Order of the color superconducting phase transition

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1. Introduction

- Color superconductivity is the generalization of ordinary superconductivity of electrons to quarks with color charge.
- Because of the attractive interaction originating from gluon exchange, the Fermi surface of quarks gets unstable against pair creation, which condense at low temperature.
- The Ginzburg-Landau effective theory, i.e. scalar QCD in three dimensions, describes the system close to the critical temperature.
- Order of the color superconducting transition is predicted to be of first order by 1-loop calculations. Same argument fails for ordinary superconductors.
- Asymptotic freedom of QCD also means the absense of infrared (IR) stable fixed points, thus no 2nd order transition can occur in $d=4-\epsilon$ dimensions. Is $\epsilon=1$ reliable? It is not in ordinary superconductivity.
- Goal: search for IR stable fixed points in the Ginzburg-Landau theory of color superconductivity directly in three dimensions.

2. Method

- The employed method is the Functional Renormalization Group (FRG).
- In the Γ_k scale dependent quantum effective action all fluctuations are incorporated beyond momentum scale k.
- A regulator function $R_k(x,y)$ is attached to the classical Lagrangian, which suppresses low momentum modes.
- Γ_k obeys the RG flow equation:
  [boundary conditions: $\Gamma_{k=\Lambda}=S_{\text{classical}}$, $\Gamma_{k=0}=\Gamma_{1PI}$]

$$\partial_k\Gamma_k=\frac{1}{2}\int_x\int_y \text{Tr}\left\{[\Gamma_k''+R_k]^{-1}(x,y)\partial_k R_k(y,x)\right\}$$

- One-loop diagrams need to be evaluated!

3. Scalar QCD

- The effective theory of color superconductivity:
  [A_i^a: gluon, ϕ_α^n: diquark, c^a: ghost, $\bar{c}^a$: antighost]

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}A_i^a\delta^{ab}\left(-\partial^2\delta_{ij}+(1-\xi^{-1})\partial_i\partial_j\right)A_j^b+\bar{c}^a(-\partial^2\delta^{ac}-gf^{abc}\partial_iA_i^b)c^c\\ &+\frac{\phi_\alpha^{\dagger n}}{2}(-\partial^2)\phi_\alpha^n+V(\phi)+igA_i^a\left(\partial_i\phi_\alpha^\dagger(\hat{T}^a\phi_\alpha)-(\hat{T}^a\phi_\alpha)^\dagger\partial_i\phi_\alpha\right)\\ &+g^2f^{abe}f^{cde}A_i^aA_i^b(\hat{T}^c\phi_\alpha)^\dagger(\hat{T}^d\phi_\alpha)+gf^{abc}\partial_iA_j^aA_i^bA_j^c\\ &+\frac{g^2}{4}f^{abe}f^{cde}A_i^aA_j^bA_i^cA_j^d\end{aligned}$$

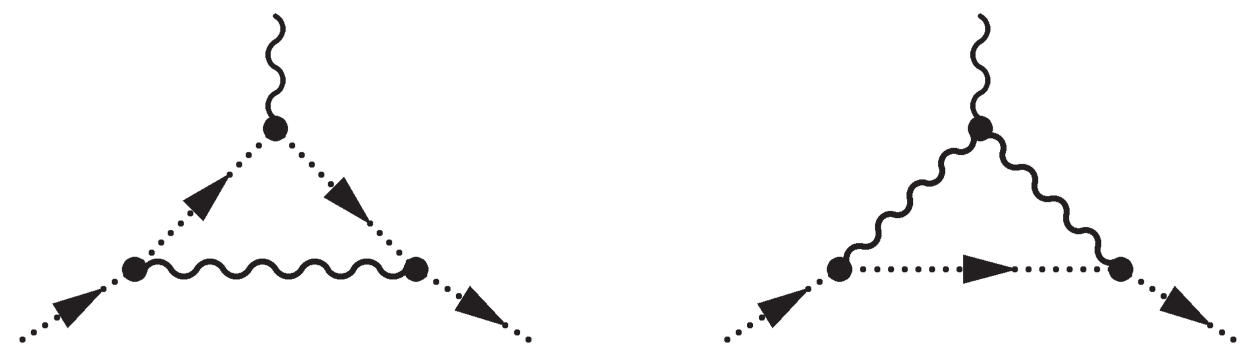
$\longrightarrow V(\phi)$ is an unspecified scalar potential

4. β-function definitions

- There are multiple ways to define the β-function of the gauge coupling, which agree due to the Slavnov-Taylor identities.
- Choice I: scalar-scalar-gauge vertex
- Choice II: scalar-scalar-gauge-gauge vertex
- Choice III: three gluon vertex
- Choice IV: four gluon vertex
- Choice V: gauge-antighost-ghost vertex
  $\longrightarrow$ simplest choice is V.
- Advantage of choice V:
  $\longrightarrow$ generalization of the QED Ward identity is automatically satisfied
  $\longrightarrow$ i.e. Z_g-Z_ϕ is matter independent (zero in QED, nonzero in QCD)
  $\longrightarrow$ corresponding diagrams are simple

5. Gluon-ghost-antighost vertex

- Two diagrams need to be evaluated:
  [$\Omega_d=\int d\Omega_d/(2\pi)^d$ is the angular integral]

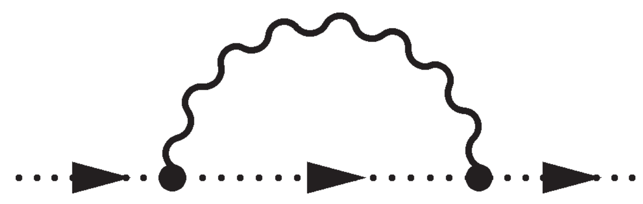


- The resulting flow:

$$k\partial_k\log(Z_{g,k}Z_{A,k}^{1/2}Z_{c,k})=g_k^2\Omega_dk^{d-4}\frac{-9\xi_k}{d+2}$$

6. Ghost wavefunction renormalization

- Here we have only one diagram:

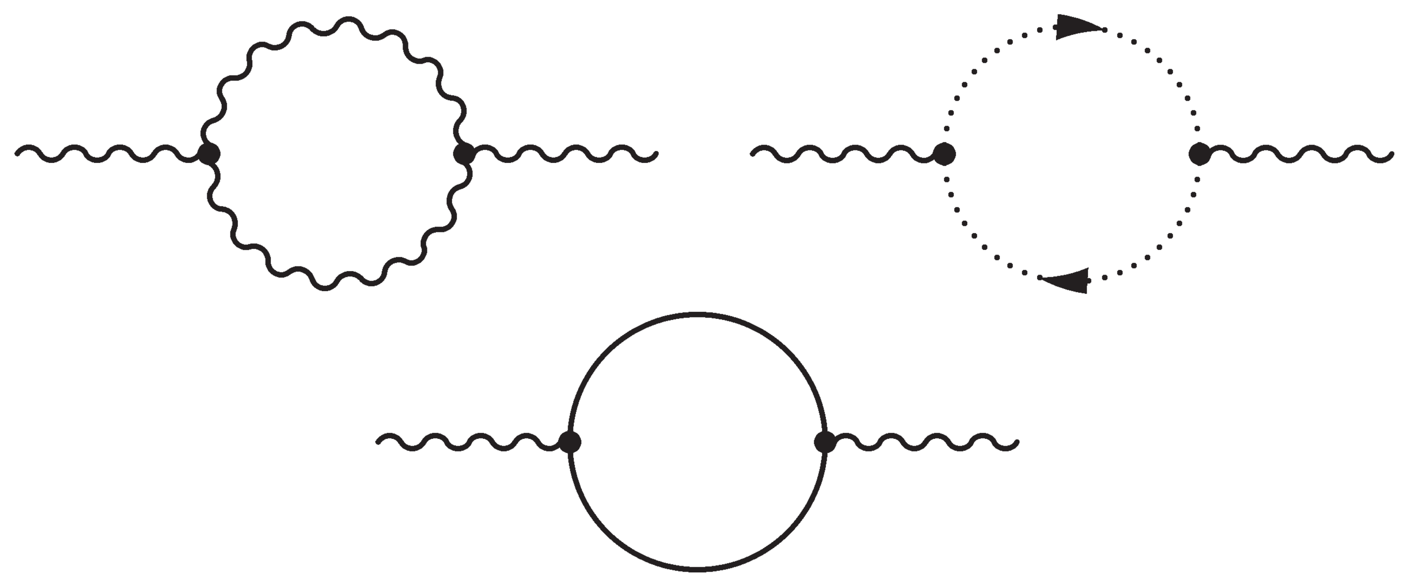


- The resulting flow:

$$k\partial_k\log Z_{c,k}=6g_k^2\Omega_dk^{d-4}\frac{2(d-1)-\xi_k(d-2)}{d^2}$$

7. Gluon wavefunction renormalization

- Three diagrams need to be evaluated:



- The resulting flow:

$$k\partial_k\log Z_{A,k}=g_k^2\Omega_dk^{d-4}\left[\frac{36d^2+42d-120-6\xi_k(4d^2-5d-20)}{d^2(d+2)}-\frac{12}{d(d+2)}\right]$$

- BUT: the gluon self energy is NOT transverse in the FRG framework!
- No problem: adjust ξ_k such that at each scale it compensates the longitudinal contribution $\Rightarrow$ $\xi_k\equiv\xi=-7/2$

8. β-function

- The β-function of the gauge couling in $d=3$ turns out to be:

$$\beta(g)|_{d=3}=-\bar{g}_k-\frac{\bar{g}_k^3}{2\pi^2}\left[\left(\frac{19}{3}+\frac{16}{15}\xi\right)-\frac{2}{5}\right]$$

$\rightarrow\beta$ is nonpositive and the only fixed point is $g_k\equiv 0$, which is UV stable
$\rightarrow$ no chance to have IR stable fixed point for any $V(\Phi)$ potential
(NO second order phase transition is found)
$\rightarrow$ vertex regularization is introducable but does not change the conclusion

9. Conclusions

- Order of the color superconducting transition is investigated via the FRG
- Calculation of the flow of the gauge coupling directly in $d=3$ shows that no IR stable fixed points exist $\Rightarrow$ 2nd order transition is not possible
- Conclusions are stable against introducing vertex regulators in the RG flow