Non-Abelian vortices in dense QCD: quark hadron continuity and non-Abelian statistics

2019/6/24@ Tokyo campus, Univ. of Tsukuba

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Keio U.(慶應義塾大学)

Topological Materials Science

Keio University
1858
CALAMVS
GLADIO
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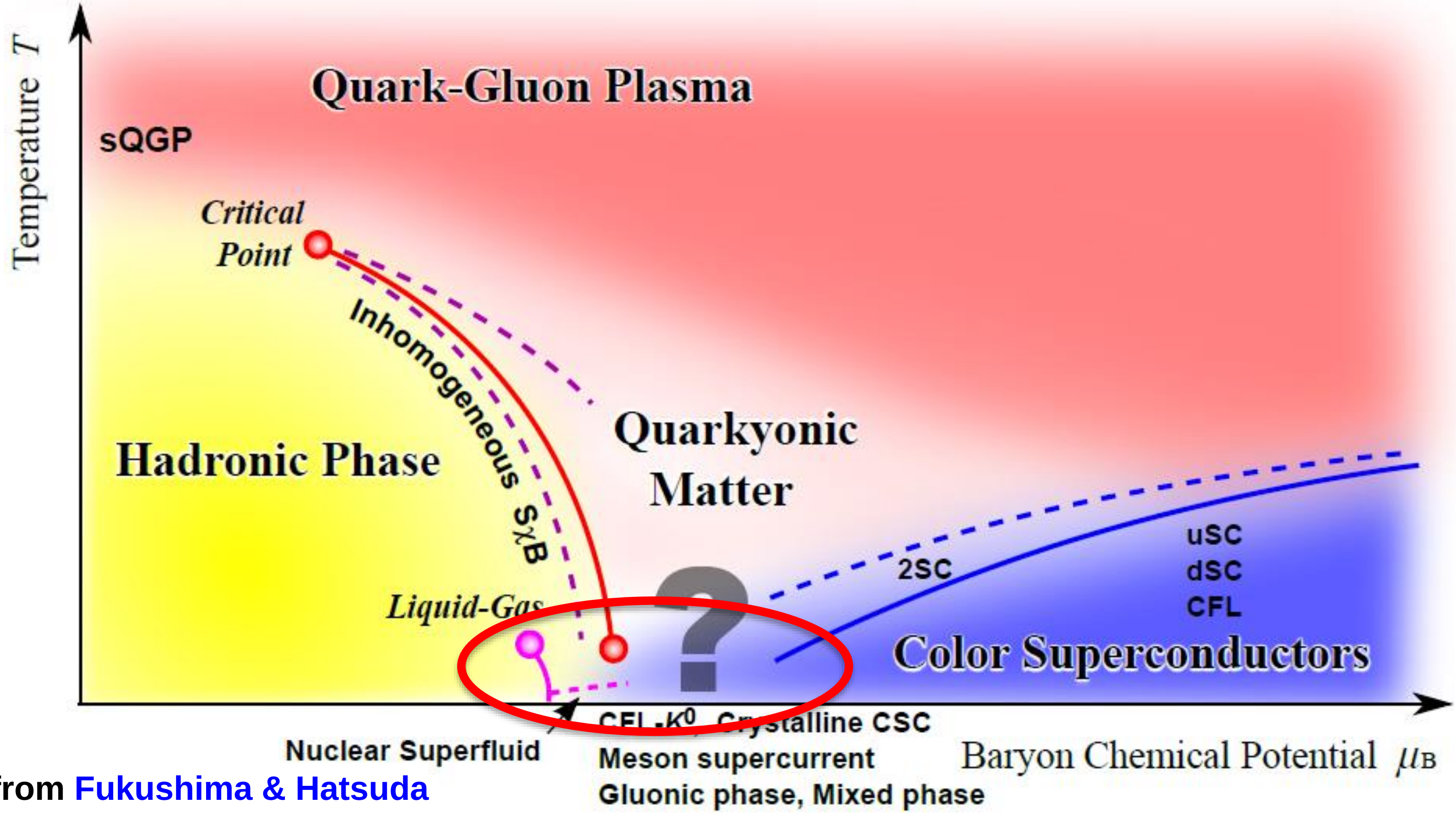
Topic in this talk



from Fukushima & Hatsuda
Rept.Prog.Phys. 74 (2011) 014001

Quantum Chromo Dynamics (QCD)

Quark matter

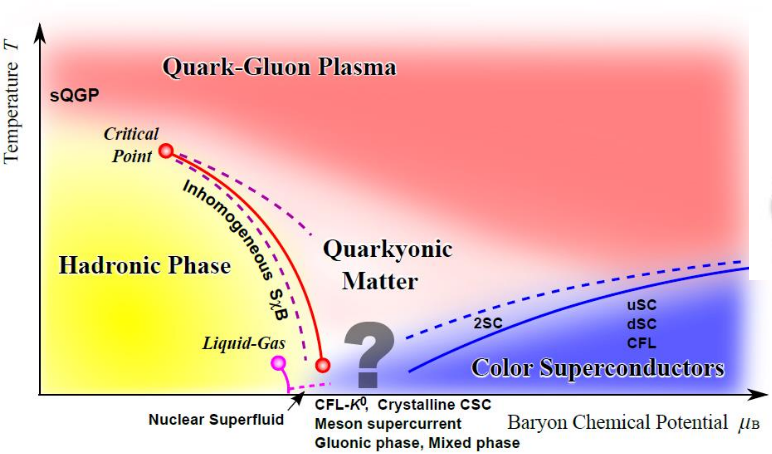
quarks

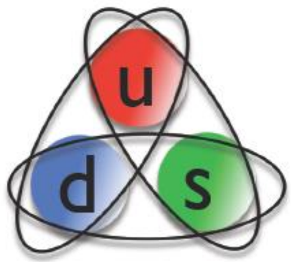
$$q^{i}_{\alpha}$$ i = u,d,s flavor(global) SU(3)
α = r,g,b color(gauge) SU(3)

Color-flavor locked (CFL) phase

"Color superconductor"

@ high density





Bailin-Love('79),
Iwasaki-Iwado('95)
Alford-Rajagopal-Wilczek('98)

$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q^{j}_{\beta} q^{k}_{\gamma} \sim \mathbf{1}_{\alpha i}$$

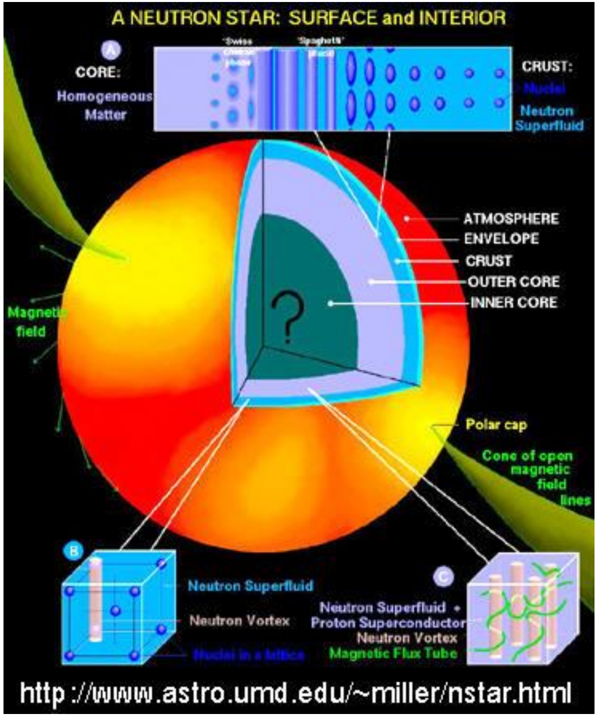
3x3 matrix

from Fukushima & Hatsuda
Rept.Prog.Phys. 74 (2011) 014001

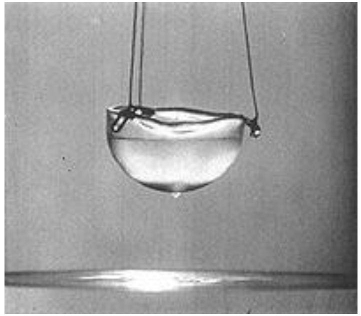
Color superconductivity
as well as *superfluidity*

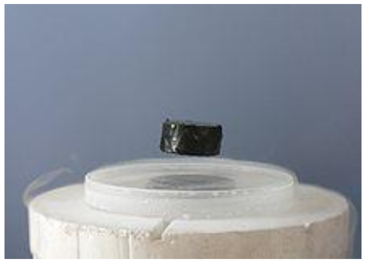
Neutron Stars

Core **Nuclear matter**



Neutron **superfluid**





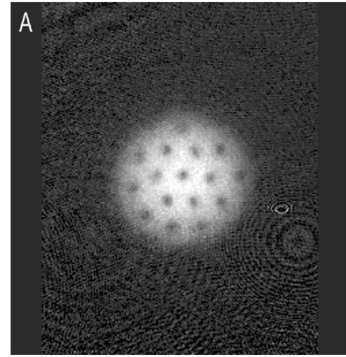
Proton **super-conductor**

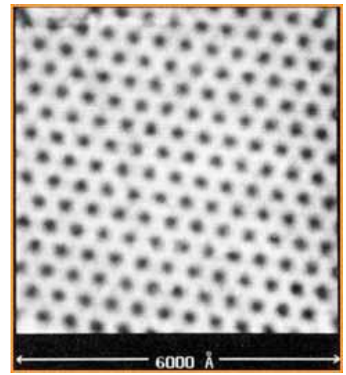
Rotation

Magnetic field

Baym&Pines ('60s)
Anderson&Itoh ('75)

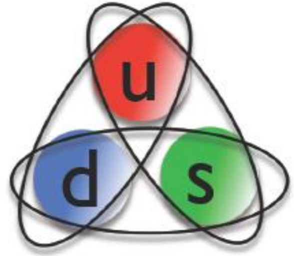
Superfluid vortices





vortices (Flux tubes)

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

$$\Phi_{\alpha i} = \begin{pmatrix} d_{[g}s_{b]} & s_{[g}u_{b]} & u_{[g}d_{b]} \\ d_{[b}s_{r]} & s_{[b}u_{r]} & u_{[b}d_{r]} \\ d_{[r}s_{g]} & s_{[r}u_{g]} & u_{[r}d_{g]} \end{pmatrix} \begin{matrix} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{matrix}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}}$$

$$\boxed{\Phi_{\alpha i} \to e^{i\alpha} g_{\mathrm{color}} \Phi_{\alpha i} g_{\mathrm{flavor}}}$$

Landau-Ginzburg model from QCD

Iida&Baym('01)
Giannakis&Ren('02)
Iida,Matsuura, Tachibana&Hatsuda('04)

$$\mathcal{L} = \mathrm{Tr}(K_0 \mathcal{D}_0 \Phi^\dagger \mathcal{D}_0 \Phi - K_3 \mathcal{D}_i \Phi^\dagger \mathcal{D}_i \Phi),$$

$$+ \frac{\varepsilon}{2} F_{0i}^2 - \frac{1}{4\lambda} F_{ij}^2 - V,$$

$$V_{\mathrm{GL}} = \mathrm{Tr}\left[\Phi^\dagger \left\{\left(\alpha + \frac{2\epsilon}{3}\right)\mathbf{1}_3 + \epsilon X_3\right\}\Phi\right]$$

$$+ \beta_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 + \beta_2 \mathrm{Tr}[(\Phi^\dagger \Phi)^2],$$

GL parameters

$$\alpha = 4N(\mu)\log\frac{T}{T_c}, \quad \beta_{1,2} = \frac{7\zeta(3)}{8(\pi T_c)^2} N(\mu) \equiv \beta,$$

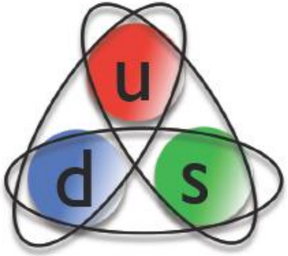
$$K_3 = \frac{1}{3}K_0 = \frac{7\zeta(3)}{12(\pi T_c)^2} N(\mu), \quad \epsilon = N(\mu)\frac{m_s^2}{\mu^2}\log\frac{\mu}{T_c},$$

density of state

$$N(\mu) = \frac{\mu^2}{2\pi^2}$$

For a while we consider high density limit, where strange quark mass can be neglected.
We also ignore E&M interaction. We can take into account these appropriately.

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

Ground state

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta & 0 & 0 \\ 0 & \Delta & 0 \\ 0 & 0 & \Delta \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

color-flavor locked (CFL)

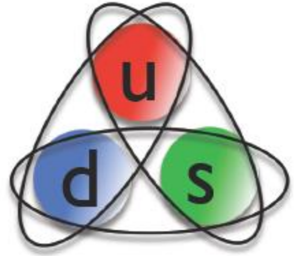
$$G = U(1)_{\mathrm{B}} \times SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{F}} \quad \cancel{U(1)_{\mathrm{B}}}$$

superfluidity

$$\to H = SU(3)_{\mathrm{C+F}} \quad g_{\mathrm{color}} = g_{\mathrm{flavor}}^{-1} \quad \cancel{SU(3)_{\mathrm{C}}}$$

color superconductivity

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^{\beta} q_k^{\gamma}$$

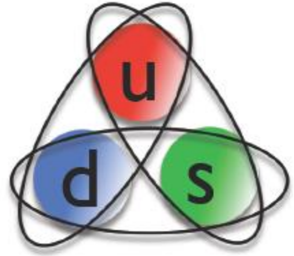
Integer quantized superfluid vortex

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

$$\begin{array}{cc} \Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix} & \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array} \\ \begin{array}{ccc} \quad \bar{u} = ds & \quad \bar{d} = sb & \quad \bar{s} = ud \end{array} & \end{array}$$

Iida & Baym, Forbes & Zhitnitsky('02)

Color superconductor



$$\Phi_{\alpha i} = \varepsilon_{\alpha\beta\gamma}\varepsilon_{ijk} q_j^\beta q_k^\gamma$$

$$\alpha = 1,2,3\ (\mathrm{r,g,b}) \quad i = 1,2,3\ (\mathrm{u,d,s})$$

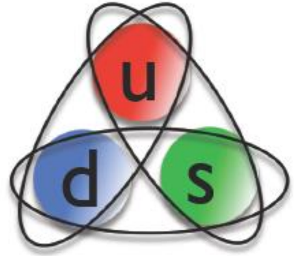
1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r) e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix} \begin{array}{l} \bar{r} = gb \\ \bar{g} = br \\ \bar{b} = rg \end{array}$$

$$\bar{u} = ds \quad \bar{d} = sb \quad \bar{s} = ud$$

Balachandran, Digal & Matsuura (BDM) ('05)
Nakano, MN & Matsuura ('07), Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underbrace{\exp\frac{i\theta}{3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_1(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

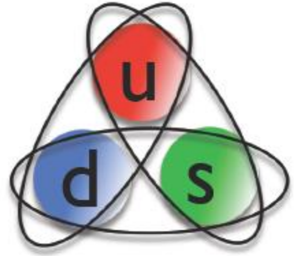
$$F_{12} \sim \oint r\,d\theta A_\theta \rightarrow$$

color flux tube

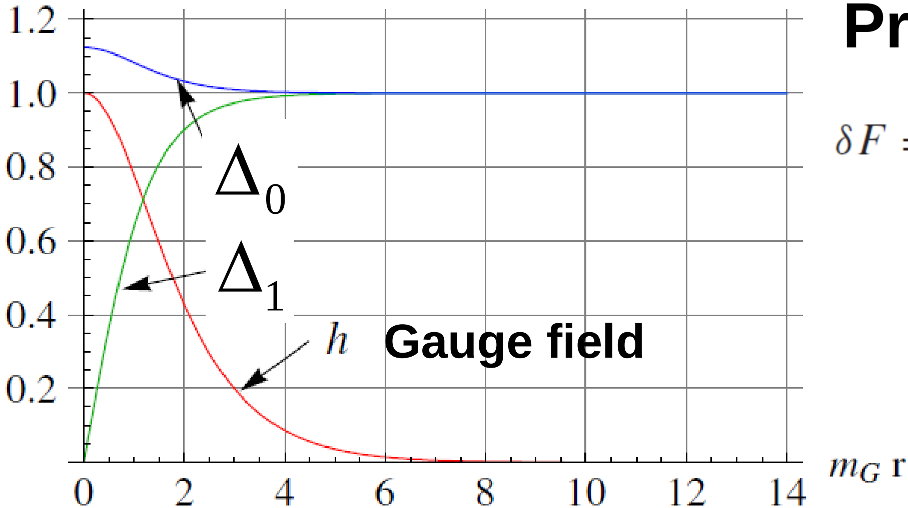
Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_1(r) e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$



Profiles

$F \equiv \Delta_1 + 2\Delta_0,$ **trace**

$G \equiv \Delta_1 - \Delta_0,$ **traceless**

$$\delta F = q_1 \sqrt{\frac{\pi}{2m_1 r}}\, e^{-m_1 r} - \frac{1}{3m_1^2 r^2} + O\left((m_1 r)^{-4}\right)$$

Long tail of a superfluid vortex

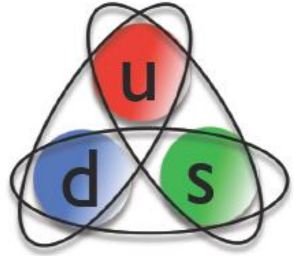
$$\delta G = q_8 \sqrt{\frac{\pi}{2m_8 r}}\, e^{-m_8 r}$$

$$\delta h = -\frac{2}{3} \frac{m_{\rm g}^2}{m_{\rm g}^2 - m_8^2} \delta G$$

confined color flux

Eto & MN ('09)

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \exp\frac{i\theta}{3}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

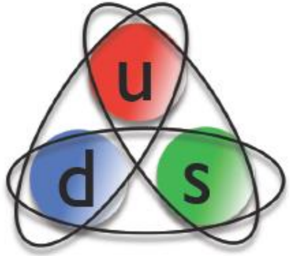
$$F_{12} \sim \oint r d\theta A_\theta$$

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex



$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \exp \frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

$$F_{12} \sim \oint r\, d\theta A_\theta$$

→ **color flux tube**

Superfluid vortex

Non-Abelian vortex

1/3 quantized vortex

$$\Phi_{\alpha i} = \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) e^{i\theta} \end{pmatrix}$$

$$= \boxed{\exp\left(\frac{i\theta}{3}\right)} \underbrace{\exp \frac{i\theta}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}}_{\text{SU(3) color}} \begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r) \end{pmatrix}$$

1/3 quantized

SU(3) color

$$F_{12} \sim \oint r\, d\theta A_\theta$$

→ color flux tube

Comment
Non-Abelian vortices were discovered earlier in the context of Supersymmetry and String theory ('03)

Superfluid vortex

Non-Abelian vortex

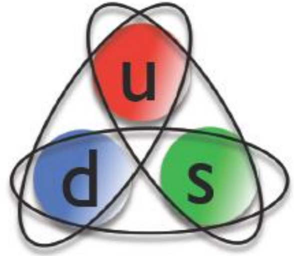
Non-Abelian vortices

Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$



Abelian vortex

No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Which are energetically favored?

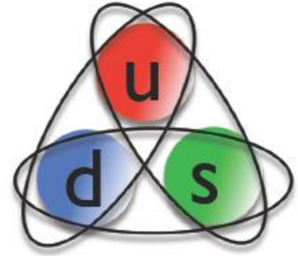
Non-Abelian vortices

Color Fluxes

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_0(r) \end{pmatrix}$$

$$\begin{pmatrix} \Delta_0(r) & 0 & 0 \\ 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$



Abelian vortex

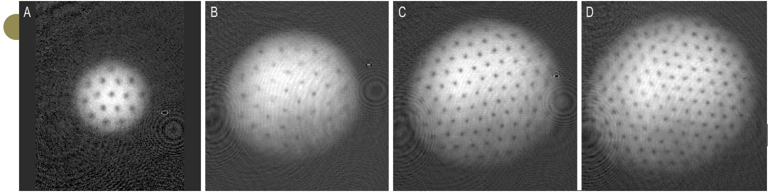
No flux

$$\begin{pmatrix} \Delta_1(r)e^{i\theta} & 0 & 0 \\ 0 & \Delta_1(r)e^{i\theta} & 0 \\ 0 & 0 & \Delta_1(r)e^{i\theta} \end{pmatrix}$$

Split

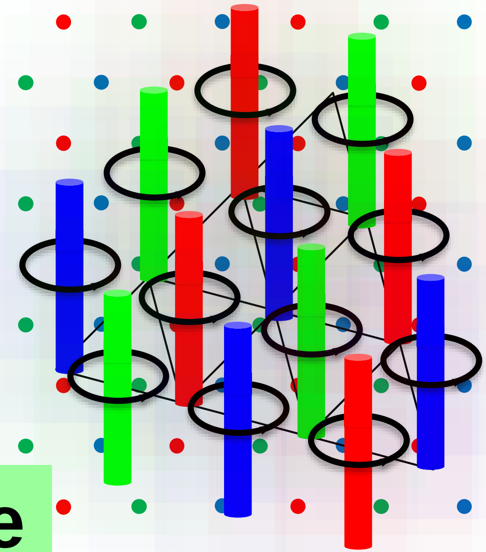
$$E[\bullet] = E[\bullet] = E[\bullet] = \frac{1}{9}E[\bigcirc]$$

Nakano, MN & Matsuura ('07), simulation by Alford et.al ('16)



Abrikosov vortex lattice

Colorful vortex lattice



$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$

$$H = SU(3)_{\mathrm{C+F}}$$
$$\downarrow$$
$$K = [SU(2)\times U(1)]_{\mathrm{C+F}}$$

@ vortex core

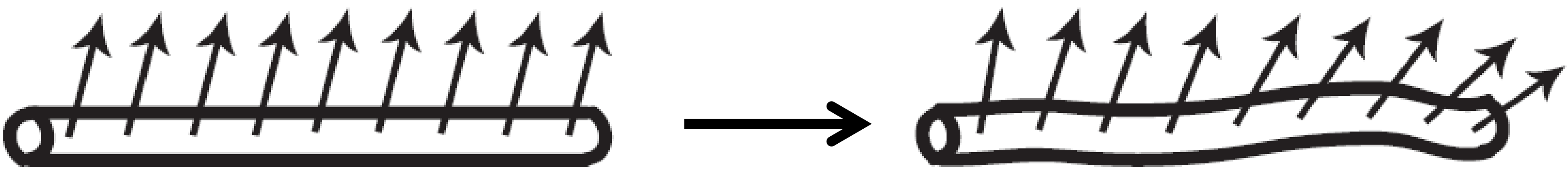
Nambu-Goldstone modes localized around the vortex

$$\boxed{\mathbf{C}\times\frac{H}{K} = \mathbf{C}\times\frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}\times\mathbf{C}P^2}$$

Continuous family of solutions exists

Eto,Nakano&MN('09)

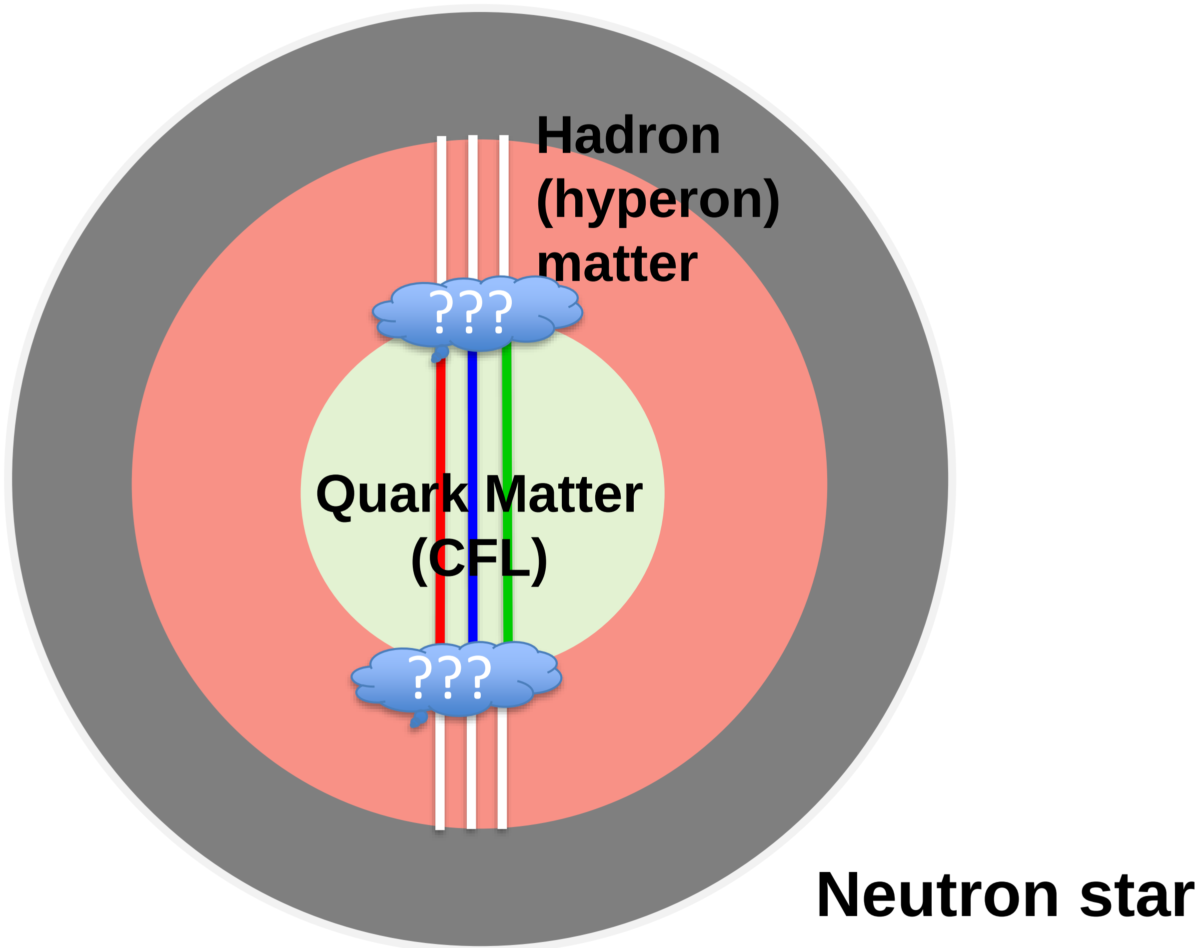
= Gapless modes propagating along the vortex line



"ground state" **1+1 dim effective theory** **fluctuations**

Quark-hadron continuity

How do vortices connect?



Continuity of Quark and Hadron Matter
Thomas Schäfer and Frank Wilczek
Phys. Rev. Lett. **82**, 3956 – Published 17 May 1999

(continuity or crossover)

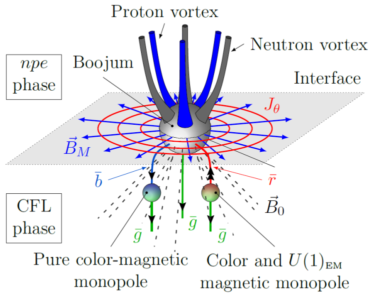
No phase transition between hadron and CFL phases

Matching of symmetries and excitations (*Nambu-Goldstone modes etc*)

Three-flavor quarks with degenerate mass

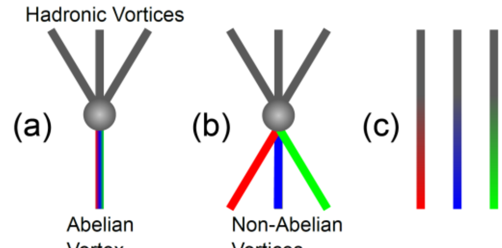
Phases	Hadron phase	Color–flavor-locked phase
	Confinement	Higgs
Quarks	Confined	Condensed
Monopoles	*Condensed?*	*Confined*
Coupling constant	Strong	Weak
Order parameters	Chiral condensate $\langle \bar{q} q \rangle$	Diquark condensate $\langle q q \rangle$
Symmetry	$SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times U(1)_{\mathrm{B}}$ $\to SU(3)_{\mathrm{L+R}}$	$SU(3)_{\mathrm{C}} \times SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times U(1)_{\mathrm{B}}$ $\to SU(3)_{\mathrm{C+L+R}}$
Fermions	**8** baryons	**8** + **1** quarks
Vectors	**8** + **1** vector mesons	**8** gluons
NG modes	**8** pions $(\bar{q} q)$	**8** + **1** pions $(\bar{q}\bar{q} q q)$
	H boson	H boson

Continuity of Quark and Hadron Matter
Thomas Schäfer and Frank Wilczek
Phys. Rev. Lett. **82**, 3956 – Published 17 May 1999

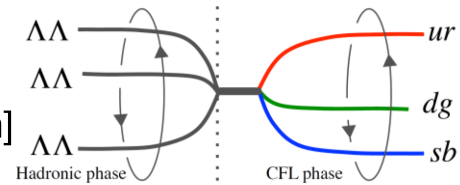


Colorful boojums at the interface of a color superconductor
Mattia Cipriani, Walter Vinci, and Muneto Nitta
Phys. Rev. D **86**, 121704(R) – Published 21 December 2012

Continuity of vortices from the hadronic to the color-flavor locked phase in dense matter
Mark G. Alford, Gordon Baym, Kenji Fukushima, Tetsuo Hatsuda, Motoi Tachibana, **Phys.Rev. D99 (2019) no.3, 036004**
e-Print: arXiv:1803.05115 [hep-ph]



Quark-hadron continuity under rotation: vortex continuity or boojum?
Chandrasekhar Chatterjee, Muneto Nitta, Shigehiro Yasui
Phys.Rev. D99 (2019) no.3, 034001, e-Print: arXiv:1806.09291 [hep-ph]



Anyonic particle-vortex statistics and the nature of dense quark matter
Aleksey Cherman, Srimoyee Sen, Laurence G. Yaffe
e-Print: arXiv:1808.04827 [hep-th]

Quark-hadron continuity beyond Ginzburg-Landau paradigm
Yuji Hirono, Yuya Tanizaki
Phys.Rev.Lett. 122 (2019) no.21, 212001, e-Print: arXiv:1811.10608 [hep-th]

What is Boojum?

Boojum trees in Arizona

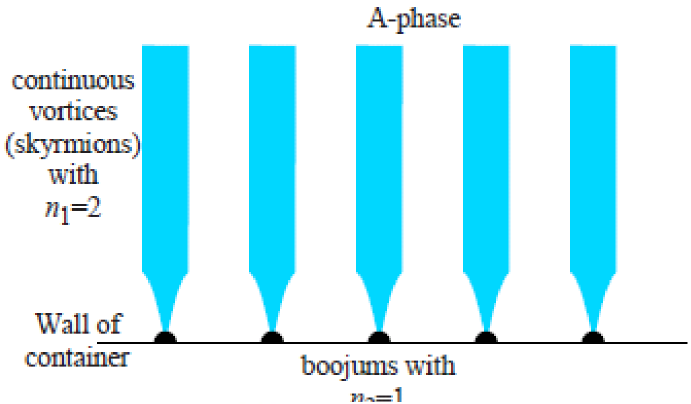
Boojum

A particularly dangerous kind of Snark

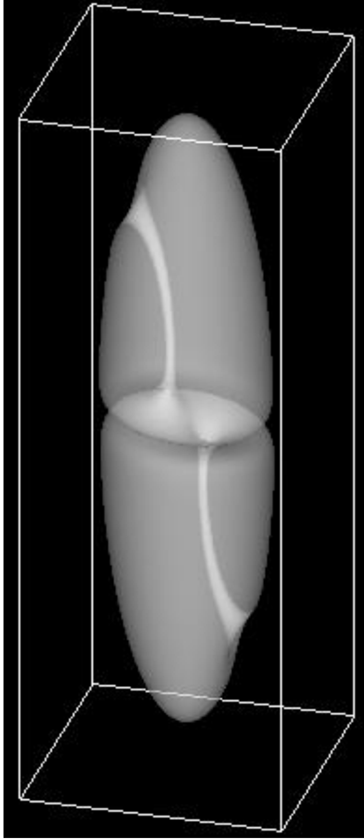
"The Hunting Of Snark"
Lewis Carroll

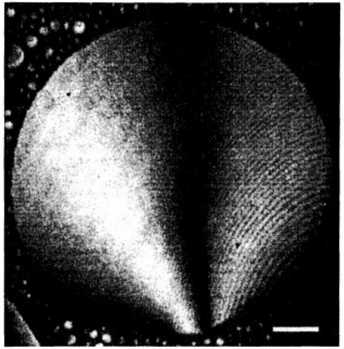
What is Boojum?

In physics, named by
D.Mermin (1976)

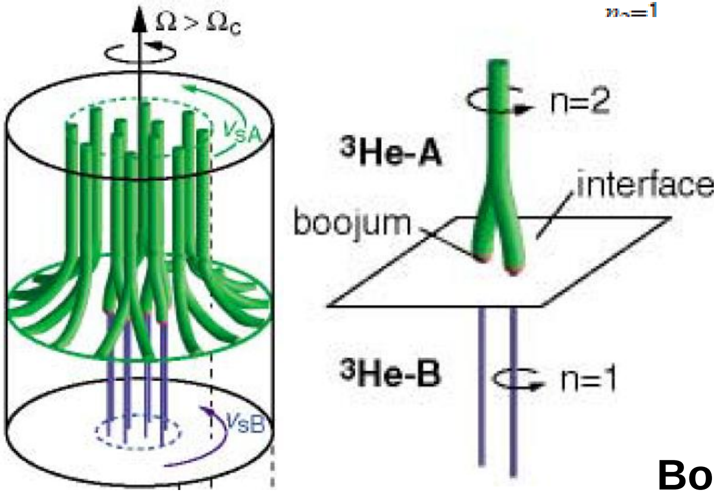


Boojums on the container wall in superfluid ^{3}He-A





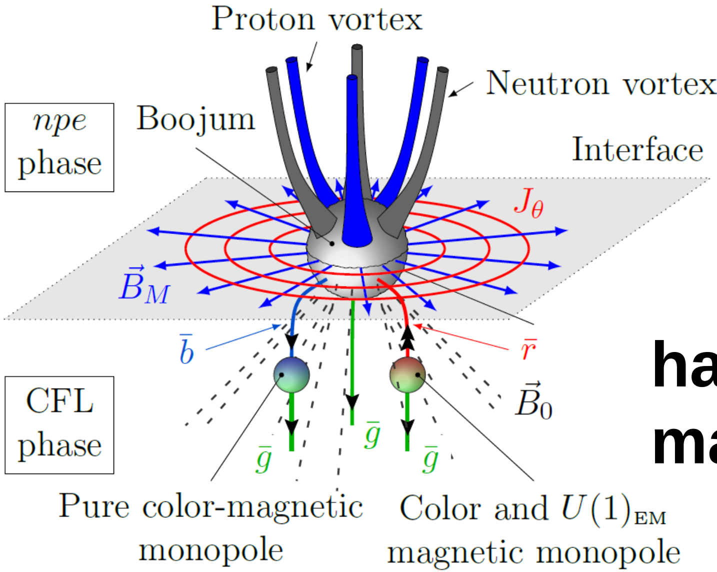
Boojum in liquid crystal

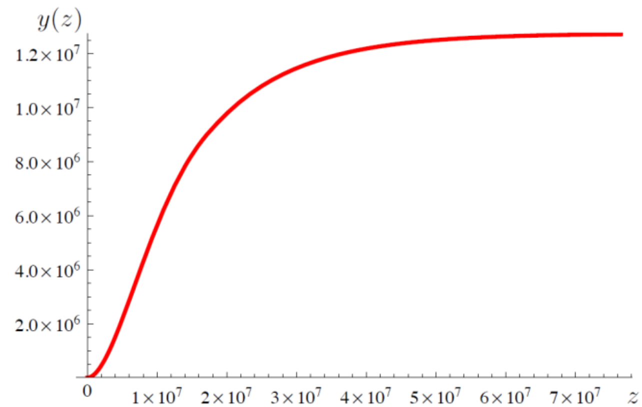


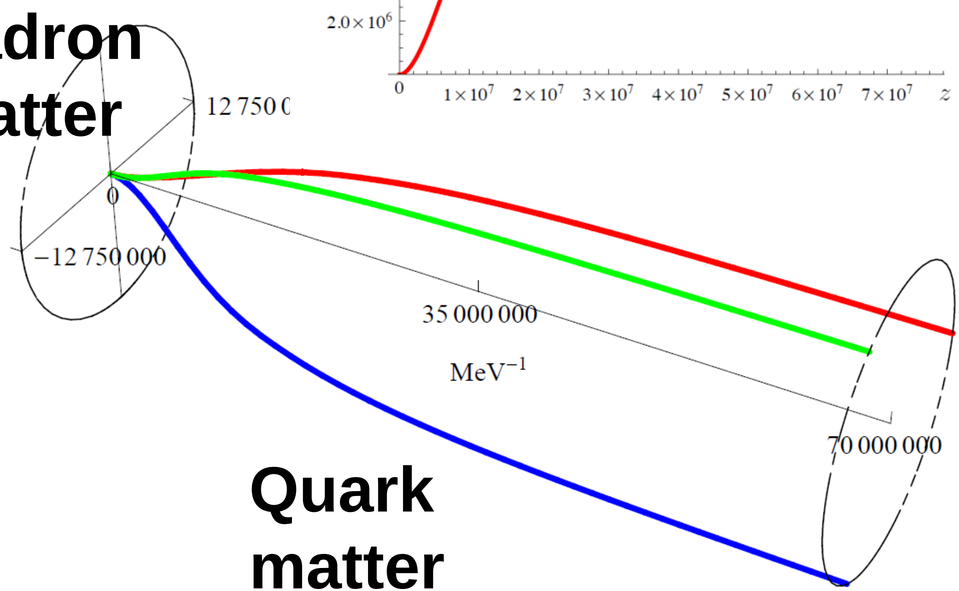
Boojums in ^{3}He A-B phase boundary

Boojum in two component BECs
Kasamatsu-Takeuchi-MN-Tsubota, JHEP1011:068,2010 [arXiv:1002.4265]

Colorful boojum at interface of quark matter







Cipriani,Vinci & MN,
Phys.Rev.D86:121704,2012
[arXiv:1208.5704]

Hadron matter with *degenerate quark mass*

⇒ Hyperon matter

Takatsuka & Tamagaki

Baryon: hyperon $\Lambda = uds$, $\Sigma = uus$, $\Xi = uss$

$$\Delta_{\Lambda\Lambda} = \langle \Lambda\Lambda \rangle \neq 0$$

8 x 8 = 1 + 8 + 8 + 10 + 10* + 27

$$-\sqrt{1/8}\,\Lambda\Lambda + \sqrt{3/8}\,\Sigma\Sigma + \sqrt{4/8}\,N\Xi$$

$U(1)_B$ spontaneous symmetry breaking

⇒ superfluidity

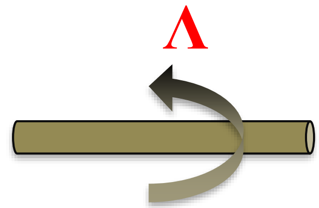
⇒ vortices under rotation

How these ΛΛ vortices connect to non-Abelian vortices in CFL phase?

Bogoliubov-de Gennes equation in hadron phase

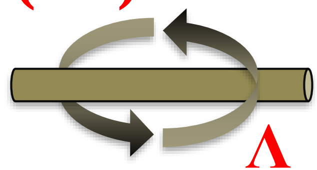
$$\begin{pmatrix} -\frac{\vec{\nabla}^2}{2m_B} - \mu_B & e^{i\theta}|\Delta_{\Lambda\Lambda}| \\ e^{-i\theta}|\Delta_{\Lambda\Lambda}| & \frac{\vec{\nabla}^2}{2m_B} + \mu_B \end{pmatrix} \begin{pmatrix} u_B \\ v_B \end{pmatrix} = \mathcal{E} \begin{pmatrix} u_B \\ v_B \end{pmatrix}$$

$$\theta \to \theta + \alpha, \quad (u, v) \to (e^{i\alpha/2}u, e^{-i\alpha/2}v)$$



Λ hyperon acquires a phase (1/2) × 2π = π around a vortex

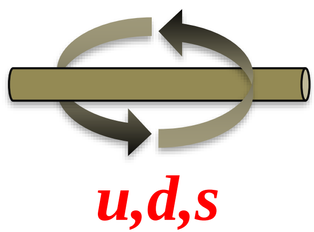
Λ=uds



At quark level…

Generalized Aharonov-Bohm phase

$$(q)_{\alpha i} = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$

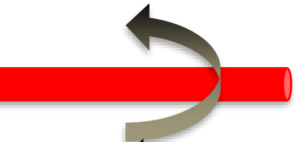


$$Q_{\Lambda\Lambda} = \frac{\pi}{3} \begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

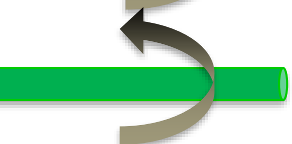
Z_6

Light (uds) quarks

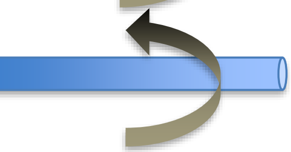
$$q \to \underbrace{e^{i\alpha/6}}_{\text{U(1)}_\text{B}} \underbrace{\mathrm{P}\exp\left(i g_s \int_\theta^{\theta+\alpha} \vec{A}\cdot \mathrm{d}\vec{l} \right)}_{\text{SU(3)}_\text{c}\ \text{AB phase}} q \qquad (q)_{\alpha i} = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$



$$q \to e^{i\alpha/6}\mathrm{diag}(e^{-i2\alpha/3}, e^{i\alpha/3}, e^{i\alpha/3})q = \mathrm{diag}(e^{-i\alpha/2}, e^{i\alpha/2}, e^{i\alpha/2})q$$

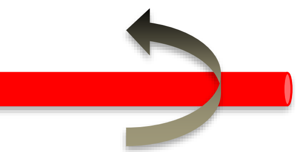


$$q \to e^{i\alpha/6}\mathrm{diag}(e^{i\alpha/3}, e^{-i2\alpha/3}, e^{i\alpha/3})q = \mathrm{diag}(e^{i\alpha/2}, e^{-i\alpha/2}, e^{i\alpha/2})q,$$



$$q \to e^{i\alpha/6}\mathrm{diag}(e^{i\alpha/3}, e^{i\alpha/3}, e^{-i2\alpha/3})q = \mathrm{diag}(e^{i\alpha/2}, e^{i\alpha/2}, e^{-i\alpha/2})q$$

heavy (cbt) quarks

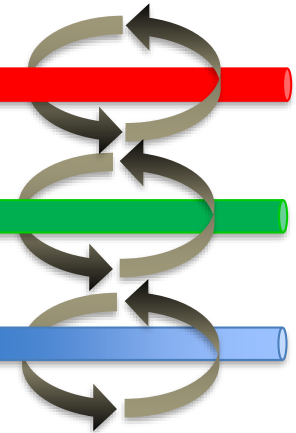


$$\psi \to \mathrm{P}\exp\left(i g_s \oint_C \vec{A}\cdot \mathrm{d}\vec{l} \right)\psi$$ **Only SU(3)$_\text{c}$ AB phase**

Light (uds) quarks

$$q \to \underbrace{e^{i\alpha/6}}_{\mathrm{U(1)_B}} \underbrace{\mathrm{P}\exp\left(i g_s \int_\theta^{\theta+\alpha} \vec{A} \cdot d\vec{l} \right)}_{\mathrm{SU(3)_c\ AB\ phase}} q \qquad (q)_{\alpha i} = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$

Complete encirclement



$$q \to \mathrm{diag.}\,(e^{-i\pi}, e^{+i\pi}, e^{+i\pi})\, q = -q$$

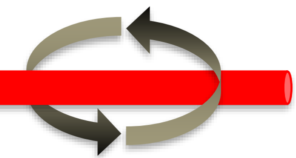
$$q \to \mathrm{diag.}\,(e^{+i\pi}, e^{-i\pi}, e^{+i\pi})\, q = -q$$

Z_2

$$q \to \mathrm{diag.}\,(e^{+i\pi}, e^{+i\pi}, e^{-i\pi})\, q = -q$$

heavy (cbt) quarks

Z_3



$$\psi \to \omega\psi \quad \text{with} \quad \omega = e^{i2\pi/3}$$

Only SU(3)$_c$ AB phase

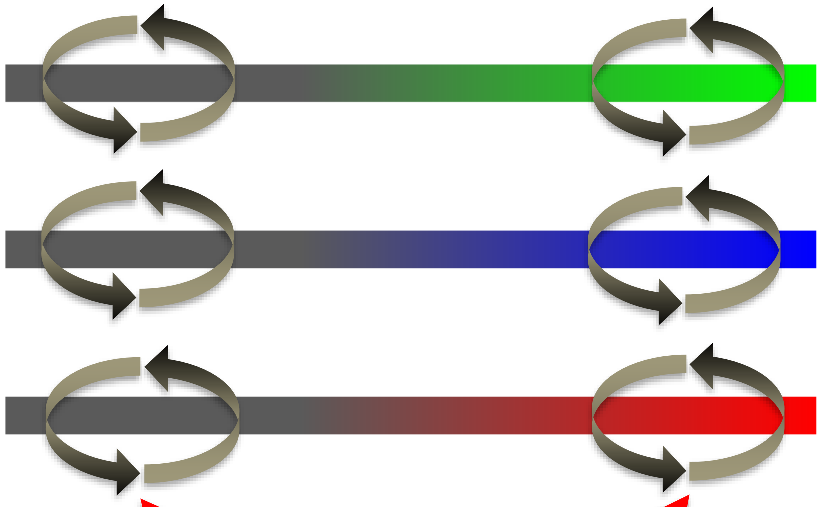
Vortex continuity doesn't work

hadron phase — **CFL phase**

$$Q_{\Lambda\Lambda} = \frac{\pi}{3}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix} \qquad Q_{ur} = \pi\begin{pmatrix} -1 & -1 & -1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

$$Q_{\Lambda\Lambda} = \frac{\pi}{3}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix} \qquad Q_{dg} = \pi\begin{pmatrix} +1 & +1 & +1 \\ -1 & -1 & -1 \\ +1 & +1 & +1 \end{pmatrix}$$

$$Q_{\Lambda\Lambda} = \frac{\pi}{3}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix} \qquad Q_{sb} = \pi\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ -1 & -1 & -1 \end{pmatrix}$$



Z_6 — Z_2

Don't match

$$\boxed{Q_{\Lambda\Lambda} \neq Q_{ur},\, Q_{dg},\, Q_{sb}}$$

Generalized Aharonov-Bohm phase matching

hadron phase

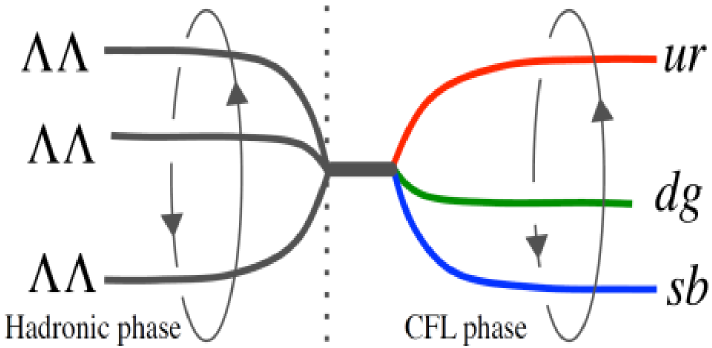
$$Q_{\Lambda\Lambda} = \frac{\pi}{3}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

$$Q_{\Lambda\Lambda} = \frac{\pi}{3}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

$$Q_{\Lambda\Lambda} = \frac{\pi}{3}\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

Z_6

$$(q)_{\alpha i} = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$



$$\pi\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

CFL phase

$$Q_{ur} = \pi\begin{pmatrix} -1 & -1 & -1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{pmatrix}$$

$$Q_{dg} = \pi\begin{pmatrix} +1 & +1 & +1 \\ -1 & -1 & -1 \\ +1 & +1 & +1 \end{pmatrix}$$

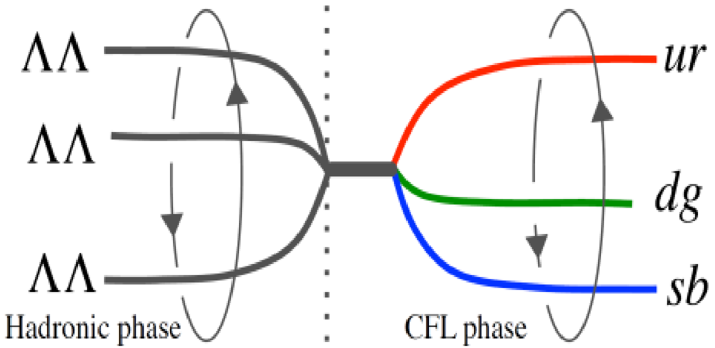
$$Q_{sb} = \pi\begin{pmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ -1 & -1 & -1 \end{pmatrix}$$

Z_2

$$\boxed{3Q_{\Lambda\Lambda} = Q_{ur} + Q_{dg} + Q_{sb}}$$

Aharanov-Bohm phase matching for heavy quarks

hadron phase | **CFL phase**

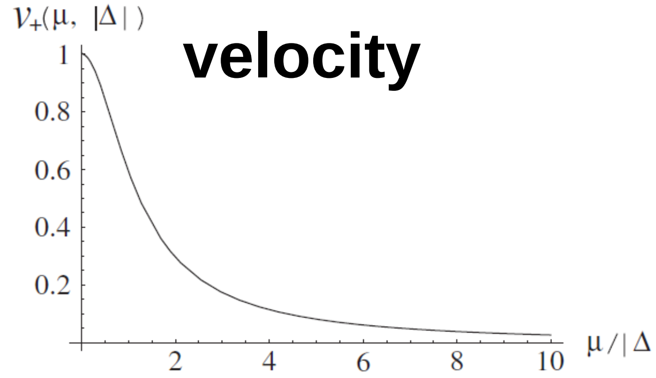


hadron phase				CFL phase
1	$\Lambda\Lambda$		ur	$\omega = e^{i2\pi/3}$
1	$\Lambda\Lambda$		dg	$\omega = e^{i2\pi/3}$
1	$\Lambda\Lambda$		sb	$\omega = e^{i2\pi/3}$
1	ψ	$\omega^3=1$	ψ	Z_3

		Hadronic phase	CFL phase	
Vortex		$\Lambda\Lambda$ vortex	Abelian vortex	NA vortex
Heavy quarks	AB phase	$\mathbf{1}$	$\mathbf{1}$	$\mathbb{Z}_3$
u, d, s	AB phase	$\mathbf{1}$	$\mathbf{1}$	$\mathbb{Z}_3$
	generalized AB phase	$\mathbb{Z}_6$	$\mathbb{Z}_2$	$\mathbb{Z}_2$

Fermions trapped inside a vortex core

$$\Phi_{\alpha i} = \left(\begin{array}{c|cc} \Delta_1(r)e^{i\theta} & 0 & 0 \\ \hline 0 & \Delta_0(r) & 0 \\ 0 & 0 & \Delta_0(r) \end{array}\right)$$



$$q = \begin{pmatrix} u_r & d_r & s_r \\ u_g & d_g & s_g \\ u_b & d_b & s_b \end{pmatrix}$$

Triplet Majorana fermion

protected by $SO(3)$

Yasui, Itakura & MN, Phys.Rev.D81,105003(2010) [arXiv:1001.3730]

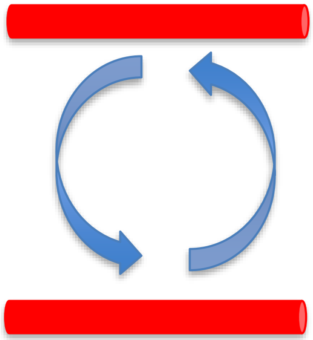
Index theorem Fujiwara,Fukui,MN&Yasui ('11)

Non-Abelian statistics Yasui,Itakuta&MN('11), Hirono,Yasui,Itakura&MN('12)

These fermions are important for transportation coeff. of quasi-particles.

Previous study

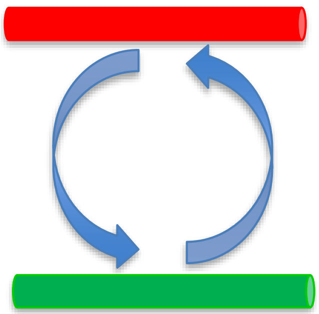
Yasui, Itakura, MN Phys.Rev.B83:134518,2011 [arXiv:1010.3331]
Yasui, Itakura, MN Nucl.Phys.B859:261-268,2012 [arXiv:1109.2755]
Hirono, Yasui, Itakura, MN Phys.Rev.B86:014508,2012 [arXiv:1203.0173]
Yasui, Hirono, Itakura, MN Phys.Rev.E87:052142,2013 [arXiv:1204.1164]



Exchange of identical vortices with Majorana fermions

→ non-Abelian anyons

Current study



Exchange of different vortices with Majorana fermions

→ novel non-Abelian anyons?

Discussion based on Ginzburg-Landau theory

hadron phase ***match*** **CFL phase**

$SU(3)$ **Bulk symmetries** $H = SU(3)_{\mathrm{C+F}}$

SSB $\downarrow$ **vortex core**

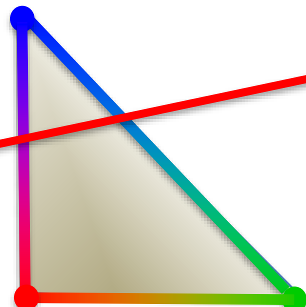
Vortex core symmetries

$SU(3)$ $K = [SU(2) \times U(1)]_{\mathrm{C+F}}$

$SU(3)$ $K = [SU(2) \times U(1)]_{\mathrm{C+F}}$

$SU(3)$ $K = [SU(2) \times U(1)]_{\mathrm{C+F}}$

Don't match



$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2) \times U(1)} = \mathbf{C}P^2$$

hadron phase ***match*** **CFL phase**

$SU(3)$ **Bulk symmetries** $H = SU(3)_{\mathrm{C+F}}$

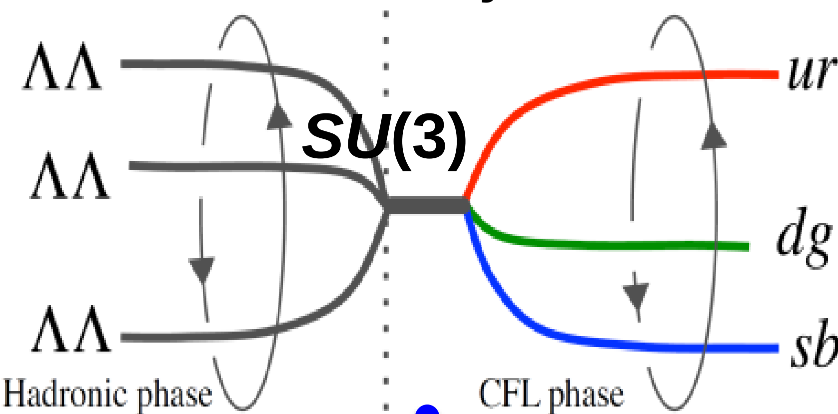
SSB $\downarrow$ **vortex core**

Vortex core symmetries

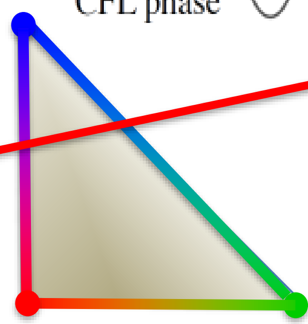
$SU(3)$ $\quad$ ur $K = [SU(2)\times U(1)]_{\mathrm{C+F}}$

$SU(3)$ $\quad$ dg $K = [SU(2)\times U(1)]_{\mathrm{C+F}}$

$SU(3)$ $\quad$ sb $K = [SU(2)\times U(1)]_{\mathrm{C+F}}$



Don't have to match



$$\frac{H}{K} = \frac{SU(3)_{\mathrm{C+F}}}{SU(2)\times U(1)} = \mathbf{C}P^2$$

Collaborators

Hadron, **HEP**, **Cond-mat**

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Takanori Fujiwara(Ibaragi), **Takahiro Fukui**(Ibaragi),
Walter Vinci（USC）, **Mattia Cipriani**（Pisa）,
Michikazu Kobayashi（Kyoto）, **Chandrasekhar Chatterjee** (Keio)

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