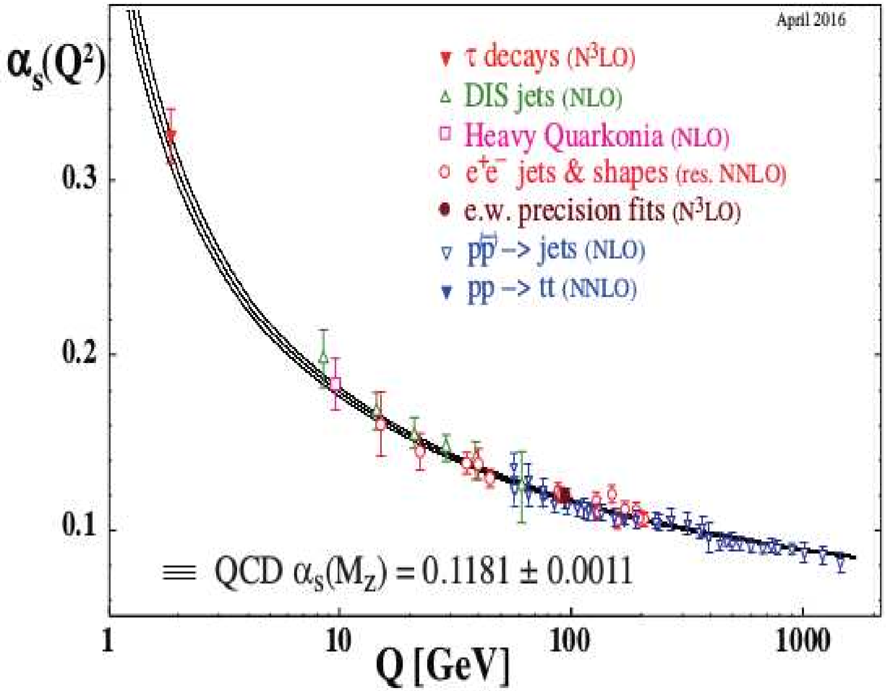

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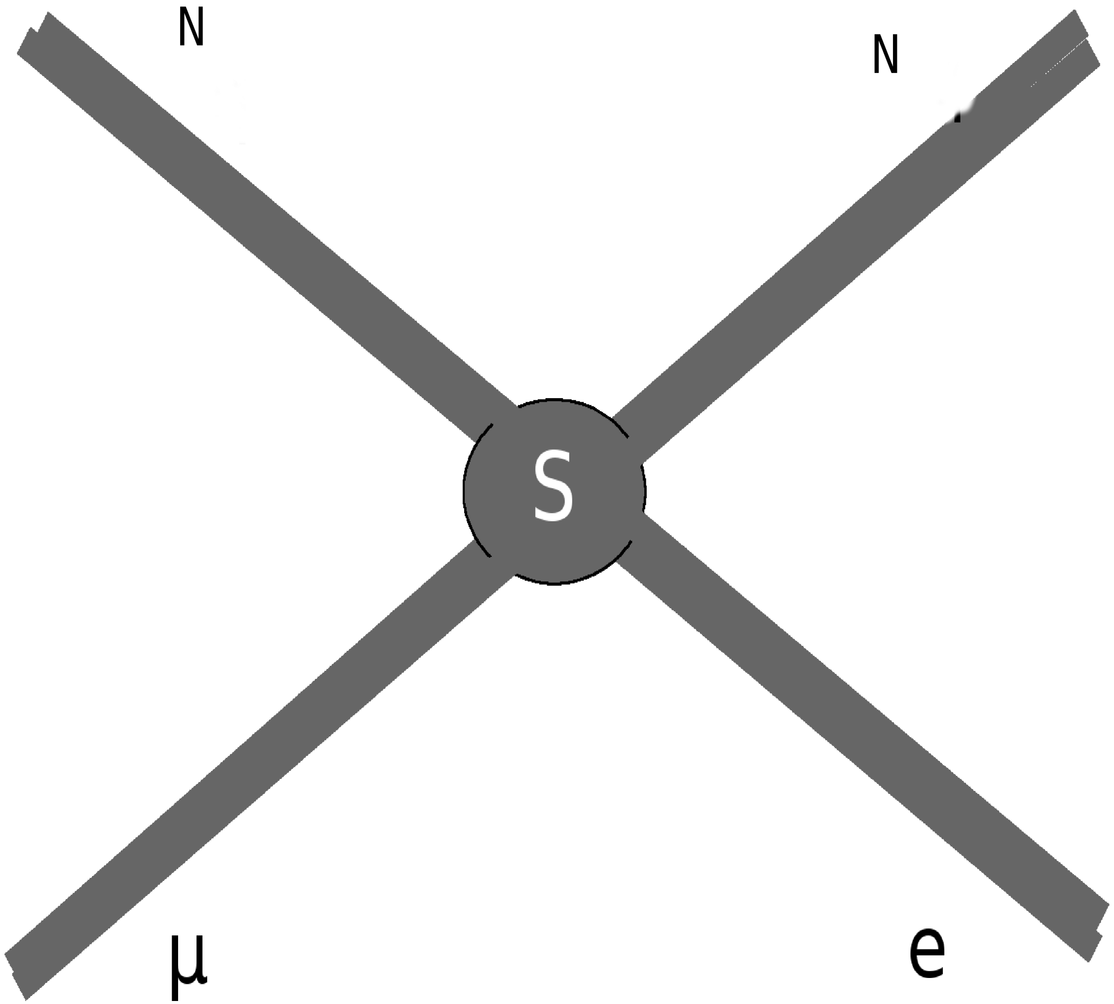
everything (as soon as do a loop calculation) :
masses , couplings, field/particle normalisation (eg:$m_b(m_h) \sim \frac{2}{3} m_b(m_b)$)

Also coefficients of LFV contact interactions can change with scale
$\Rightarrow$ climbing the mountain = evolve operator coefficients up in scale.

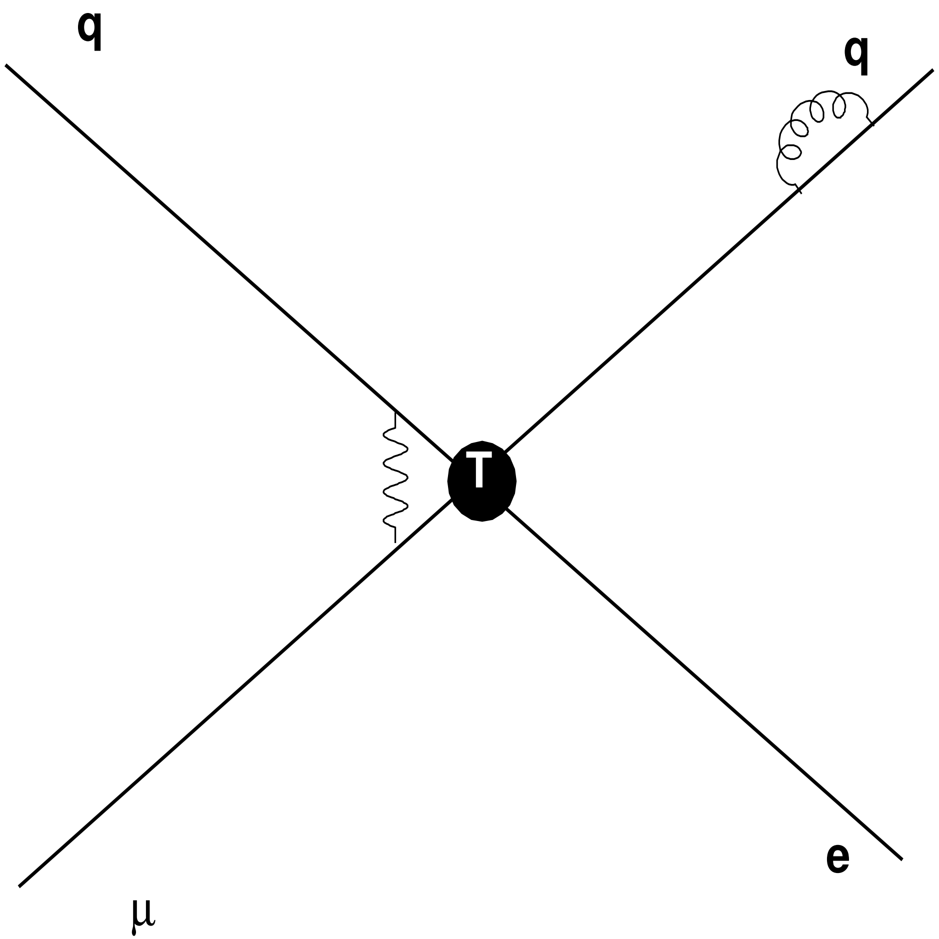
Peeling off the SM loop corrections

expt measures operator coefficient $\widetilde{C}(\mu_{exp})$, at exptal energy scale $\sim m_\mu \to m_\tau$, among external legs at same scale...



Peeling off SM loops

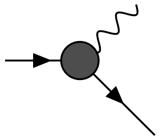
But if I look on shorter distance scale ($\sim 1/m_W$) I might see



loops can mix one interaction into another, not just rescale couplings

But surely loops effects in LFV are negligeable?

Two dipole operators contribute to $\mu \to e\gamma$:



$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R}\overline{\mu_R}\sigma^{\alpha\beta}e_L F_{\alpha\beta} + C_{D,L}\overline{\mu_L}\sigma^{\alpha\beta}e_R F_{\alpha\beta}\right)$$

$$BR(\mu \to e\gamma) = 384\pi^2(|C_{D,R}|^2 + |C_{D,L}|^2) < 5.7 \times 10^{-13}$$

$$\Rightarrow |C_X^D| \lesssim 10^{-8}$$

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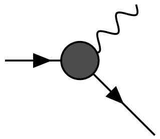

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How big does one expect $C_{D,X}$ to be? Suppose operator coefficient

			$n=1$	$n=2$
$\frac{m_\mu}{v^2} C_{D,X} \sim \frac{e m_\mu}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 100$ TeV	10 TeV
$\frac{m_\mu}{v^2} C_{D,X} \sim \frac{e v}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 3000$ TeV	300 TeV

$\Rightarrow \mu \to e$ expts probe multi-loop effects in NP theories with $\Lambda_{NP} \gg$ reach of LHC

Why are the LFV bounds so good?

Current $\mu \to e$ Branching Ratios $\lesssim 10^{-12}$. Normalised to *weak* muon decay

$$BR(\mu \to e\bar{e}e) \equiv \frac{\Gamma(\mu \to e\bar{e}e)}{\Gamma(\mu \to e\bar{\nu}\nu)} \quad , \quad \Gamma(\mu \to e\bar{\nu}\nu) = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{m_\mu^5}{1536\pi^3 v^4} \simeq \frac{1}{2 \times 10^{-6}\text{sec}}$$

...so if $\Gamma(\mu \to e\bar{e}e) \simeq \frac{m_\mu^5}{1536\pi^3\Lambda_{LFV}^4}$ then $BR \lesssim \begin{cases} 10^{-12} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 200 \text{ TeV} \\ 10^{-16} \Rightarrow \Lambda_{LFV} \sim 10^3 v \simeq 2000 \text{ TeV} \end{cases}$

Compare to $\frac{(g-2)_\mu}{2} \equiv a \simeq \alpha_{em}/\pi$ (measure Eqns o Motion: QED *amplitude*):

torque $\vec{\tau} = \vec{\mu} \times \vec{B}$; $\vec{\mu} = g\frac{e}{2m}\vec{S}$

$$\begin{aligned} \Delta a &\equiv a^{SM} - a^{exp} \simeq 3 \times 10^{-9} \\ &\sim \frac{m_\mu^2}{16\pi^2\Lambda_{NP}^2} \end{aligned}$$

$\Rightarrow \Lambda_{NP} \sim m_t$.

because BR is ratio to weak decays

But QED loops are $\mathcal{O}(\alpha/4\pi)$... surely negligeable correction to tree?

Work top-down = suppose a model that gives only tensor operator at m_W:

$$2\sqrt{2}G_F\ C_T(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$$

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1: forget RGEs Match to nucleons $N \in \{n,p\}$ as $\widetilde{C}_T^{NN} \simeq \langle N|\bar{u}\sigma u|N\rangle C_T^{uu} \lesssim \frac{3}{4}C_T^{uu}$

$$\Rightarrow \boldsymbol{BR(\mu A \to eA) \approx BR_{SD} \approx \frac{1}{2}|C_T|^2}$$

nuclear matrix elements: EngelRTO, KlosMGS

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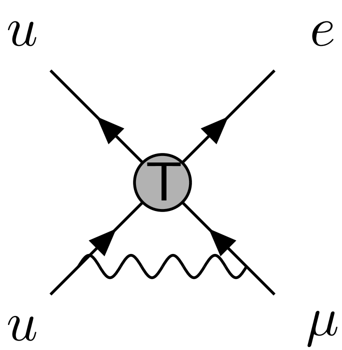
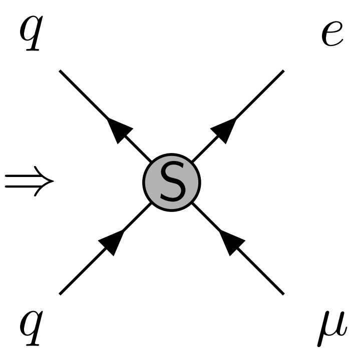
$C_T^{uu}(\overline{u}\sigma u)(\overline{e}\sigma P_Y \mu)$



$+ \ldots \quad \Rightarrow$



$$64\frac{\alpha_e}{4\pi}\log\frac{m_W}{m_\tau}\ C_T^{uu}\ (\overline{u}u)(\overline{e}P_Y\mu)$$

$$\Delta C_S^{uu}(m_\tau) \sim \frac{1}{7}C_T^{uu}(m_W)$$

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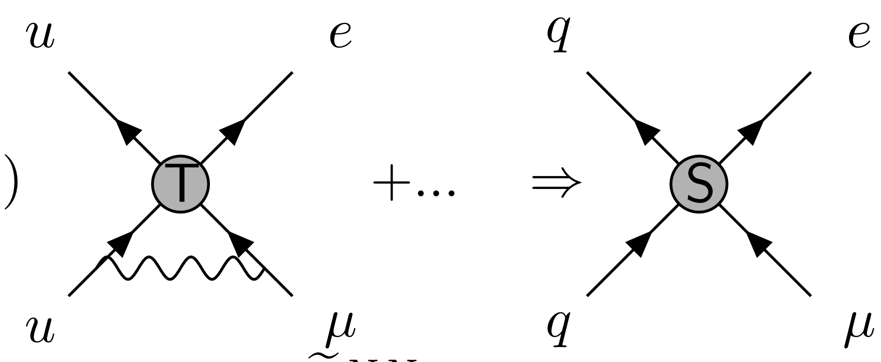

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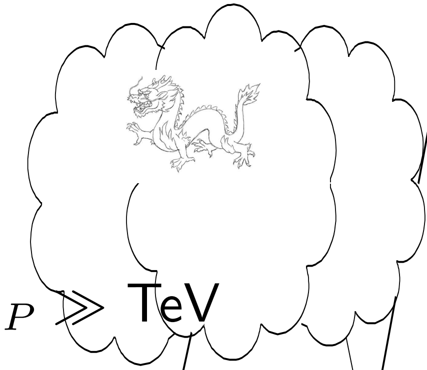
Then match to nucleons: $\widetilde{C}_S^{NN} = \langle N|\bar{u}u|N\rangle \Delta C_S^{uu} \sim C_T^{uu}$ so $\widetilde{C}_S^{pp} \gtrsim \widetilde{C}_T^{pp}$,

$$\boldsymbol{BR(\mu A \to eA) \approx BR_{SI} \sim Z^2|2C_T^{uu}|^2 \sim 10^3 BR_{SD}}$$

loops can change Lorentz structure/external legs ⇒ different operator whose coefficient better constrained

Loops matter...are SM; how to include?



$\Lambda_{NP} \gg$ TeV

$\{Z, W, \gamma, g, h, t, f\}$

$m_W \sim m_h \sim m_t$

$\{\gamma, g, e, \mu, \tau, u, d, c, s, b\}$

RGEs: $\mu \frac{\partial}{\partial \mu} \vec{C} = \vec{C} \mathbf{\Gamma}$

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GeV $\sim m_c, m_b, m_\tau$

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data

(why do SM loops in EFT? Its non-renormalisable! Should calculate in models?

In a renormalisable model for LFV:
+ calculate any observable to arbitrary order in couplings as a function of finite number of Lagrangian inputs
+ there are public codes!
But:

1. SM loop calns are hard :
   multiple perturbative expansions — gauge , loops, (hierarchical) Yukawas, and mixing angles
   ex: dominant contribution of $y_{\mu e}\bar{\mu}eH$ to $\mu \to$eγ is at 2-loop (Barr-Zee)
   Bjorken+Weinberg,1977
   multitude of subsequent model papers use subdominant 1-loop contribution.

   doing loop integrals with many massive particles is an art

2. and there is a *plethora* of models. Each with maaany unknown parameters.

How/why to do loop calculations in EFT ?

loop calculations diverge in all theories (in 4-d)

$\Rightarrow 1.$ "regularise" : $\infty \to N$

$2.$ "renormalise" :subtract/hide Ns in Lagrangian parameters

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CLFV operators $\propto 1/\Lambda_{NP}^2$ are "non-renormalisable"

$=$ there are some loop diagrams it is not interesting to compute,

because generate operators not-in-list (eg $\propto 1/\Lambda_{NP}^4$), $\Rightarrow$ more coefficients

can compute SM loops decorating CLFV vertex

has to reproduce results of all heavy models (= that satisfy input assumptions)

(otherwise: why bother doing EFT? And quark flavour people use it...)

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1. Loop calculation *simpler* in EFT: no heavy masses in loops

2. 1 loop EFT resums $\left(\frac{1}{16\pi^2}\log\right)^n$...so get a piece of 1-loop + piece of 2-loop +...

3. only do the loop calculations once (then works for all heavy New Physics models)

drawback: **cannot** depend on regularisation/renormalisation for operators, because they are not there in models. Requires care. *eg* can only get log divs, because in dim reg, there are only log divs.

lets admit to do SM loops in EFT...

Including the loops in EFT

line up all operator coefficients in row vector $\vec{C}$, satifies $\mu\frac{\partial}{\partial\mu}\vec{C} = \vec{C}\mathbf{\Gamma}$. Solution:

$$\vec{C}(m_\mu) \quad = \quad \vec{C}(m_W)\boldsymbol{G}$$

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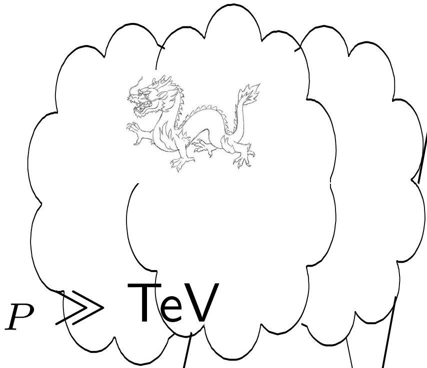
$$\begin{aligned} C_{D,X}(m_\mu) = \; & C_{D,X}(m_W)\left(1 - 16\frac{\alpha_e}{4\pi}\ln\frac{m_W}{m_\mu}\right) \\ & -\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{m_\mu}\left(-8\frac{m_\tau}{m_\mu}C^{\tau\tau}_{T,XX} + C^{\mu\mu}_{S,XX} + C_{2loop}\right) \\ & +16\frac{\alpha_e^2}{2e(4\pi)^2}\ln^2\frac{m_W}{m_\mu}\left(\frac{m_\tau}{m_\mu}C^{\tau\tau}_{S,XX}\right) \\ & -8\lambda^{a_T}\frac{\alpha_e}{4\pi e}\ln\frac{m_W}{2\text{ GeV}}\left(-\frac{m_s}{m_\mu}C^{ss}_{T,XX} + 2\frac{m_c}{m_\mu}C^{cc}_{T,XX} - \frac{m_b}{m_\mu}C^{bb}_{T,XX}\right)f_{TD} \\ & +16\frac{\alpha_e^2}{3e(4\pi)^2}\ln^2\frac{m_W}{2\text{ GeV}}\left(\sum_{u,c}4\frac{m_q}{m_\mu}C^{qq}_{S,XX} + \sum_{d,s,b}\frac{m_q}{m_\mu}C^{qq}_{S,XX}\right) \end{aligned}$$

$\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

Now what?



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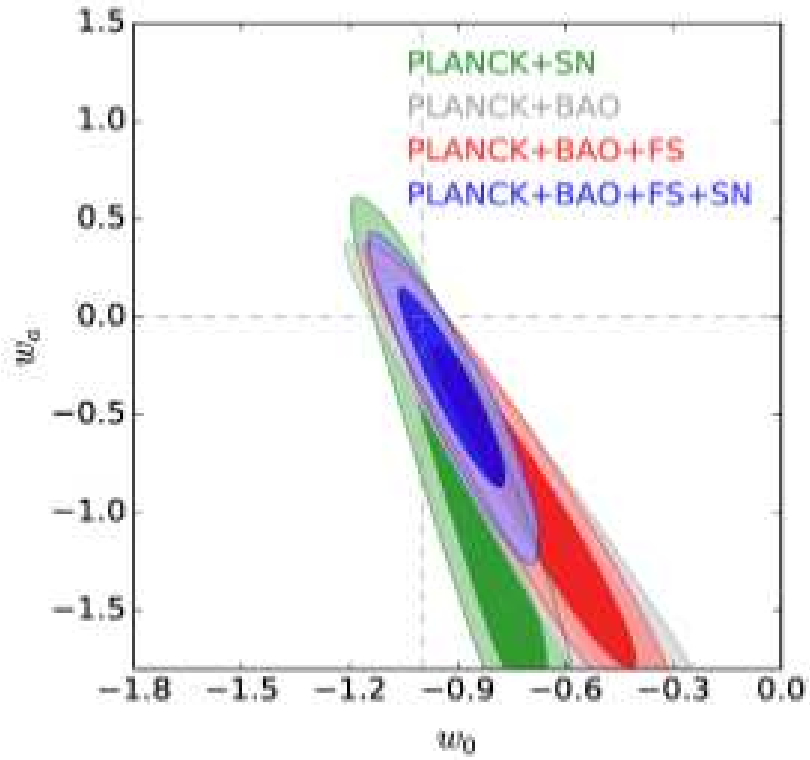
Do we get ellipses in parameter space?



Ideally: more constraints than parameters, build models that sit in overlap of ellipses

⇒How many constraints on how many parameters for $\mu \leftrightarrow e$ contact interactions?

(Reconstructing coeffs with/without signal not same. Here suppose ellipses centered at zero = no detection. With signal, can displace ellipse, get ring...like unitarity triangle fits)

count operator coefficients, vs exptal constraints on them

At Λ_{expt}: operator basis $\mu - e$ conv., $\mu \to e\bar{e}e$, $\mu \to e\gamma$ Kuno Okada

μ interaction with nucleon $N \in \{n, p\}$ parametrised by 20 4-f operators :

$$
\begin{array}{llll}
S,\ V & \overline{e}P_X\mu\overline{N}N & \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha N & X \in \{L, R\} \\
A,\ T & \overline{e}\gamma^\alpha P_X\mu\overline{N}\gamma_\alpha\gamma_5 N & \overline{e}\sigma^{\alpha\beta}P_X\mu\overline{N}\sigma_{\alpha\beta}N & \\
P & \overline{e}P_X\mu\overline{N}\gamma_5 N & &
\end{array}
$$

and 2 dipoles

$$
D \qquad \overline{e}\sigma^{\alpha\beta}P_X\mu F_{\alpha\beta}
$$

which also contribute in $\mu \to e\gamma$, $\mu \to e\bar{e}e$. For $\mu \to e\bar{e}e$

$$
\begin{array}{lll}
V & (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) & (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e) \\
S & (\overline{e}P_Y\mu)(\overline{e}P_Y e) &
\end{array}
$$

chiral basis for the lepton current (relativistic e), but not for the non-rel. nucleons.

28 operators

The operator basis $m_\tau \to m_W$: 82 operators

Add QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and *flavour-diagonal* combination of γ, g, u, d, s, c, b. $Y \in L, R$.

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad \textit{dim } 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad \textit{dim } 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{\mu}\gamma_\alpha P_X\mu)$$

$$(\overline{e}P_Y\mu)(\overline{\mu}P_Y\mu)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_Y f) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{f}\gamma_\alpha P_X f)$$

$$(\overline{e}P_Y\mu)(\overline{f}P_Y f) \qquad (\overline{e}P_Y\mu)(\overline{f}P_X f) \qquad f \in \{u, d, s, c, b, \tau\}$$

$$(\overline{e}\sigma P_Y\mu)(\overline{f}\sigma P_Y f)$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad \textit{dim } 7$$

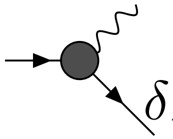
$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

...zzz...but 82 coeffs!

$(P_X, P_Y = (1 \pm \gamma_5)/2)$, all operators with coeff $-2\sqrt{2}G_F C$.

Constraints from 3 processes: $\mu \to e\gamma, \mu \to e\bar{e}e, \mu A \to eA$

Two dipole operators contribute to $\mu \to e\gamma$:



$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R} \overline{\mu_R} \sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_{D,L} \overline{\mu_L} \sigma^{\alpha\beta} e_R F_{\alpha\beta} \right)$$

$$BR(\mu \to e\gamma) = 384\pi^2 (|C_{D,R}|^2 + |C_{D,L}|^2) < 5.7 \times 10^{-13} \Rightarrow |C_X^D| \lesssim 10^{-8}$$

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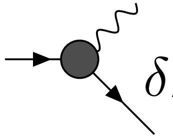
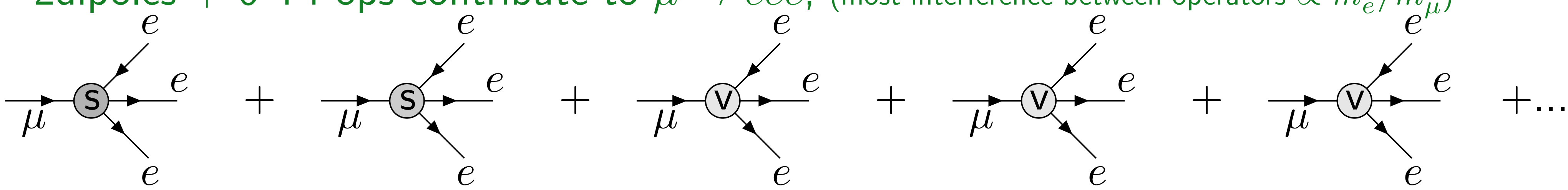


$$\delta\mathcal{L}_{meg} = -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_{D,R}\overline{\mu_R}\sigma^{\alpha\beta}e_L F_{\alpha\beta} + C_{D,L}\overline{\mu_L}\sigma^{\alpha\beta}e_R F_{\alpha\beta} \right)$$

$$BR(\mu \to e\gamma) = 384\pi^2(|C_{D,R}|^2 + |C_{D,L}|^2) < 5.7 \times 10^{-13} \Rightarrow |C_X^D| \lesssim 10^{-8}$$

2dipoles + 6 4-f-ops contribute to $\mu \to e\bar{e}e$, (most interference between operators $\propto m_e^2/m_\mu^2$)



$(\overline{e}P_L\mu)(\overline{e}P_Le)$ $(\overline{e}P_R\mu)(\overline{e}P_Re)$ $(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_Le)$ $(\overline{e}\gamma P_R\mu)(\overline{e}\gamma P_Re)$ $(\overline{e}\gamma P_L\mu)(\overline{e}\gamma P_Re)$

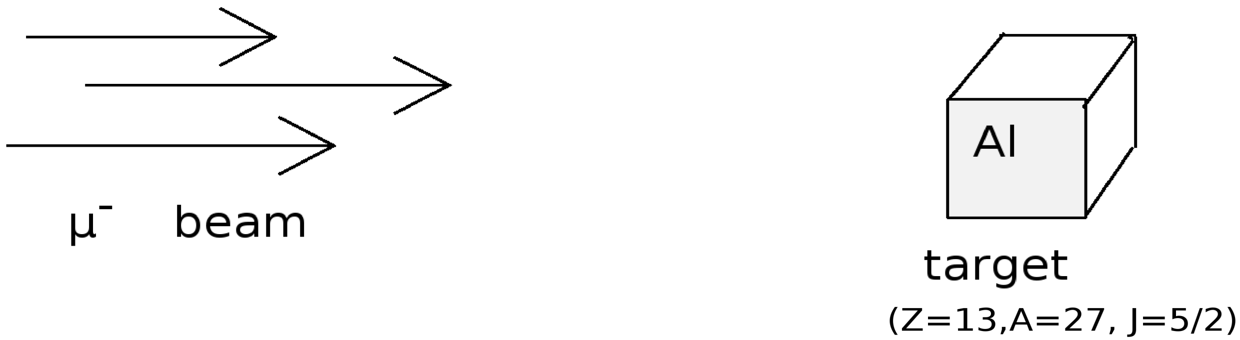
$$BR(\mu \to e\bar{e}e) = \frac{|C_{S,LL}|^2 + |C_{S,RR}|^2}{8} + 2|C_{V,RR}|^2 + 2|C_{V,LL}|^2 + |C_{V,LR}|^2 + |C_{V,RL}|^2$$

$$\leq 10^{-12} \quad \Rightarrow \quad \boldsymbol{|C_X| \lesssim 10^{-6}\sqrt{BR/10^{-12}}}$$

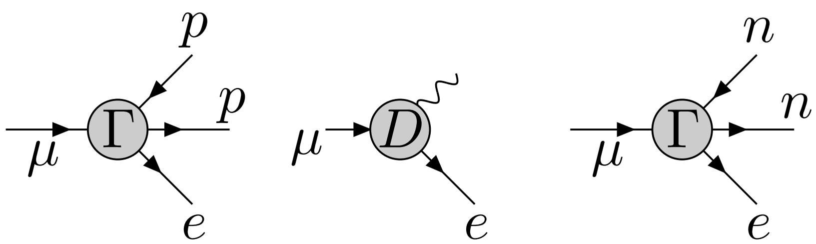
neglected dipole contribution; constrained by $\mu \to e\gamma$

see nothing in $\mu \to e\gamma$, $\mu \to e\bar{e}e$, $\Rightarrow$ all 8 Cs small

$\mu \to e$ conversion



- μ^- captured by Al nucleus, tumbles down to $1s$. $(r \sim Z\alpha/m_\mu \gtrsim r_{Al})$
- in SM: muon capture $\mu + p \to \nu + n$
- bound μ interacts with nucleus, converts to e $(E_e \approx m_\mu)$



$$\Gamma = \{I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma\}$$
$$\Gamma = \{S, P, V\ ,\ A\ ,\ T\}$$

$\approx$ WIMP scattering on nuclei

1) "Spin Independent" rate $\propto A^2$ (amplitude $\propto \sum_N \propto A$)

2) "Spin Dependent" rate $\sim \Gamma_{SI}/A^2$ (sum over nucleons $\propto$ spin of only unpaired nucleon)

Constraints on the nucleon operators from $\mu - e$ conv.

DavidsonKunoSaporta

DavidsonKunoYamanaka

$$\mathrm{BR}_{SD}(A\mu \to Ae) \sim \left|\widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R}\right|^2 + \left|\widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L}\right|^2 \qquad (N \text{ odd})$$

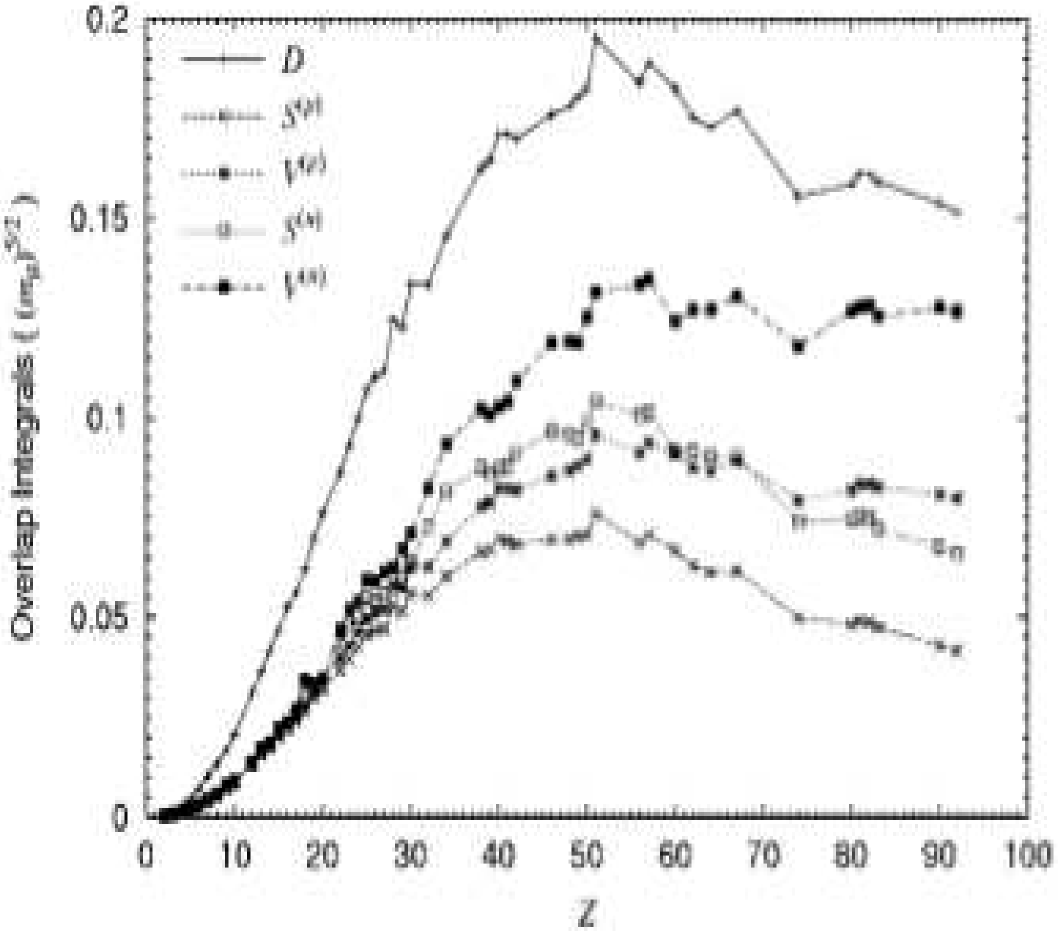
$$\mathrm{BR}_{SI}(A\mu \to Ae) \propto |\widetilde{C}^{pp}_{V,R} V^{(p)}_A + \widetilde{C}'^{pp}_{S,L} S^{(p)}_A + \widetilde{C}^{nn}_{V,R} V^{(n)}_A + \widetilde{C}'^{nn}_{S,L} S^{(n)}_A + C_{D,L} D|^2 + \{L \leftrightarrow R\}$$

Can distinguish SD vs SI, L vs R. But if observe SI conversion, how to know if is due to scalar/vector operator on n or p?

KitanoKoikeOkada

$$S^{(p)}_A, V^{(p)}_A \sim \int d^3x \widetilde{\psi}^{1s}_\mu |f_p(x)|^2 \widetilde{\psi}^*_e (\bar{p}\{1, \gamma_0\} p)$$

The overlap integrals of Kitano, Koike, Okada



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$$\sim \quad Z^2\left|\vec{C}_R \cdot \hat{v}_A\right|^2 + Z^2\left|\vec{C}_L \cdot \hat{v}_A\right|^2 \qquad \vec{v}_A \equiv \left(V^{(p)}_A, S^{(p)}_A, V^{(n)}_A, S^{(n)}_A, D_A\right)$$

Can distinguish SD vs SI, L vs R. But if observe SI conversion, how to know if is due to scalar/vector operator on n or p?

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$$S^{(p)}_A, V^{(p)}_A \sim \int d^3x \widetilde{\psi}^{1s}_\mu |f_p(x)|^2 \widetilde{\psi}^*_e (\bar{p}\{1, \gamma_0\}p)$$

different "target vectors" $\vec{v}_A$ for different nuclear targets

target vectors "live" in coefficient space, like $\vec{C} = (\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S, (D))$

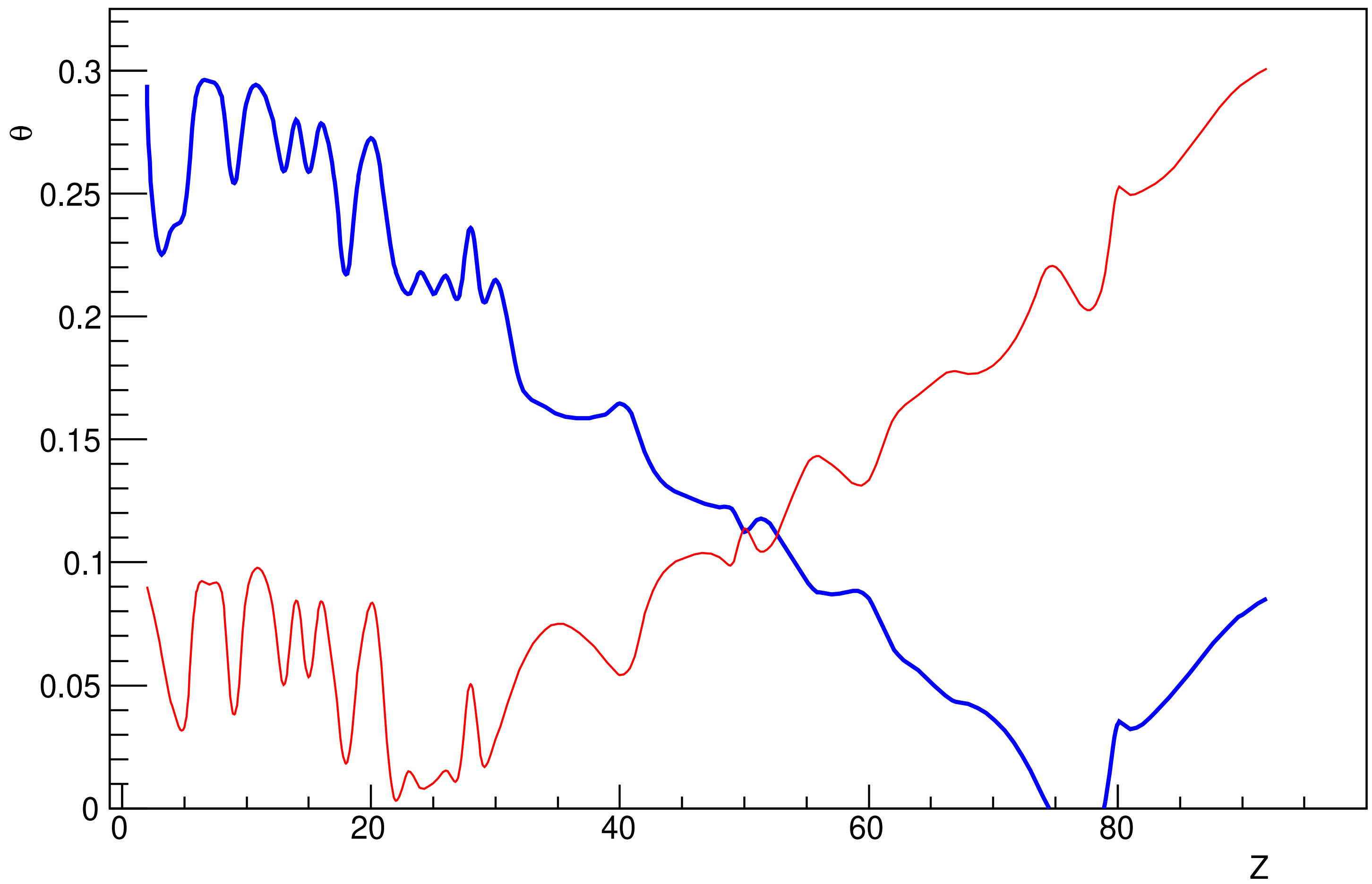
1. 1st exptal search (eg Gold) probes $\vec{C} \parallel \vec{v}_{Au}$

2. next target, suff large component $\perp$ Gold

Current data+ theory uncertainty $\sim 10\%$: two targets give $\Delta\theta > 0.2$

$$\boldsymbol{BR(\mu Au \to eAu) \leq 7 \times 10^{-13}} \qquad (Au : Z = 79)$$

$$BR(\mu Ti \to eTi) \leq 4.3 \times 10^{-12} \qquad (Ti : Z = 22)$$



$\vec{v}_A = (V_A^{(p)}, S_A^{(p)}, V_A^{(n)}, S_A^{(n)})$, and $BR \propto |\vec{v}_A \cdot \vec{C}|^2$

$\vec{v}_{Au} \cdot \vec{v}_Z \equiv |\vec{v}_{Au}||\vec{v}_Z| \cos\theta$ ...plot θ on vertical axis

Constraints on the nucleon operators from $\mu - e$ conv.

DavidsonKunoSaporta
DavidsonKunoYamanaka

$$\mathrm{BR}_{SD}(A\mu \to Ae) \sim \left|\widetilde{C}^{NN}_{A,L} + 2\widetilde{C}^{NN}_{T,R}\right|^2 + \left|\widetilde{C}^{NN}_{A,R} + 2\widetilde{C}^{NN}_{T,L}\right|^2 \qquad (N \text{ odd})$$

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1. 1st exptal search (eg Gold) probes $\vec{C} \parallel \vec{v}_{Au}$

2. next target, suff large component $\perp$ Gold

$\Rightarrow$ three (suitable) nuclear targets (+improve theory caln) could probe 3 combinations of $\{\widetilde{C}^{pp}_V, \widetilde{C}^{pp}_S, \widetilde{C}^{nn}_V, \widetilde{C}^{nn}_S\}$

In the future...with a 5% theory uncertainty:

First target of Mu2e, COMET: Aluminium (Z=13, A=27)

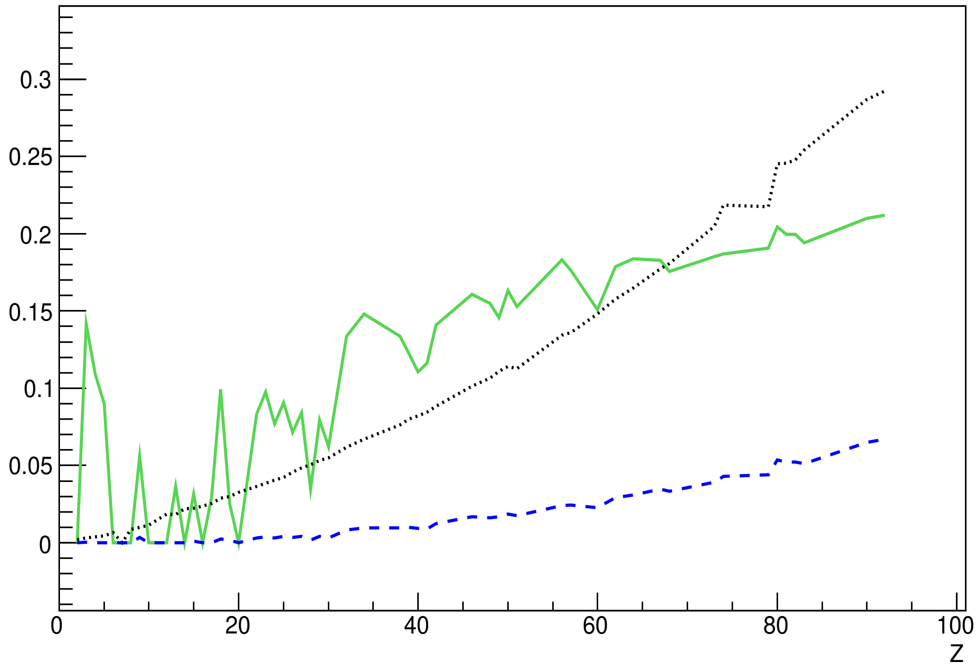
$$\hat{v}_{Al} \approx \frac{1}{2}(1,1,1,1)$$ (recall $\tilde{C}_V^{pp}, \tilde{C}_S^{pp}, \tilde{C}_V^{nn}, \tilde{C}_S^{nn}$)

basis of three other "directions":

$$\hat{v}_{np} \equiv \frac{1}{2}(-1,-1,1,1)$$

$$\hat{v}_{VS} \equiv \frac{1}{2}(1,-1,1,-1)$$

$$\hat{v}_{IsoSV} \equiv \frac{1}{2}(-1,1,1,-1)$$



probe 3 combinations of SI coeffs

What to learn at Λ_{exp}: setting constraints from $\mu A \to eA$, $\mu \to e\gamma$ + $\mu \to e\bar{e}e$

parametrise with 20 nucleon ops (8 SI: S,V) $+$ (12 SD: P,A,T)
$+$2 dipole operators
$+$6 four-lepton operators

1. constrain 2 dipoles $+$6 4ℓ coeffs with $\mu \to e\gamma + \mu \to e\bar{e}e$

2. Spin Indep now: constrain 4 combinations of 8 $\{S, V\}$ coefficients

SI future: constrain 6 combinations of 8 $\{S, V\}$ coefficients

3. Spin-Dependent, now: (?) 2 counstraints? (Ti?)

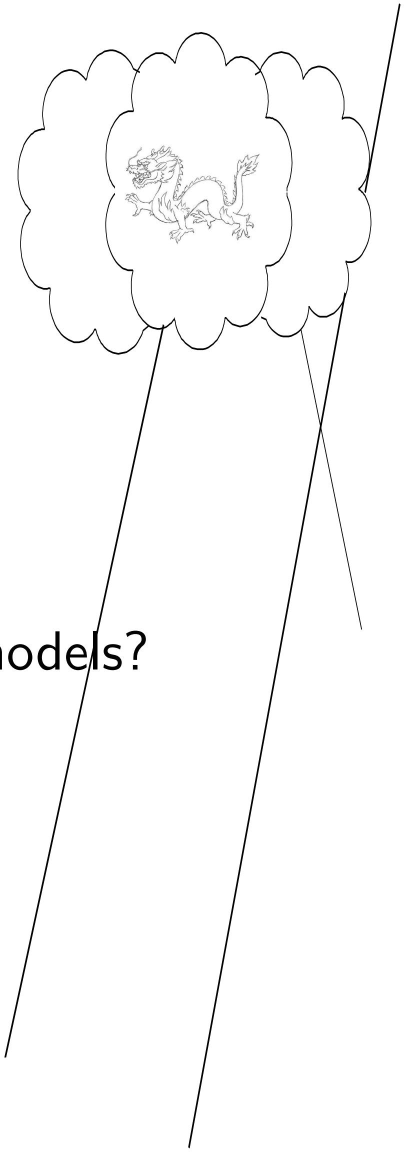
future: $4 \to 8$ constraints ?

n vs p by comparing odd-p, A vs T vs P $\Leftrightarrow$ dedicated nucl.caln.)

$$\Rightarrow \text{28 coefficients,} \left\{ \begin{array}{ll} \text{now} & 12 \to 14 \\ \text{future} & 18 \to 22 \end{array} \right\} \text{constraints}$$

...so what to do?
(no ellipse in coeff space even at exptal scale)

EFT as a tool to travel in scale



1. why is what dependent on which "scale"?
2. But loops are small: surely negligeable?
   - data is sensitive to loop effects
   - tree level is not always the dominant contribution
3. in practise — how does EFT allow to translate from data to models?
   $\Leftrightarrow$ loop calculations in EFT
4. **results: constraints and sensitivities**+***questions:***
   what is useful for you?

Can still calculate sensitivities...

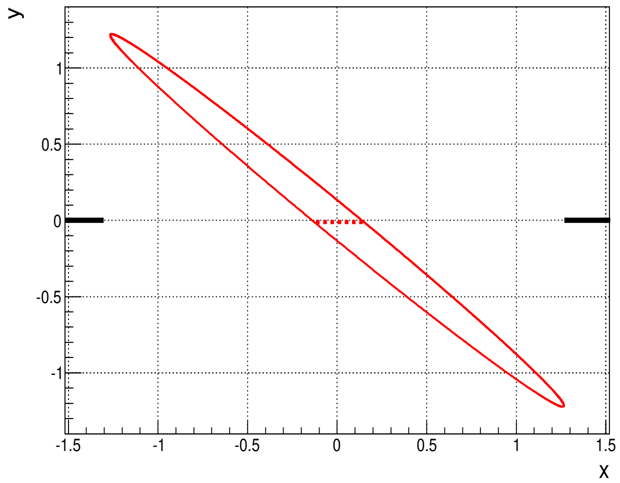
sensitivity: "one at a time bound" = value below which a parameter is to small to be seen in expt.
But it could be larger than this, if it is cancelled by another contribution.

coefficient	$\mu \to e\gamma$	$\mu \to e\bar{e}e$	$\mu - e$ conv.
$\lvert C_{D,X} \rvert$	1.12×10^{-8}	4.30×10^{-7}	2.35×10^{-7}
$\lvert C^{ee}_{V,XX} \rvert$	1.10×10^{-4}	7.80×10^{-7}	1.86×10^{-5}
$\lvert C^{ee}_{V,XY} \rvert$	2.55×10^{-4}	9.34×10^{-7}	3.77×10^{-5}
$\lvert C^{ee}_{S,XX} \rvert$	1.73×10^{-4}	2.8×10^{-6}	(3.64×10^{-3})
$\lvert C^{\mu\mu}_{V,XX} \rvert$	1.10×10^{-4}	5.60×10^{-5}	1.85×10^{-5}
$\lvert C^{\mu\mu}_{V,XY} \rvert$	2.56×10^{-4}	1.12×10^{-4}	3.77×10^{-5}
$\lvert C^{\mu\mu}_{S,XX} \rvert$	8.24×10^{-7}	(1.58×10^{-5})	(1.73×10^{-5})
$\lvert C^{\tau\tau}_{V,XX} \rvert$	3.80×10^{-4}	1.95×10^{-4}	1.24×10^{-5}
$\lvert C^{\tau\tau}_{V,XY} \rvert$	4.40×10^{-4}	1.91×10^{-4}	1.25×10^{-5}
$\lvert C^{\tau\tau}_{S,XX} \rvert$	5.33×10^{-6}	1.02×10^{-4}	1.12×10^{-4}
$\lvert C^{\tau\tau}_{S,XY} \rvert$	—	—	—
$\lvert C^{\tau\tau}_{T,XX} \rvert$	1.10×10^{-8}	(4.20×10^{-7})	(2.30×10^{-7})

Table 1: Current sensitivities of $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu - e$ conv. to the coefficients, at m_W, of QCD×QED-invariant 2- and 4-lepton operators. $X, Y \in \{L, R\}$, $X \neq Y$.

But sensitivity = "how small could you see"...what about constraints?

constraint = limit on coefficient beyond which it is incompatible with data. Irrespective of other coefficients.



What to do?

• argue some cancellations can only be accidental($\approx$reduce $A + B \to [20\%][A + B]$)

$\Leftrightarrow$ *what is "natural" in EFT?*

eg coupling g_2 cannot cancel log(M/m)

coefficients which run under QCD cannot cancel against those who do not...

allows to increase number of constraints (not to 82)

• additional observables who can set restrictive constraints?

Can give experimental bounds in terms of high-scale coefficients ...

recall: SI $\mu - e$ conv. constrains at tree level/at 2 GeV 14 quark coefficients

(+dipoles and di-gluonsnot written here)

$$\sqrt{\frac{BR^{exp}_{Al}}{33}} \gtrsim \left| 3C^{uu}_{V,L} + 3C^{dd}_{V,L} + 11C^{uu}_{S,R} + 11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R} \right|$$

also constraint on coeffs with $L \leftrightarrow R$ (the chirality of e)

matched nucleon and quark coefficients at 2 GeV using lattice $\{G^{Nq}_S\}$

at one loop, 44 (2 dipoles+2digluons) of 82 operators contribute to $\mu - e$ conv.

$$\sqrt{\frac{BR^{exp}_{Al}}{33}} \gtrsim \Big| 3C^{uu}_{V,L} + 3C^{dd}_{V,L} + \frac{\alpha}{\pi}\Big[3C^{dd}_{A,L} - 6C^{uu}_{A,L}\Big]\log$$

$$+\frac{\alpha}{3\pi}[C^{ee}_{V,L} + C^{\mu\mu}_{V,L}]\log - \frac{\alpha}{3\pi}[C^{ee}_{A,L} + C^{\mu\mu}_{A,L}]\log$$

$$-\frac{2\alpha}{3\pi}\Big[2(C^{uu}_{V,L} + C^{cc}_{V,L}) - (C^{dd}_{V,L} + C^{ss}_{V,L} + C^{bb}_{V,L}) - (C^{ee}_{V,L} + C^{\mu\mu}_{V,L} + C^{\tau\tau}_{V,L})\Big]\log$$

$$+\lambda^{-a_S}\Big(11C^{uu}_{S,R} + 11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R}\Big)$$

$$+\lambda^{-a_S}\frac{\alpha}{\pi}\Big[\frac{13}{6}(11C^{uu}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R}) + \frac{5}{3}(11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R})\Big]\log$$

$$-\lambda^{a_T} f_{TS}\frac{8\alpha}{\pi}\Big[22C^{uu}_{T,R} + \frac{8m_N}{27m_c}C^{cc}_{T,R} - 11C^{dd}_{T,R} - 0.84C^{ss}_{T,R} - \frac{4m_N}{27m_b}C^{bb}_{T,R})\Big]\log\Big|$$

also constraint on coeffs with $L \leftrightarrow R$ (the chirality of e)
quark coefficients at m_W
$\log \equiv \log(m_W/2\text{GeV}) \simeq 3.7$,
$\lambda = \alpha_s(m_W)/\alpha_s(2\text{GeV}) \simeq 0.44$, $f_{TS} \simeq 1.45$, $a_S = 12/23$, $a_T = -4/23$.

a public code?
what would you like it to do? top-down? bottom-up? ?

Summary

Loops are important for Lepton Flavour Change, because they allow to transform contact interactions that are difficult to probe experimentally, into interactions that are stringently constrained.

If the ratio m_{SM}/Λ_{NP} is “small enough” EFT is an ideal tool to account for SM loop effects.
“small enough” $=$ such that dimension eight operators can be neglected, and the log-enhanced part of loops is the dominant part
EFT is ideal because the loop calculations are trivial, and only have to be calculated once, for all heavy theories.

An EFT analysis can give the “sensitivity” of any selected experimental process to any high-scale operator coefficient (at some chosen scale) —is a paper/webpage giving these numbers useful?
(sensitivity $=$ “one-at-a-time” bound)
An EFT analysis can also give the constraints on high-scale coefficients arising from experimental constraints — are these interesting? If so, in what format? Code/formulae/webpage/...?

BackUp

What is Effective Field Theory?

- EFT $=$ recipe to study observables at scale ℓ
  1. choose *appropriate* variables to describe *relevant* dynamics
  2. 0th order interactions, by sending all parameters $\left\{ \begin{array}{ll} L \gg \ell & \to \infty \\ \delta \ll \ell & \to 0 \end{array} \right.$
  3. then perturb in ℓ / L and δ / ℓ

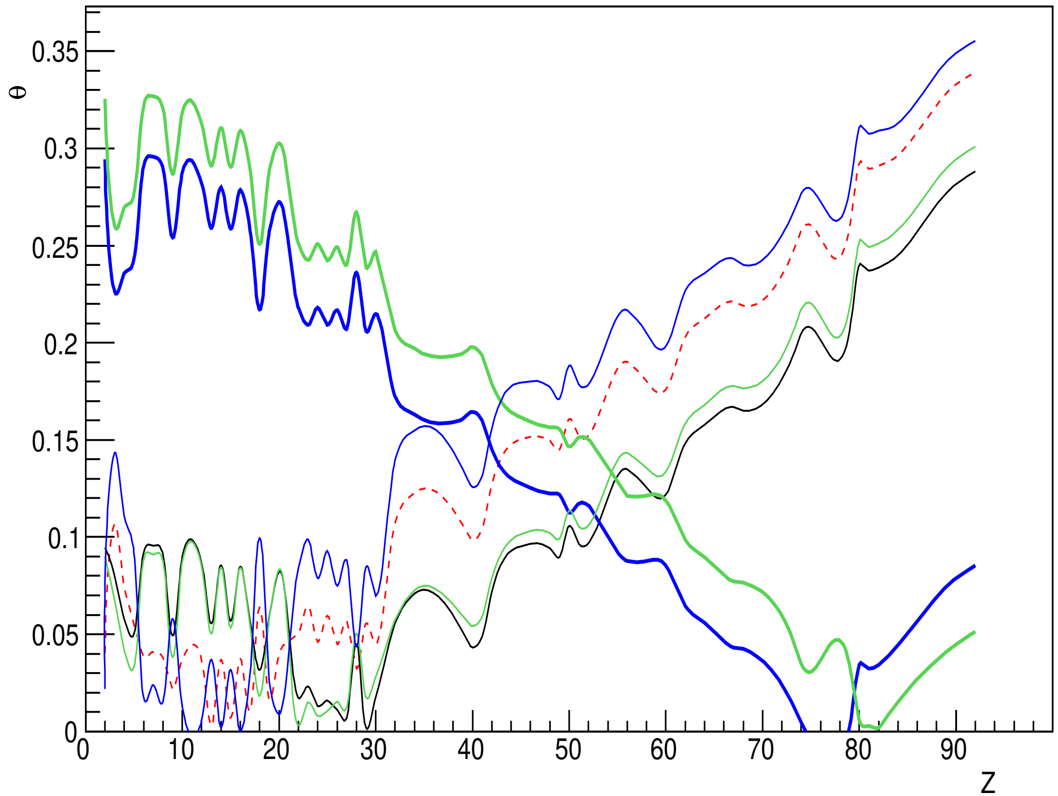
Example : interactions in the early Universe of age τ_U ($\tau_U \sim 10^{-24}$ sec)
$\star$ processes with $\tau_{int} \gg \tau_U$ ...neglect!
$\star$ processes with $\tau_{int} \ll \tau_U$ ...assume in thermal equilibrium!
$\star$ processes with $\tau_{int} \sim \tau_U$ ...calculate this dynamics
$\star$ can then do pert. theory in slow interactions and departures from thermal equil.

Example : low energy LFV due to heavy New Particles ($\Lambda_{NP} \gg m_W \gg$ GeV)
$\star$ SM particles (all masses $\ll m_W$) are dynamical variables
$\star$ renormalisable interactions $=$ QCD*QED
$\star$ can include small SM masses in pert. theory (eg m_e / m_μ), $+$ heavy particle effects as conotact interactions (Fermi interaction, LFV contact interactions...)

Some LFV processes and bounds

some processes	current constraints on BR	future sensitivities
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$	6×10^{-14} (MEG)
$\mu \to e\bar{e}e$	$< 1.0 \times 10^{-12}$(SINDRUM)	10^{-16} (2018, Mu3e)
$\mu A \to eA$	$< 7 \times 10^{-13}$ Au, (SINDRUM)	10^{-16} (Mu2e,COMET)
		10^{-18} (PRISM/PRIME)
$\overline{K_L^0} \to \mu\bar{e}$	$< 4.7 \times 10^{-12}$ (BNL)	
$K^+ \to \pi^+\bar{\mu}e$	$< 1.3 \times 10^{-11}$ (E865)	10^{-12} (NA62)
$\tau \to \ell\gamma$	$< 3.3, 4.4 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$\tau \to 3\ell$	$< 1.5 - 2.7 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II, LHCb?)
$\tau \to e\phi$	$< 3.1 \times 10^{-8}$	few$\times 10^{-9}$ (Belle-II)
$h \to \tau^\pm e^\mp$	$< 6.9 \times 10^{-3}$	
$Z \to e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$	

target misalignment without dipole



Gold , **Pb** , Sulfur(Z=16) , Ti (Z=22) , Copper (Z=29) Al

target misalignment with dipole