24th, June XQCD2019

Rotating neutron star in strong magnetic fields and the MR relations

CHINATSU WATANABE, KOTA YANASE*, AND NAOTAKA YOSHINAGA

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- Neutron stars (NS)
- Motivation

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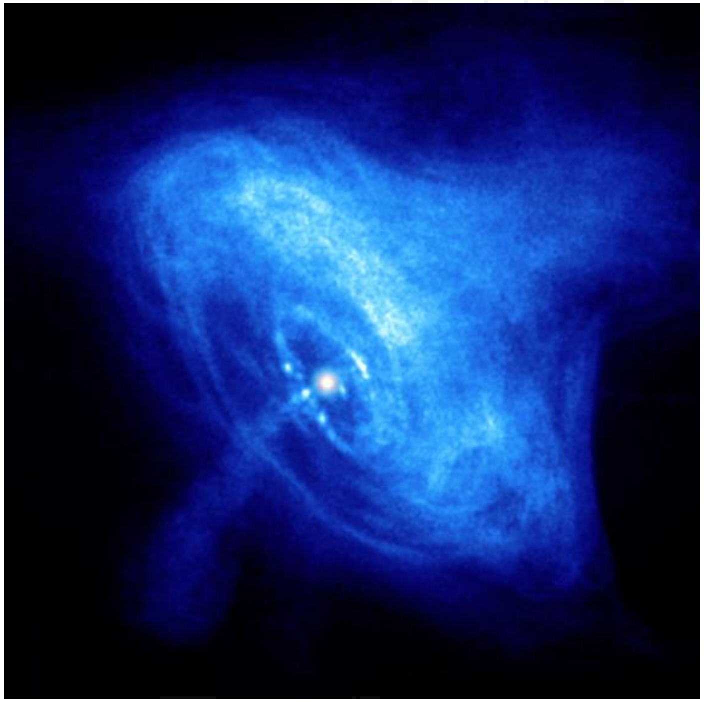
- Mass-Radius (MR) relations for rotating and magnetic field NS
- Summary

Background

Neutron Stars (NS)



The Crab Nebula and pulsar, Chandra X-ray scope
Credit: NASA/CXC/ASU/J. Hester et al.

Mass 1.4 $M_\odot$

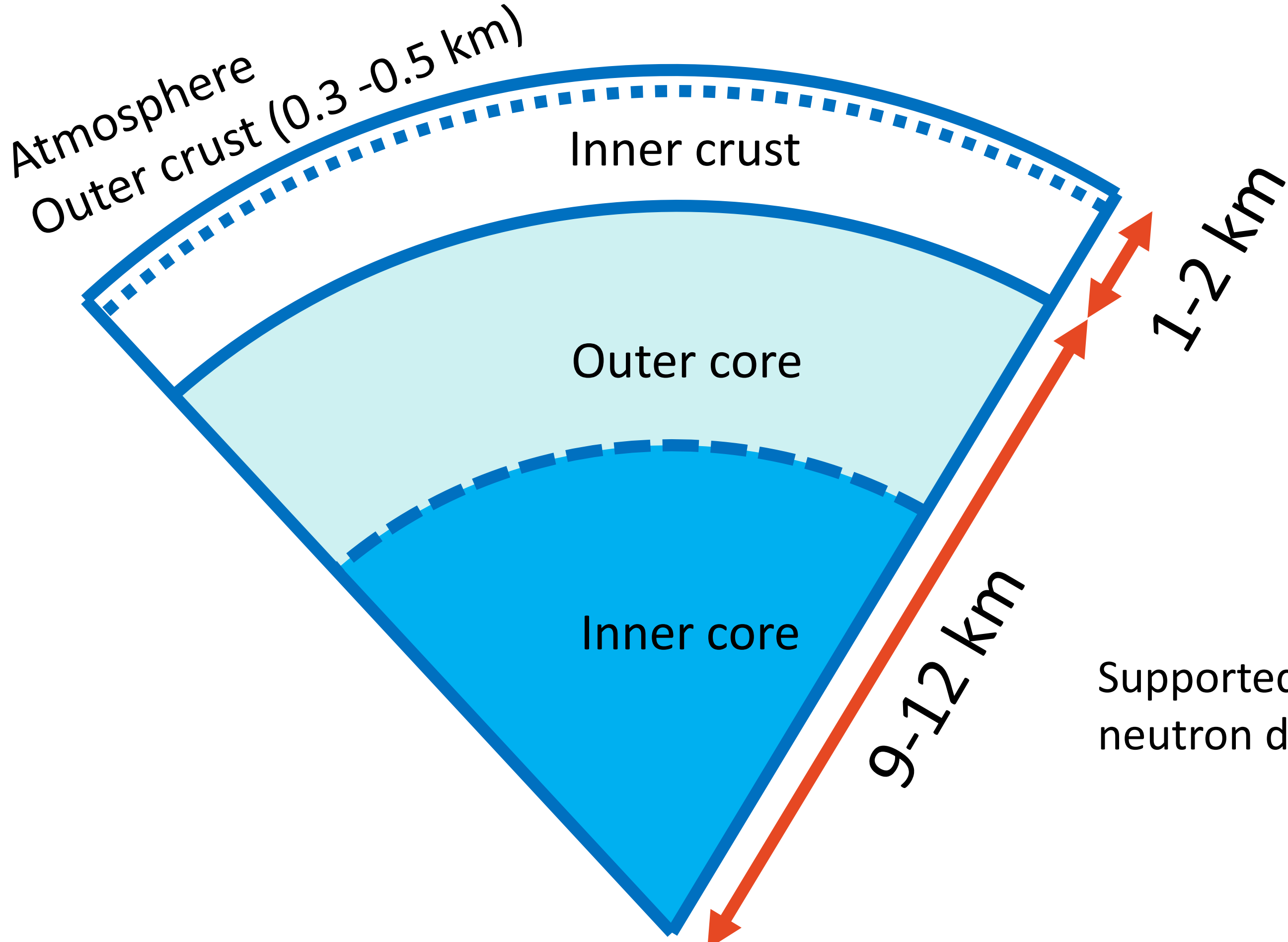
Radius 10 km

Central density $\rho = 10\ \rho_0$

$\rho_0 \simeq 0.15$ nucleons/fm^3
(normal nuclear density)

P. Haensel, A.Y. Potekhin, and D.G. Yakovlev
"*NEUTRON STARS 1*" , ASSL, (2007)

Structure of NS



Supported by
neutron degeneracy pressure

Motivation

Two solar mass problem

2 neutron stars with around twice the solar mass.

$$\text{J1614-2230} \;:\; 1.97 \pm 0.04 M_{\odot}$$

B. Demorest *et al.*
Nature **467**, (2010) 1081-1083

$$\text{J0348+0432} \;:\; 2.01 \pm 0.04 M_{\odot}$$

John Antoniadis *et al.*
Science **340**, (2013) 1233232

Two solar mass problem

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⇨ Such a very heavy neutron star gives a strong limit on the equation of state.

How can we explain $2M_\odot$ NS ?

We consider magnetic fields or/and rotation.

NS with strong magnetic field or/and rapid rotation may have such a large mass.

$$\mathrm{B} \sim 2 \times 10^{15} \mathrm{Gauss}$$

S. A. Olausen and V. M. Kaspi
APJ Supplement Series,212:6 (22pp) (2014)

$$\mathrm{Rotation} = 716\ \mathrm{Hz}$$

J.Hessels *et al*.
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observation

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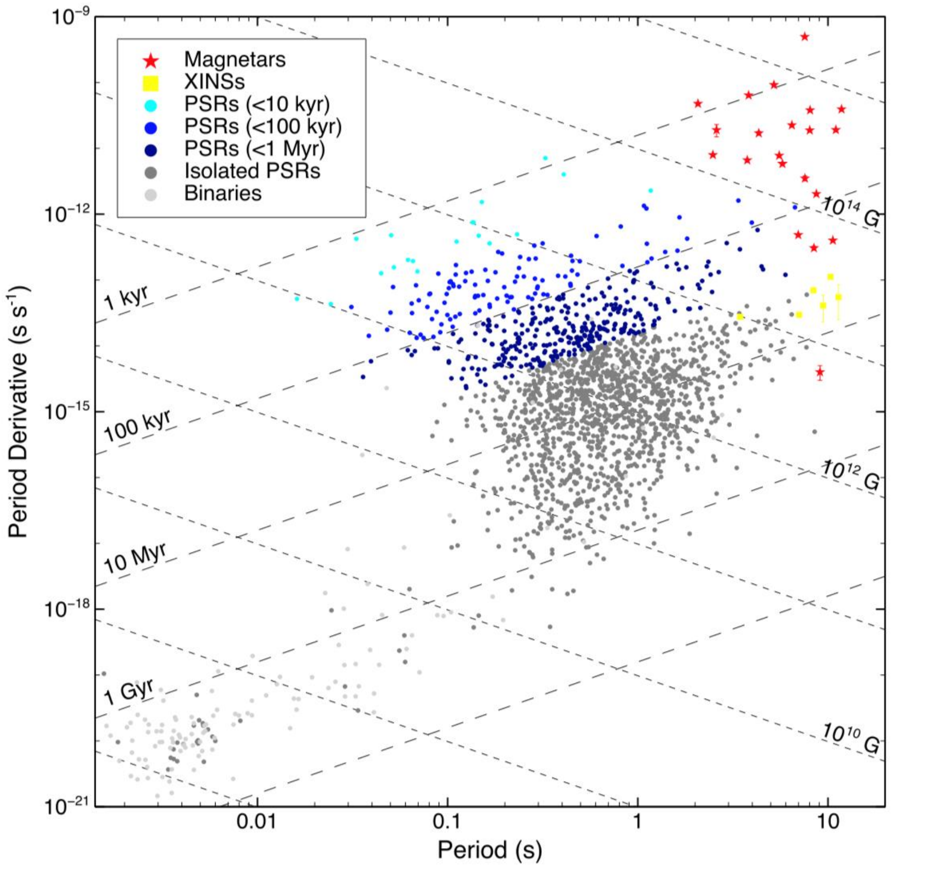
Theory of magnetized neutron stars

P-$\dot{P}$ diagram



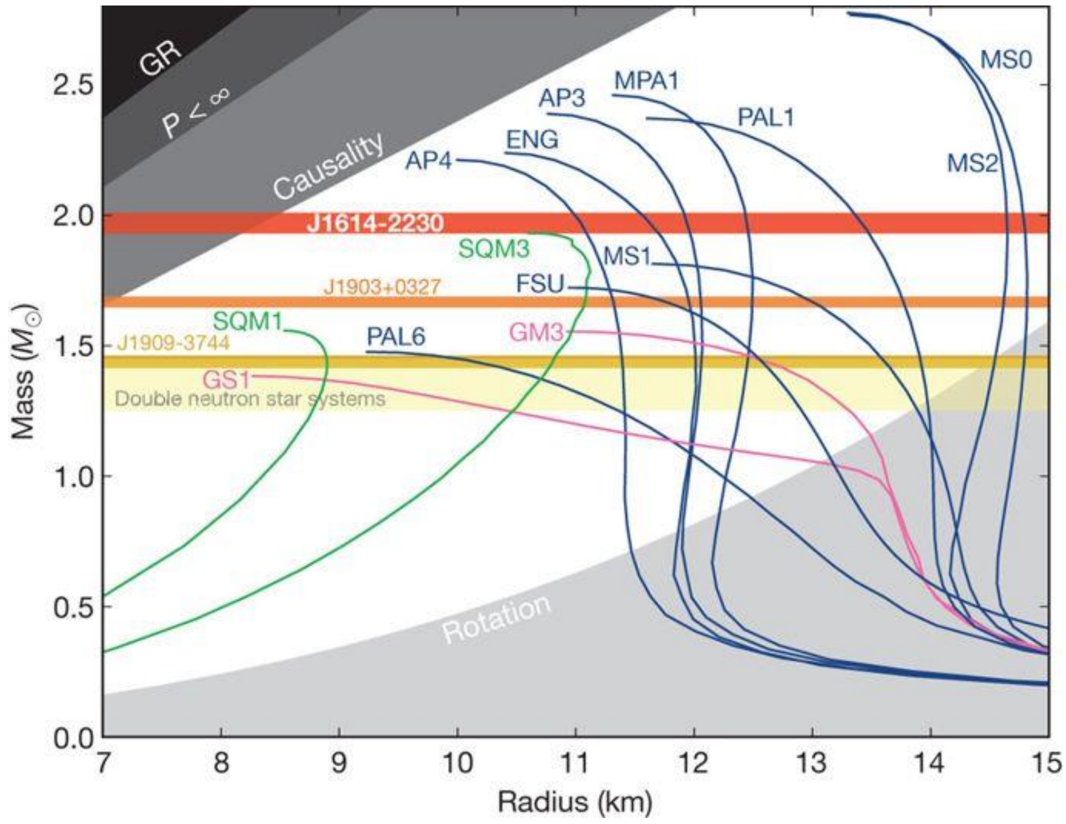
S. A. Olausen and V. M. Kaspi
APJ **212**, 22(2014)

About Radius

We can limit the radius from the observation.
The radii suggested by observational considerations.

Mass ($M_{\odot}$)	Radius (km)	method
0.86−2.42	> 7.6−10.4	Black body from surface Guillot *et al.* (2013)
1.2−1.7	< 9.0−13.2	Eddington limit Zamfir *et al.* (2012)
@ 1.4	> 6.6	The absorption line red shift Waki *et al.* (1984)
@ 1.4	$\lesssim$ 13.6	Gravitational wave Annala *et al.* (2018)

Equation of state (EoS) and MR relation



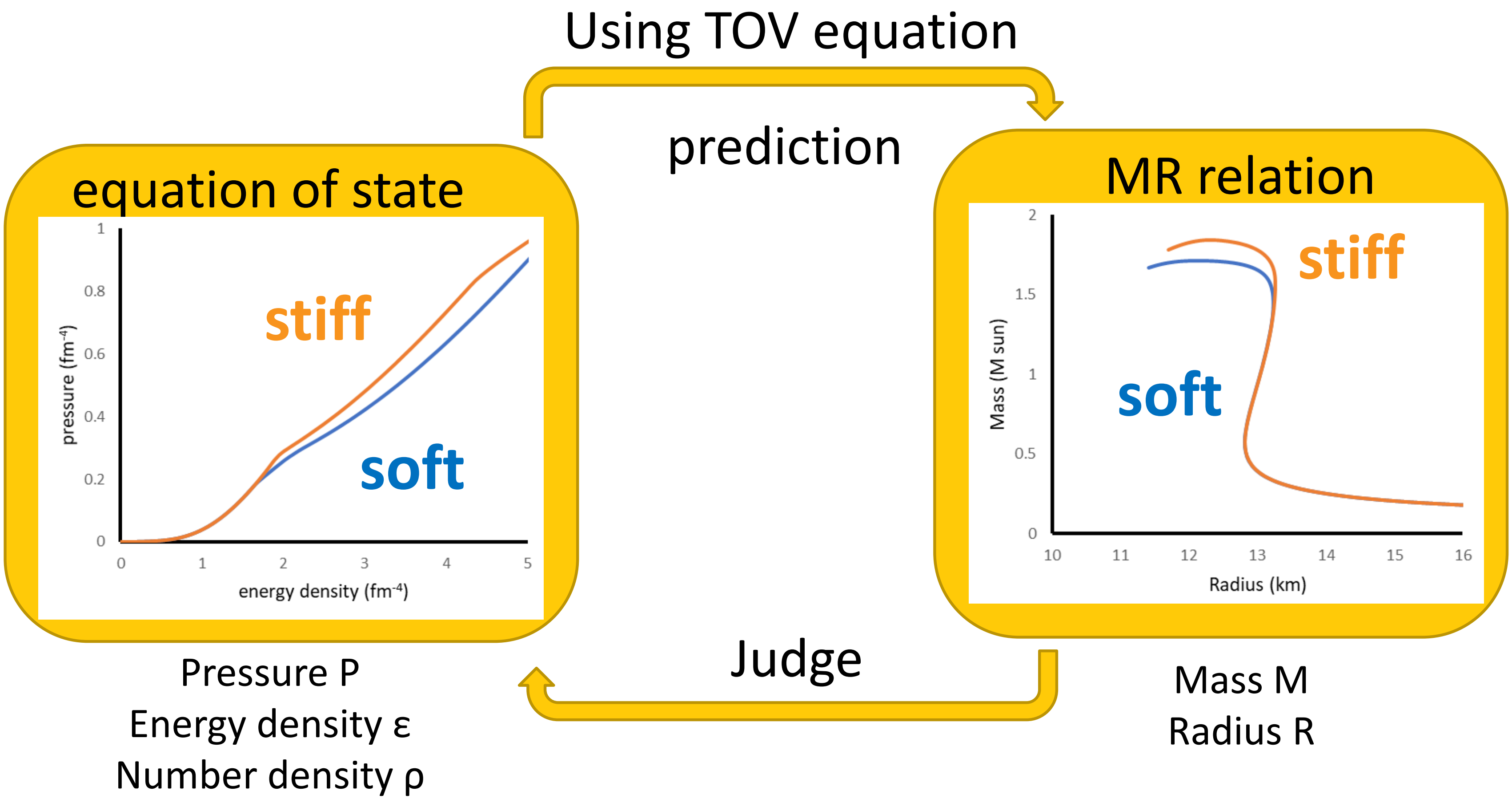
Various kinds of EoSs to have been proposed to describe NS.

- nucleons
- nucleons + hyperons
- strange quark matter

Masses do not surpass twice the solar mass, once hyperons are included.

P. B. Demorest et al.
Nature 467, 1081(2010)

How to Judge



Using TOV equation
prediction
equation of state
stiff
soft
pressure (fm-4)
energy density (fm-4)
0
0.2
0.4
0.6
0.8
1
0
1
2
3
4
5
MR relation
stiff
soft
Mass (M sun)
Radius (km)
0
0.5
1
1.5
2
10
11
12
13
14
15
16
Judge
Pressure P
Energy density ε
Number density ρ
Mass M
Radius R

Formulation

Metric of slowly rotating neutron star considering axial deformation in GR.

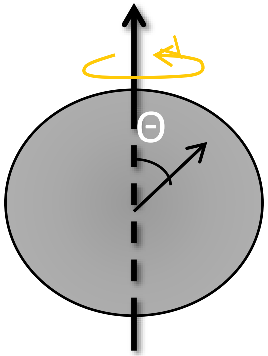
$$ds^2 = -e^{2\nu_0}\left[1 - 2h_0(r) + 2h_2(r)P_2(\cos\theta)\right]dt^2$$

$$+e^{2\lambda_0}\left\{1 + \frac{e^{2\lambda_0}}{r}\left[2m_0(r) + 2m_2(r)P_2(\cos\theta)\right]\right\}dr^2$$

$$+r^2\left[1 + 2k_2(r)P_2(\cos\theta)\right]\left\{d\theta^2 + \left[d\phi - \omega(r)dt\right]^2 \sin^2\theta\right\}$$

ω : angular velocity

$P_2(\cos\theta)$: Legendre's polynomial of order 2



S. Chandrasekhar & John C. Miller,
Mon. Not. R. astr. Soc., 167, 63-79 (1974)

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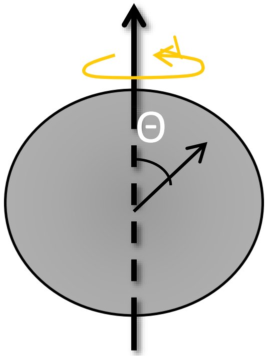
$$+r^2\left[1 + 2k_2(r)P_2(\cos\theta)\right]\left\{d\theta^2 + \left[d\phi - \omega(r)\,dt\right]^2\sin^2\theta\right\}$$

$$\frac{dM_0}{dr} = 4\pi r^2\varepsilon(p_0),$$

$$\frac{dp_0}{dr} = -\frac{\left(\varepsilon(p_0) + p_0\right)\left(M_0 + 4\pi r^3 p_0\right)}{r\left(r - 2M_0\right)},$$



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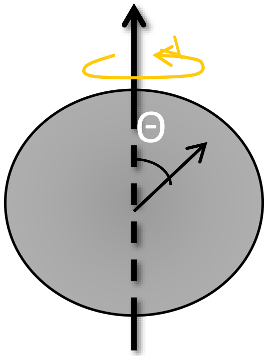


S. Chandrasekhar & John C. Miller, Mon. Not. R. astr. Soc., 167, 63-79 (1974)

Hartle Equations

To calculate the additional mass for slowly rotating neutron star.

$$\frac{1}{r^3}\frac{d}{dr}\left(r^4 j\frac{d\varpi}{dr}\right)+4\frac{dj}{dr}\varpi=0$$

$$-\frac{d}{dr}\delta P_0+\frac{1}{3}\frac{d}{dr}\left(r^2e^{-2\nu_0}\varpi^2\right)=m_0e^{4\lambda_0}\left(\frac{1}{r^2}+8\pi p_0\right)-\frac{1}{12}e^{2\lambda_0}r^3j^2\left(\frac{d\varpi}{dr}\right)^2+4\pi re^{2\lambda_0}\left(\varepsilon+p\right)\delta P_0$$

$$\frac{dm_0}{dr}=4\pi r^2\left(\varepsilon+p\right)\frac{d\varepsilon}{dp}\delta P_0+\frac{1}{12}r^4j^2\left(\frac{d\varpi}{dr}\right)^2-\frac{1}{3}r^3\varpi^2\frac{dj^2}{dr}$$

$$\frac{dv_2}{dr}=-2\frac{d\nu_0}{dr}h_2+\left(\frac{1}{r}+\frac{d\nu_0}{dr}\right)\left[\frac{1}{6}r^4j^2\left(\frac{d\varpi}{dr}\right)^2-\frac{1}{3}r^3\varpi^2\frac{dj^2}{dr}\right]$$

$$\frac{dh_2}{dr}=-\frac{2v_2}{r(r-2M)d\nu_0/dr}+\left\{-2\frac{d\nu_0}{dr}+\frac{r}{2(r-2M)d\nu_0/dr}\left[8\pi\left(\varepsilon+p\right)-\frac{4M}{r^3}\right]\right\}h_2$$
$$+\frac{1}{6}\left[r\frac{d\nu_0}{dr}-\frac{1}{2(r-2M)d\nu_0/dr}\right]r^3j^2\left(\frac{d\varpi}{dr}\right)^2-\frac{1}{3}\left[r\frac{d\nu_0}{dr}+\frac{1}{2(r-2M)d\nu_0/dr}\right]r^2\varpi^2\frac{dj^2}{dr}$$

S. Chandrasekhar & John C. Miller,
Mon. Not. R. astr. Soc., 167, 63-79 (1974)

Relativistic Mean Field (RMF) Theory

We use the following Lagrangian which includes interactions between baryon octet and σ, ω, ρ, σ*,and ϕ mesons.

$$
\begin{aligned}
\mathcal{L} = \sum_b \Big(& \overline{\psi}_b \left(i\gamma_\mu \partial^\mu - m_b + g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* - g_{\omega b}\gamma_\mu \omega^\mu - g_{\phi b}\gamma_\mu \phi^\mu - g_{\rho b}\gamma_\mu \boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu \right. \\
& \left. - q_b \gamma_\mu A^\mu - \kappa_b \sigma_{\mu\nu} F^{\mu\nu} \right) \psi_b \Big) \\
& + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) + \frac{1}{2}\left(\partial_\mu \sigma^* \partial^\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}\right) \\
& + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu \phi^\mu - \frac{1}{4}\Phi_{\mu\nu}\Phi^{\mu\nu} \\
& + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \frac{1}{4}\boldsymbol{P}^{\mu\nu}\cdot\boldsymbol{P}_{\mu\nu} \\
& - \frac{1}{3} b m_n \left(g_\sigma \sigma\right)^3 - \frac{1}{4} c \left(g_\sigma \sigma\right)^4 + \frac{1}{4!}\xi \left(g_\omega^2 \omega_\mu \omega^\mu\right)^2 + \Lambda_\omega \left(g_\omega^2 \omega_\mu \omega^\mu\right)\left(g_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu\right) \\
& + \sum_l \Big(\overline{\psi}_l \left(i\gamma_\mu \partial^\mu - q_l \gamma_\mu A^\mu - m_l \right) \psi_l \Big) \\
& - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}.
\end{aligned}
$$

Nuclear properties

★Bethe-Weizsäcker formula (1935)

Nuclear binding energy can be described roughly by a liquid-drop model.

Radius of nuclei : $R=r_0A^{1/3}$ r_0~0.15fm

$$B(A,Z) \equiv Zm_p + (A-Z)m_n - M(A,Z)$$

$$= a_{vol}A - a_{surf}A^{2/3} - a_{coul}\frac{Z^2}{A^{1/3}} - a_{sym}\frac{(N-Z)^2}{A} + \delta(A).$$

a_{vol}: $\propto \frac{4\pi}{3}R^3$ volume 16.2MeV

a_{surf}: $\propto 4\pi R^2$ surface 19.0MeV

a_{coul}: $\propto \frac{Q^2}{R}$ coulomb 0.76MeV

a_{sym}: $\frac{1}{2}\left(\frac{\partial^2}{\partial t^2}\frac{\varepsilon}{\rho}\right)_{\rho=\rho_0}$ $t=(\rho_n-\rho_p)/\rho$ symmetry energy 23.5MeV

$\delta(A)$: Pairing energy 1MeV

Nuclear properties

Binding energy per nucleon

$$B(A,Z)/A = a_{vol} = 16.2\ \text{MeV}$$

Nucleon number density

$$\rho_0 = \frac{a}{4\pi R^3/3} = 0.15\,\text{nucleon}/\text{fm}^3$$

Symmetry energy

$$a_{sym} = \frac{1}{2}\left(\frac{\partial^2}{\partial t^2}\frac{\varepsilon}{\rho}\right)_{\rho=\rho_0} \quad \left(t = (\rho_n - \rho_p)/\rho \right)$$

23.5MeV

Incompressibility

$$K = \left[k^2 \frac{d^2}{dk^2}\left(\frac{\varepsilon}{\rho}\right)\right]_{k=k_F} = 9\left[\rho^2 \frac{d^2}{d\rho^2}\left(\frac{\varepsilon}{\rho}\right)\right]_{\rho=\rho_0}$$

Symmetry-energy slope parameter

$$L = 3\rho_0 \left(\frac{dS}{d\rho}\right)_{\rho_0}$$

$a_{sym} = S(\rho_0)$

incompressibility of symmetry energy

$$K = 9\rho_0^2 \left(\frac{d^2S}{d\rho^2}\right)_{\rho_0}$$

Properties of various EoSs

EoS	B/A MeV	ρ_0 fm^{-3}	a_{sym} MeV	K MeV	L MeV	K_{sym} fm^{-3}
GM1(*1)	16.3	0.153	32.5	300	94.4	18.1
TM1-a(*2)	16.3	0.146	36.9	281.2	111.2	33.8
TM1-b(*2)	16.3	0.146	36.9	281.2	111.2	33.8
TM2-ωρ-a(*2)	16.4	0.146	32.1	281.7	54.8	-70.5
TM2-ωρ-b(*2)	16.4	0.146	32.1	281.7	54.8	-70.5

(*1)N.Glendenning & S.Moszkowski
Phy.Rev.Letter, vol.67, Num.18(1991)

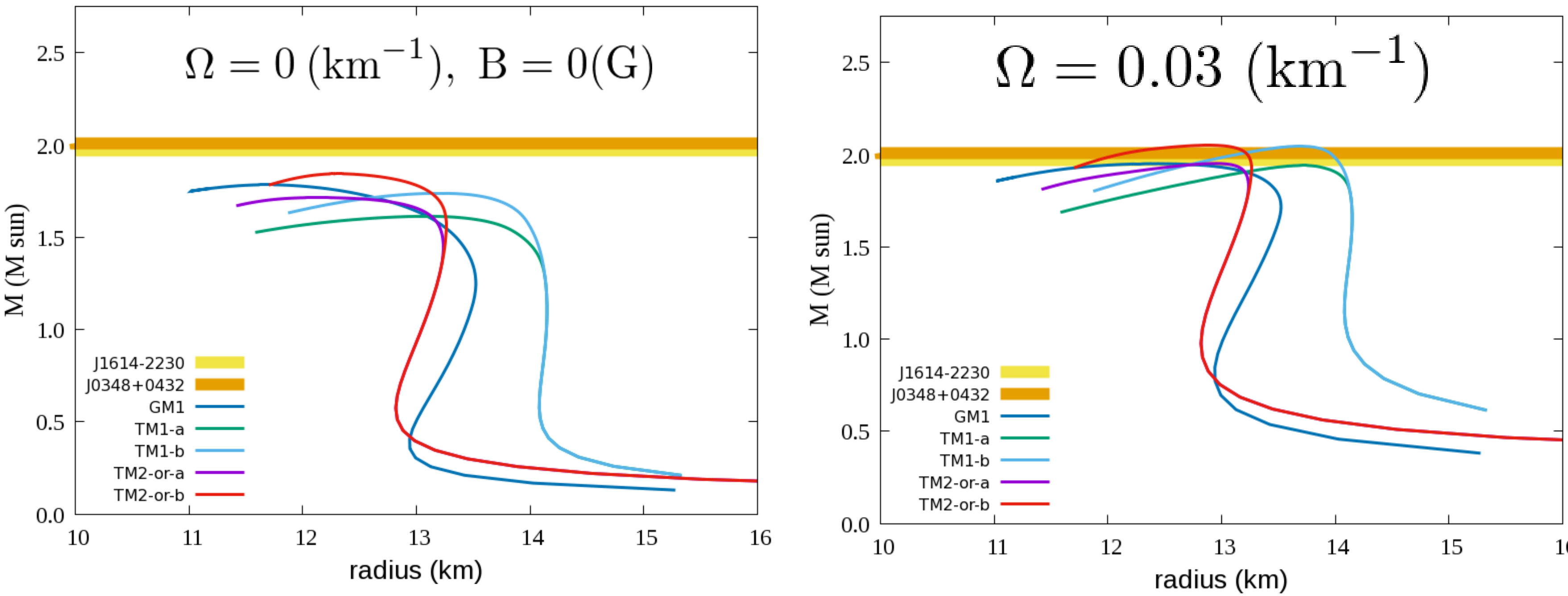
(*2) M.Fortin et al.,
Physical Review C95, 065803 (2017)

Results and Summary

COMPARISON OF VARIOUS EOSs

ROTATION AND MAGNETIC FIELDS

Comparison of rotating NS masses (5 EoSs)



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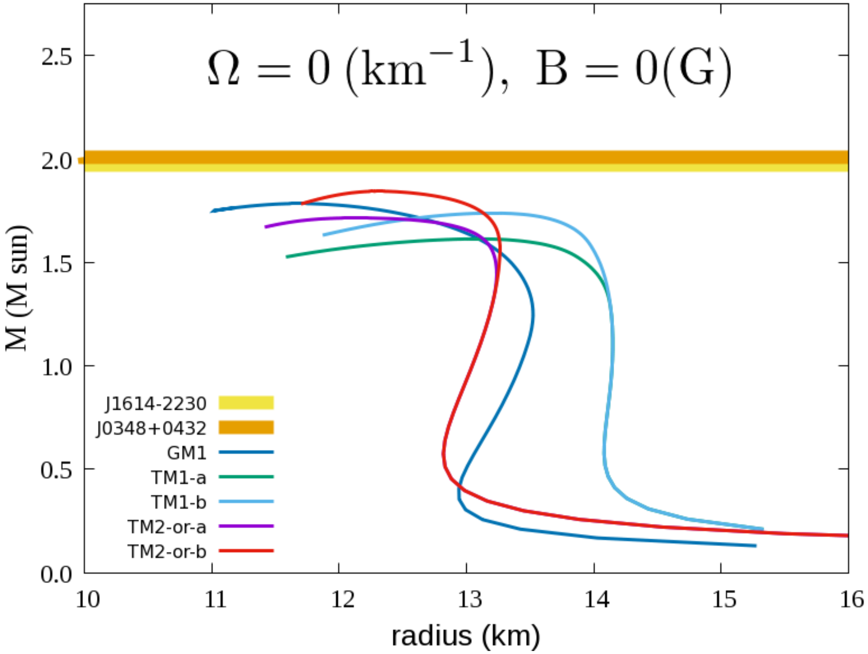
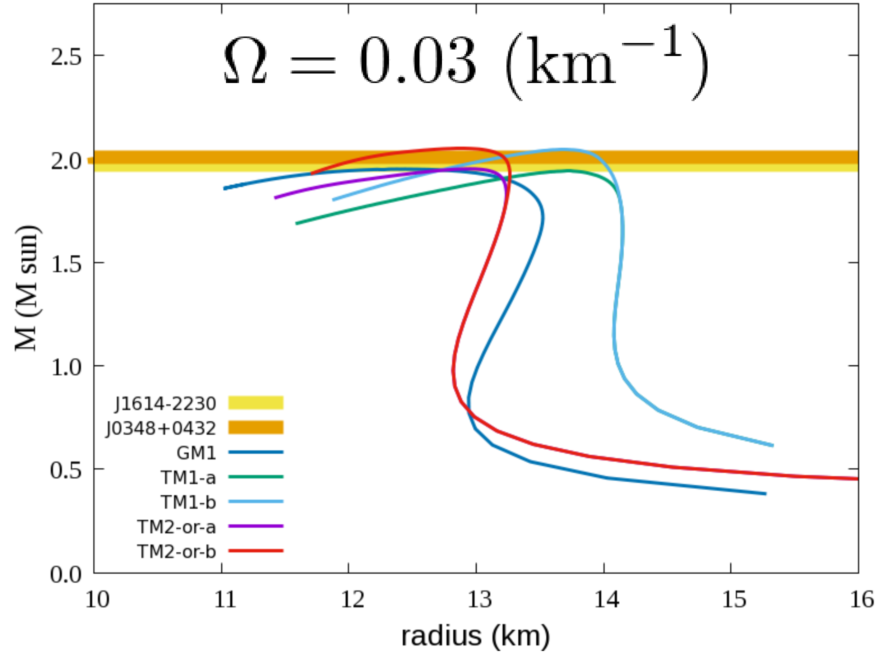
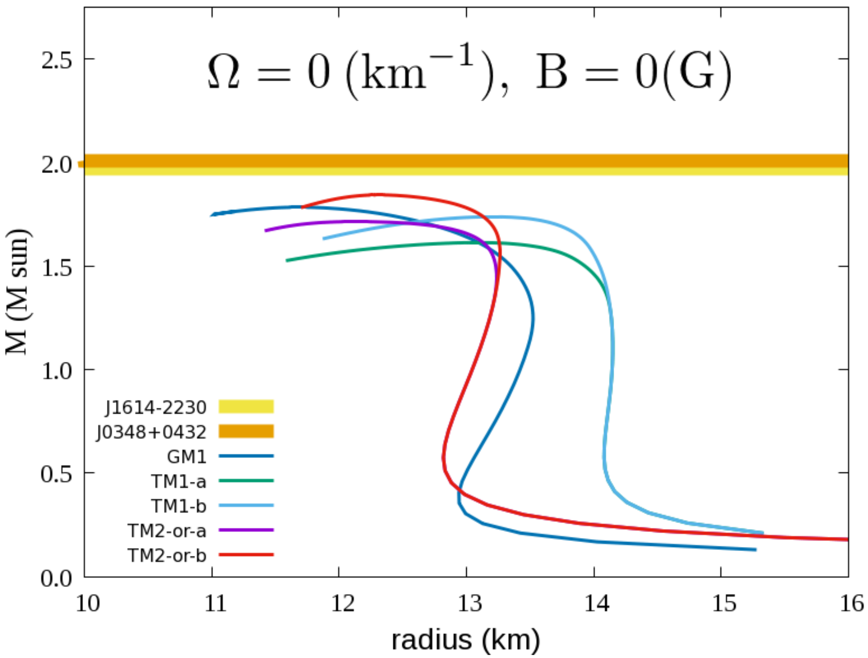


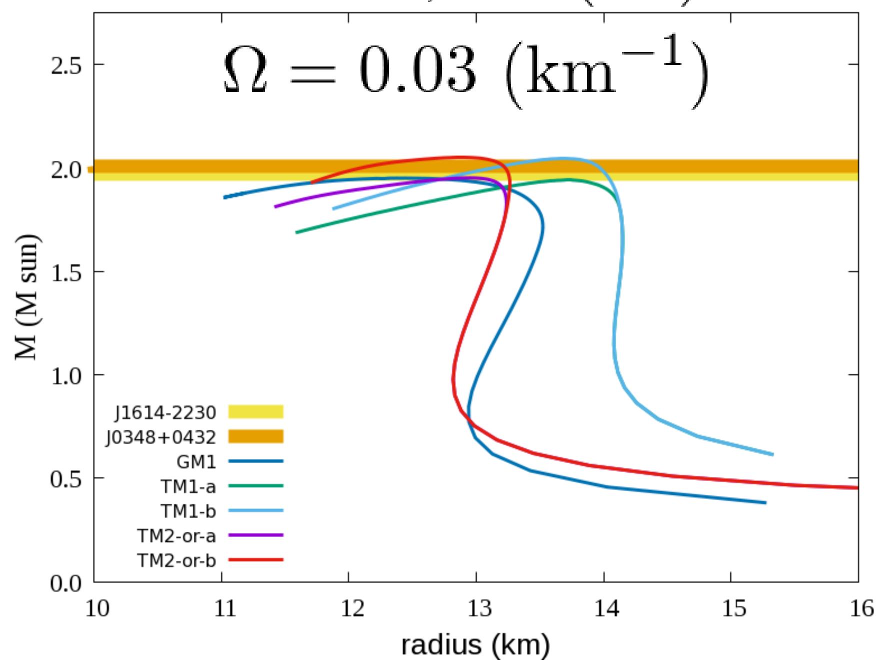


TM1-b and TM2-ωρ-b EoSs give over twice the solar mass.

Comparison of rotating NS masses (5 EoSs)



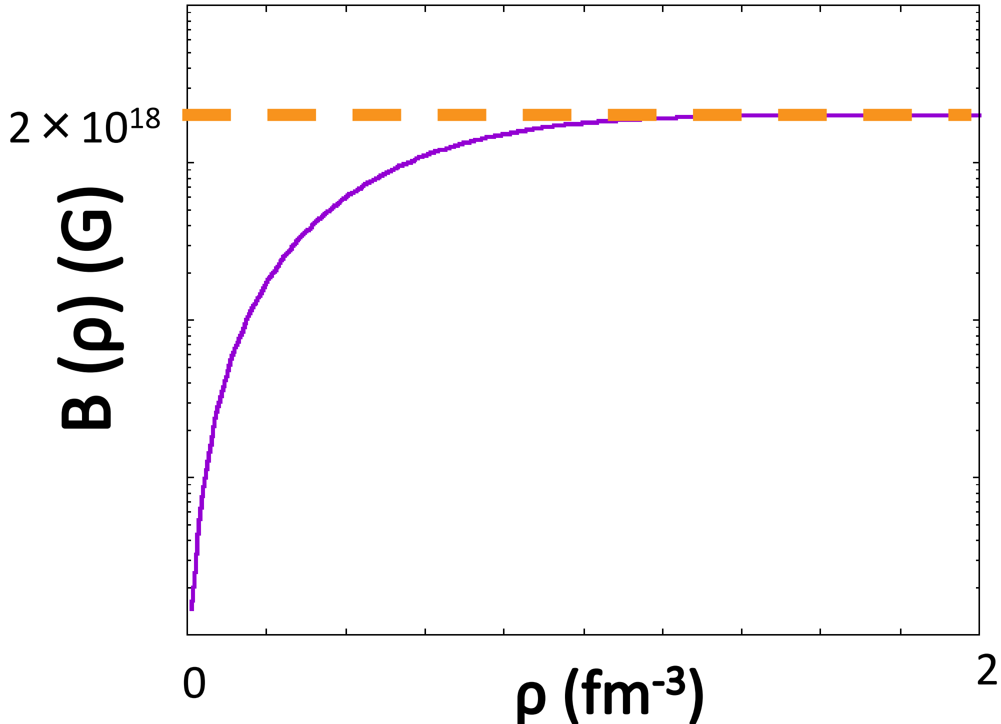
$\Omega \sim 13,700\ (\mathrm{Hz})$



TM1-b and TM2-ωρ-b EoSs give over twice the solar mass.

Magnetic field B(ρ)

For magnetic field, we used following $B(\rho)$
(Spherically Symmetric Magnetic Pressure)



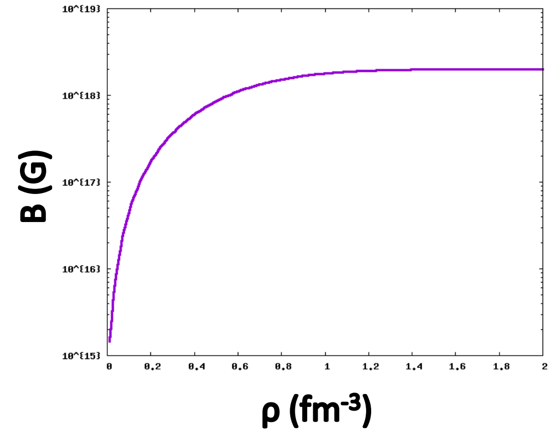
Magnetic field B(ρ)

For magnetic field, we used following $B(\rho)$ (Spherically Symmetric Magnetic Pressure)

$$B(\rho) = B_s + B_0 \left[1 - \exp\left\{ -\alpha \left(\frac{\rho}{\rho_0} \right)^{\gamma} \right\} \right]$$

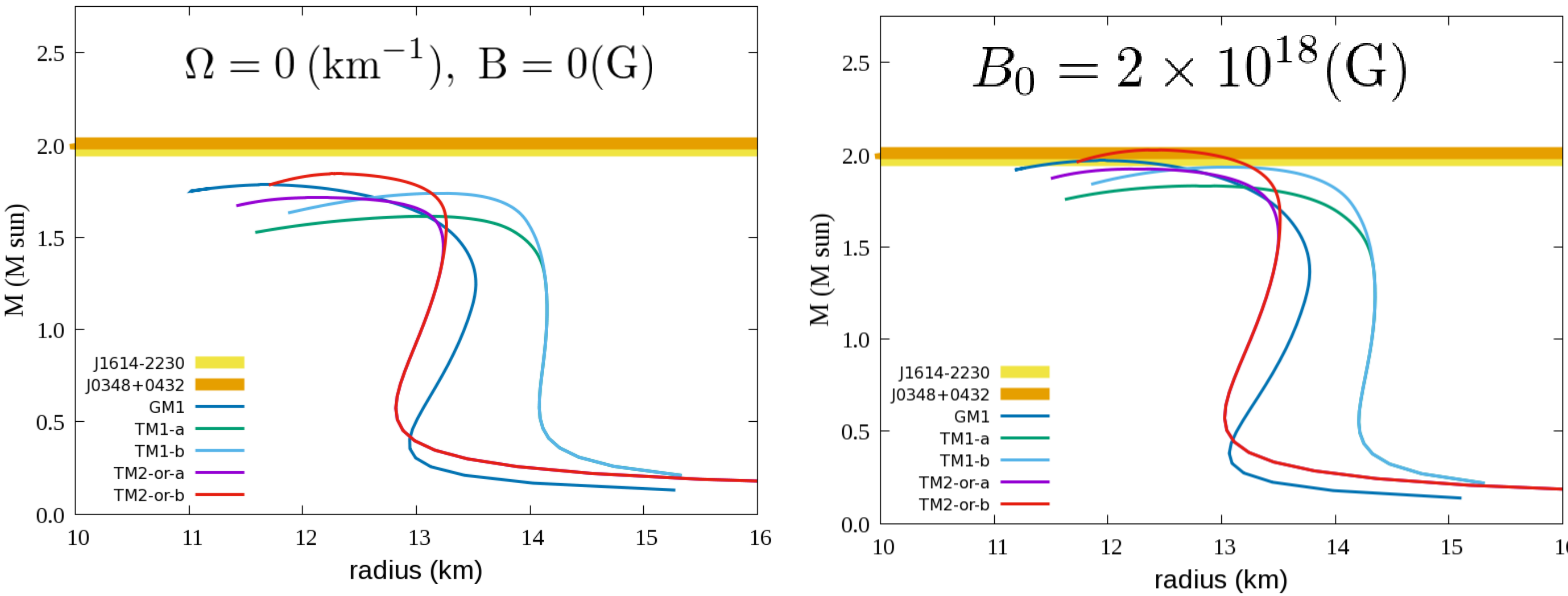
$$B_s = 1 \times 10^{15} (G) \qquad \alpha = 0.05, \gamma = 2$$

$$B_0 = 2 \times 10^{18} (\mathrm{G})$$



R.H.Castro , L.B.Castro & D.P.Menezes,
Phy. Rev. C 89, 015805 (2014)

Comparison of magnetized NS masses (5 EoSs)



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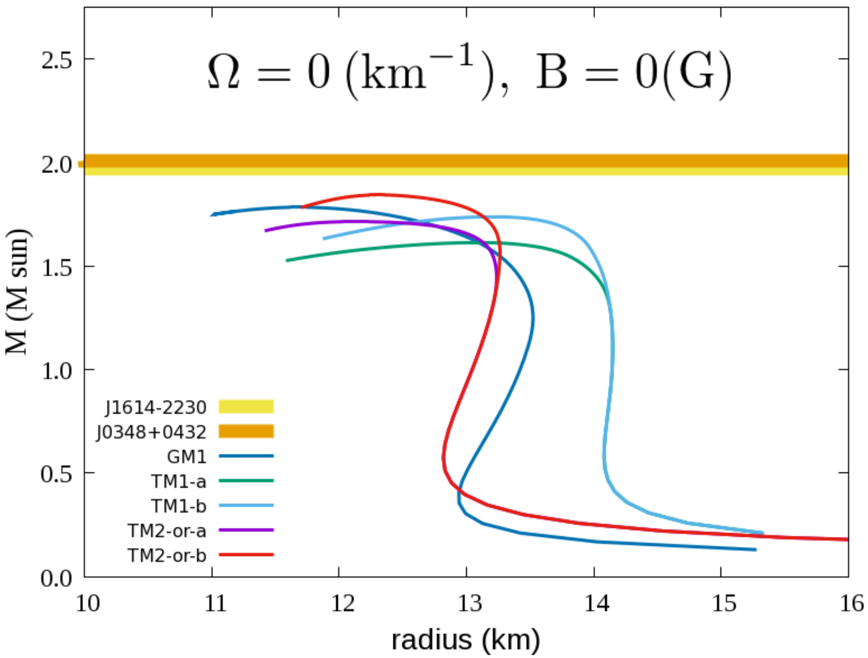
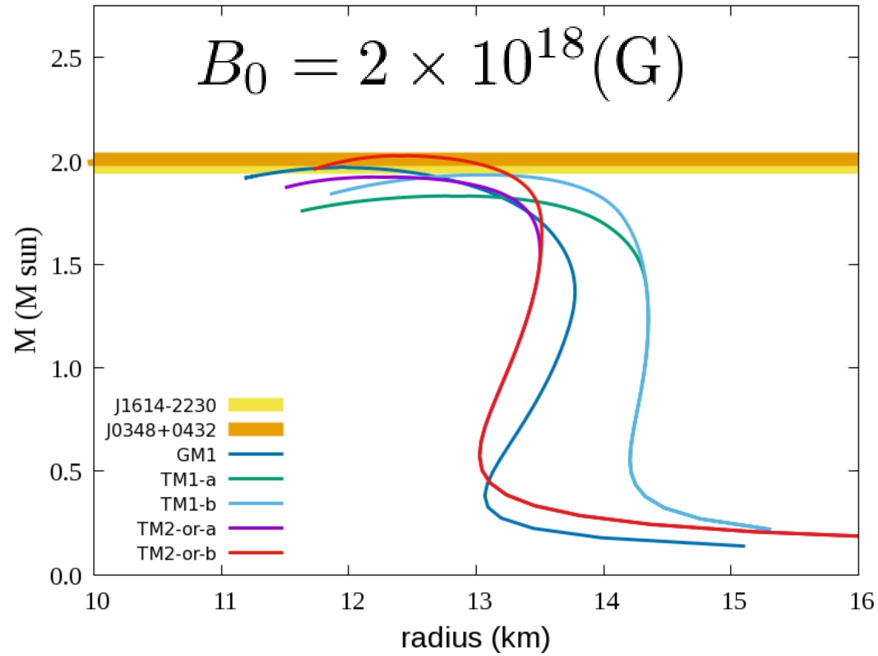
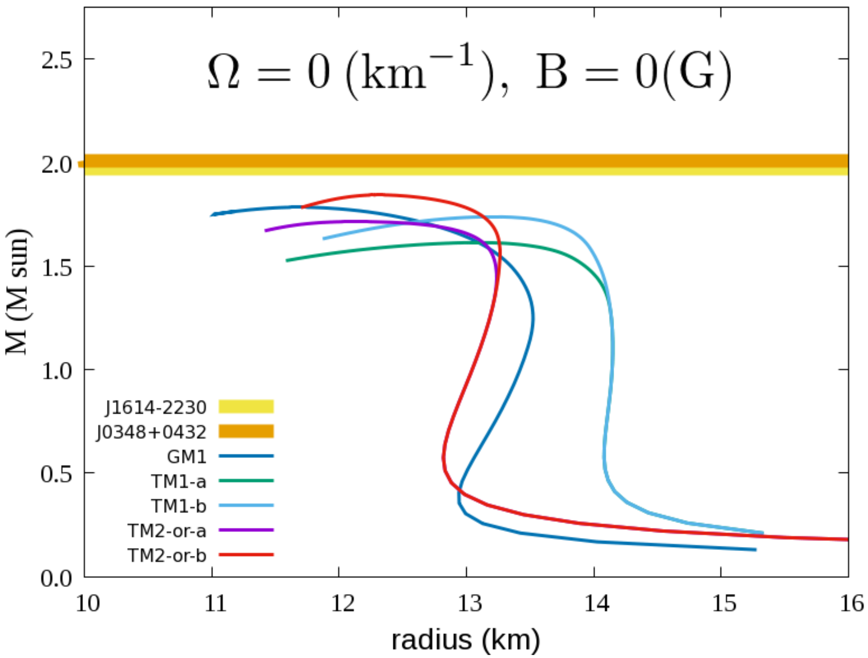
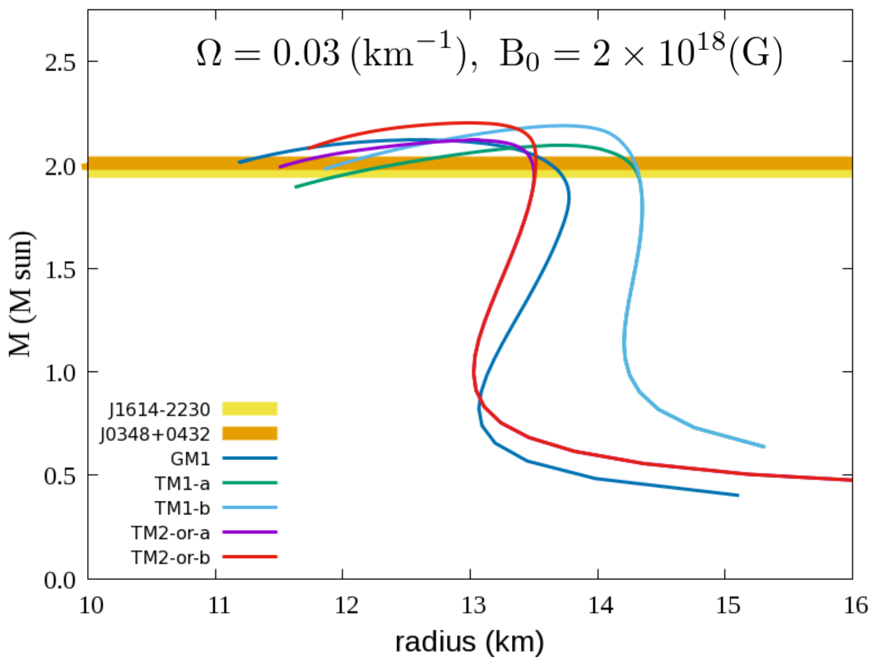




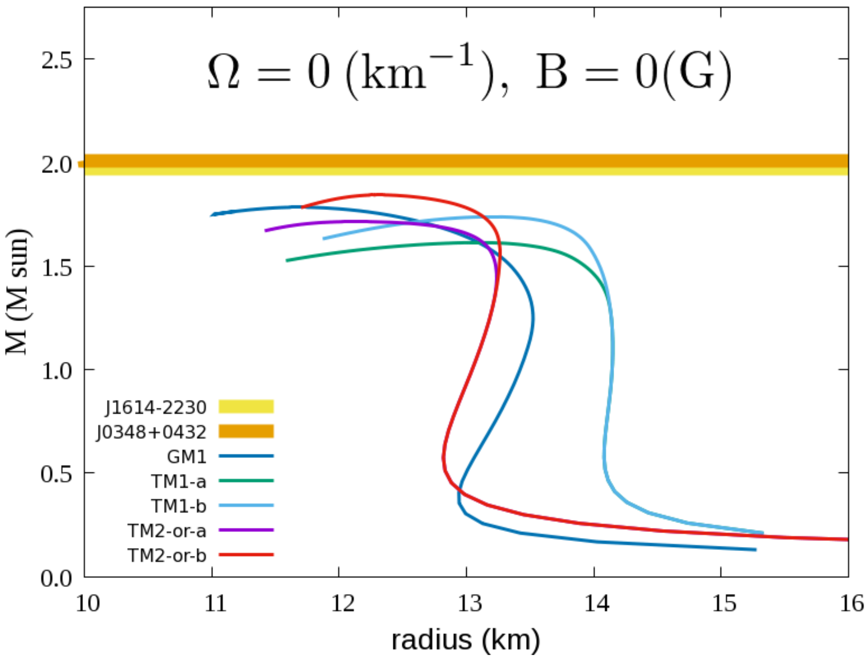
Only TM2-ωρ-b EoS gives over twice the solar mass.

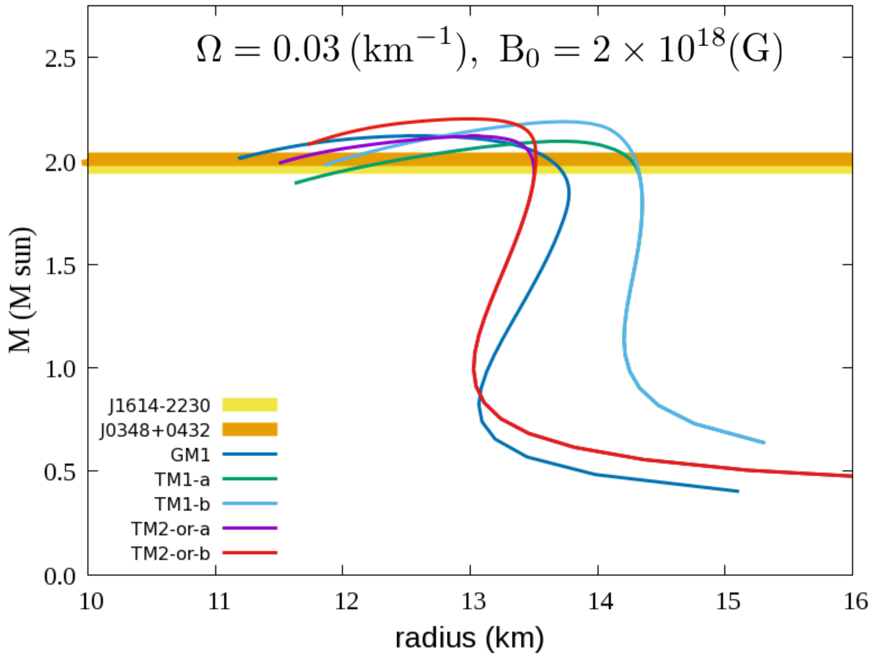
Comparison of rotating and magnetized NS (5 EoSs)





Comparison of rotating and magnetized NS (5 EoSs)





All 5 EoSs give over twice the solar mass.

Radius for $1.4M_\odot$ NS

EoS	R km	$R_{(rot)}$ km	$R_{(mag)}$ km	$R_{(rot\&mag)}$ km
GM1	13.46	13.34	13.77	13.49
TM1-a	14.07	14.12	14.33	14.26
TM1-b	14.10	14.12	14.33	14.26
TM2-ωρ-a	13.23	13.02	13.47	13.21
TM2-ωρ-b	13.24	13.02	13.47	13.21

$\Omega=0.03\mathrm{km}^{-1}$ $B_0=2\times10^{18}\mathrm{G}$

Observation
$6.6 < R \leqq 13.6$ km

Waki *et al.* (1984)

Annala *et al.* (2018)

Radius for $1.4M_{\odot}$ NS

EoS	R km	$R_{(rot)}$ km	$R_{(mag)}$ km	$R_{(rot\&mag)}$ km
GM1	13.46	13.34	13.77	13.49
TM1-a	14.07	14.12	14.33	14.26
TM1-b	14.10	14.12	14.33	14.26
TM2-ωρ-a	13.23	13.02	13.47	13.21
TM2-ωρ-b	13.24	13.02	13.47	13.21

Observation
$6.6 < R \leqq 13.6$ km

Approximately
GM1 &
TM2-ωρ-a &
TM2-ωρ-b
are in the range.

$\Omega = 0.03\,\mathrm{km}^{-1}$ $B_0 = 2 \times 10^{18}$ G

Radius for $1.4M_{\odot}$ NS

EoS	R km	$R_{(rot)}$ km	$R_{(mag)}$ km	
GM1	13.46	13.34	13.77	[illegible]
TM1-a	14.07	14.12	14.33	[illegible]
TM1-b	14.10	14.12	14.33	[illegible]
TM2-ωρ-a	13.23	13.02	13.47	13.21
TM2-ωρ-b	13.24	13.02	13.47	13.21

$\Omega = 0.03 km^{-1}$ $B_0 = 2 \times 10^{18} G$

EoS	L MeV
GM1[*1]	94.4
TM1-a[*2]	111.2
TM1-b[*2]	111.2
TM2-ωρ-a[*2]	54.8
TM2-ωρ-b[*2]	54.8

m

GM1 &
TM2-ωρ-a &
TM2-ωρ-b
are in the range.

Summary

- ✓ We calculated the mass-radius relations for magnetized and rotating neutron stars using various kinds of EoSs.
- ✓ We obtained neutron stars with masses more than 2 $M_{\odot}$ both with strong magnetic fields and in rapid rotations for 5 hadronic EoSs.
- ✓ TM1-a and TM1-b EoSs have larger radius than expected. They are out of bounds.

Future work

- To calculate MR relations for hybrid stars (mixture of quarks and hadrons) under the circumstance of rotation and magnetic fields.

- If the rotational axis and the deformation axis are different, gravitational waves might occur (wobbling motion). We are planning to look into that.

Some part of our work will be published in PTEP soon.

Thank you for your attention.

Back up

BACK UP

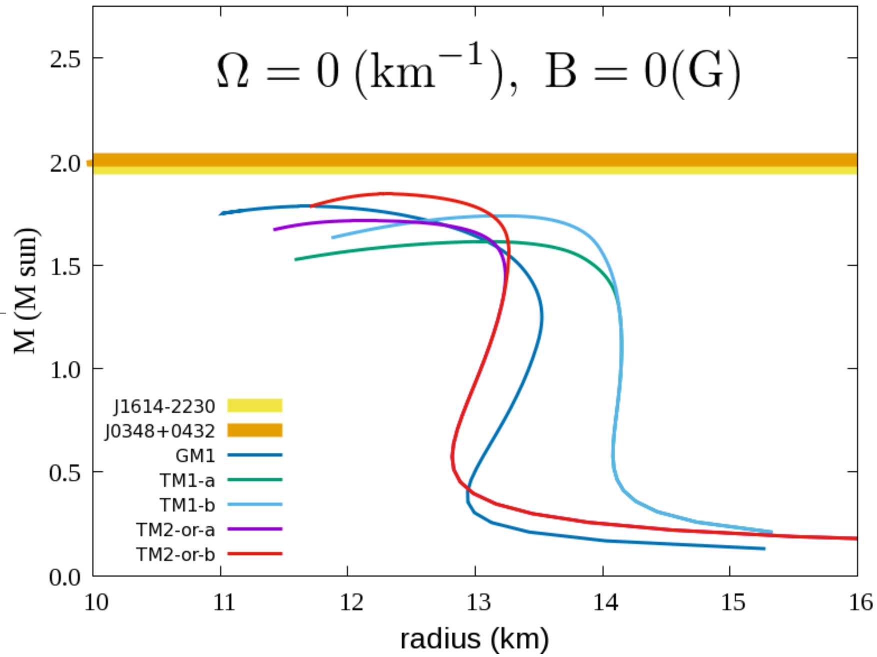
Including mesons

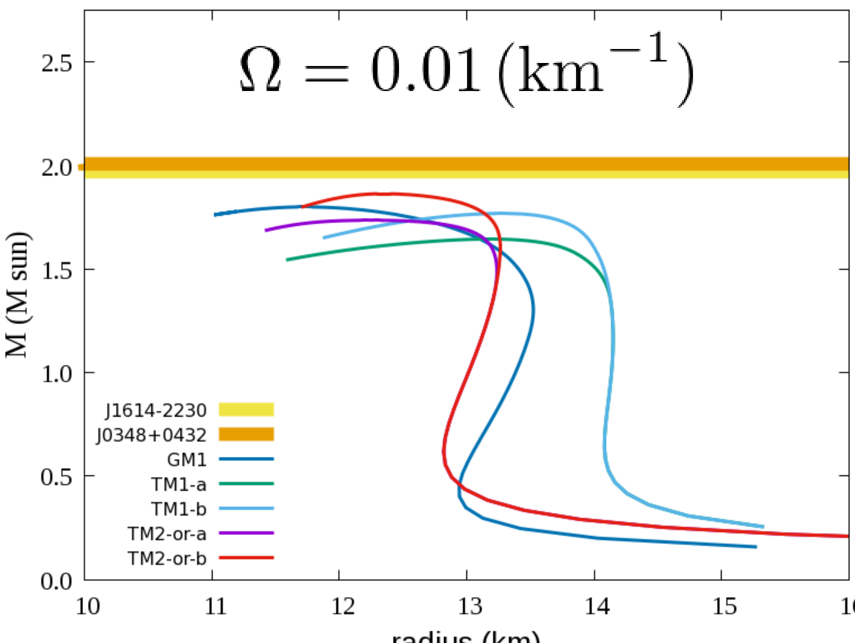
EoS	σ	ω	ρ	σ*	ϕ
GM1	○	○	○	-	-
TM1-a	○	○	○	○	○
TM1-b	○	○	○	○	○
TM2ωρ-a	○	○	○	○	○
TM2ωρ-b	○	○	○	○	○

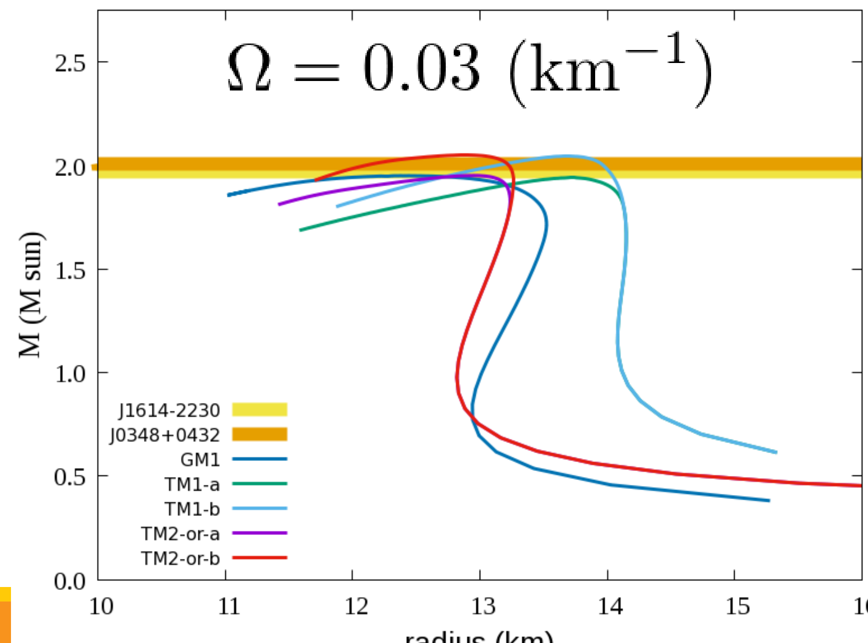
	p	n	Λ	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-
m_b	938.3	939.6	1116	1189	1193	1197	1314	1321
κ_b	1.79	-1.91	-0.61	1.67	1.61	-0.38	-1.25	0.06
q_b	$+1$	0	0	$+1$	0	-1	0	-1

Parameters for EoSs

	GM1	TM1	TM2$\omega\rho$
g_σ	8.895	10.03	9.998
g_ω	10.61	12.61	12.50
g_ρ	8.195	9.264	11.30
$b \times 10^3$	2.947	-1.508	-1.763
$c \times 10^3$	-1.070	0.061	-0.790
ξ	0	0.0169	0.0113
Λ_ω	0	0	0.03


$\Omega = 0\ (\mathrm{km}^{-1}),\ \mathrm{B} = 0(\mathrm{G})$
M (M sun)
radius (km)
J1614-2230
J0348+0432
GM1
TM1-a
TM1-b
TM2-or-a
TM2-or-b


$\Omega = 0.01\,(\mathrm{km}^{-1})$
M (M sun)
radius (km)
J1614-2230
J0348+0432
GM1
TM1-a
TM1-b
TM2-or-a
TM2-or-b


$\Omega = 0.03\ (\mathrm{km}^{-1})$
M (M sun)
radius (km)
J1614-2230
J0348+0432
GM1
TM1-a
TM1-b
TM2-or-a
TM2-or-b