Quark mass generation by monopole condensation

Chiral symmetry is spontaneously broken ? or
It is explicitly broken even in the chiral limit ?

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Chiral symmetry in QCD

standard idea

Spontaneous symmetry breaking

Pions are Nambu-Goldstone bosons ? ?

our idea

Explicit symmetry breaking

Pions are not massless in the chiral limit

Abelian dominance in SU(2) gauge theory (strong coupled regions)

Relevant degrees of freedom to strong coupled QCD

Abelian gauge field A_μ^3

Monopole Φ

Massless quark doublet $\begin{pmatrix} q^+ \\ q^- \end{pmatrix}$

color magnetic charge $g_m \equiv \dfrac{1}{2g}$

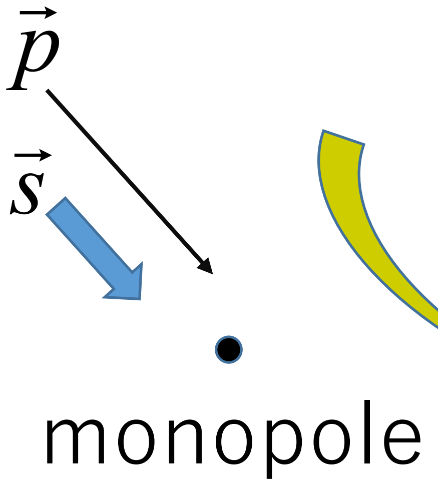
color charge $\begin{pmatrix} +1 \\ -1 \end{pmatrix} g$

Monopoles in SU(2) gauge theory

the low energy scattering of **massless quark doublet** and a monopole with mass

Note the conserved angular momentum

$$\vec{J} = \vec{L} + \vec{S} \boxed{- g g_m \frac{\vec{r}}{r}}$$



The change of quantum numbers before and after the scattering must satisfy

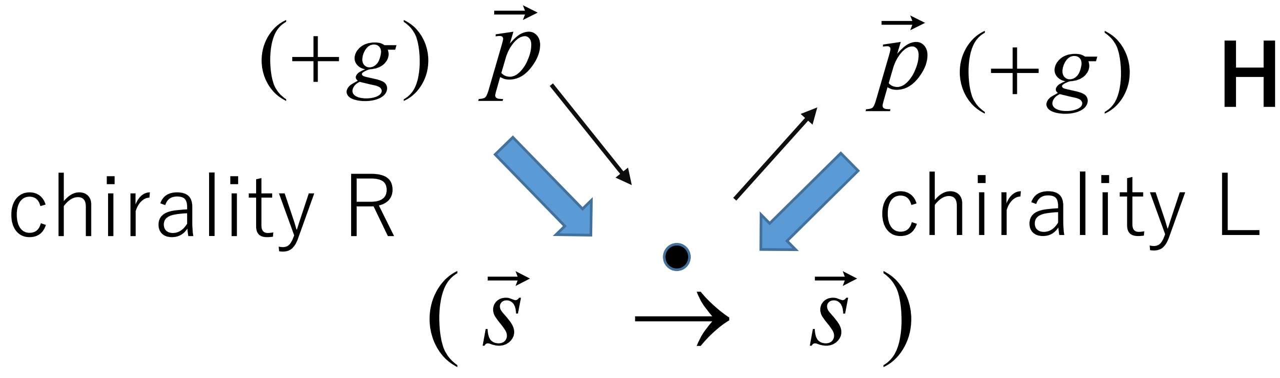
$$\Delta(\vec{r} \cdot \vec{J}) = \Delta(\vec{r} \cdot \vec{S}) - \Delta(g g_m) r = 0$$

Angular momentum conservation

$$\Delta(\vec{r}\cdot\vec{S})-\Delta(gg_m)r=0$$

$\vec{s} \to \vec{s}$

When an incoming fermion does not flip its charge $(+g \to +g)$ after the scattering, its chirality must flip .



Helicity flips (=chirality flip)

Another case;
charge flip but chirality not flip

Which one ?

? Charge conservation
? Chirality conservation

charge conservation ← gauge symmetry
(**<u>chirality non conservation</u>**)

V.A. Rubakov 1982, 1988
Z.F Ezawa and A. I 1983

$$\begin{pmatrix} \pm g \\ R(L) \\ flavor = i \end{pmatrix} \Rightarrow \begin{pmatrix} \pm g \\ L(R) \\ flavor = i \end{pmatrix}$$

<u>flavor conserved</u>
(no monopoles
with SU(2) flavor)

We need to impose a boundary condition

Y. Kazama,etal. 1977

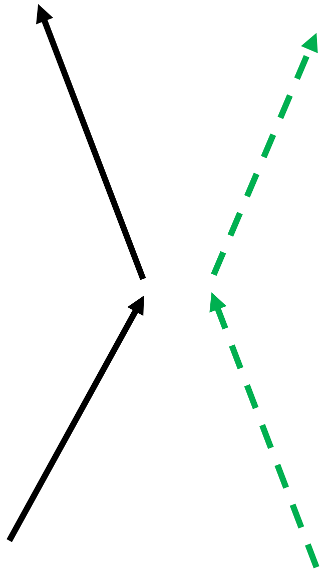
Boundary condition for quarks
at the location of monopole

Chirality non conserved boundary condition

$$q^{\pm}_{R(L),i}(\vec{r}=0) = q^{\pm}_{L(R),i}(\vec{r}=0) \quad on \quad monopole \quad at\ \vec{r}=0$$

$$L_{\mathrm{int}} = h(\bar{q}_R q_L + \bar{q}_L q_R)\Phi^* \Phi$$

How weak is chiral SU(2) breaking monopole quark interaction?



quark
g

monopole
$g_m = g/2$

strength of monopole quark interaction

$$g \times g_m = 1/2$$

strength of quark gluon interaction

$$g \times g = 4\pi\alpha_s$$

The ratio $\dfrac{g \times g_m}{4\pi\alpha_s} \approx 0.08 \quad for \quad \alpha(1GeV) \cong 0.5$

The chiral symmetry breaking interaction is 10 times smaller than strong interactions

$$L_{\text{int}} = h(\bar{q}_R q_L + \bar{q}_L q_R)\Phi^* \Phi$$

How large h is ?

$$h \approx \left(\frac{g \times g_m}{g^2(\Lambda)} \right) \frac{1}{\Lambda} \cong \frac{1}{8\pi\alpha_s(1GeV) \times 1GeV} \approx \frac{1}{12.5GeV}; \quad \begin{array}{l} \Lambda = 1GeV \\ \alpha_s(1GeV) \cong 0.5 \end{array}$$

Quark mass generation by monopole condensation

$< \Phi > \approx 175MeV$ M.N. Chernodub, 2000

$$h\bar{q}q < \Phi >^2 = h(175MeV)^2 \bar{q}q \approx 2.5MeV\,\bar{q}q$$

Constituent quark mass $m_q \approx 2.5MeV$

The weak chiral SU(2) breaking is compatible with results from chiral perturbation ?

Gell-Mann-Oakes-Renner relation

$$m_\pi^2 f_\pi^2 = 2m_{ud} < \bar{q}q > + \underline{m_{\pi 0}^2 f_\pi^2}$$

$$m_{ud} \equiv \frac{m_u + m_d}{2}$$

$m_{\pi 0}$; bare pion mass in the chiral limit

vanishing or non vanishing

To what extent is the formula examined in lattice gauge theories ?

Are pions Nambu-Goldstone bosons?

$$m_\pi^2 f_\pi^2 = 2m_{ud} < \bar{q}q > + m_{\pi 0}^2 f_\pi^2$$

$$< \bar{q}q > \cong (270MeV \pm 5MeV)^3, \quad m_{ud} \cong 3.4MeV \pm 0.1MeV$$

（**H. Fukaya, etal. PTEP 2016 (2016) no.9, 093B06**）

Banks-Casher relation

（**Particle data group 2019**）

(Lattice gauge theory)

$f_\pi \cong 130MeV$ （**Particle data group 2012**）

$\cong 110MeV$ **in the limit** $m_{ud} \to 0$

From the uncertainty in quark mass $\delta m_{ud} = \pm 0.1MeV$ there is a possibility that the pion mass $\boxed{0 < m_{\pi 0} < 15MeV}$ is non vanishing in the chiral limit $m_{ud} \to 0$

consistent with constituent quark mass~2MeV

Conclusion

Chiral symmetry is explicitly broken by monopole quark interaction.

The strength of the interaction is 10 times smaller than the strong interactions
（ it produces constituent quark mass, the order of 1MeV ）

Pions are not massless even in the chiral limit
（ the non vanishing mass is less than 15MeV ）

Prediction

Decay width of monopole to hadrons is much small

(monopole quark interaction is 10 times smaller than the strong interactions)

Chiral condensate arises simultaneously with quark confinement

(monopole condensation generates constituent quark mass m_q . It leads to $<\bar{q}q> \neq 0$)

Hadron mass decreases in dense nuclear matter

(monopole condensate $<\Phi> \propto m_q$ decreases in the matter)

Chiral magnetic effect does not arise

(monopole quark interaction washes out chiral chemical potential $\mu_5 = 0$)