Relativistic quantum molecular dynamics for heavy-ion collisions at high baryon density region

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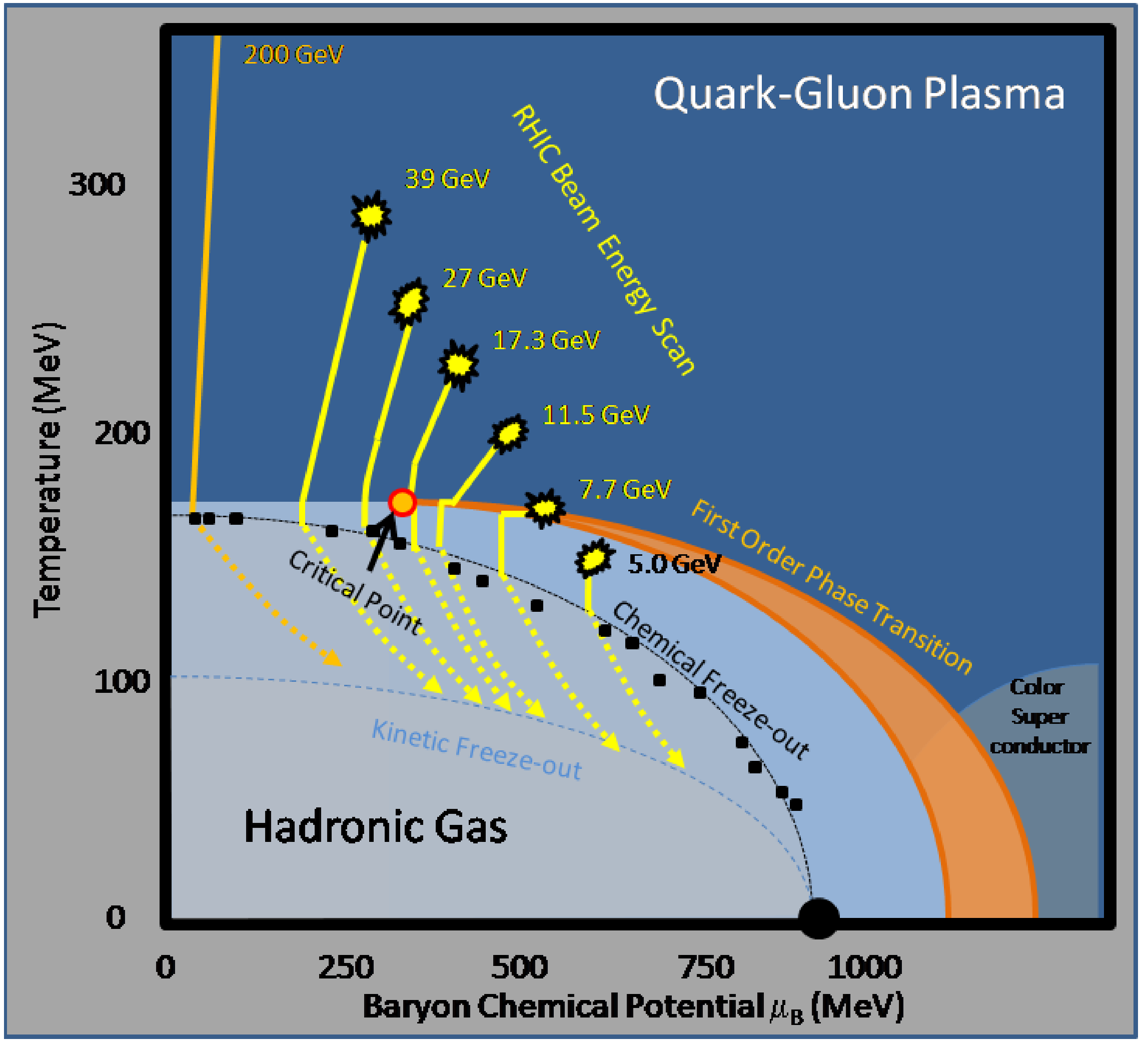
- Introduction
- RQMD with relativistic mean-field theory (RMF)
  effects of delta matter transition
- Flow, cluster formation, and Baryon number fluctuations

XQCD 2019 (The 17th International Conference on QCD in Extreme Conditions
Tokyo Campus, University of Tsukuba, 24-26 June 24-26

Search for the QCD equation of state (EoS) by the beam energy scan



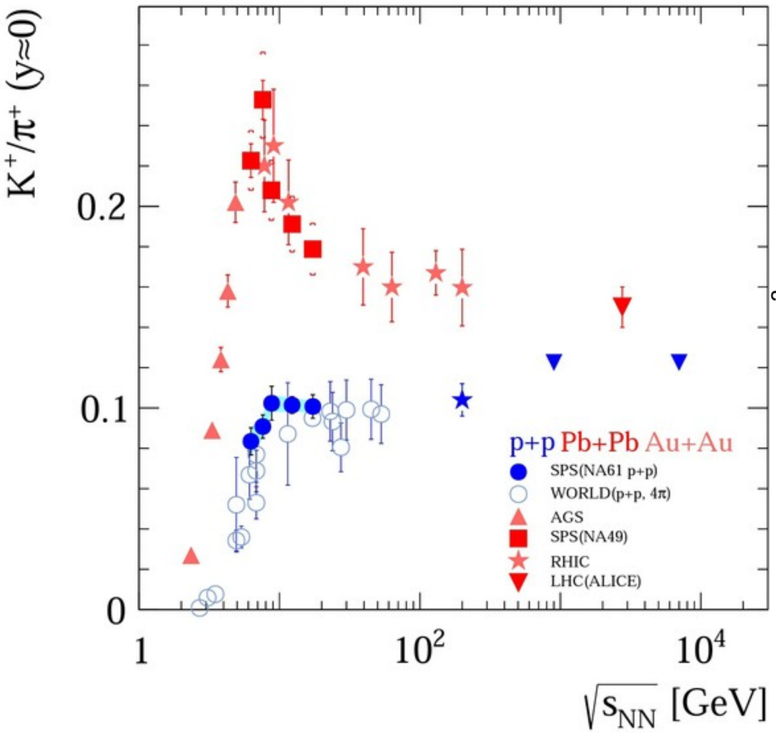
RHIC BES and NA61/SHINEprovides valuable information for the QCD phase structure at high baryon densities. New experiments such as FAIR, J-PARC, NICA is planning.

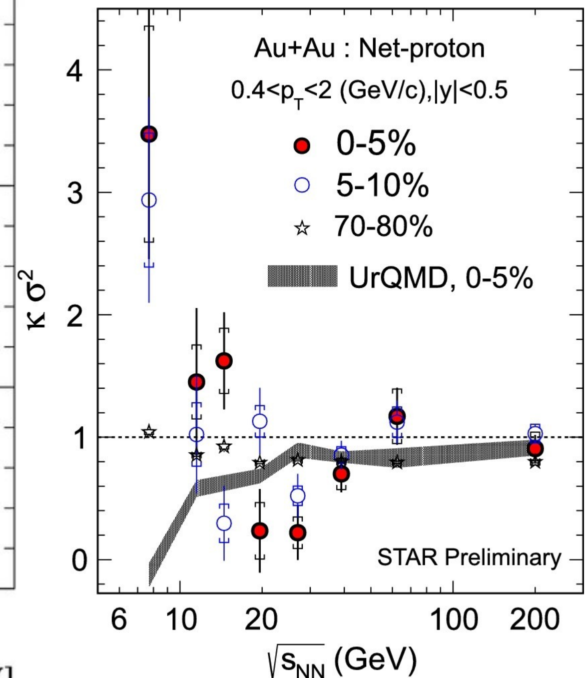
At RHIC/LHC energies, there is significant progress in developing dynamical models to simulate space time evolution of heavy-ion colliions.

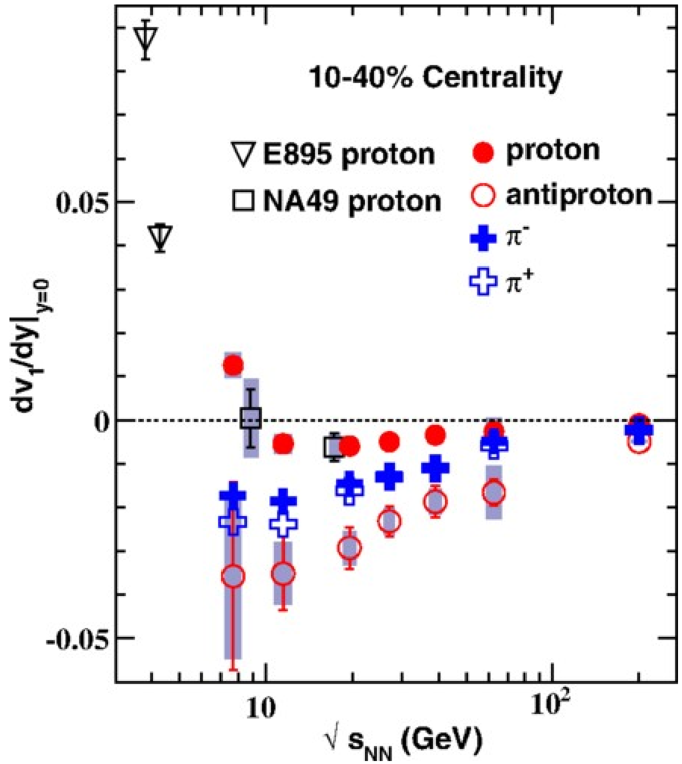
How do we construct dynamical models which can simulate heavy-ion collisions at high baryon density?

Lattice QCD has not covered the J-PARC, FAIR, NICA energy regions.

Non-monotonic structures in beam energy dependence







NA49

X. Luo QM15

Onset of de-confinement?

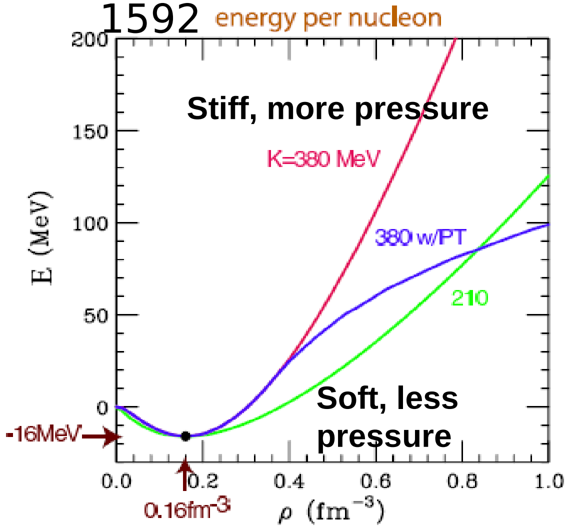
First-order phase transition?
End point?

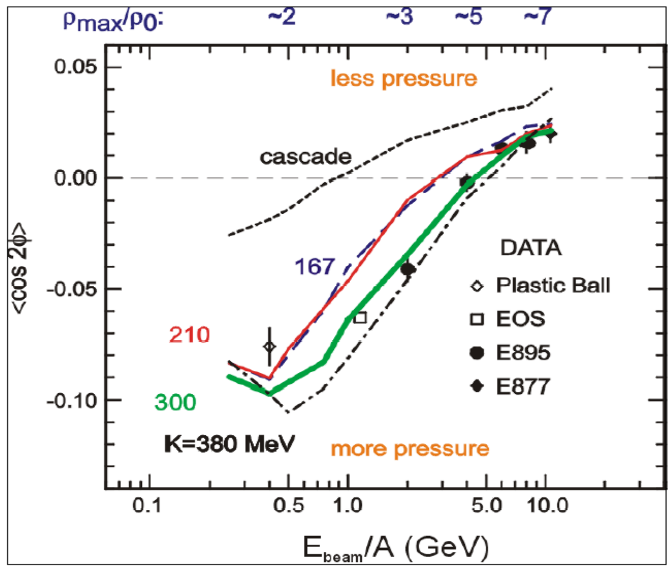
$$v_1 = \langle \cos\phi \rangle = \left\langle \frac{p_x}{\sqrt{p_x^2 + p_y^2}} \right\rangle$$

L. Adamczyk et al. (STAR Collaboration)
Phys. Rev. Lett. 112, 162301 – Published 23 April 2014

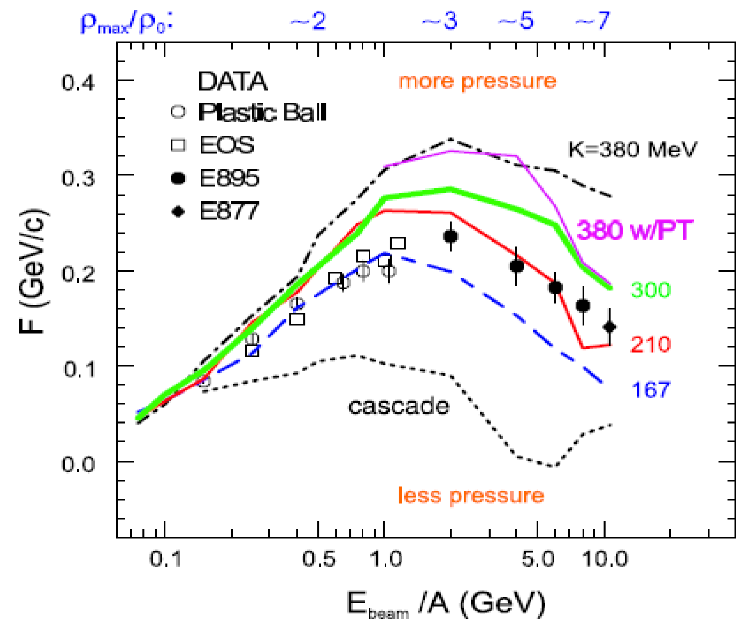
Determination of EOS at high density from an anisotropic flow in heavy ion collisions

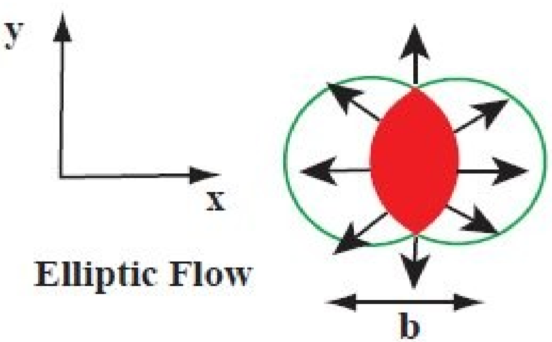
P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002) 1592



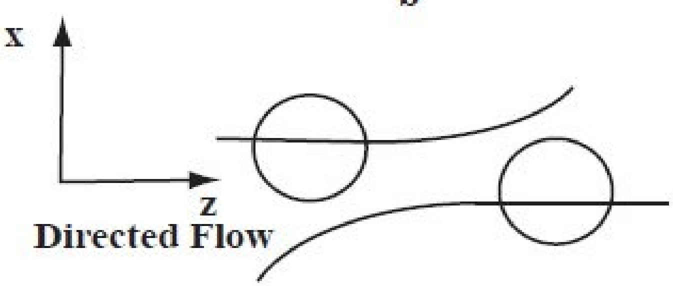


In-plane flow, v1





$$v_2 = \langle \cos 2\phi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \qquad F = \left. \frac{d\langle p_x/A \rangle}{d(y/y_{cm})} \right|_{y/y_{cm}=1}$$



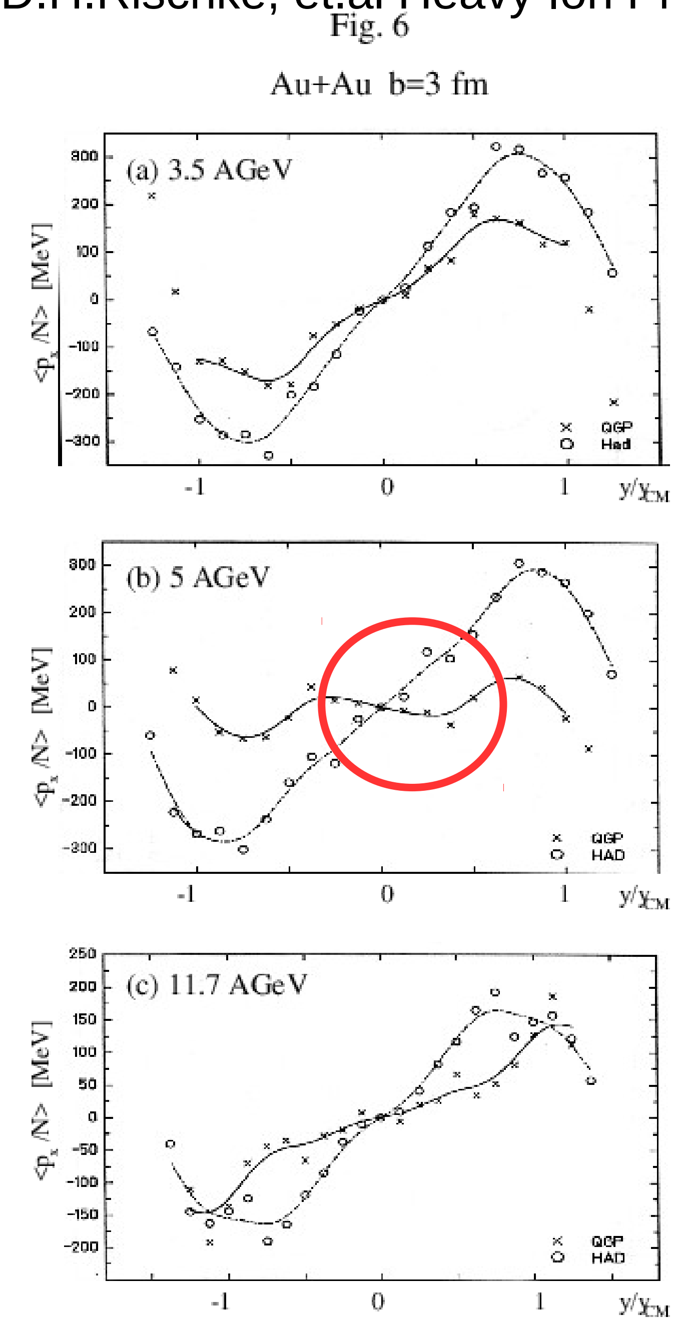
BUU Transport model predicts strong sensitivities of EOS on the directed and elliptic flows.

See recent work on v3:P.Hillmann, J.Phys.G45(2018)085101

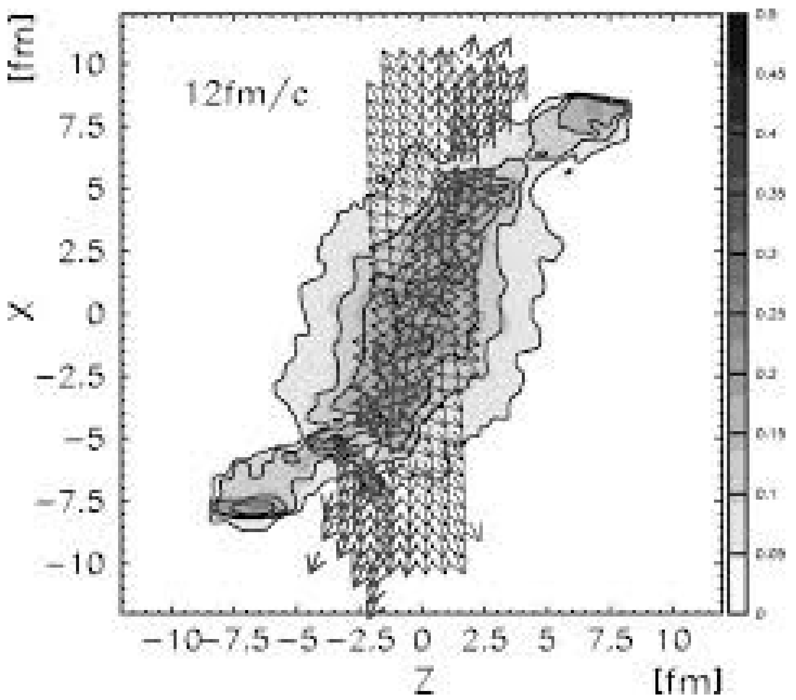
Negative slope: 1st order phase transiton signal?

D.H.Rischke, et.al Heavy Ion Phys.1, 309(1995)

L. P. Csernai, D. Röhrich, PLB 45 (1999), 454.



J.Brachmann,et.al. Phys.Rev.C61 (2000)



EoS with a a first-order phase transition yields tilted ellipsoid in hydro

1st P.T. EoS predicts negative sloope in hydro

How to develop dynamical model?

$\sqrt{s_{NN}} > 30\,\text{GeV}$

Chun Shen and Bjorn Schenke, Phys.Rev. C97 (2018) no.2, 024907

Initial conditions

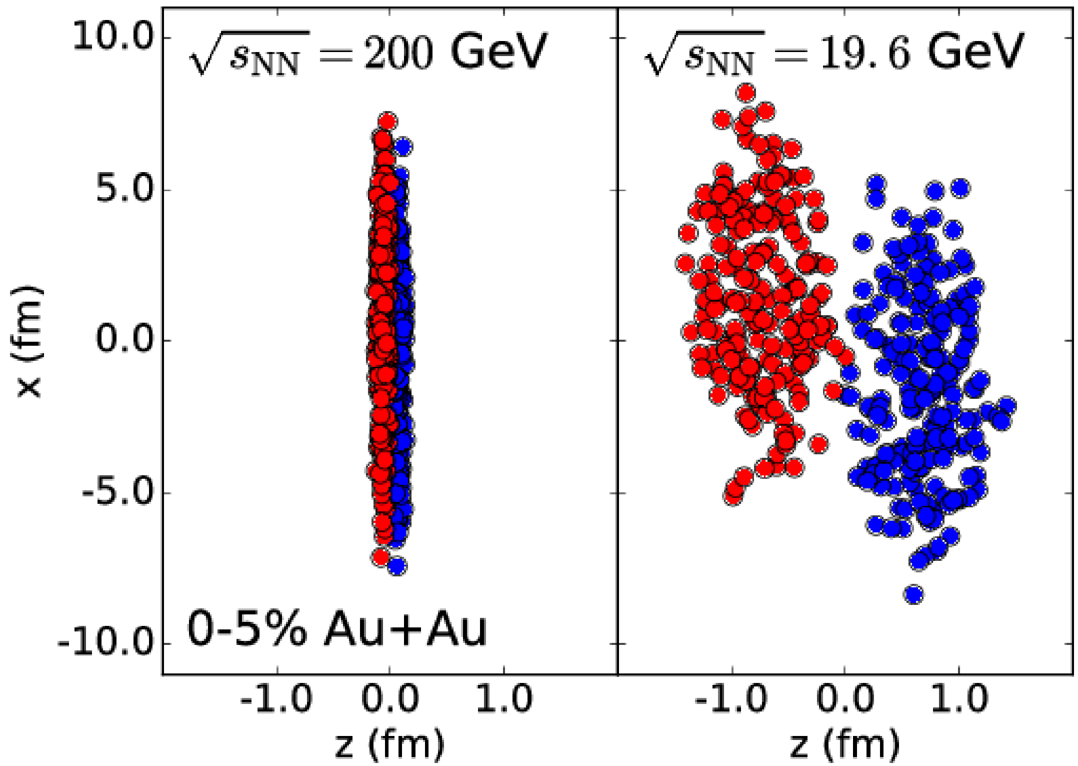
↓

hydrodynamics

↓

Hadron transport

time



$\sqrt{s_{NN}} < 30\,\text{GeV}$

Initial conditions ⟷ hydrodynamics ⟷ Hadron transport

Space-time evolution of all these has to be solved simultaneously.

New dynamically integrated transport model

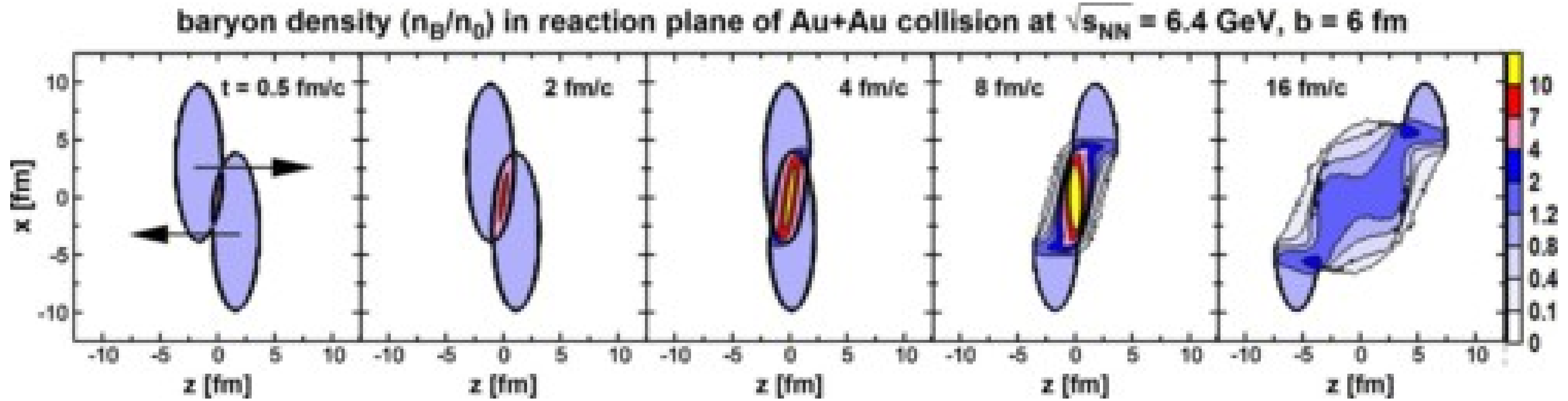
Solve the space-time evolution of both particles and fluids through the source term:

$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_B^\mu = \rho_B \quad p_\mu \partial^\mu f(x,p) = I_{coll} + S$$

Y. Akamatsu, M. Asakawa, T. Hirano, M. Kitazawa,K. Morita, K. Murase, Y. Nara, C. Nonaka, A. Ohnishi, Phys.Rev. C98 (2018) no.2, 024909

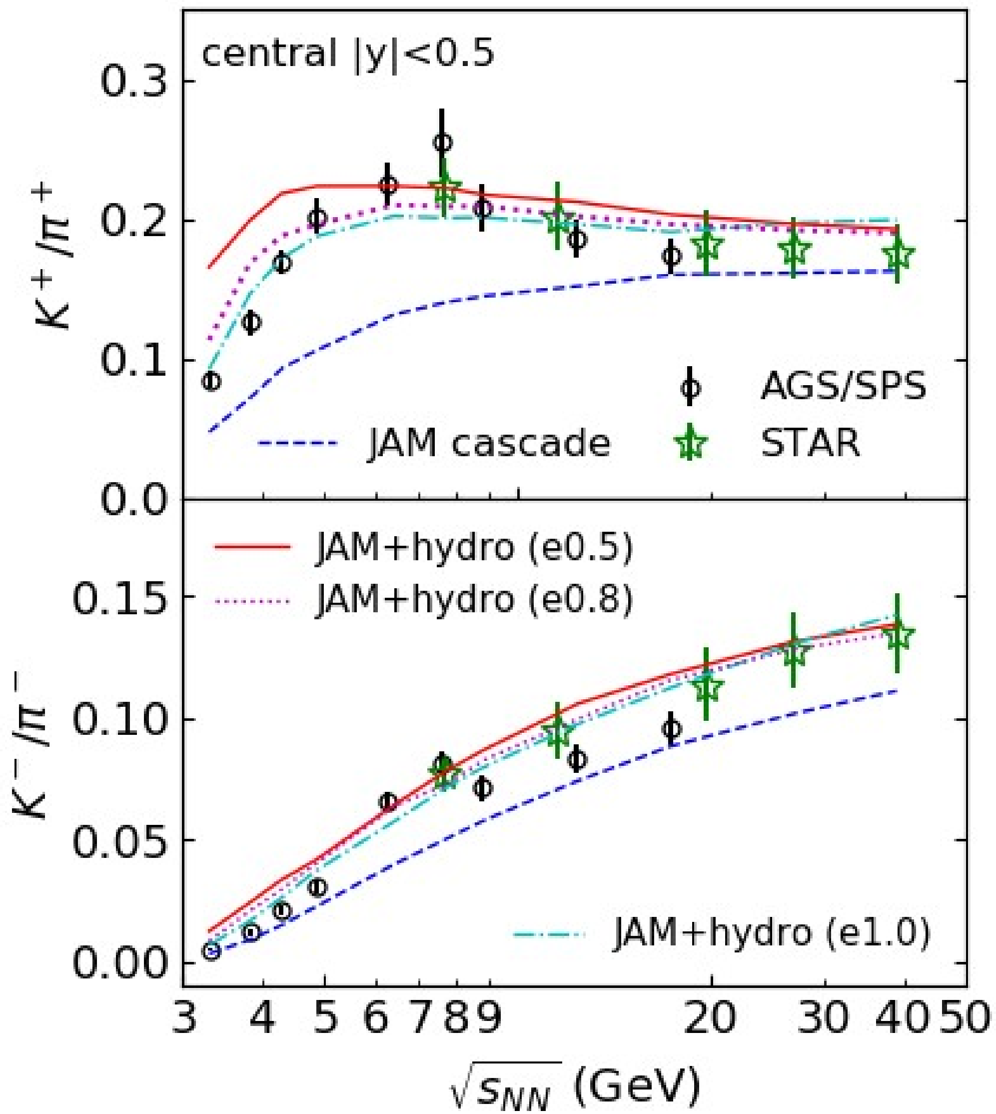
Low density: hadronic transport model JAM

High density: hydrodynamics



Picture from 3FD model: P. Batyuk et.al. PRC94(2016)044817

Beam energy dependence of K/pi ratios from JAM+hydrodynamics



Incomplete thermalization of the system is important for the description of K/pi ratio.

Improvement of the hybrid model

EoS in the hadronic cascade is inconsistent with the EoS in the hydrodynamcis
We need to include consistent EoS in non-equilibrium dynamics.

Hydrodynamics + Cascade model

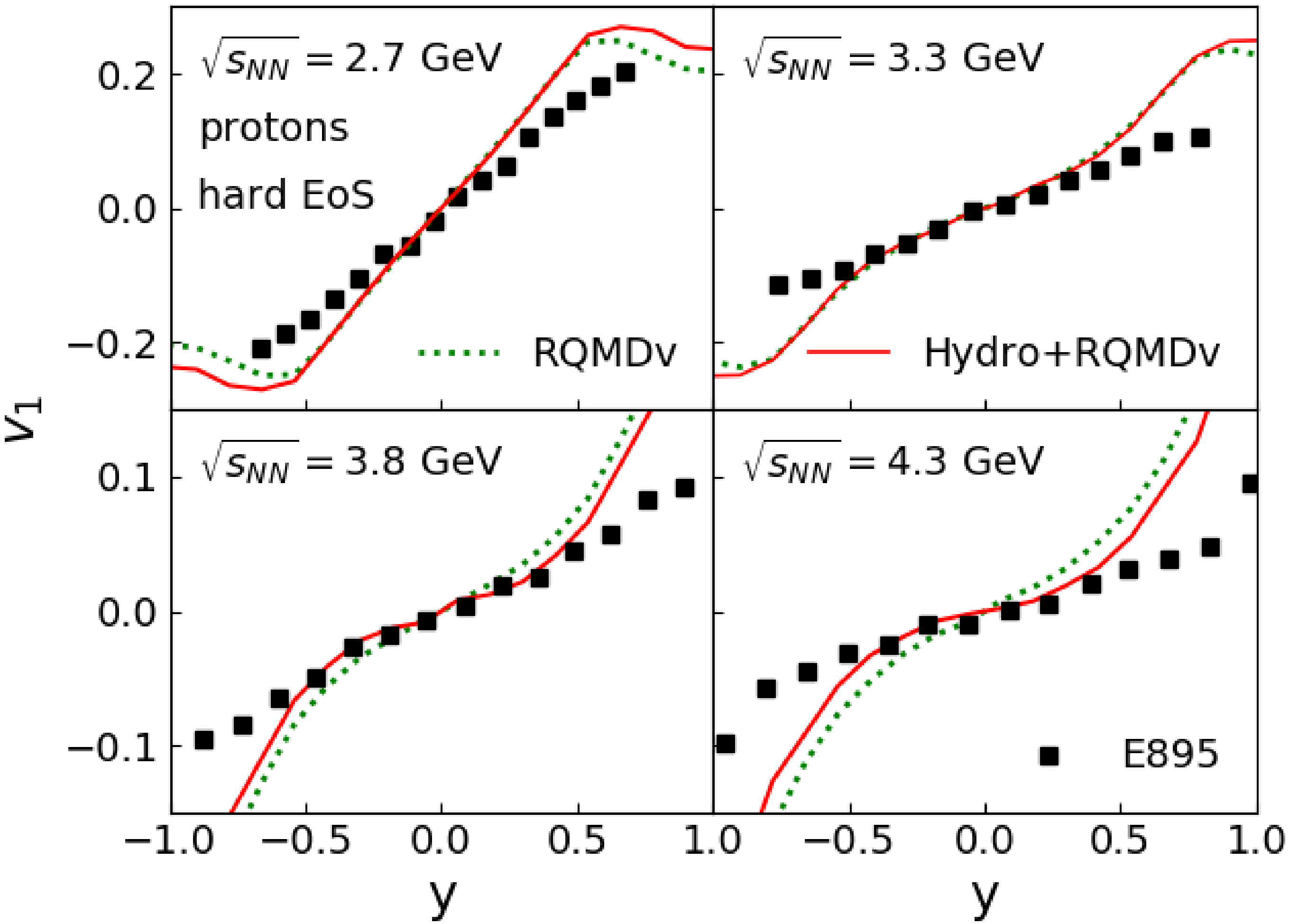
EOS: Hadron Gas

Hydrodynamics + Quantum Molecular Dynamics (QMD)

QMD: N-body non-equilibrium microscopic transport approach

Directed flow from Hydro + QMD

Results from Hydro + RQMDv in which potentials are implemented as a vector.



EOS: 1st-order PT

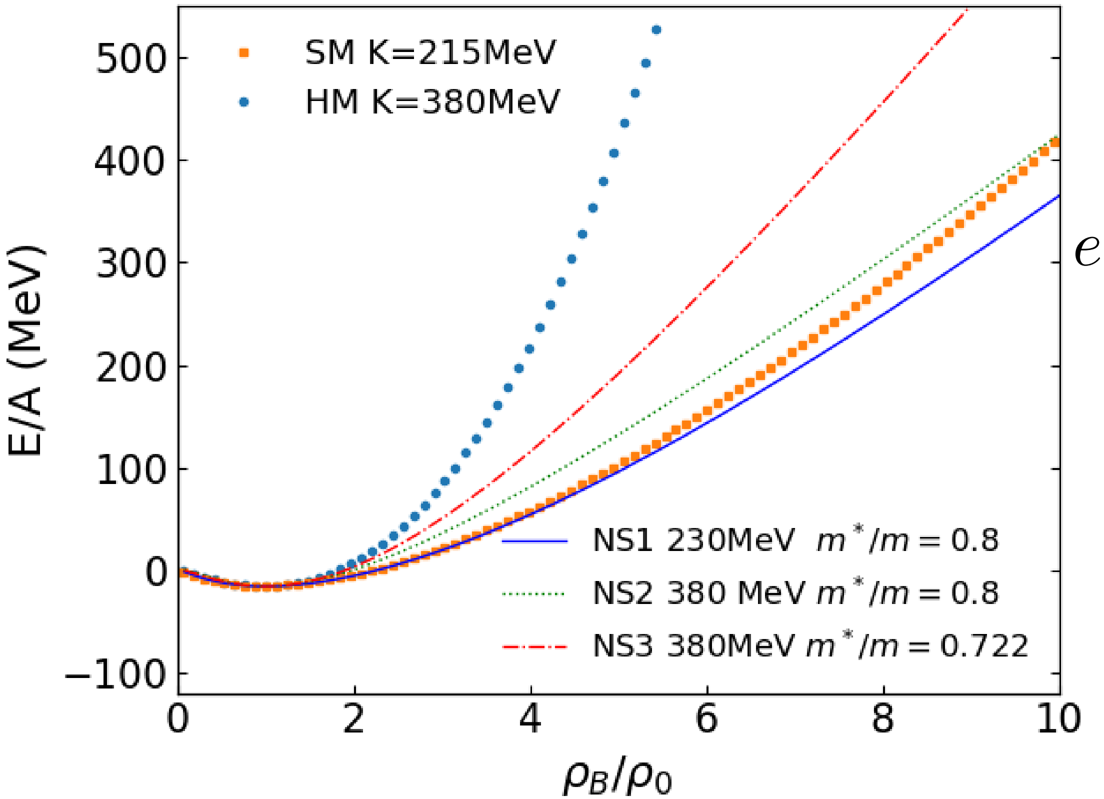
proton flow is positive even if first-order phase transition is included.

Mean-field in the particle phase is very important for the flow.

EOS from the relativistic mean-field theory

The RMF is first employed by RVUU transport models by
C. M. Ko, Q. Li and R. C. Wang, Phys. Rev. Lett. 59, 1084 (1987)
B. Blattel, V. Koch, W. Cassing and U. Mosel, Phys. Rev. C 38, 1767 (1988)

$$[p^*_\mu \partial^\mu_x + (p^*_\nu F^{\mu\nu} + m^* \partial^\nu_x m^*)\partial^{p^*}_\mu] f(x, p^*) = C_{\text{coll}}$$



Non-linear sigma-omega model

$$e = \int d^3p E^* f(p) + \frac{1}{2}\frac{g_v^2}{m_\omega^2}\rho_B^2 + U(\sigma)$$

$$m^* = m - g_s\sigma$$

$$U(\sigma) = \frac{m_\sigma^2}{2}\sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4$$

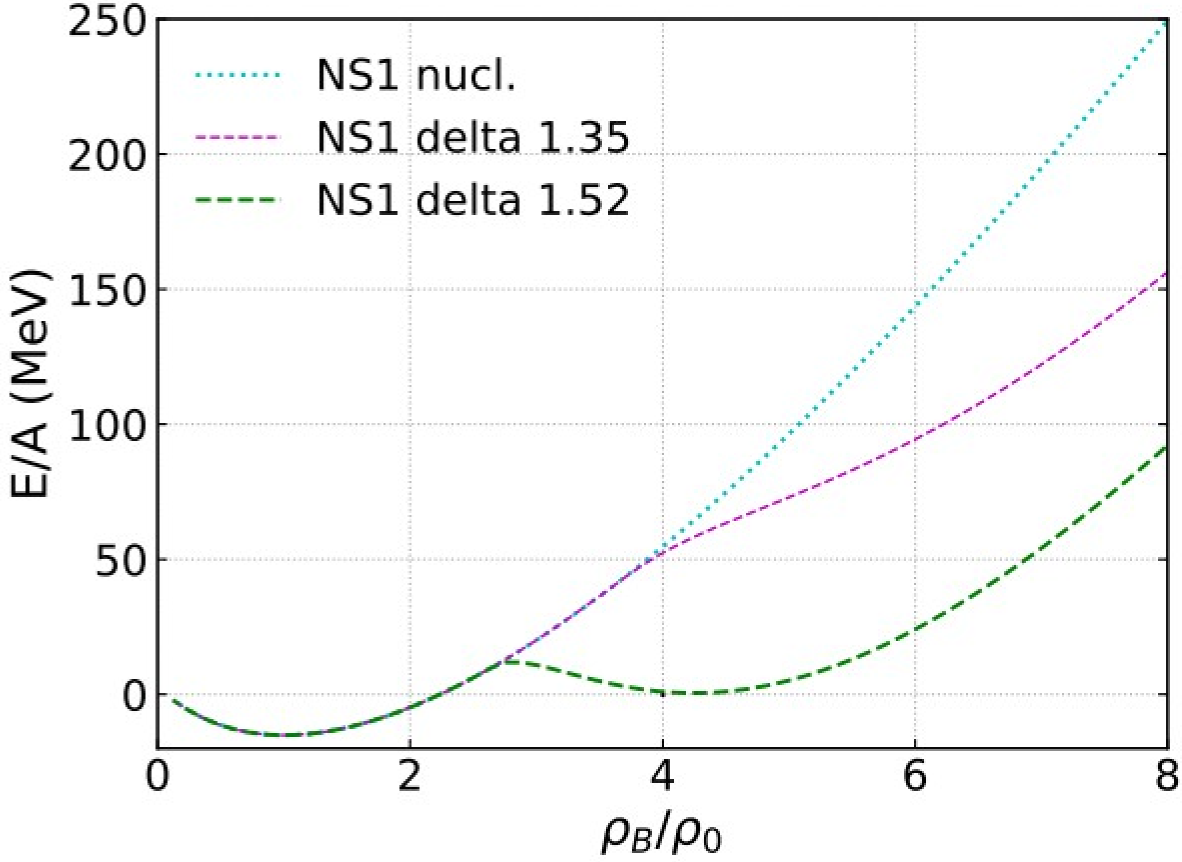
Delta-isomer state in RMF

J. Boguta, Phys. Lett.B109 (1982)251,
B. M.Waldhauser,et. al, PRC36(1987) 1019

$$g_{\Delta\omega} = g_{N\omega}$$

$$g_{\Delta\sigma} = \alpha g_{N\sigma}$$

$$\alpha = 1.35, 1.52$$



The Quantum Molecular Dynamics

Quantum molecular dynamics (QMD) approach is a N-body non-equilibrium theory to describe heavy ion collisions.
J. Aichelin and H. Stoecker, Phys. Lett.B176 (1986)14,
J. Aichelin, Phys. Rep.202 (1991) 233.

Particles are represented by a Gaussian wave packet. $\langle H \rangle = \langle \Phi \left| H \right| \Phi \rangle$

$$\frac{d\boldsymbol{r}_i}{dt} = \frac{\partial \langle H \rangle}{\partial \boldsymbol{p}_i} \quad \frac{\boldsymbol{p}_i}{dt} = -\frac{\partial \langle H \rangle}{\partial \boldsymbol{r}_i}$$

$$\langle H \rangle = \sum_i \frac{\boldsymbol{p}_i^2}{2m_i} + \sum_{i \neq j} \langle V_{ij} \rangle$$

Mean-fields are simulated by the potential interactions,
Collision term is also included to simulate Boltzmann type collisions kernel.

Relativistic quantum molecular dynamics (RQMD) approach

RQMD was developed based on the constrained Hamiltonian dynamics:
H. Sorge, H. Stoecker, W. Greiner, Ann. Phys. 192, 266 (1989).

T. Maruyama, et. al. Prog. Theor. Phys. 96, 263 (1996).

Manifestly covariant way: four-vectors $q_i^\mu, \quad p_i^\mu \quad (i = 1, N)$

For the description of N-particle system, we have 8N dimension.
In order to reduced the dimension from 8N to 6N, we need 2N constraints.

Hamiltonian is a linear combinations of the constraints, and equations of motion are given by

$$H = \sum_i \lambda_i \phi_i \qquad \frac{dq_i}{d\tau} = \{H, q_i\}, \quad \frac{dp_i}{d\tau} = \{H, p_i\}$$

2N constraints: On-mass shell condition and time fixation.

$$\phi_i = (p_i - V_i)^2 + (m_i - S_i)^2 = p_i^{*2} + m_i^{*2} = 0, \quad (i = 1, \cdots, N)$$

JAM Mean-field mode summary

$$H = \sum_i^N \sqrt{(\boldsymbol{p}_i - \boldsymbol{V}_i)^2 + (m_i - S_i)^2} + V_i^0$$

$$\dot{\boldsymbol{x}}_i = \frac{\boldsymbol{p}_i^*}{p_i^{*0}} + \sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial \boldsymbol{p}_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial \boldsymbol{p}_i} \right), \quad \dot{\boldsymbol{p}}_i = -\sum_j \left(\frac{m_j^*}{p_j^{*0}} \frac{\partial m_j^*}{\partial \boldsymbol{r}_i} + v_j^{*\mu} \frac{\partial V_{j\mu}}{\partial \boldsymbol{r}_i} \right)$$

V_i^μ: ω-field S_i: σ-field

$$m_i^* = m_i - g_s \sigma_i, \quad V^\mu = g_v \omega_i^\mu$$

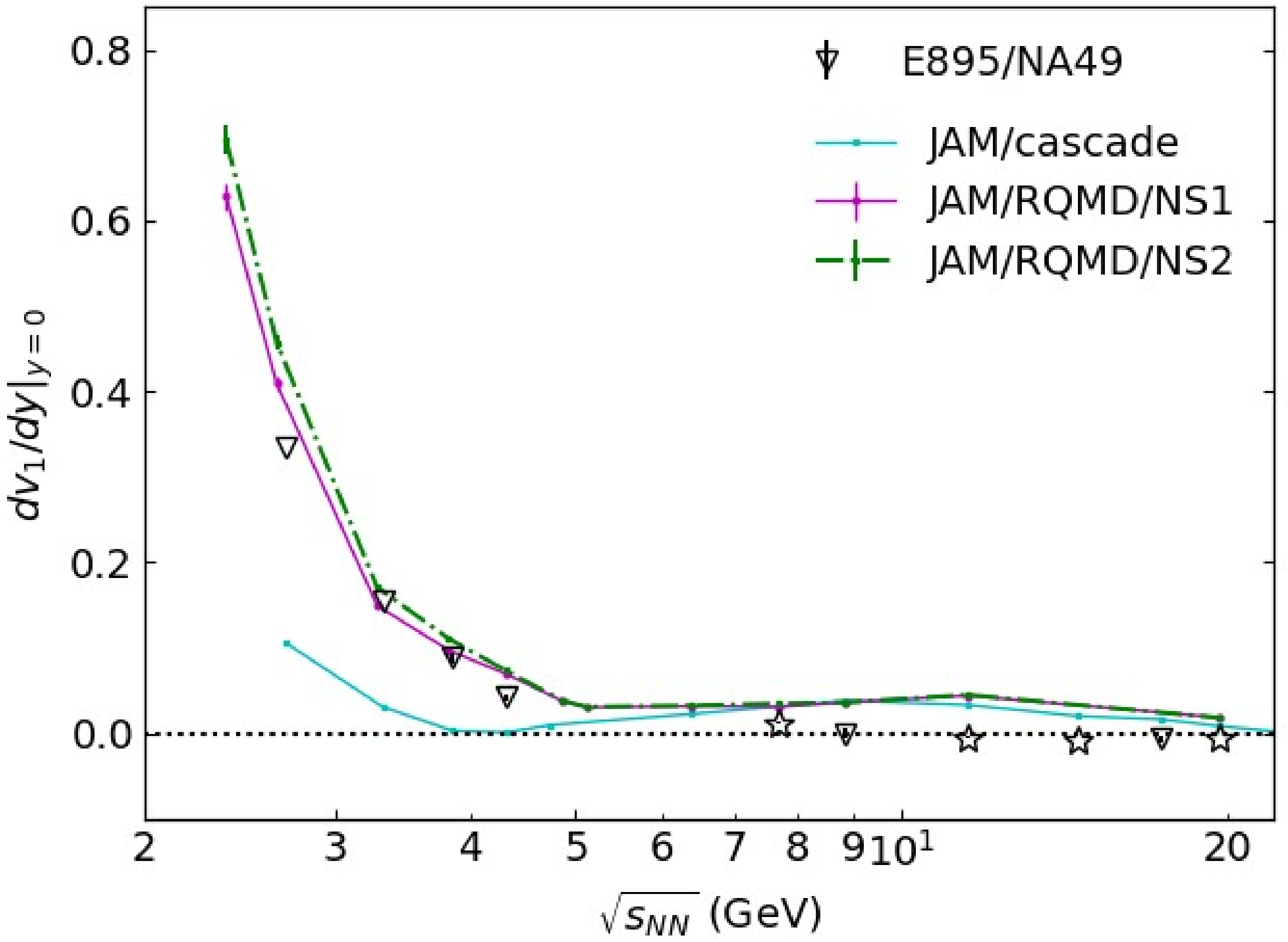
$$m_\sigma^2 \sigma + g_2 \sigma^2 + g_3 \sigma^3 = g_s \rho_s(i) \qquad m_\omega^2 \omega^\mu = g_v J_B^\mu(i)$$

$$\rho_s(i) = \sum_{i \neq j} \frac{m_j^*}{p_j^{0*}} \rho_{ij}, \quad J_B^\mu(i) = \sum_{i \neq j} B_j \frac{p_j^{*\mu}}{p_j^{0*}} \rho_{ij}$$

$$\rho_{ij} = \frac{\gamma_{ij}}{(4\pi L)^{3/2}} \exp(q_{Tij}^2 / 4L)$$

V1 from JAM/RQMDsv mode

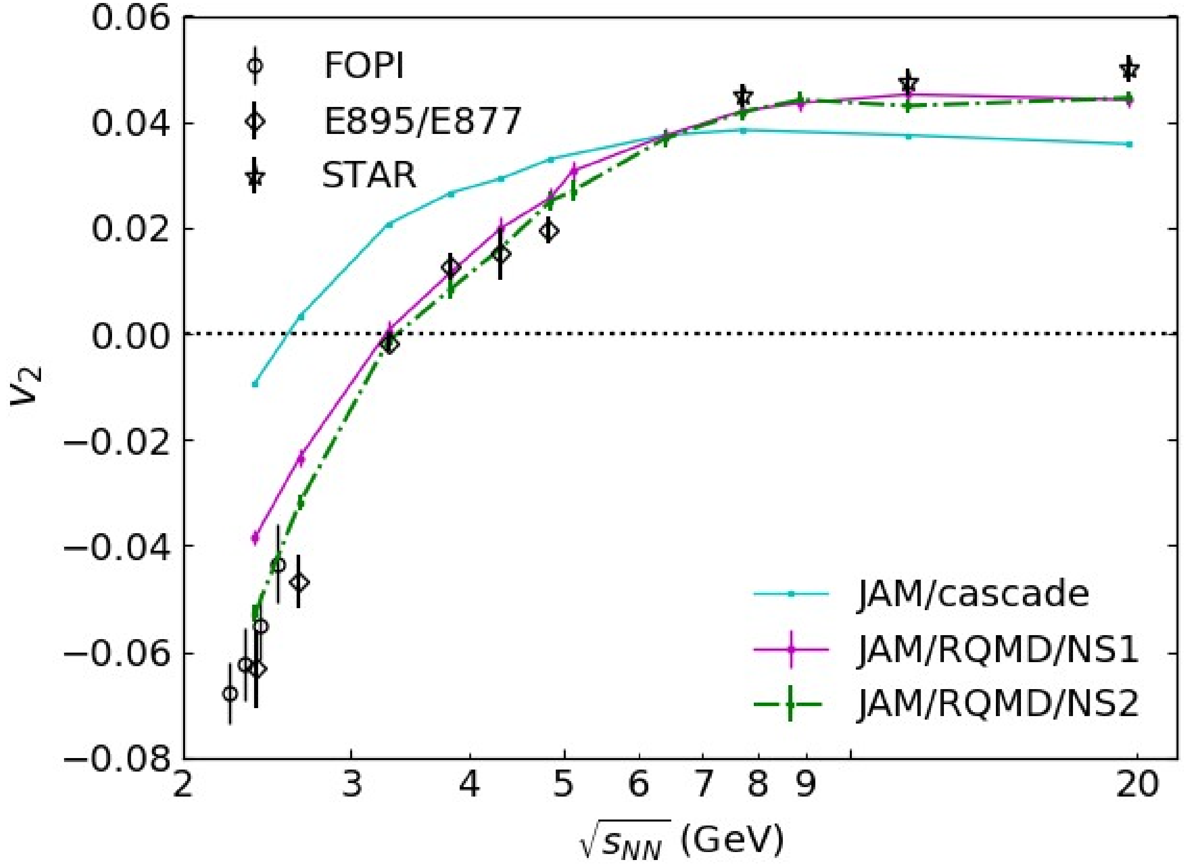
RQMD with the sigma-omega model



Y. Nara and H. Stoecker, arXiv:1906.03537 [nucl-th]

V2 from JAM/RQMDsv

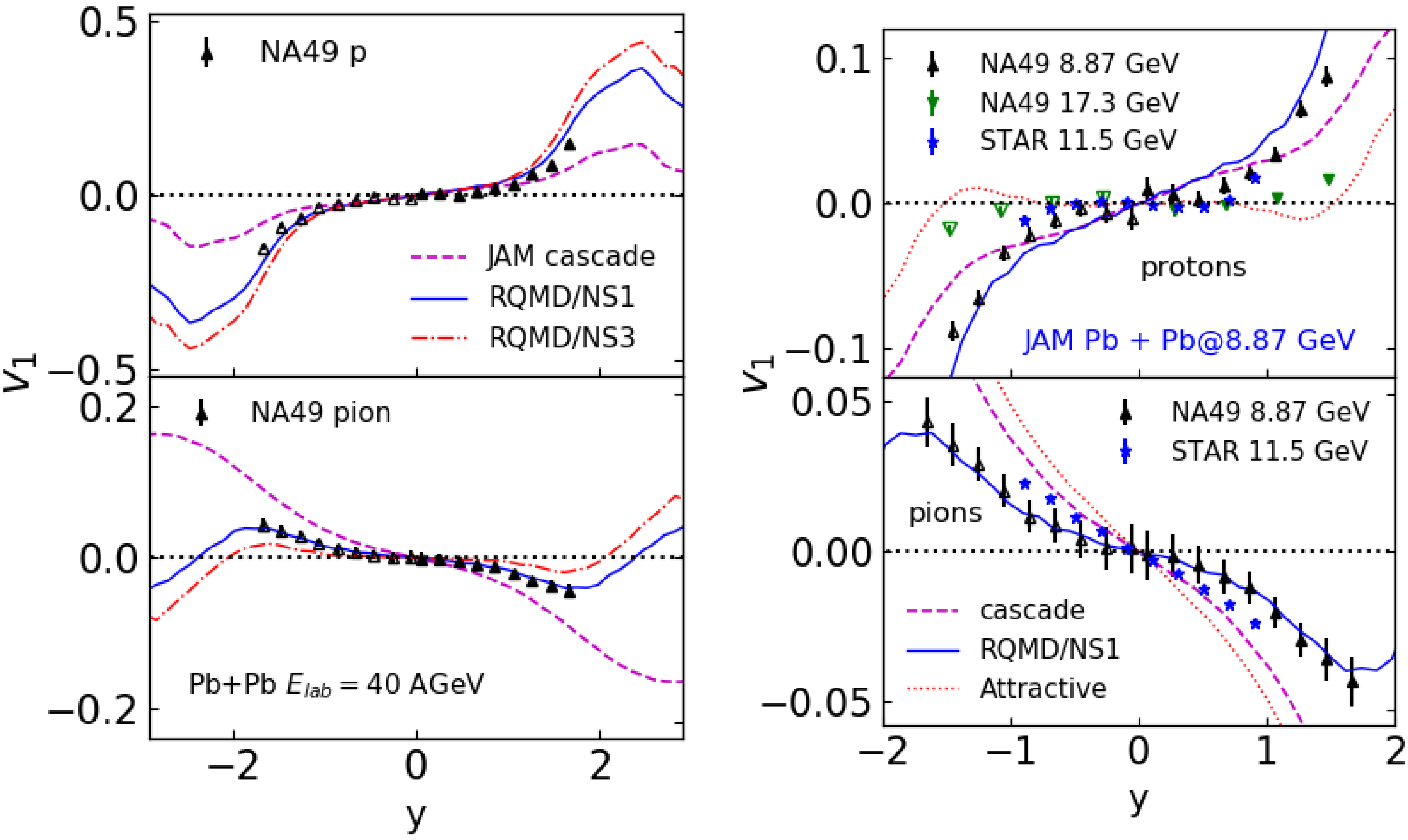
RQMD with the sigma-omega model



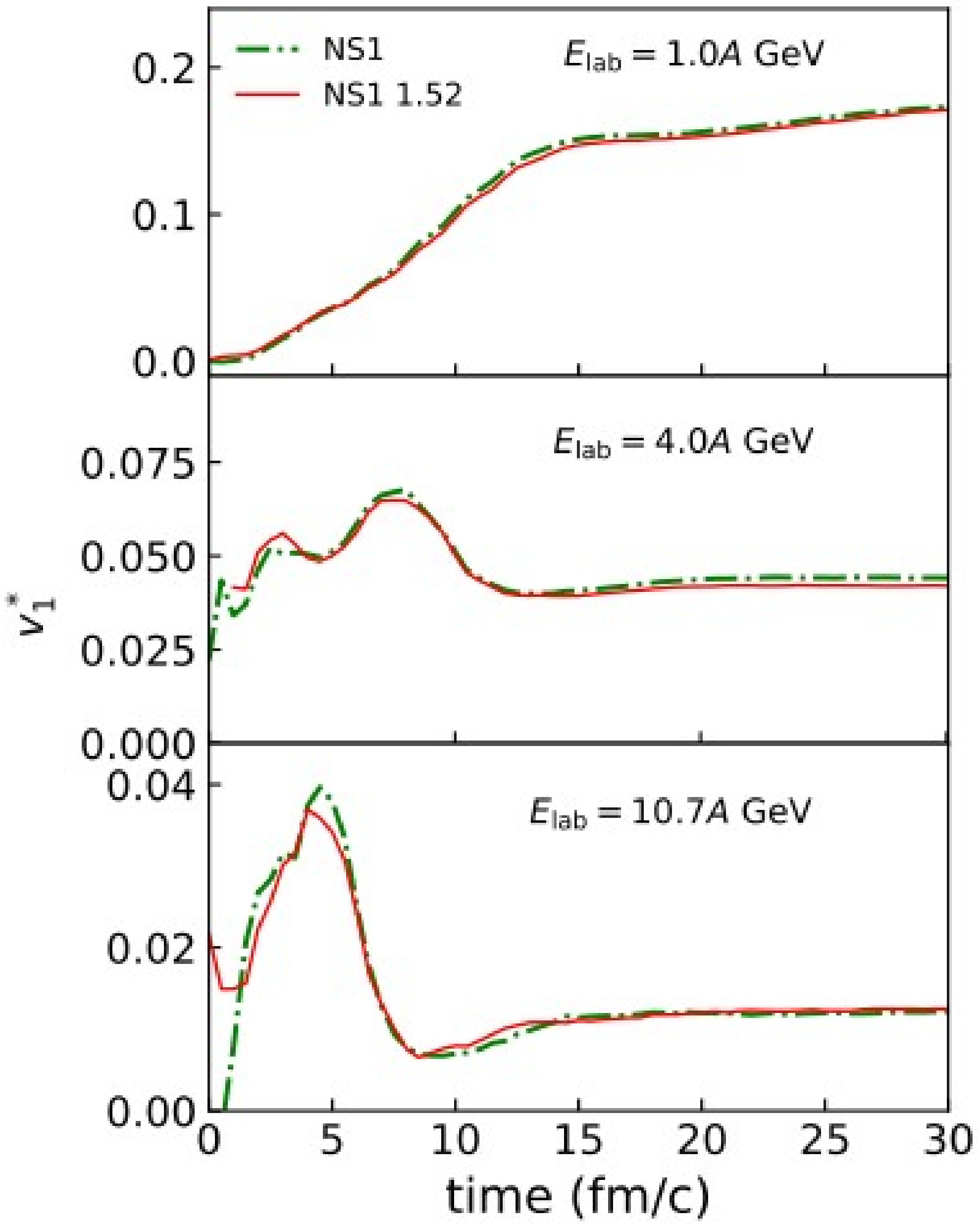
Y. Nara and H. Stoecker, arXiv:1906.03537 [nucl-th]

Rapidity dependence of V1 at SPS

JAM/RQMD with the sigma-omega model



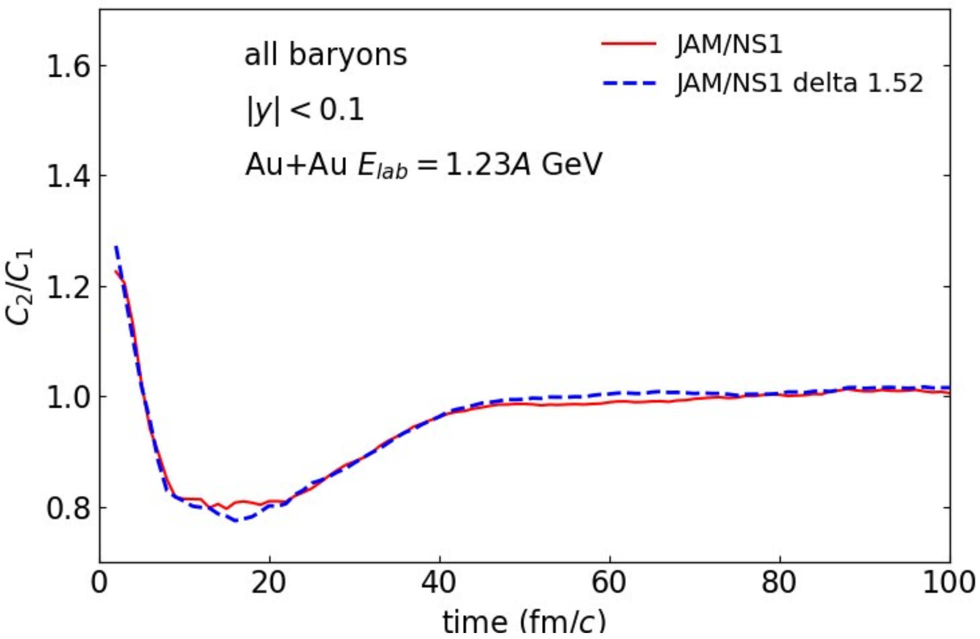
Time evolution of directed flow

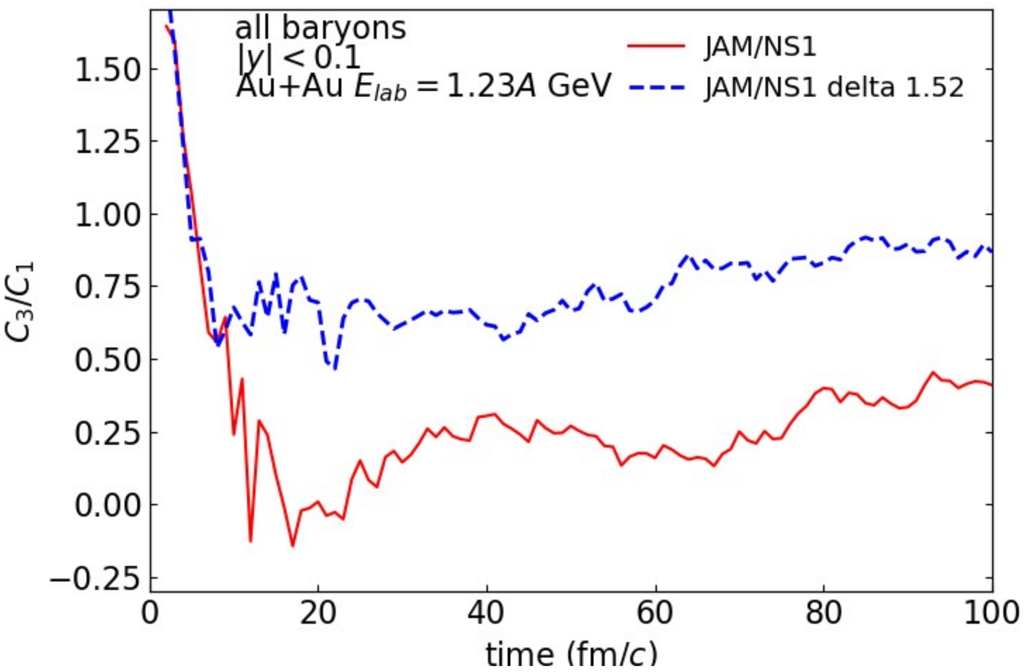


No effects of delta-isomer on the directed flow v1.

In principle, we should also modify collision term for the consistency with the mean-field part.

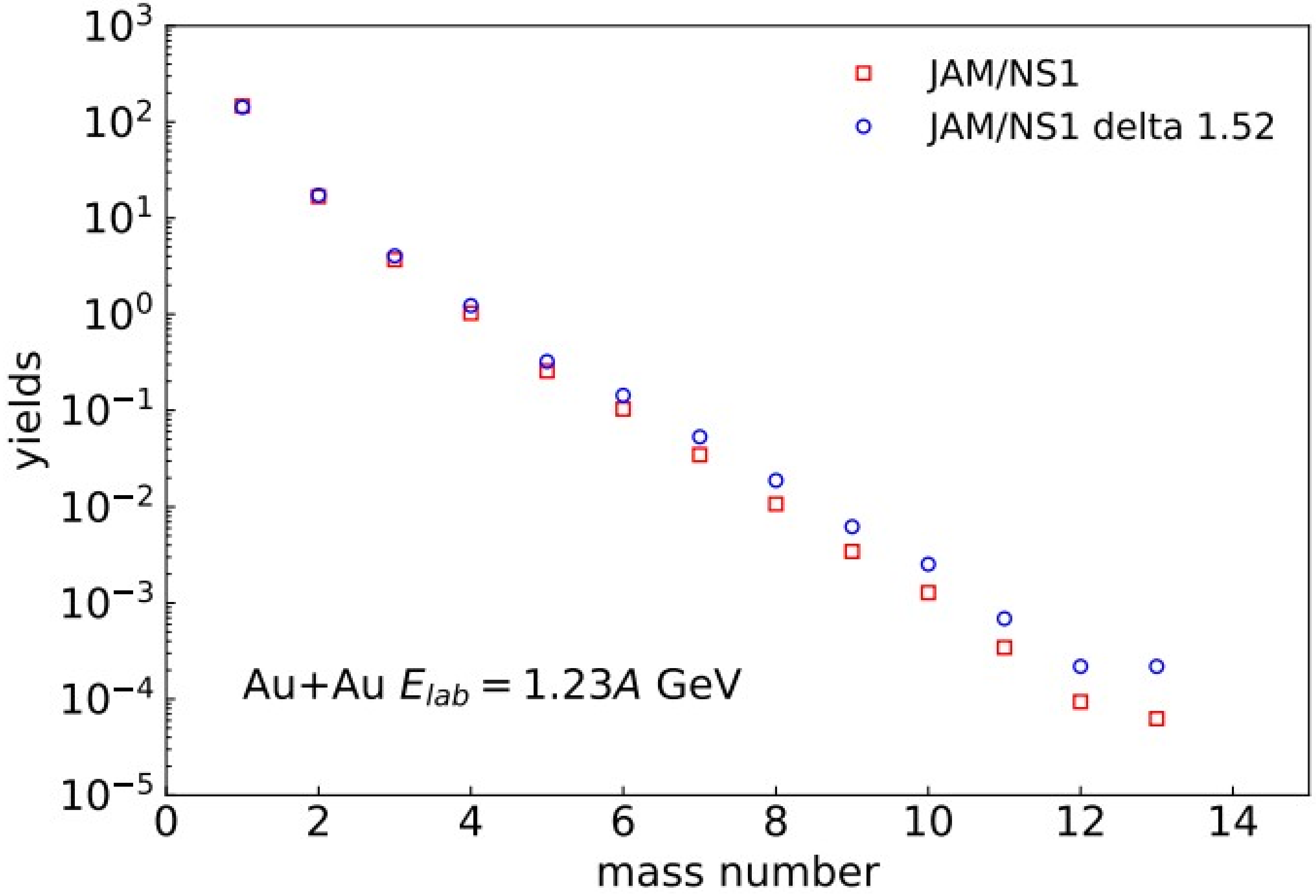
Time evolution of c2/c1 and c3/c1





$$C_1 = \langle N \rangle,\ C_n = \langle (N - \langle N \rangle)^n \rangle$$

Nuclear cluster formation



Summary

- We extend the JAM+hydro approach by including the EoS effects in the non-equilibrium phase within the QMD approach: HyQMD.

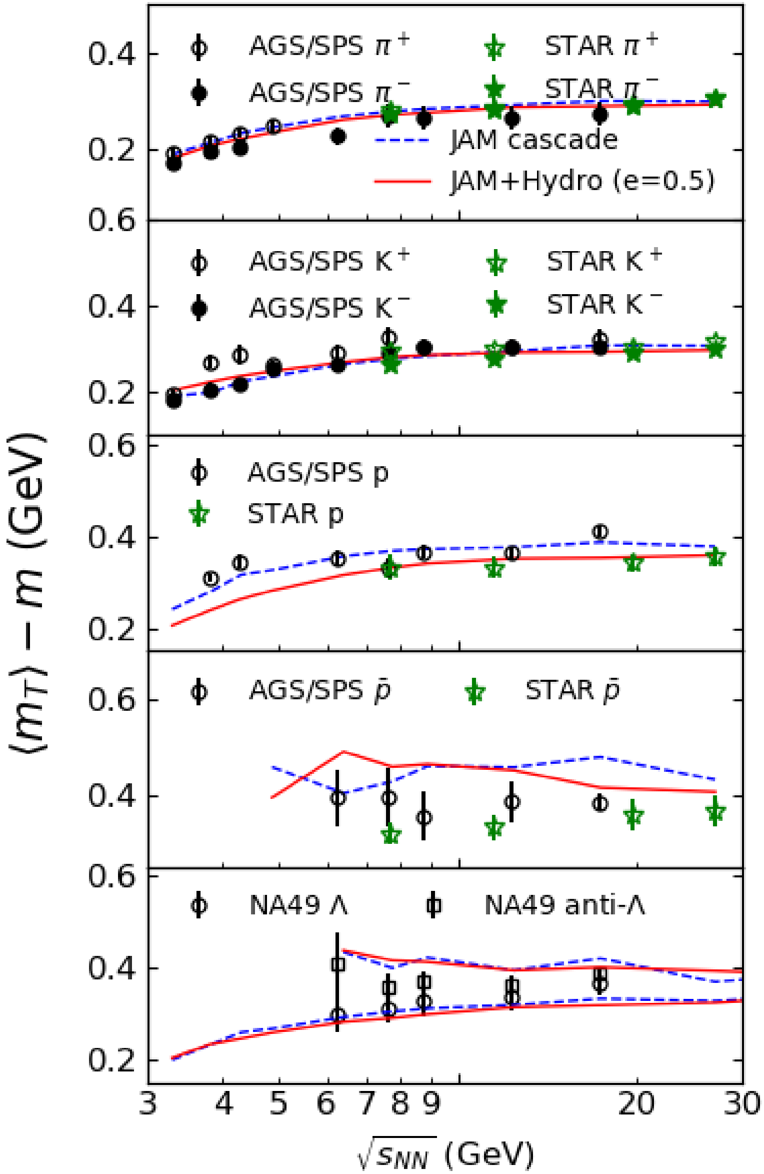
- Relativistic quantum molecular dynamics in JAM is extended by implementing the sigma-omega interactions.

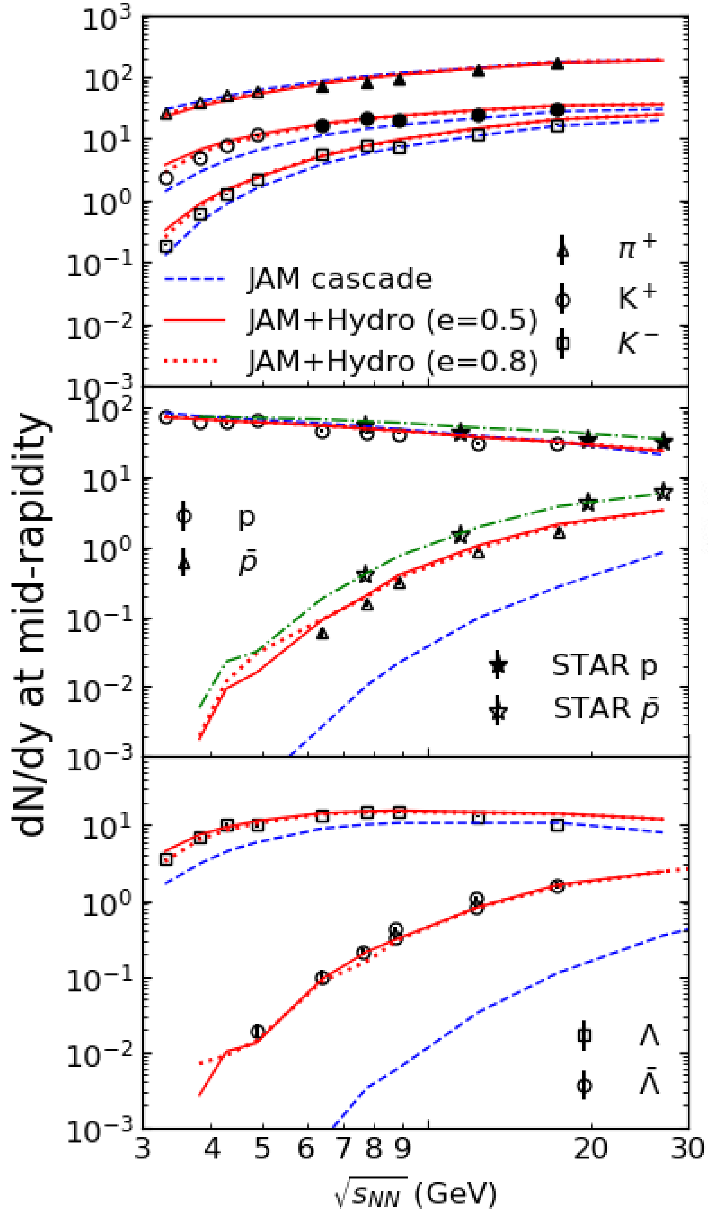
- Description of collective flows are significantly improved over non-relativistic Skyrme type potential.

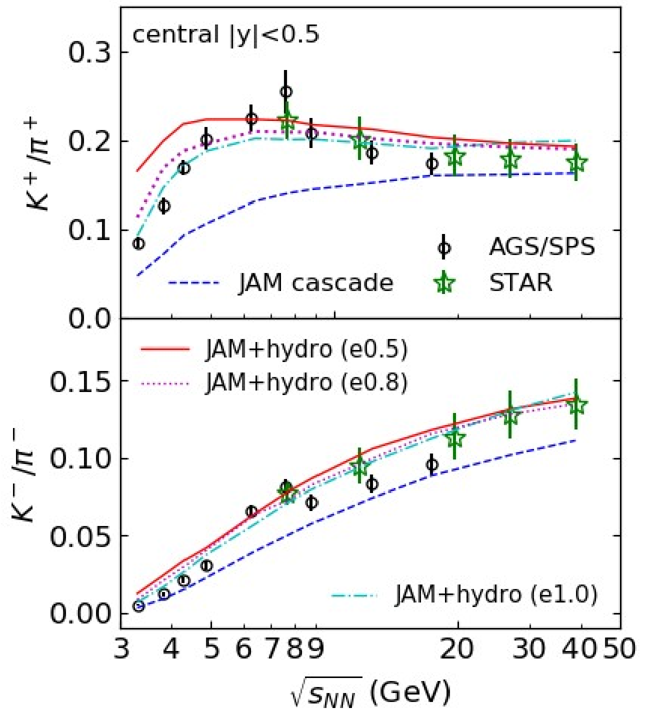
- Effects of Delta-isomer state on the flow, cluster formation, and baryon number fluctuations are studied within RQMD.

- RQMD can be applied for the description of final hadron gas stage at RHIC/LHC energeis.

Beam energy dependence of transverse mass and multiplicities from a new integrated model



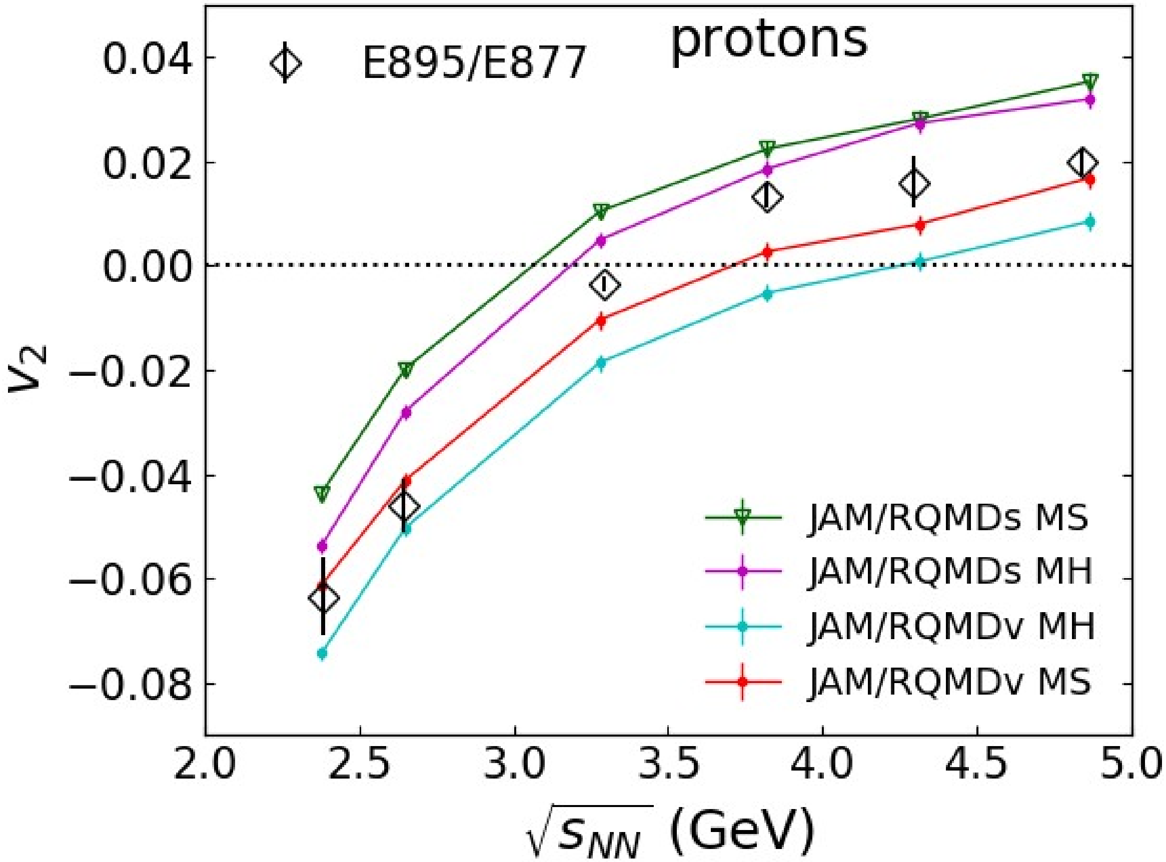




Significant improvements of strangeness and anti-baryon productions.

EoS dependence on v2

Mom. Dep. Soft EoS (K=270MeV), hard (K=370 MeV)

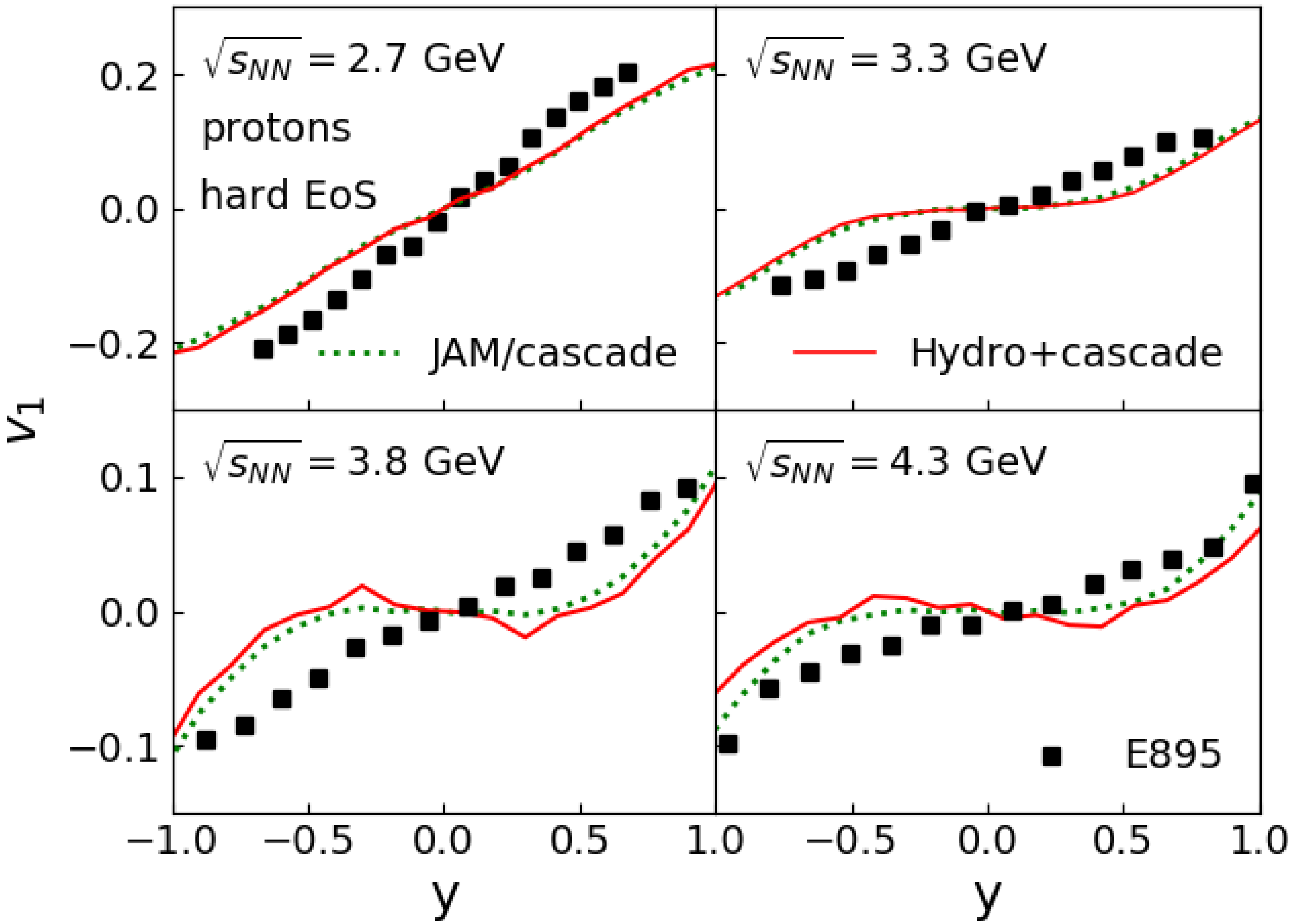


Skyrme type potential cannot explain the excitation function of v2.

$$V_i = \frac{\alpha}{2\rho_0}\rho_i + \frac{\beta}{(1+\gamma)\rho_0^\gamma}\rho_i^\gamma + \sum_{k=1,2}\frac{C_{ex}^{(k)}}{2\rho_0}\sum_{j\neq i}\frac{1}{1+[p_{ij}/\mu_k]^2}\rho_{ij}$$

V1 from the Hydro + JAM/cascade model

V1 from the Hydro+JAM/cascade mode is the same as that of cascade calculations.



Single particle potential:

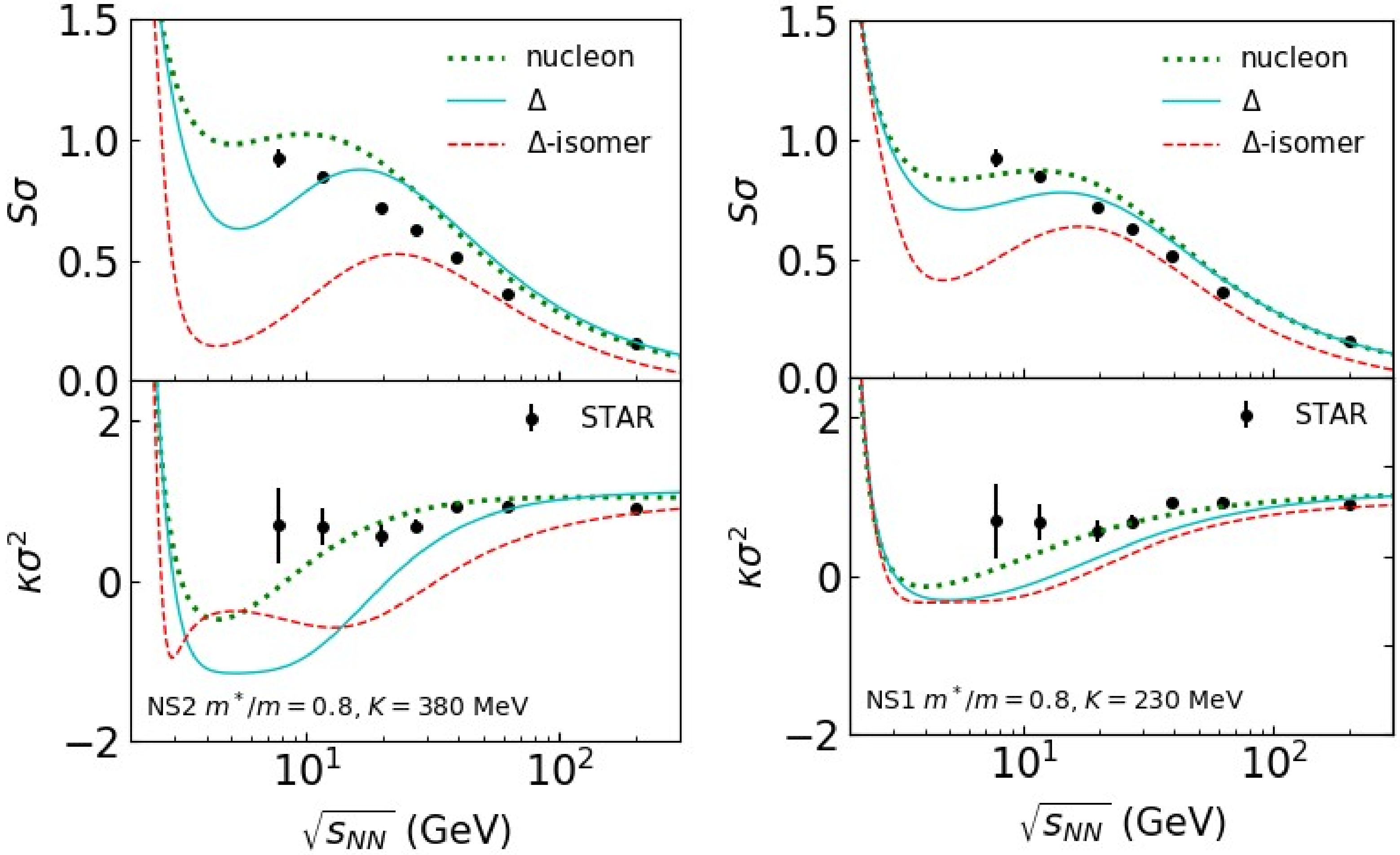
$$V_i = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Hard EoS: K=380MeV
+ first-order PT (Bag model)

Effects of Delta-isomer state on kurtosis

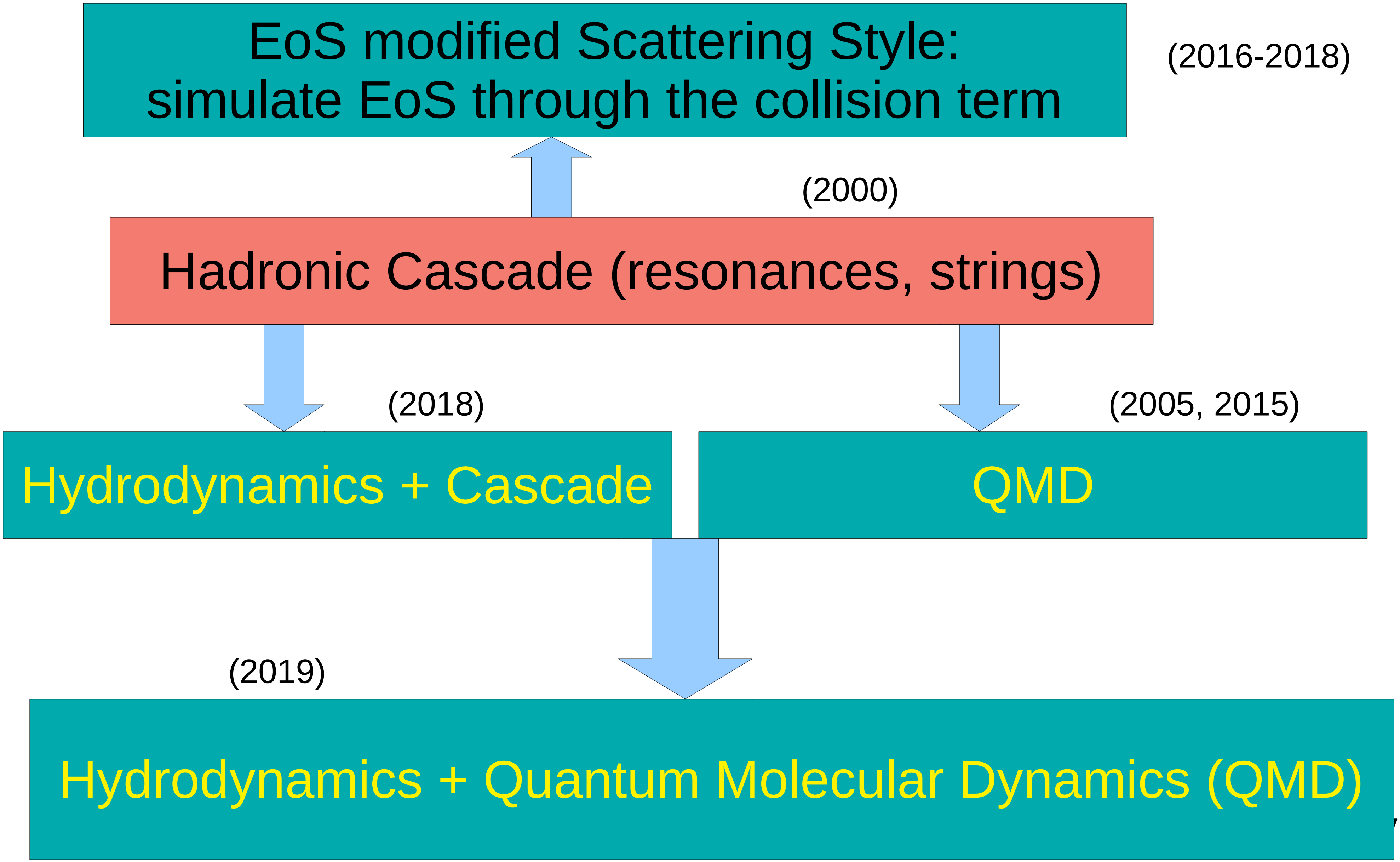
Compute baryon number fluctuations according to
K. Fukushima, PRC91 (2015) 044910

large EoS dependence !



Delta and its isomer state has large effects on the Net-baryon number fluctuations.
What about the dynamical effect? We can do it by RQMD.

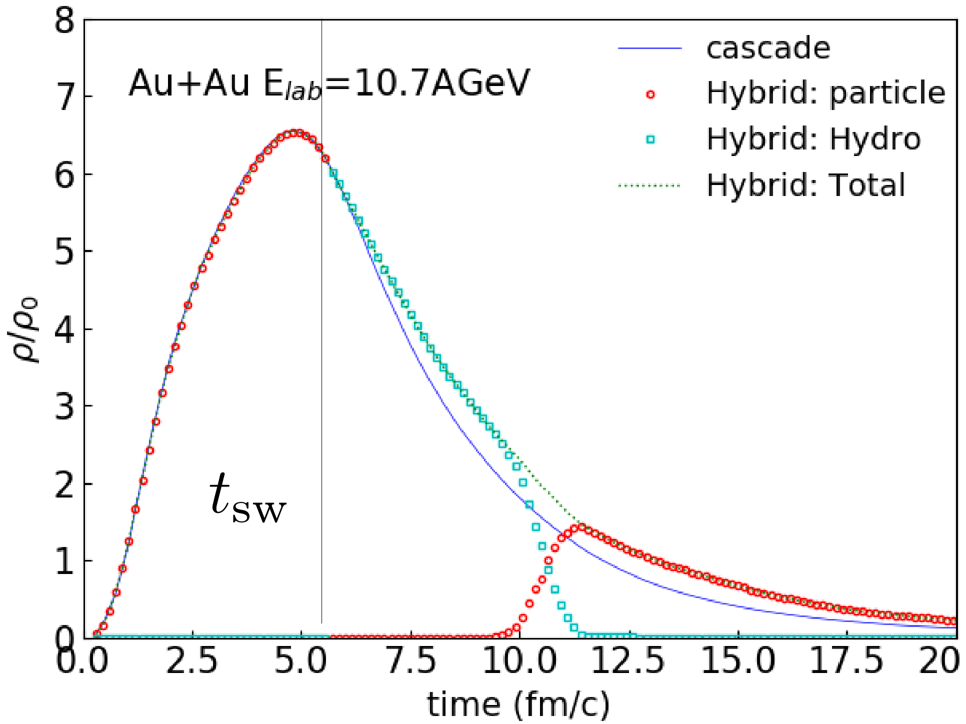
Recent developments in JAM



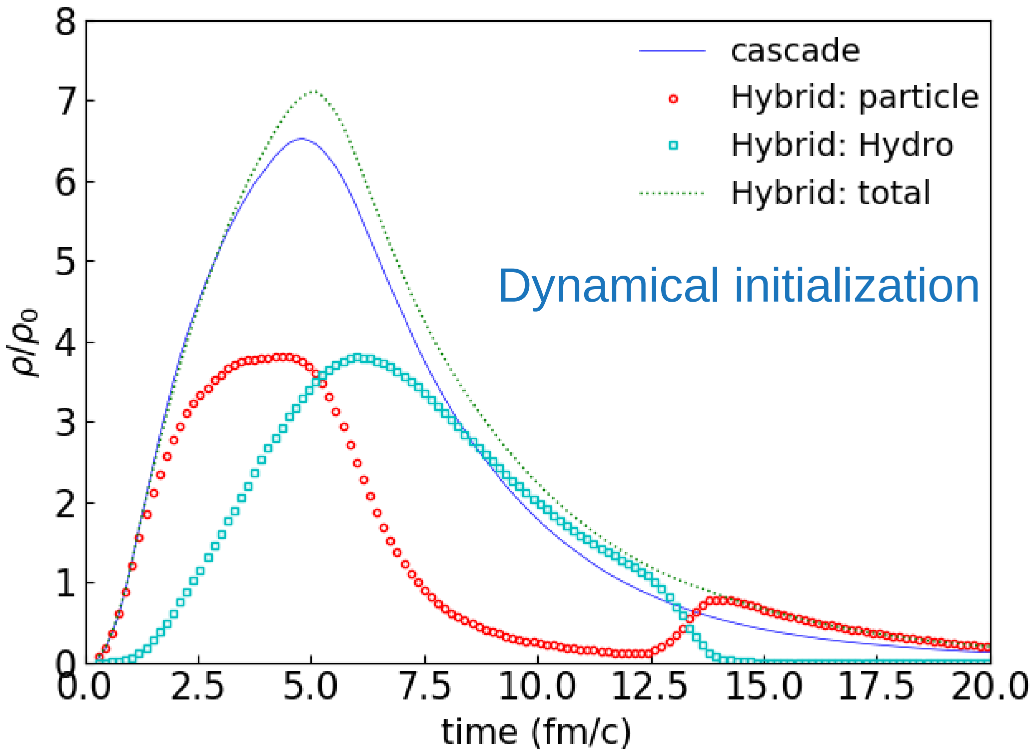
JAM microscopic transport model

- space-time propagation of particles based on cascade method
- Resonance (up to 2GeV) and string excitation and decays
- Re-scattering among all hadrons
- DPM type string excitation law as in HIJING.
- Use Pythia6 for string fragmentation
- Nuclear cluster formation and its statistical decay
- Propagation by the hadronic mean-fields within relativistic quantum molecular dynamics (RQMD/S) (2005, 2016)
- EoS controlled collision term (2017)
- Dynamical coupling of Fluid dynamics through source terms (2018) (Hydro + hadronic cascade)
- RQMD with scalar and vector potentials based on RMF (2019)
- Hydrodynamic Quantum Molecular Dynamics (HyQMD) approach (2019)

Hybrid model for AGS and SPS energies



Switch to hydro evolution
after two nuclei pass each other.



Hadronic EoS is used

Switch to hadron transport below a critical energy density.

It is important to take into account potential effect in the Cooper-Fry formula
to ensure smooth transition from fluid to particles.

$$\mu = B\mu_B + S\mu_S \rightarrow B(\mu_B - V(\rho_B)) + S\mu_S$$

Phys.Rev. C98 (2018) no.2, 024909

A new approach: JAM+hydro model

Dynamical coupling of fluids through source terms

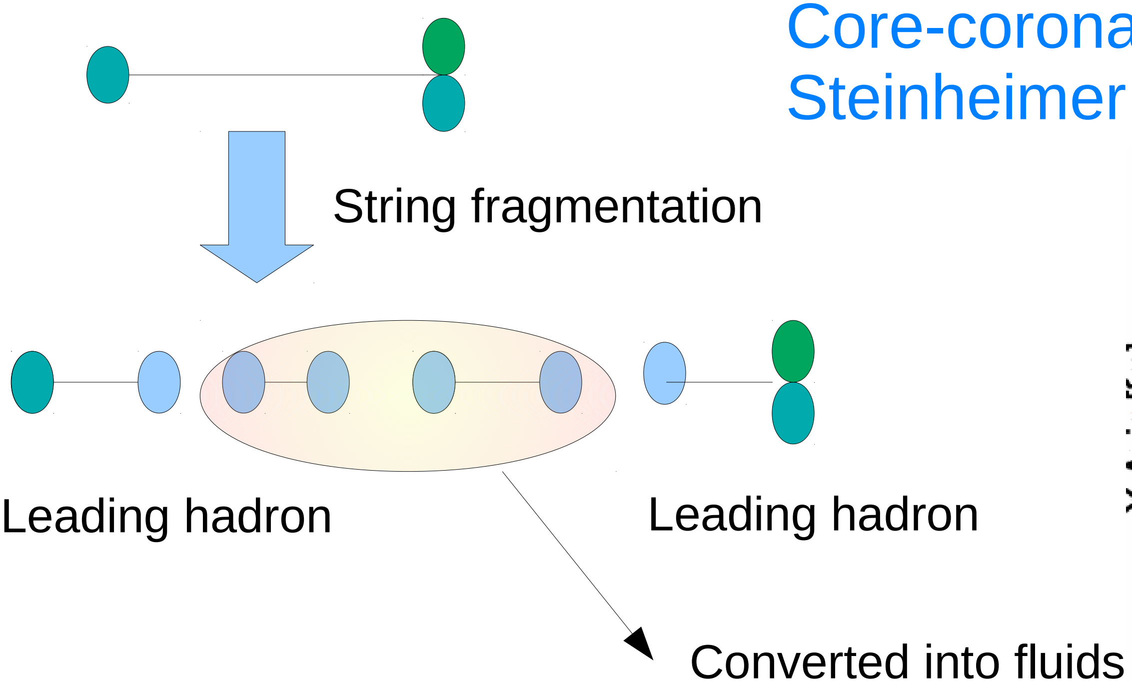
$$\partial_\mu T_f^{\mu\nu} = J^\nu, \quad \partial_\mu N_B^\mu = \rho_B$$

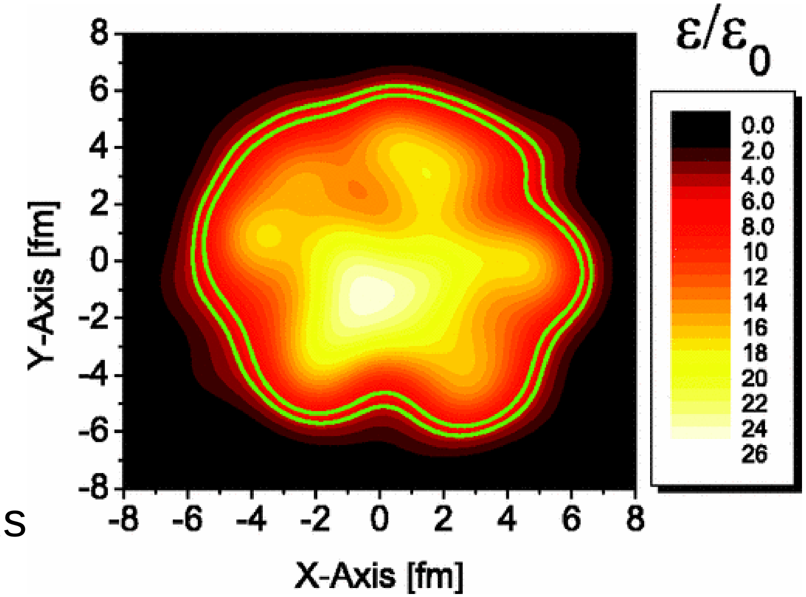
Dynamical initial condition
for hydrodynamics M. Okai, et. al
Phys. Rev C 95, 054914 (2017)

Time dependent Core-corona separation

Put Hadrons from string or resonance decay into fluids after their formation time
except leading hadrons when local energy density exceeds a hydronization energy density

Core-corona separation (K. Werner, 2007)
Steinheimer and Bleicher (2011)





Model parameters

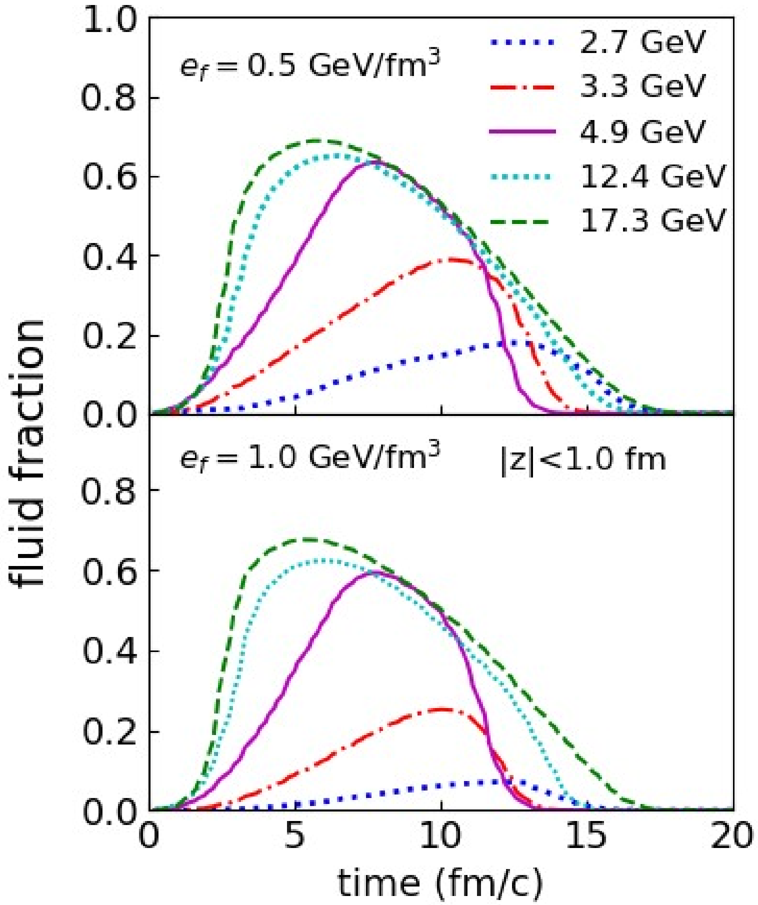
1) fluidization energy density

$$e_f = 0.5 - 1.0 \text{ GeV/fm}^3$$

2) particlization energy density

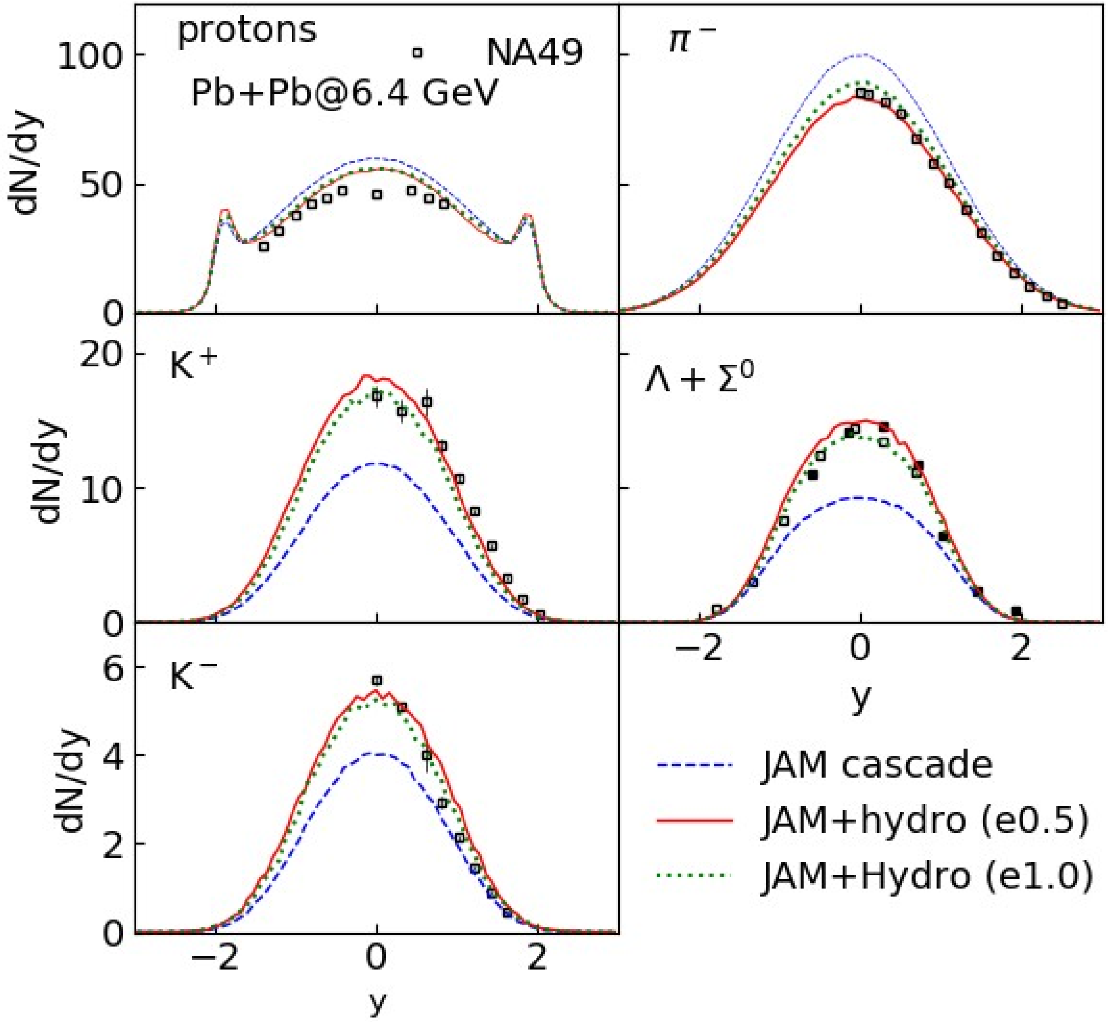
$$e_p = 0.5 \text{ GeV/fm}^3$$

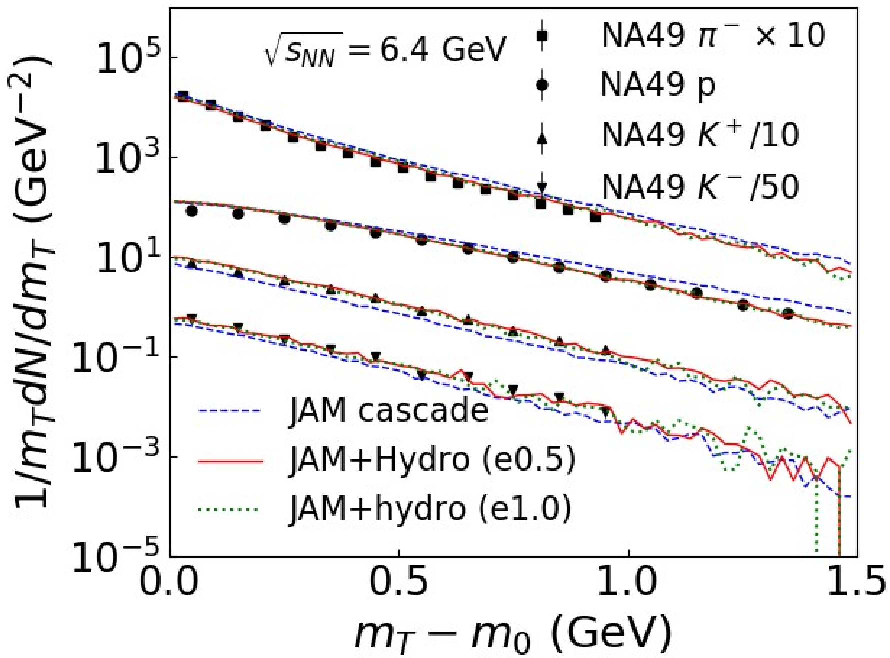
3) equation of state: EOS-Q
- first-order phase transition
- Bag model B=235MeV^4
- hadronic resonances up to 2GeV
- baryon density dependent
- repulsive potential for baryons



Fraction of fluid energy at central region is about 70% at top SPS energy.

Particle spectra from a new hybrid model in Pb+Pb at Elab=20AGeV





Fluidization energy density 0.5 or 1.0

Beam energy dependence of Lambda/pi ratios from a new hybrid model.