Status of Standard Model prediction for muon g-2

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Introduction — Breakdown of SM prediction for muon g-2

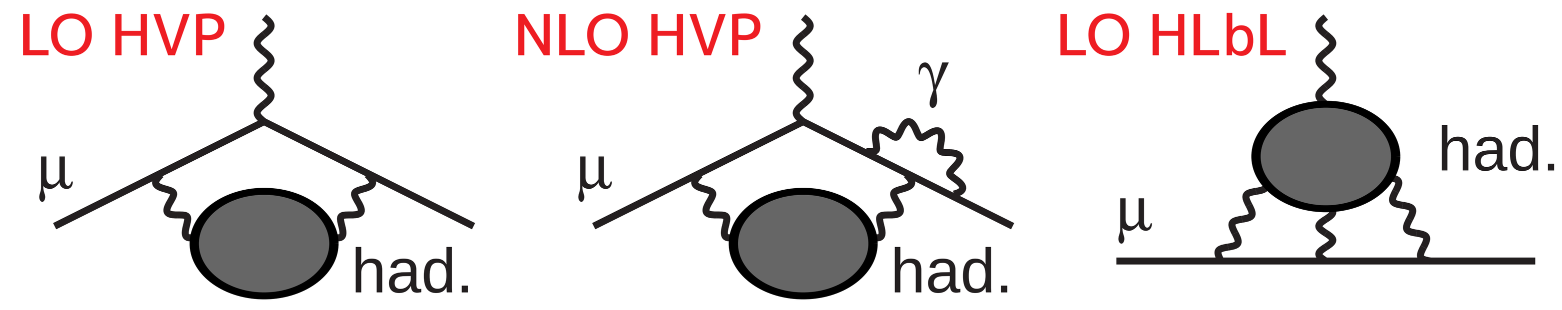
	2011		2018
QED	11658471.81 (0.02)	⟶	11658471.90 (0.01) [arXiv:1712.06060]
EW	15.40 (0.20)	⟶	15.36 (0.10) [Phys. Rev. D 88 (2013) 053005]
LO HLbL	10.50 (2.60)	⟶	9.80 (**2.60**) [EPJ Web Conf. 118 (2016) 01016]
NLO HLbL			0.30 (0.20) [Phys. Lett. B 735 (2014) 90]
	HLMNT11		KNT18
LO HVP	694.91 (4.27)	⟶	693.27 (**2.46**) this work
NLO HVP	-9.84 (0.07)	⟶	-9.82 (0.04) this work
NNLO HVP			1.24 (0.01) [Phys. Lett. B 734 (2014) 144]
Theory total	11659182.80 (4.94)	⟶	11659182.05 (3.56) this work
Experiment			11659209.10 (6.33) world avg
Exp - Theory	26.1 (8.0)	⟶	27.1 (7.3) this work
Δa_μ	3.3σ	⟶	3.7σ this work

(HVP: Hadronic Vacuum Polarization)
(HLbL: Hadronic Light-by-Light)

(Numbers taken from KNT18, Phys. Rev. D97 (2018) 114025)

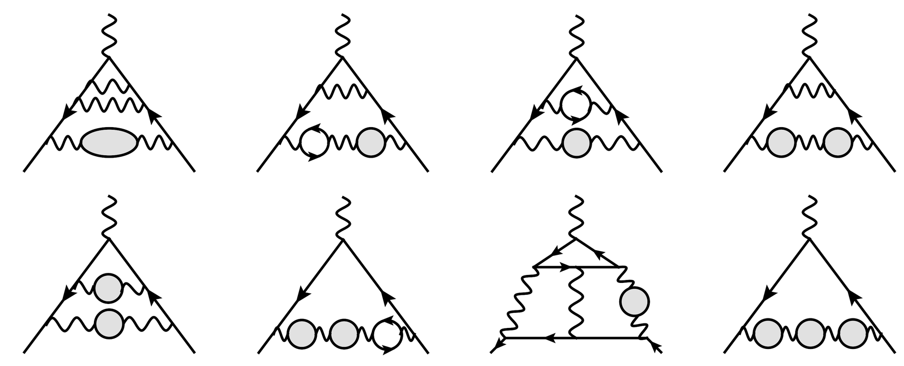
Slide by A. Keshavarzi (Liverpool) at 'Muon $g-2$ Workshop' at Mainz, June 18-22, 2018

where the hadronic contributions are:



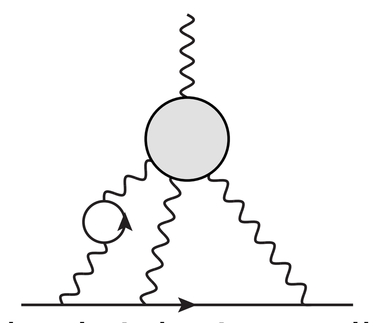
LO HVP: Leading Order (LO) Hadronic Vacuum Polarization (HVP) contribution
NLO HVP: Next-to-Leading Order (NLO) HVP contributions
LO HLbL: LO Hadronic Light-by-Light contributions

NNLO HVP



NNLO Hadronic Vacuum Polarization contributions

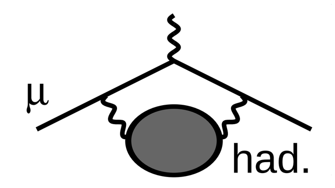
NLO HLbL



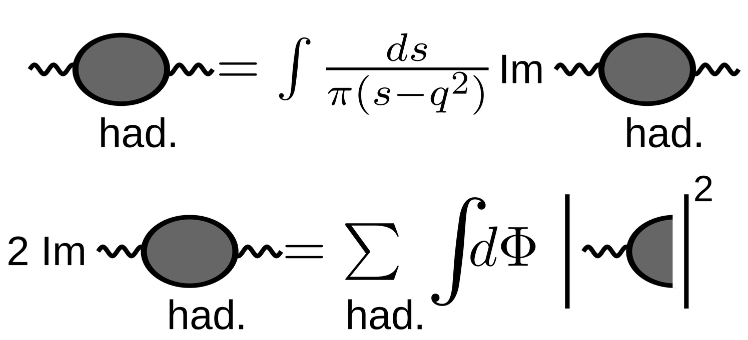
NLO Hadronic L-by-L contributions

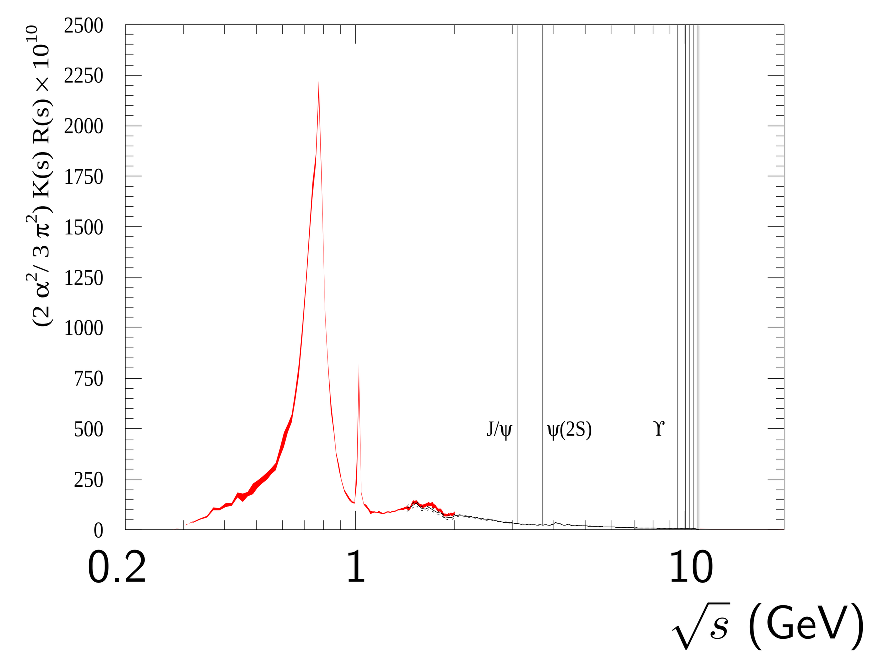
LO hadronic contribution

The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.





$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^\infty ds\ \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

Channel	Energy range [GeV]	$a_\mu^{\rm had,LO\,VP} \times 10^{10}$	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \times 10^4$	New data
	Chiral perturbation theory (ChPT) threshold contributions			
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	⋯
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	⋯
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	⋯
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	⋯
	Data based channels ($\sqrt{s} \le 1.937$ GeV)			
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{{\rm no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	⋯
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{{\rm no}\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	⋯
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{{\rm no}\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	⋯
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to\pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	⋯
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	⋯
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to{\rm npp})K\bar{K}_{{\rm no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
	Estimated contributions ($\sqrt{s} \le 1.937$ GeV)			
$(\pi^+\pi^-3\pi^0)_{{\rm no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	⋯
$(\pi^+\pi^-4\pi^0)_{{\rm no}\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	⋯
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	⋯
$\omega(\to{\rm npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	⋯
$\omega(\to{\rm npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	⋯
$\omega(\to{\rm npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	⋯
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	⋯
	Other contributions ($\sqrt{s} > 1.937$ GeV)			
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	⋯	6.26 ± 0.19	7.07 ± 0.22	⋯
ψ'	⋯	1.58 ± 0.04	2.51 ± 0.06	⋯
$\Upsilon(1S-4S)$	⋯	0.09 ± 0.00	1.06 ± 0.02	⋯
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	⋯
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	⋯

Table from KNT18, Phys. Rev. D97 (2018) 114025

Breakdown of contributions to a_μ(had, LO VP) from various hadronic final states

We have included new data sets from $\sim$ 30 papers, in addition to those included in the HLMNT11 analysis

We have included $\sim$ 30 hadronic final states

At $2 \lesssim \sqrt{s} \lesssim 11$ GeV, we use inclusively measured data

At higher energies $\gtrsim 11$ GeV, we use pQCD

d'Agostini bias and unbiased data combination

When combining data with non-negligible normalization errors, we must be very careful!

χ^2 vs normalization error: d'Agostini bias

G. D'Agostini, Nucl. Instrum. Meth. A346 (1994) 306

Let us consider an observable x whose true value is 1, as an example.
Suppose that there is an experiment which measures x and whose normalization uncertainty is 10%.
Now, assume that this experiment measured x twice:

$$\text{1st result:}\quad 0.9 \pm 0.1_{\rm stat} \pm 10\%_{\rm syst}\ ,$$
$$\text{2nd result:}\quad 1.1 \pm 0.1_{\rm stat} \pm 10\%_{\rm syst}\ .$$

Taking the systematic errors 0.09 and 0.11, respectively, the covariance matrix and the χ^2 function are

$$({\rm cov.}) = \begin{pmatrix} 0.1^2+0.09^2 & 0.09\cdot 0.11 \\ 0.09\cdot 0.11 & 0.1^2+0.11^2 \end{pmatrix}\ ,$$
$$\chi^2 = \begin{pmatrix} x-0.9 & x-1.1 \end{pmatrix} ({\rm cov.})^{-1} \begin{pmatrix} x-0.9 \\ x-1.1 \end{pmatrix}\ .$$

χ^2 takes its minimum at $x = 0.98$: Biased downwards!

d'Agostini bias (2): improvement by iterations

What was wrong? We interpreted the syst. errors of the data,

$$\text{1st result:}\quad 0.9 \pm 0.1_{\rm stat} \pm 10\%_{\rm syst}\ ,$$
$$\text{2nd result:}\quad 1.1 \pm 0.1_{\rm stat} \pm 10\%_{\rm syst}\ ,$$

as 0.09 and 0.11, respectively, which introduced the downward bias. Instead, we should take 10% of some estimator $\bar{x}$ as the syst. errors. Then,

$$({\rm cov.}) = \begin{pmatrix} 0.1^2+(0.1\bar{x})^2 & (0.1\bar{x})^2 \\ (0.1\bar{x})^2 & 0.1^2+(0.1\bar{x})^2 \end{pmatrix}\ ,$$
$$\chi^2 = \begin{pmatrix} x-0.9 & x-1.1 \end{pmatrix} ({\rm cov.})^{-1} \begin{pmatrix} x-0.9 \\ x-1.1 \end{pmatrix}\ .$$

χ^2 takes its minimum at $x = 1.00$: Unbiased!
In more general cases, we use iterations: we find an estimator for the next round of iteration by χ^2-minimization. R.D.Ball et al, JHEP 1005 (2010) 075.

Integration over real data

$\sigma(e^+e^- \to \pi^+\pi^-)$ data

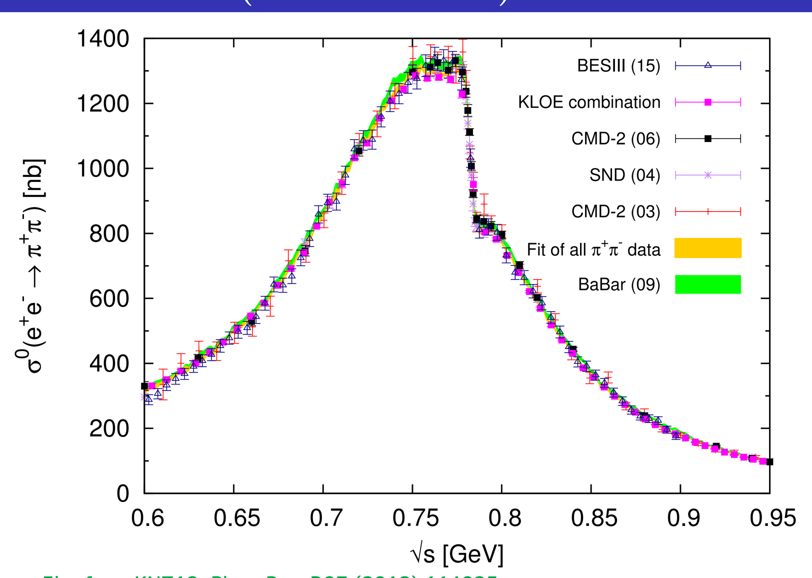


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

$\sigma(e^+e^- \to \pi^+\pi^-)$: ρ-ω interference region

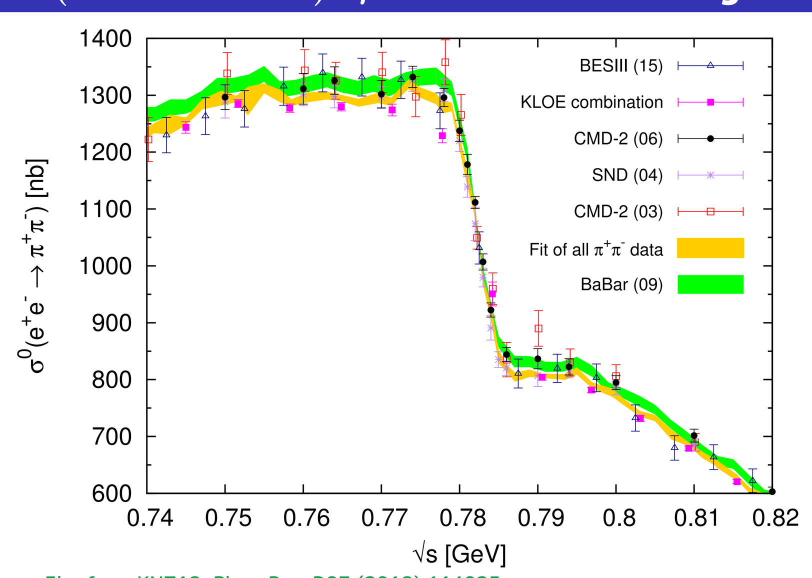


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

$\sigma(e^+e^- \to \pi^+\pi^-)$: relative differences

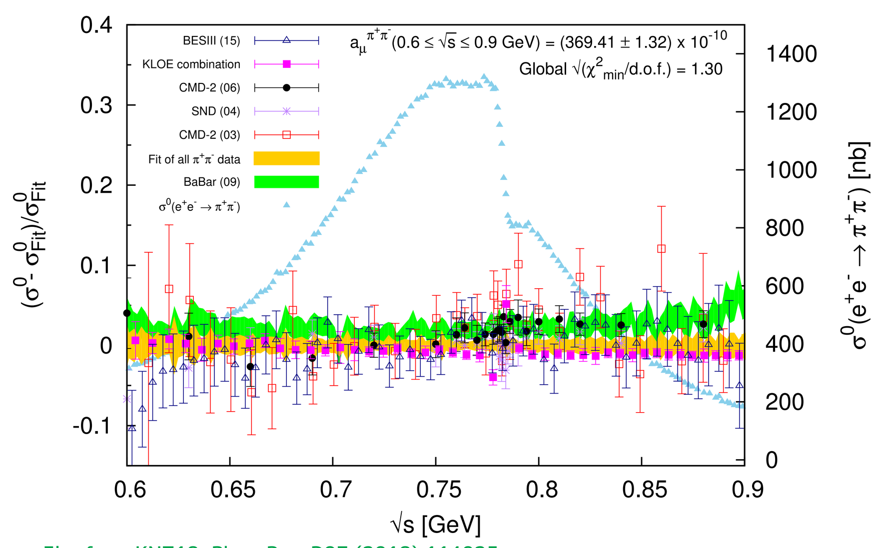


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Contribution to $(g-2)_\mu$ from $\pi^+\pi^-$ channel

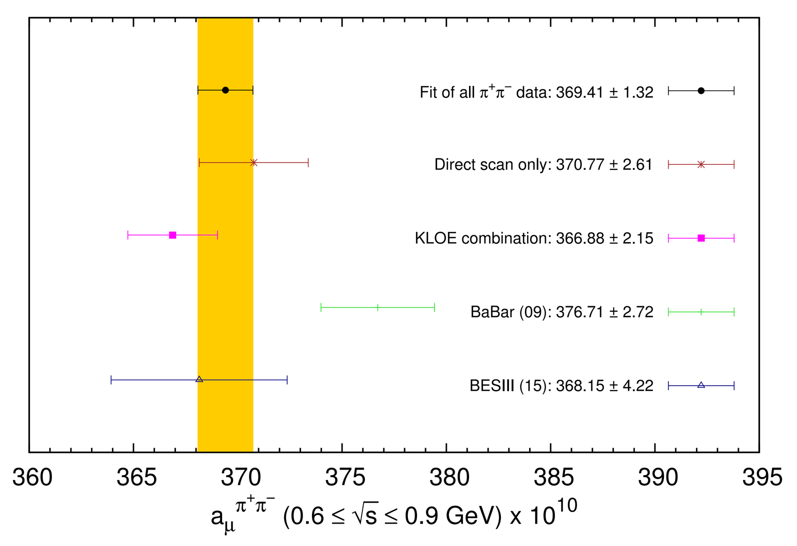
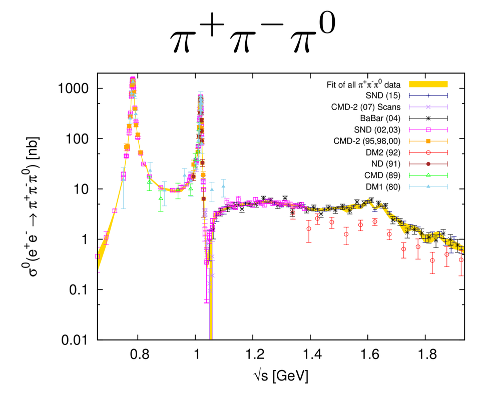


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

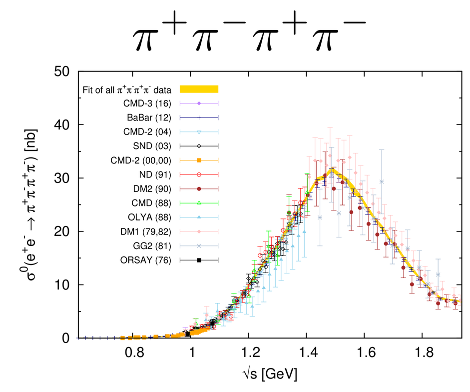
Other notable channels

$\pi^+\pi^-\pi^0$



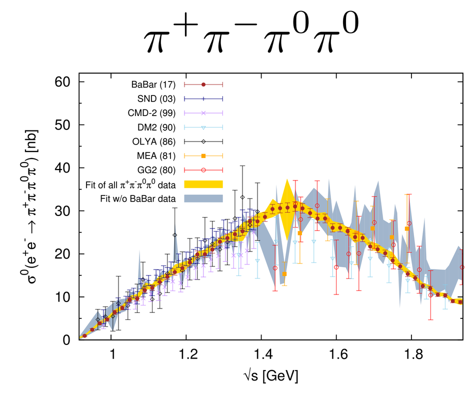
HLMNT11: 47.51 ± 0.99
KNT18: 47.92 ± 0.89

$\pi^+\pi^-\pi^+\pi^-$



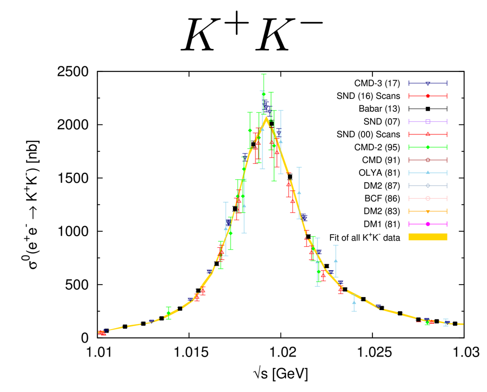
HLMNT11: 14.65 ± 0.47
KNT18: 14.87 ± 0.20

$\pi^+\pi^-\pi^0\pi^0$



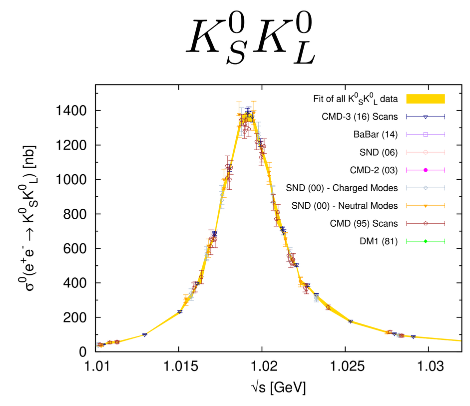
HLMNT11: 20.37 ± 1.26
KNT18: 19.39 ± 0.78

K^+K^-



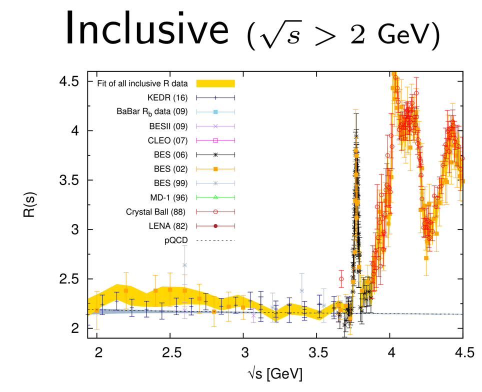
HLMNT11: 22.15 ± 0.46
KNT18: 23.03 ± 0.22

$K_S^0K_L^0$



HLMNT11: 13.33 ± 0.16
KNT18: 13.04 ± 0.19

Inclusive ($\sqrt{s} > 2$ GeV)



HLMNT11: 41.40 ± 0.87
KNT18: 41.27 ± 0.62

Conclusions

Results | KNT18 update

KNT18 $a_\mu^{\rm had,\,VP}$ update

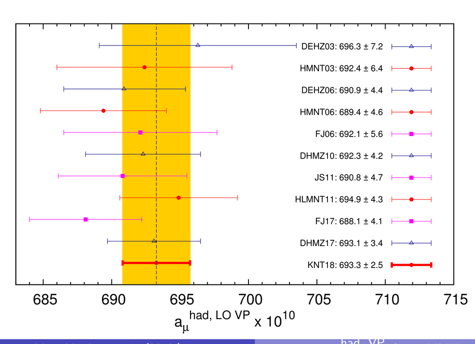
HLMNT(11): 694.91 ± 4.27

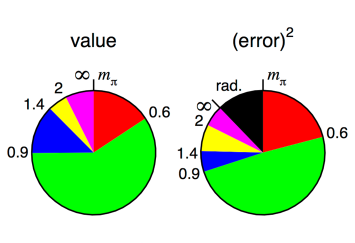
↓

This work: $a_\mu^{\rm had,\,LO\,VP} = 693.27 \pm 1.19_{\rm stat} \pm 2.01_{\rm sys} \pm 0.22_{\rm vp} \pm 0.71_{\rm fsr}$
$= 693.27 \pm 2.34_{\rm exp} \pm 0.74_{\rm rad}$
$= 693.27 \pm 2.46_{\rm tot}$

$a_\mu^{\rm had,\,NLO\,VP} = -9.82 \pm 0.04_{\rm tot}$

⇒ Accuracy better then 0.4% (uncertainties include all available correlations)





⇒ 2π dominance

Alex Keshavarzi (UoL) | $a_\mu^{\rm had,\,VP}$ from KNT18 | 12th February 2018 14 / 22

Exp. value of muon g-2 vs SM prediction

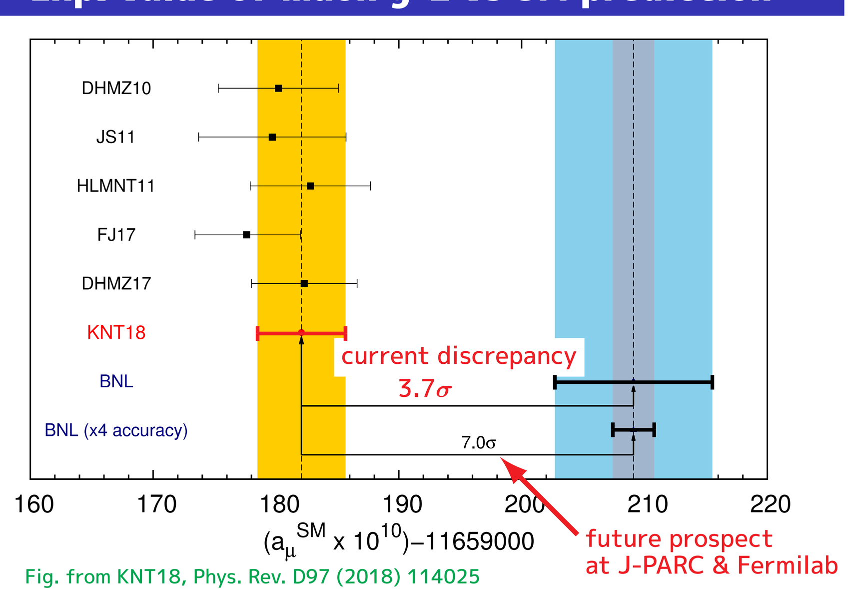


Fig. from KNT18, Phys. Rev. D97 (2018) 114025

Our main conclusion:
The SM prediction is deviated from the experimental value by 3.7σ.

Reference

A. Keshavarzi, DN and T. Teubner, Phys. Rev. **D97** (2018) 114025