Quark-hadron continuity beyond Ginzburg-Landau paradigm

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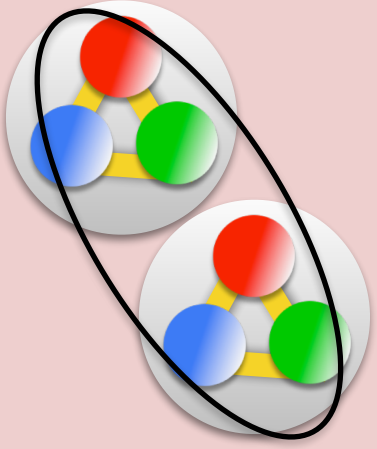
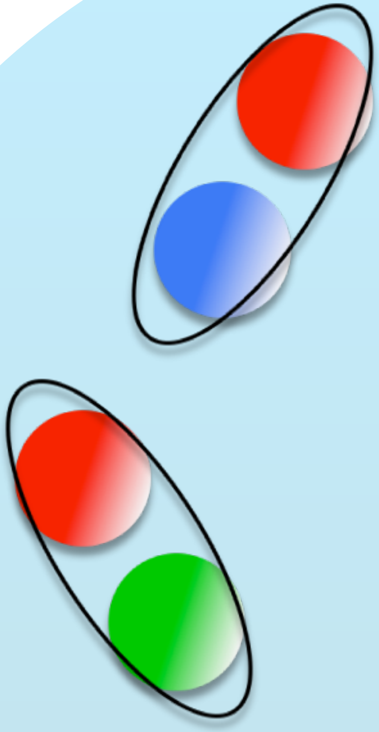
[arXiv:1811.10608]

with Yuya Tanizaki (NCSU)

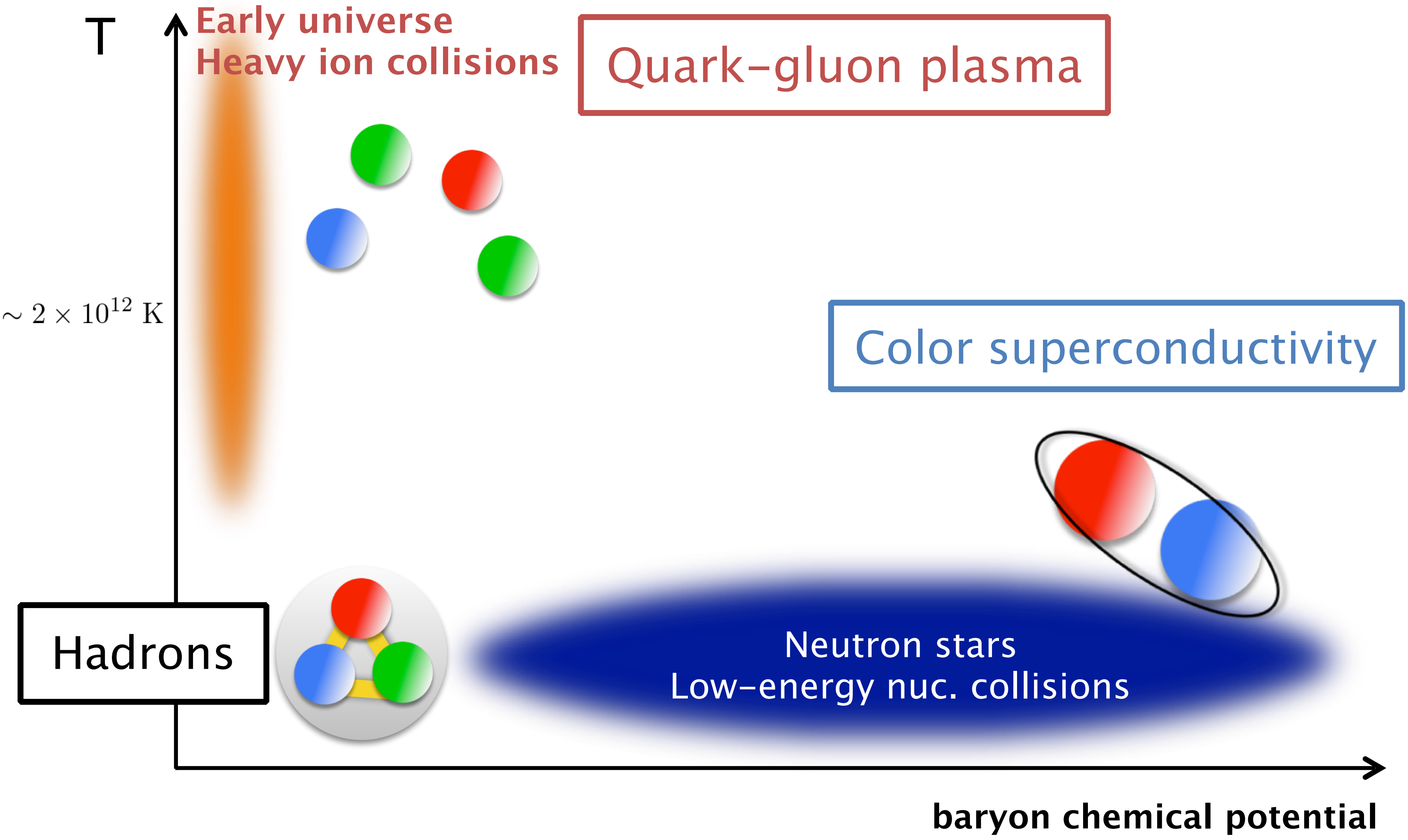




Yuji Hirono

apctp

Phases of QCD matter

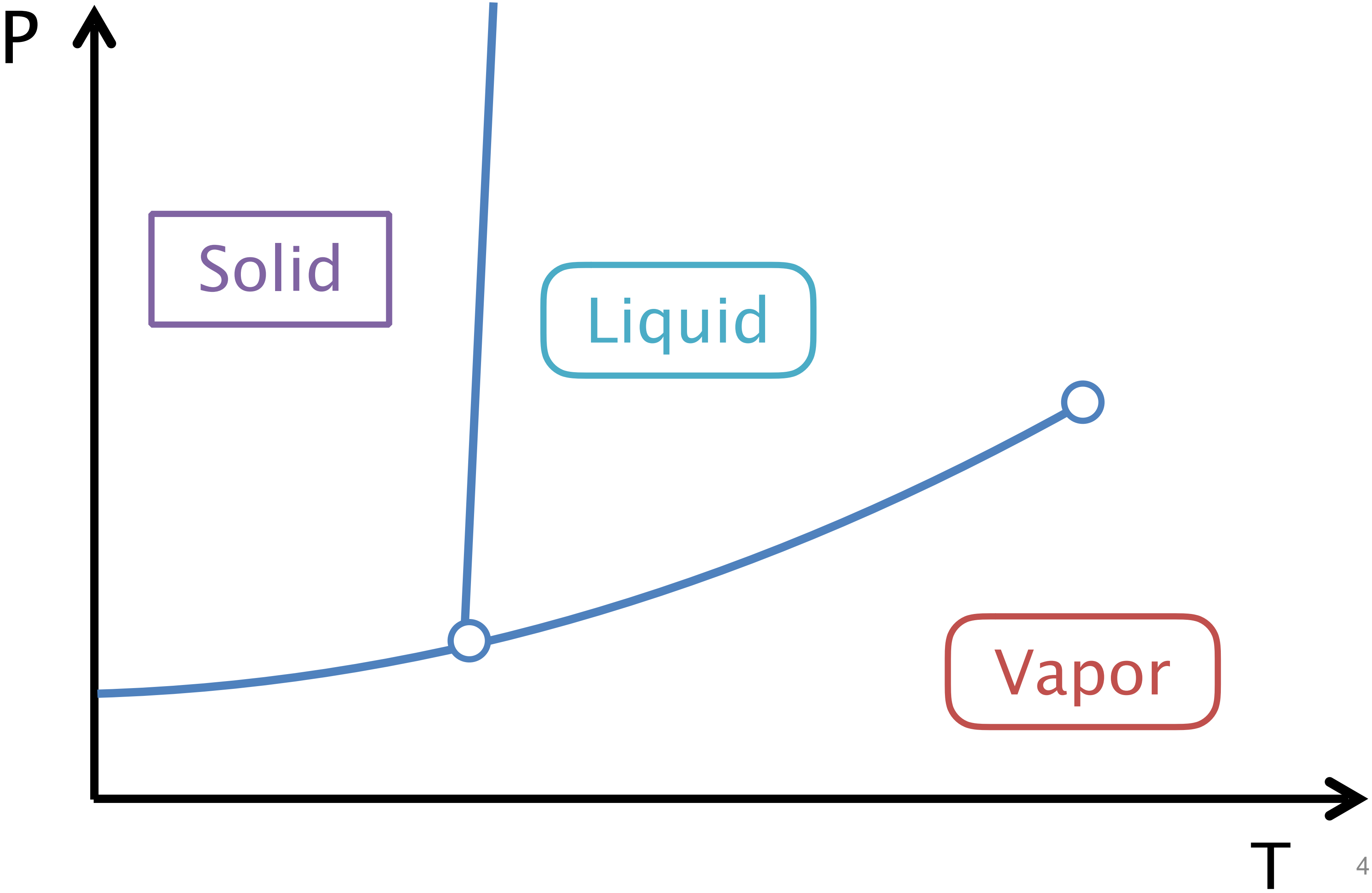


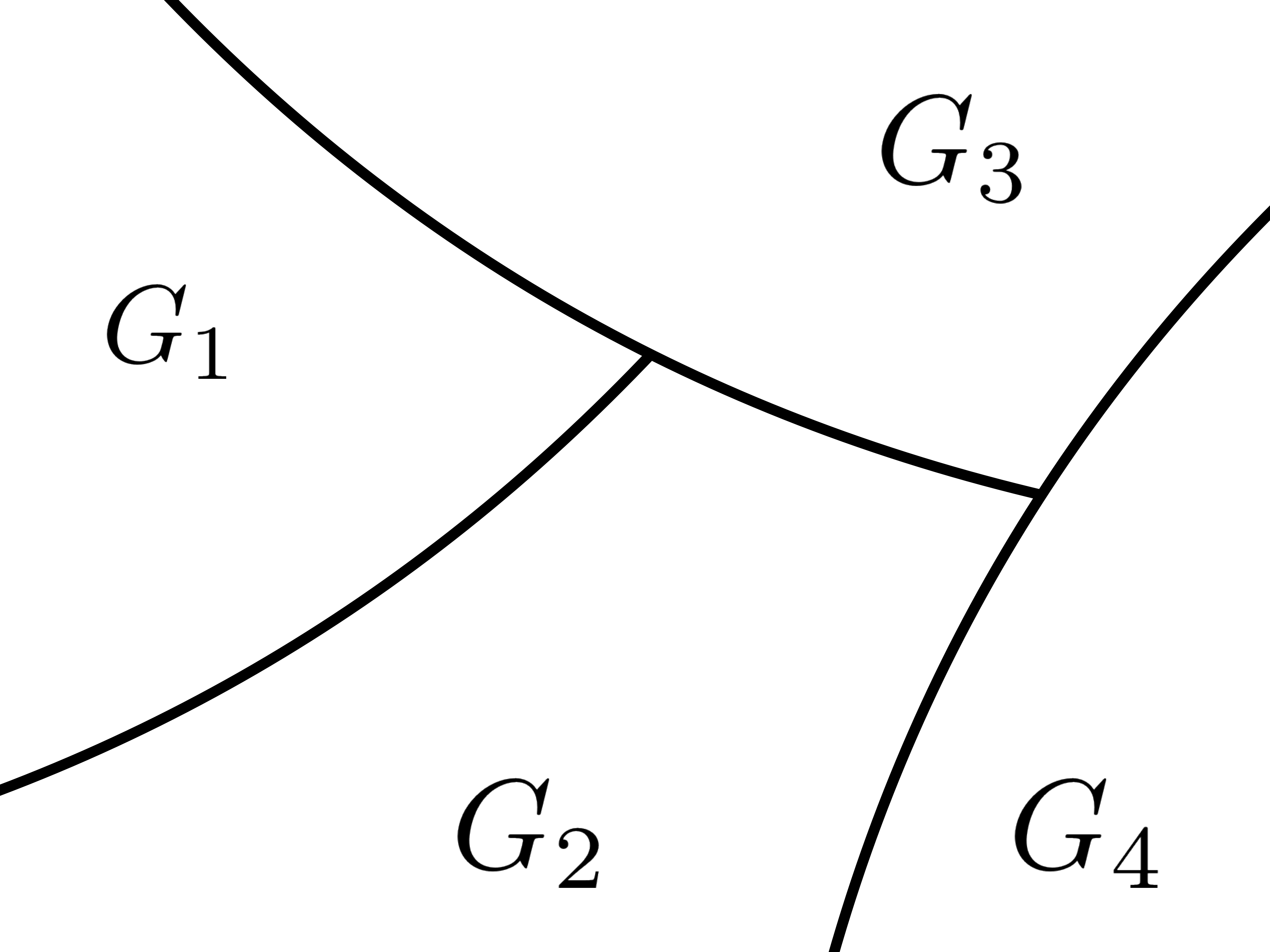
Classification of phases

- Ginzburg–Landau theory
  - Classification of phases by **symmetry breaking patterns**

- Ex) Water
  - Liquid, vapor: continuous translational symmetry
  - Solid: discrete translational symmetry

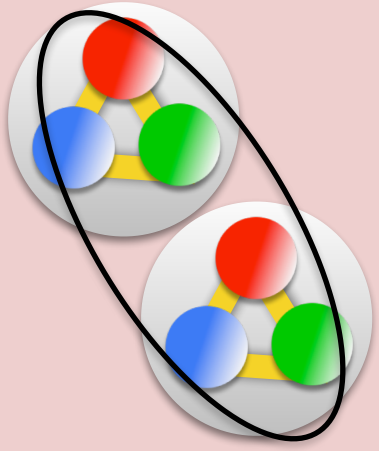
Phases of water



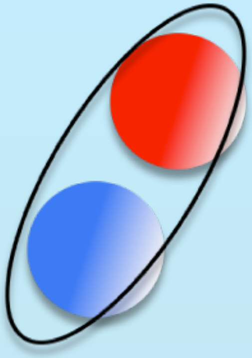

G_3
G_1
G_2
G_4

“Quark–hadron continuity”

[Schafer, Wilczek '99]



Nucleon superfluidity



Color superconductor
“CFL phase”

Color superconductivity

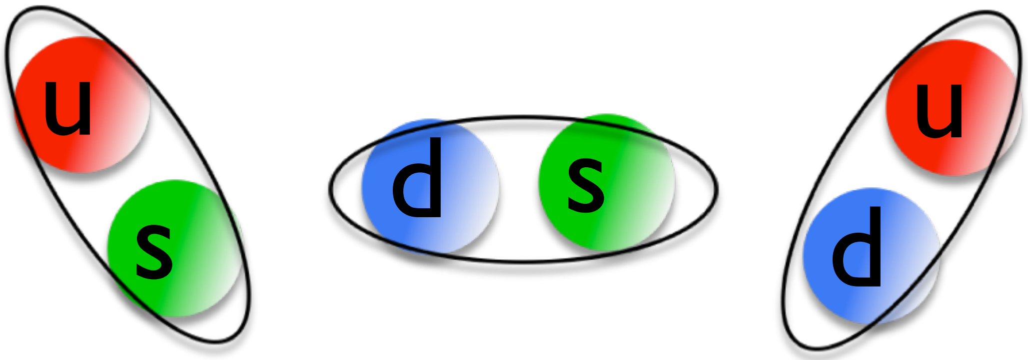
- SU(3) gauge theory with light quarks
  - up, down, strange
- Order parameter: diquark condensate

$$\Phi_{\underset{\substack{\nearrow\\\text{color}}}{\alpha}\underset{\substack{\nwarrow\\\text{flavor}}}{i}} = \epsilon_{\alpha\beta\gamma}\epsilon_{ijk}\langle q^T_{\beta j}\, i\gamma_0\gamma_2\, q_{\gamma k}\rangle$$

Color–flavor locked phase

- At large densities, the most stable pairing is

$$\Phi_{\alpha i} = \Delta \delta_{\alpha i}$$



- All the gluons are gapped: **color SC**
- SSB of global U(1): **superfluidity**

“Quark–hadron continuity”

[Schafer, Wilczek '99]

- Symmetry breaking pattern

$$SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times U(1)_{\mathrm{B}} \to SU(3)_{\mathrm{L+R}}$$

Nucleon superfluidity

CFL phase

“Quark–hadron continuity”

U(1) phonons ↔ **U(1) phonons**

Pions ↔ **CFL pions**

Baryons ↔ **Quarks**

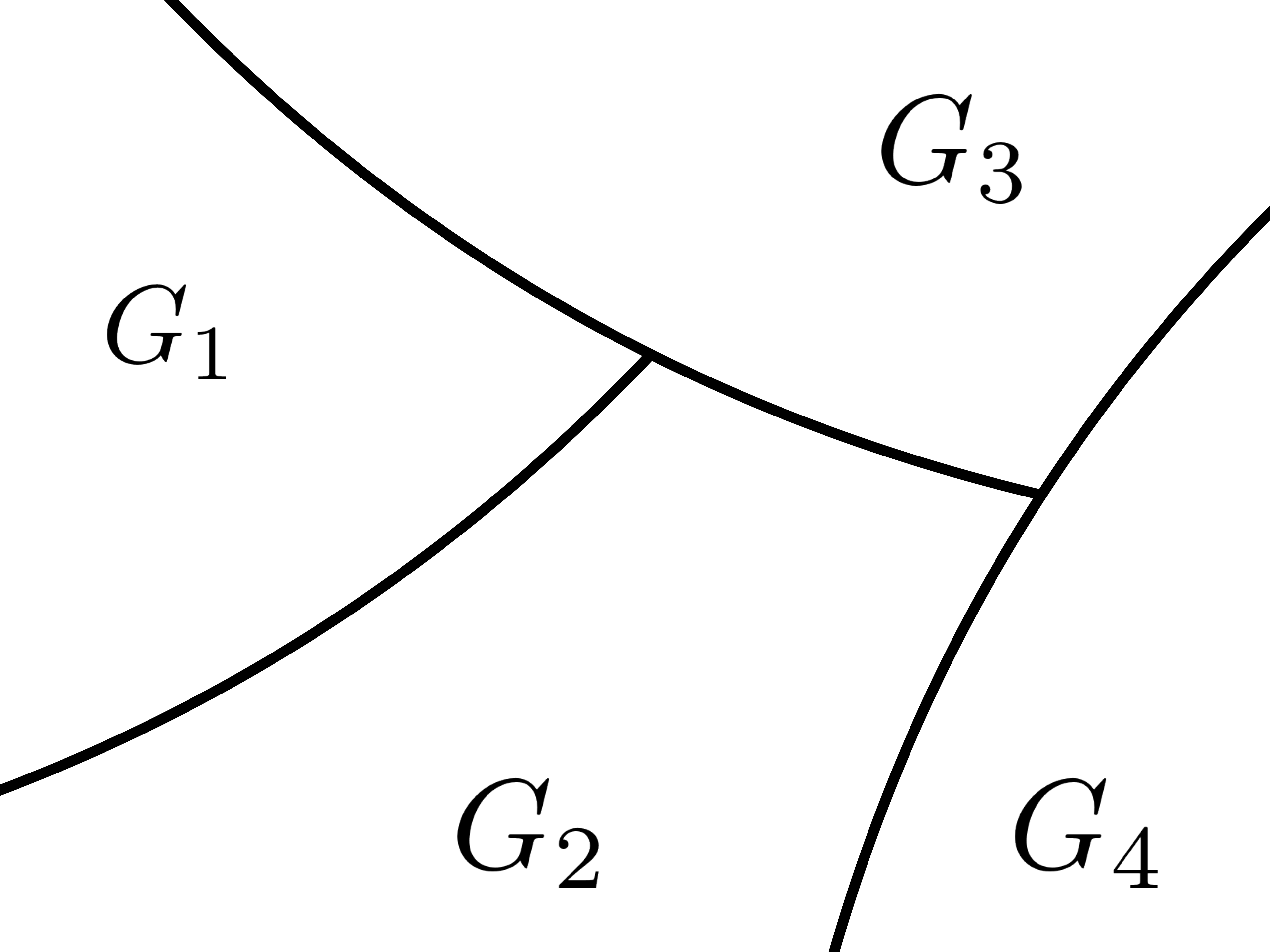
Vector mesons ↔ **Gluons**

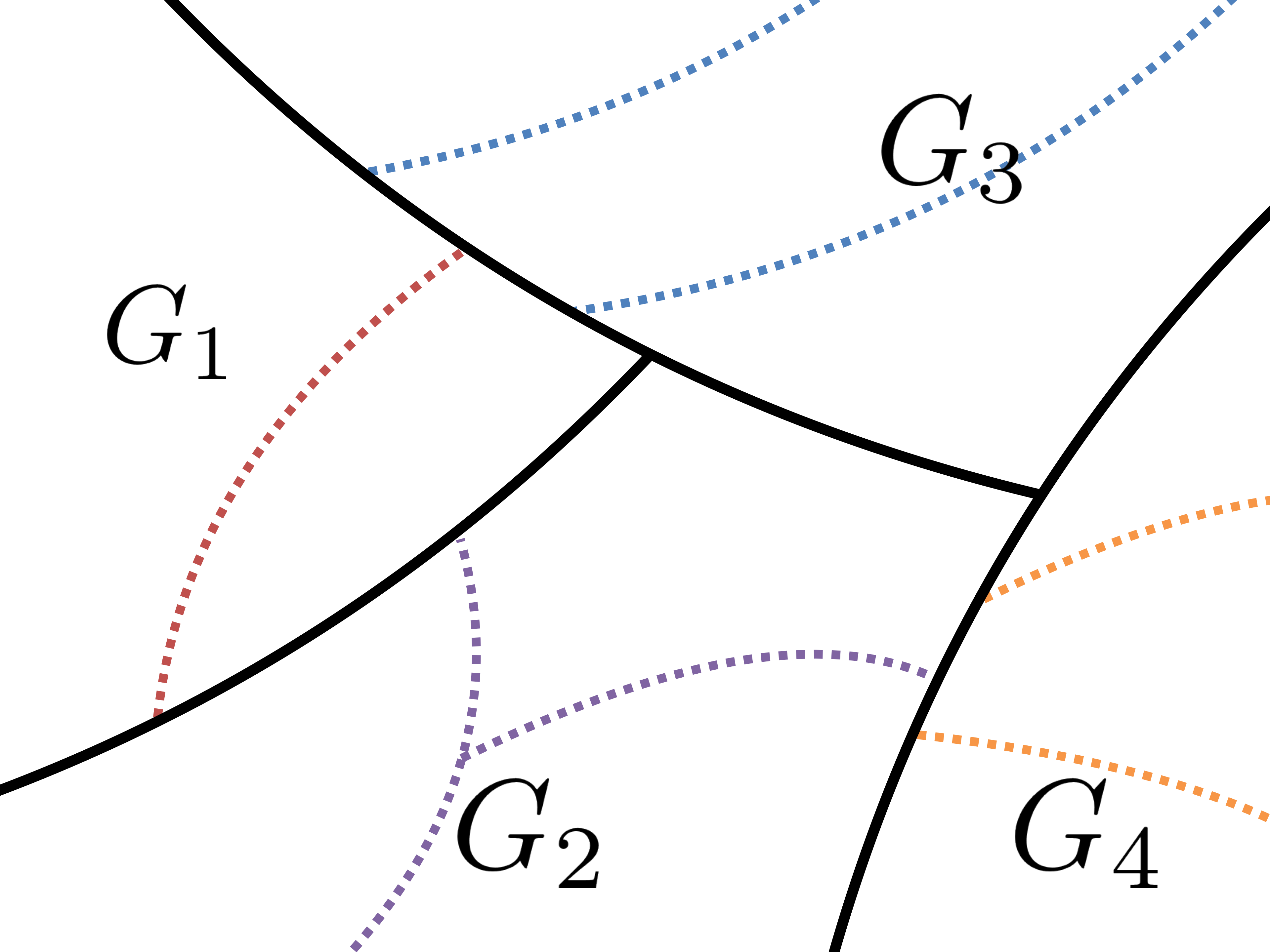
Exceptions in GL classification

- Fractional quantum Hall effect
  - Distinct phases without change of symmetry

- Now understood as *topological order* **[X. G. Wen '89]**

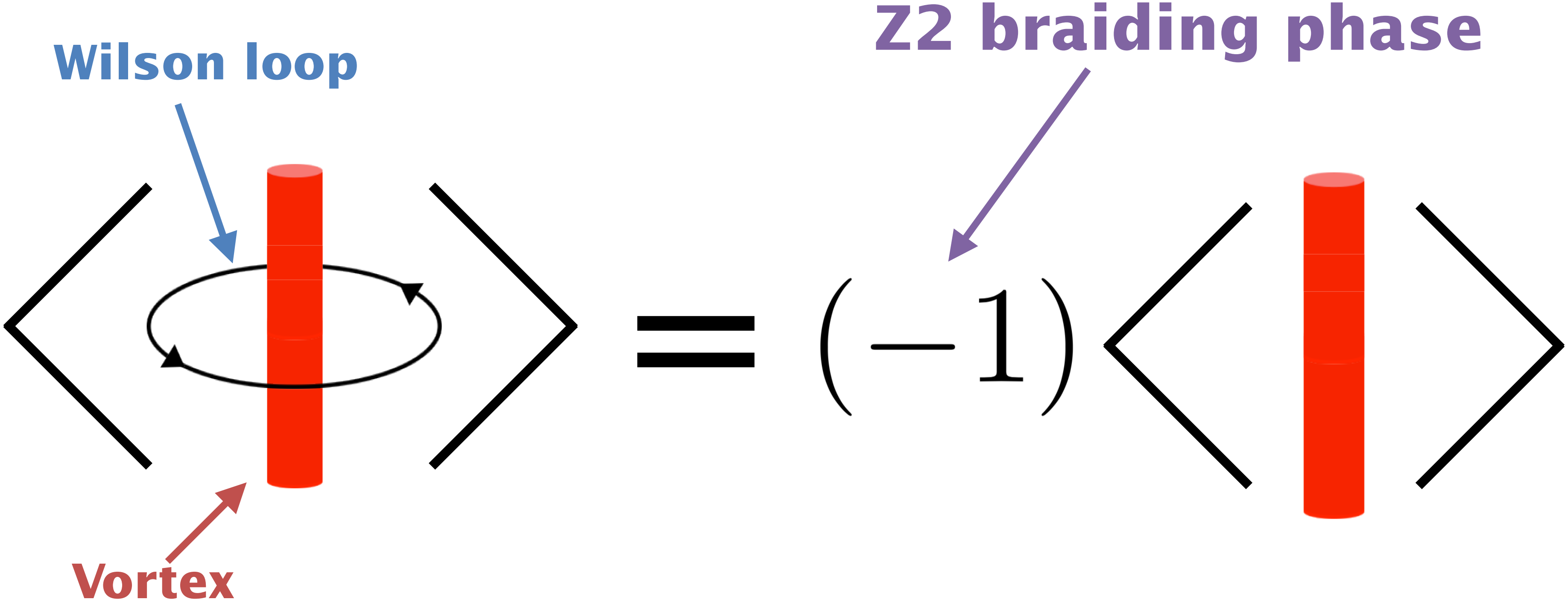
- Features of topologically ordered states
  - Fractional statistics (anyons)
  - Degenerate ground states depending on the spacetime topology
  - Description by topological QFT


G_3
G_1
G_2
G_4


G_1
G_2
G_3
G_4

s–wave superconductivity

- Topologically ordered
- Fractional braiding phase of vortex & particle

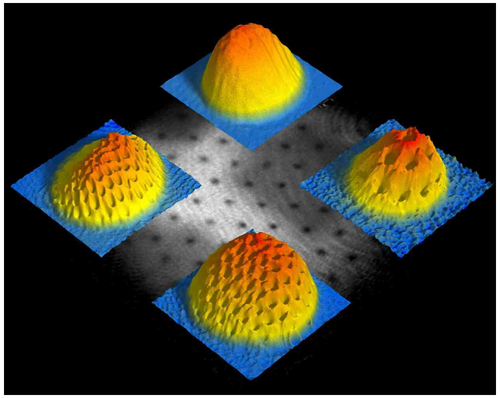


Vortices in CFL

[Balachandran, Digal, Matsuura '06]

$$\Phi \simeq \Delta \mathrm{diag}\left(e^{i\theta},1,1\right)$$

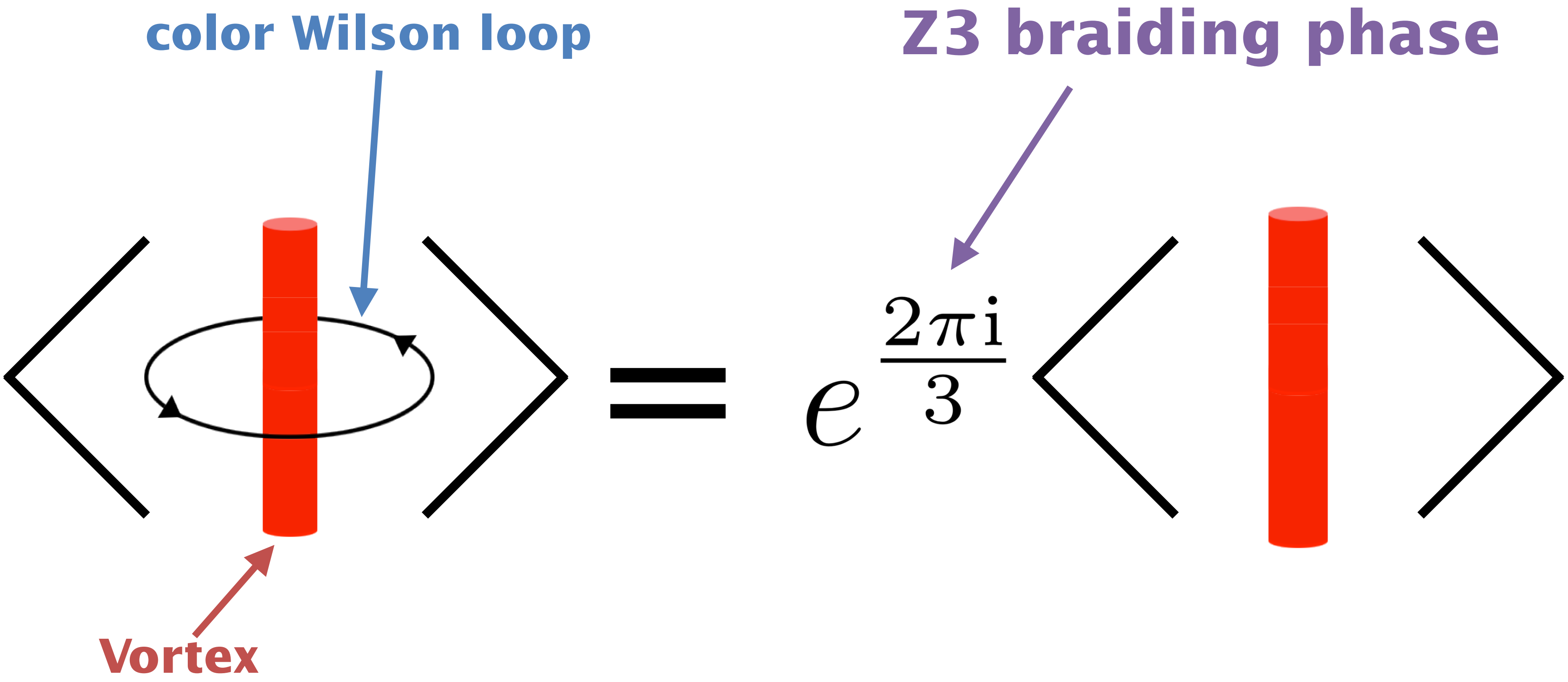
- Quantized (1/3) superfluid circulation
- Color magnetic flux
- Rotating neutron star



[http://cua.mit.edu/ketterle_group/Nice_pics.htm]

Fractional statistics of vortices & particles

[Cherman, Sen, Yaffe 1808.04827]



color Wilson loop

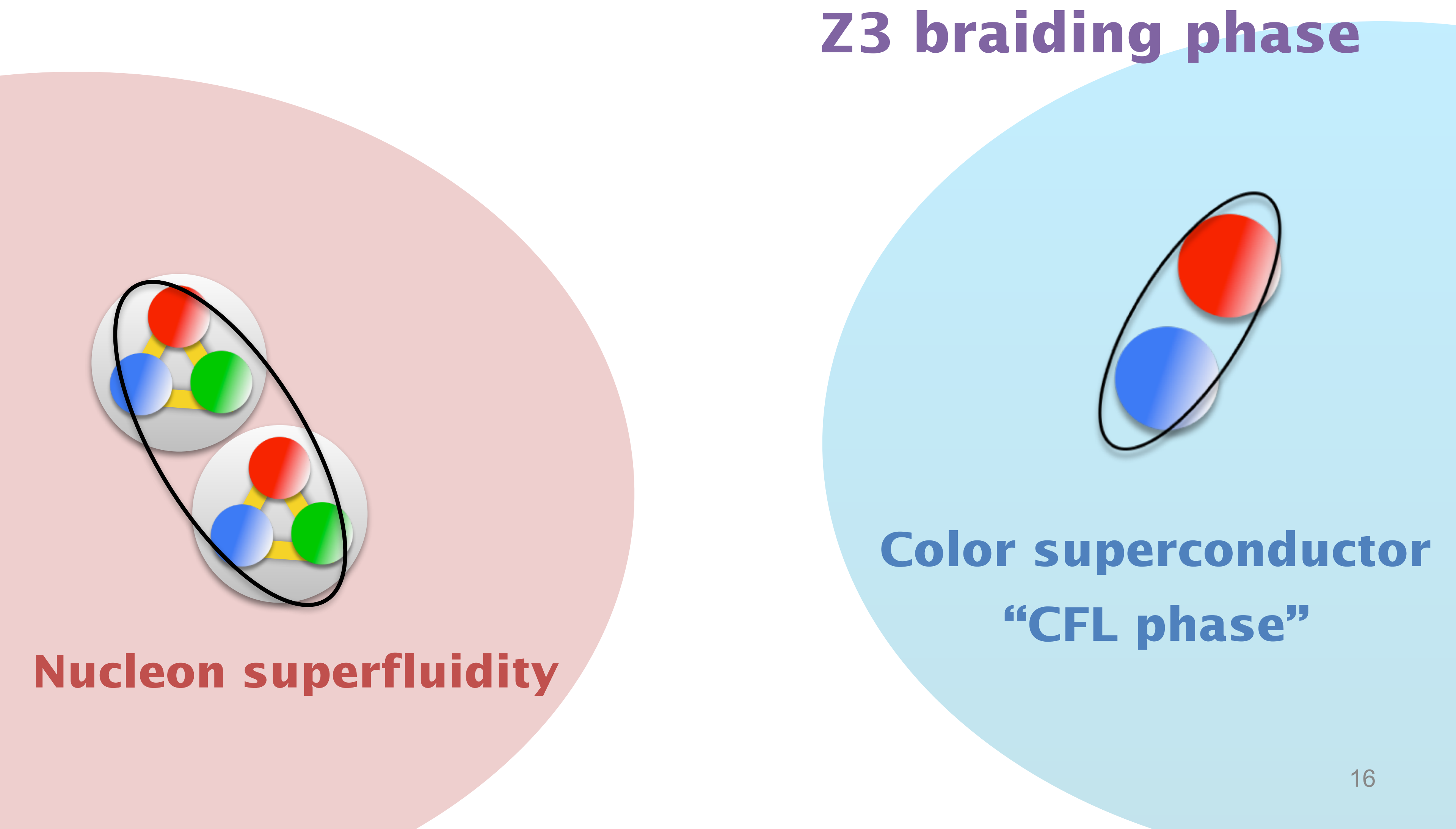
Z3 braiding phase

$$= e^{\frac{2\pi i}{3}}$$

Vortex

Fractional statistics of vortices & particles

[Cherman, Sen, Yaffe 1808.04827]



Z3 braiding phase

Nucleon superfluidity

Color superconductor

“CFL phase”

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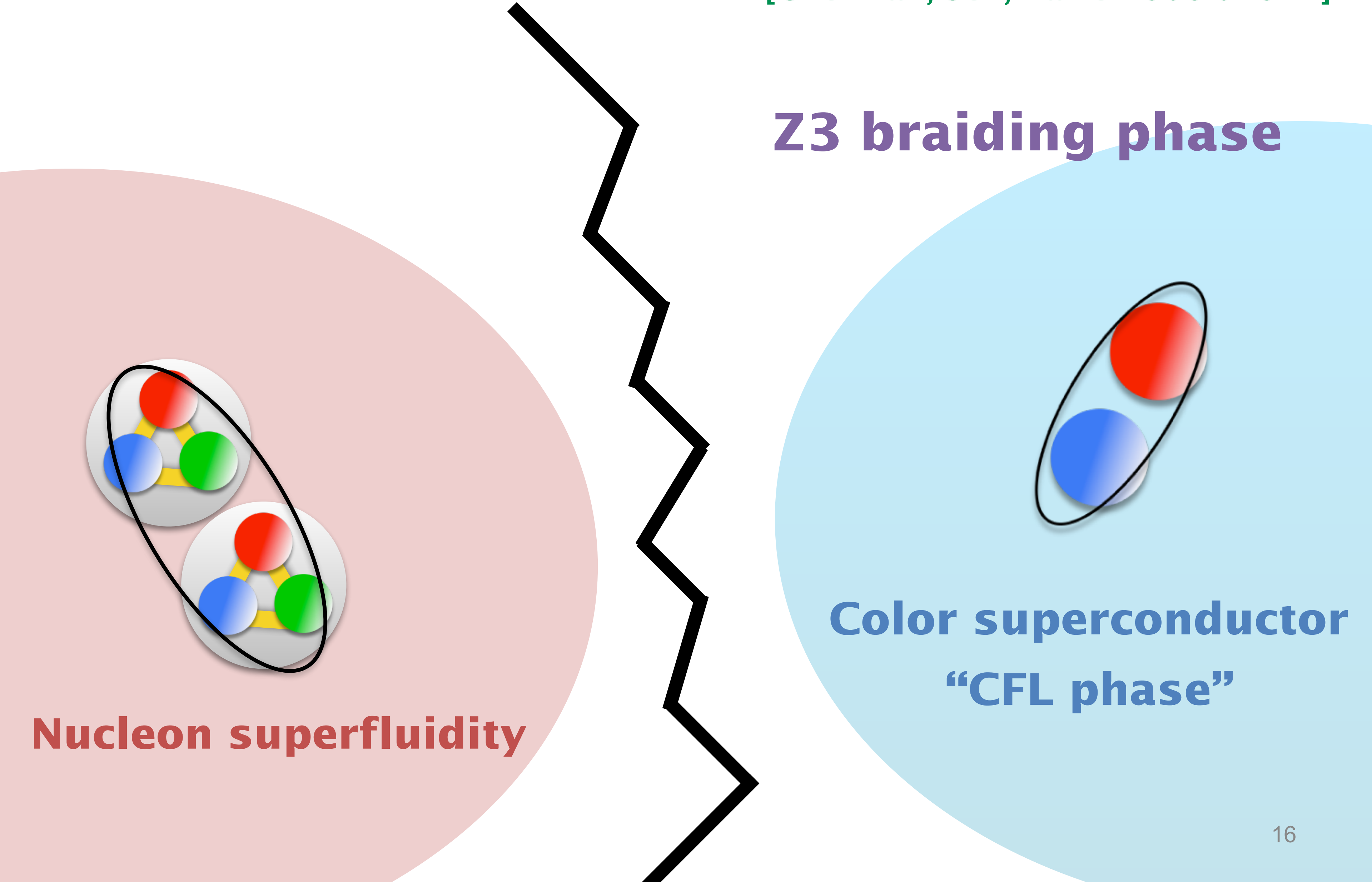


Z3 braiding phase

Nucleon superfluidity

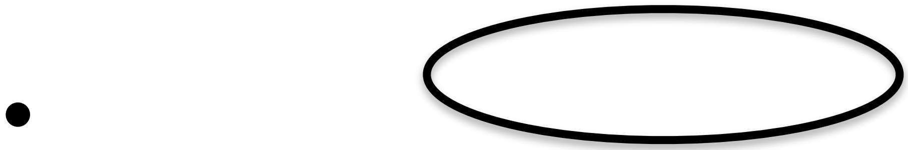
Color superconductor
“CFL phase”

How to characterize topological order

- Topological order: SSB of higher–form symmetry
  - A generalization of global symmetry

[Gaiotto, Kapustin, Seiberg, Willett ’15]

- Charged objects are *extended* : Wilson loop, etc
  - “n–form symmetry”



$$\psi(x) \qquad\qquad W(C) = e^{\mathrm{i}\int_C a}$$

How to characterize topological order

- Ex) U(1) gauge theory without matter

$$W(C) = e^{\mathrm{i}\int_C a} \quad U_\alpha W(C) U_\alpha^{-1} = e^{\mathrm{i}\alpha} W(C)$$

How to characterize topological order

- Ex) U(1) gauge theory without matter

$$W(C) = e^{\mathrm{i}\int_C a} \quad U_\alpha W(C) U_\alpha^{-1} = e^{\mathrm{i}\alpha} W(C)$$

- Noether's theorem
  - Ex.) conservation of electric & magnetic flux
- SSB of continuous HF symmetry
  - Nambu–Goldstone boson (ex. photon)
- SSB of discrete HF symmetry
  - topological order
    - s-wave SC: SSB of Z2 one-form symmetry

Topological order of s-wave superconductors

Low-energy theory for SC

$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi} \int b \wedge \mathrm{d}a$$

We consider SC in 2+1 dimensions.

At low energies below SC gap, the system is described by the so-called BF theory at level 2

$$k = 2 \qquad a = a_\mu \mathrm{d}x^\mu \qquad b = b_\mu \mathrm{d}x^\mu$$

$$\int b \wedge \mathrm{d}a = \int \mathrm{d}^3x \, \epsilon^{\mu\nu\rho} b_\mu \partial_\nu a_\rho$$

Low-energy theory for SC

$$S_{\mathrm{BF}} = \frac{\mathrm{i}k}{2\pi}\int b \wedge \mathrm{d}a$$

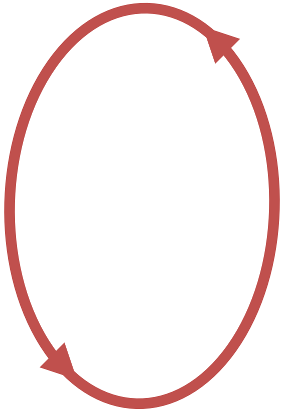
This theory describes

- Fractional statistics of vortices & particles
- Ground-state degeneracy depending of space-time topology

Physical observables

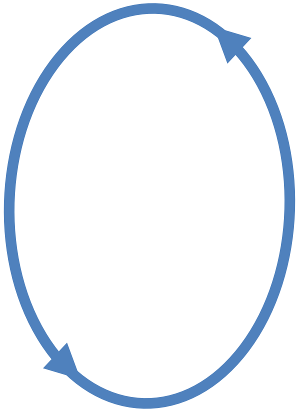
Wilson loop operator

$$W(C) = \exp i \int_C a$$



Vortex operator

$$V(C) = \exp i \int_C b$$



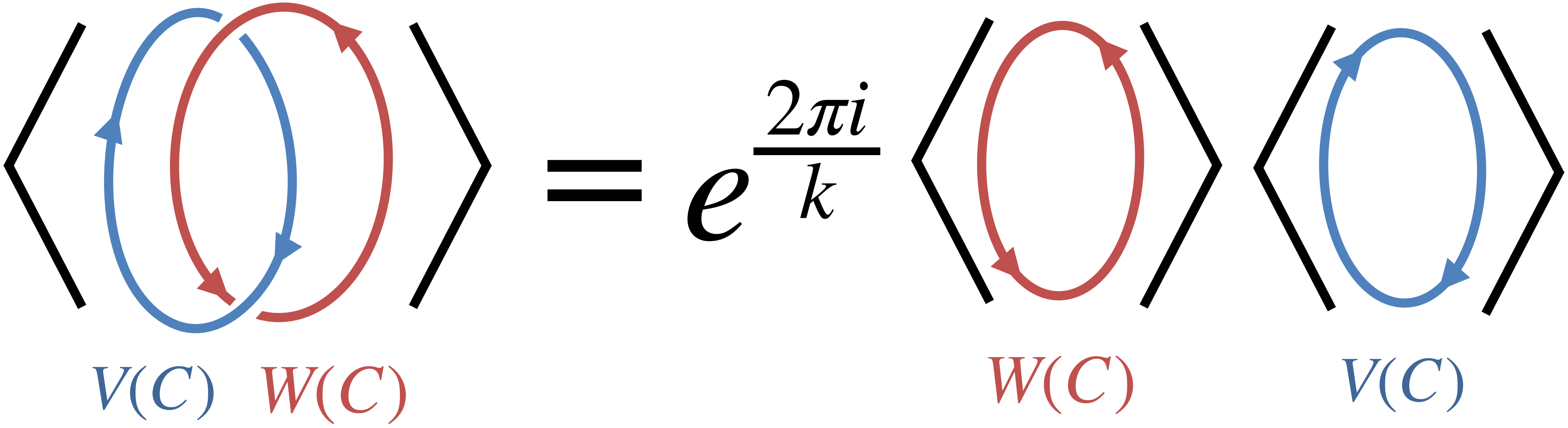
represents pair-creation/annihilation process of charged particles/vortices

Higher–form symmetries

- Two emergent $\mathbb{Z}_k$ one–form symmetries
- Charged objects are Wilson loops & vortex loops

$$W(C) \mapsto e^{\frac{2\pi i}{k}} W(C) \qquad V(C) \mapsto e^{\frac{2\pi i}{k}} V(C)$$

- Fractional statistics

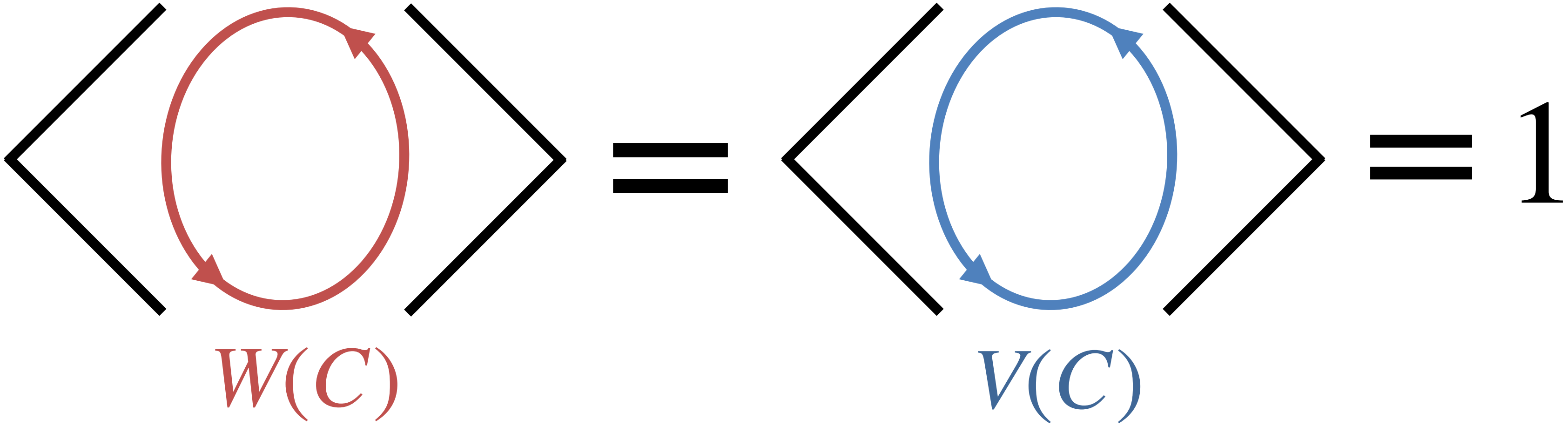


Higher-form symmetries

- Two emergent $\mathbb{Z}_k$ one-form symmetries
- Charged objects are Wilson loops & vortex loops

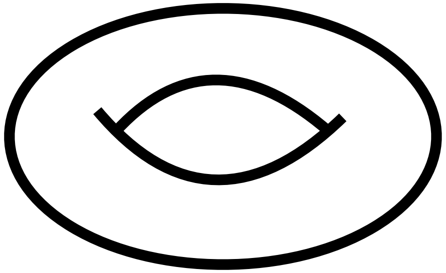
$$W(C) \mapsto e^{\frac{2\pi i}{k}} W(C) \qquad V(C) \mapsto e^{\frac{2\pi i}{k}} V(C)$$

- Both symmetries are spontaneously broken

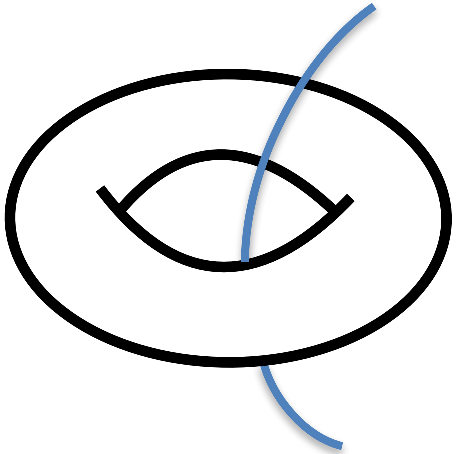


Topological ground state degeneracy

$$|\Omega\rangle =$$



$$V(C)|\Omega\rangle =$$



Those states have the same energy, because vortex operator is topological (it commutes with Hamiltonian)

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

- To study the topological nature, we analyze the higher-form symmetry of CFL

- We consider degenerate masses for u, d, s
- Massless degrees of freedom:

U(1) phonons

- Fractional statistics

- Correlation of **U(1) circulation**

& **color holonomy**

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

- Gauged GL Lagrangian

$$\mathscr{L} = \frac{1}{2g^2}|G|^2 + |(\mathrm{d} + ia_{SU(3)})\Phi|^2 + V_{\mathrm{eff}}[\Phi]$$

$$G = \mathrm{d}a_{SU(3)} + i(a_{SU(3)})^2$$

- Fix the gauge so that $\Phi = \Delta\,\mathrm{diag}\left[e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}\right]$
- The resulting Lagrangian is

$$\mathscr{L} = \frac{1}{2g_0^2}\left(|\mathrm{d}\phi_1 + a_1|^2 + |\mathrm{d}\phi_2 - a_1 + a_2|^2 + |\mathrm{d}\phi_3 - a_2|^2\right)$$

where Cartan generators are taken as

$$\tau_1 = \mathrm{diag}\,(1, -1, 0) \qquad \tau_2 = \mathrm{diag}\,(0, 1, -1)$$

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

$$S = S_{\mathrm{phonon}}[b_i] + \frac{i}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

Phonons

BF term

$i = 1, 2, 3 \qquad A = 1, 2$

$b_i = \frac{1}{2}(b_i)_{\mu\nu} dx^\mu \wedge dx^\nu$:2-form fields dual to ϕ_i

- Topological BF theory coupled with massless superfluid phonons

$$K = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}$$

- not square
- dim coker K
  = (# of massless phonons)

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

$$S = S_{\mathrm{phonon}}[b_i] + \frac{i}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

Phonons

BF term

$i = 1, 2, 3 \qquad A = 1, 2$

$$S_{\mathrm{phonon}}[b_i] = \frac{g_0^2}{8\pi^2} \int \mathrm{d}(b_0)_i \wedge \star \mathrm{d}(b_0)_i$$

$$(b_0)_i = P_{ij} b_j \qquad P_{ij} = \delta_{ij} - [KK^+]_{ij}$$

K^+_{Ai} is the Moore-Penrose inverse of K_{iA}

$$K^+ = \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ 1/3 & 1/3 & -2/3 \end{pmatrix}$$

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

$$S = S_{\text{phonon}}[b_i] + \frac{i}{2\pi} K_{iA} \int b_i \wedge \mathrm{d}a_A$$

- Physical observables

$$W_{\boldsymbol{q}}(C) = \exp \mathrm{i}\, q_A \int_C a_A$$

color Wilson loops

$$V_{\boldsymbol{p}}(S) = \exp \mathrm{i}\, p_i \int_S b_i$$

vortex operators

S : worldsheet of a vortex

Phonons

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

- The system has a $\mathbb{Z}_3$ two-form symmetry

$$b_i \mapsto b_i + K^+_{Ai}\lambda \qquad \mathrm{d}\lambda = 0,\ \int_S \lambda \in 2\pi\mathbb{Z}$$

- Rotate the phase of vortex operators by $\mathbb{Z}_3$ phase

$$V_{\boldsymbol{p}}(S) \mapsto \mathrm{e}^{\frac{2\pi\mathrm{i}\sum_i p_i}{3}} V_{\boldsymbol{p}}(S)$$

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

- Braiding phase of particles & vortices

$$\frac{\langle W_{\boldsymbol{q}}(C) V_{\boldsymbol{p}}(S)\rangle}{\langle W_{\boldsymbol{q}}(C)\rangle \langle V_{\boldsymbol{p}}(S)\rangle} = \exp\left[2\pi\mathrm{i}\; q_A K^{+}_{Ai}\, p_i\; \mathrm{link}(C,S)\right]$$

[Hirono, Tanizaki, arXiv:1904.08570]

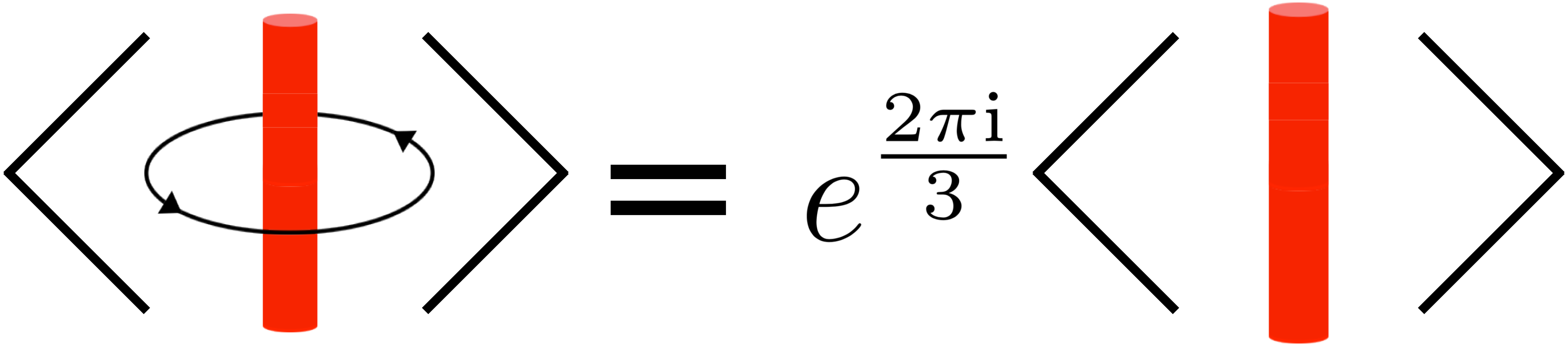
- Noting that $\langle W_{\boldsymbol{q}}(C)\rangle = 1$

$$\langle W_{\boldsymbol{q}}(C) V_{\boldsymbol{p}}(S)\rangle = \mathrm{e}^{2\pi\mathrm{i} q K^{+} p\, \mathrm{link}(C,S)} \langle V_{\boldsymbol{p}}(S)\rangle$$

Wilson loops are the generators of Z3 symmetry

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL’19]



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$$\langle W_{\boldsymbol{q}}(C) V_{\boldsymbol{p}}(S)\rangle = \mathrm{e}^{2\pi \mathrm{i} q K^{+} p \, \mathrm{link}(C,S)} \langle V_{\boldsymbol{p}}(S)\rangle$$

Wilson loops are the generators of Z3 symmetry

Low-energy effective theory for CFL

[Hirono, Tanizaki, PRL'19]

- Z3 2-form symmetry is **unbroken**

$$\langle V(S)\rangle \xrightarrow[\text{large } S]{} 0$$

- Z3 2-form symmetry $\subset U(1)$ 2-form symmetry
- Continuous 2-form symmetry cannot be broken in 4D (Coleman-Mermin-Wagner theorem)

Physical consequences

- Braiding phase is because of an emergent Z3 two–form symmetry
- This symmetry is not spontaneously broken
- CFL phase is not topologically ordered
- Nucleon superfluidity is not either:

  Continuity nucleon SF & color SC including higher-form symmetries

Summary

- GL-classification has exceptions: topological order
- Spontaneous breaking of a (discrete) higher-form symmetry leads to topological order
- To test the “quark-hadron continuity”, we analyzed the symmetries of low-energy EFT of CFL phase including higher-form symmetries
- Z3 braiding phase of color Wilson loop & vortex
  = consequence of an emergent Z3 symmetry
- Z3 2-form symmetry is unbroken: no T.O. in CFL
- Quark-hadron continuity is still a consistent scenario