Dyon in pure $SU(2)$ Yang–Mills theory with a gauge-invariant mass toward confinement/defonfinement phase transition[⋆]

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Introduction

The Kraan–van Baar–Lee–Lu (KvBLL) calorons [1] are extensively used to understand the confinement/deconfinement phase transition in the Yang–Mills theory at finite temperature [2]. The KvBLL caloron is a topological soliton solution of the self-dual equation of the $SU(2)$ Yang–Mills theory on $S^1\times\mathbb{R}^3$ space with instanton charge, which consists of BPS dyons having both electric and magnetic charges with nontrivial holonomy at spatial infinity.

Recently, we have found a novel dyon solution as a non-BPS solution of (non self-dual) field equations of a gauge-scalar model with the radially fixed scalar field in the adjoint representation. This dyon solution of the gauge-scalar model is identified with the topological field configuration of the Yang–Mills theory with a gauge-invariant gluon mass term without scalar field, which is regarded as the low-energy effective model of the Yang–Mills theory with mass gap. This follows from the gauge-independent Higgs mechanism [3] which does not rely on the spontaneous breaking of gauge symmetry. Our dyon has the nonvanishing asymptotic value corresponding to the nontrivial holonomy at spatial infinity to be comparable with the KvBLL caloron. Thus we can propose another scenario for reproducing confinement/deconfinement phase transition in the Yang–Mills theory at finite temperature based on our dyon solution.

In this poster, we show the existence of such dyons and discuss the characteristic properties, especially the asymptotic holonomy.

From the "complementary" gauge-scalar model to the massive Yang–Mills theory

We introduce the $SU(2)$ gauge scalar model on $S^1\times\mathbb{R}^3$ space with a periodicity T^{-1}

$$S_{\rm gs}=\int_0^{T^{-1}}d\tau\int d^3\boldsymbol{x}\,{\rm tr}\left[\frac{1}{2}\mathscr{F}_{\mu\nu}\mathscr{F}_{\mu\nu}+\left(\mathscr{D}_\mu[\mathscr{A}]\phi\right)\left(\mathscr{D}_\mu[\mathscr{A}]\phi\right)\right],\quad(1)$$

where

$$\mathscr{F}_{\mu\nu}=\mathscr{F}^A_{\mu\nu}T_A=\partial_\mu\mathscr{A}_\nu-\partial_\nu\mathscr{A}_\mu+ig[\mathscr{A}_\mu,\mathscr{A}_\nu],\quad(2)$$

$$\mathscr{D}_\mu[\mathscr{A}]\phi=\left(\mathscr{D}_\mu[\mathscr{A}]\phi\right)^AT_A=\partial_\mu\phi+ig[\mathscr{A}_\mu,\phi].\quad(3)$$

Here we have chosen the Hermitian basis of $\mathfrak{su}(2)$ by using the Pauli matrices σ_A as

$$T_A:=\frac{\sigma_A}{2}.\quad(A=1,2,3)\quad(4)$$

The radial degree of the scalar field $\phi(x)$ is fixed

$$2{\rm tr}\left(\phi(x)\phi(x)\right)=v^2,\quad v>0.\quad(5)$$

We define the normalized scalar field $\hat{\phi}(x)$ for latter convenience

$$\hat{\phi}(x):=\frac{1}{v}\phi(x),\quad 2{\rm tr}\left(\hat{\phi}(x)\hat{\phi}(x)\right)=1.\quad(6)$$

To begin with, we construct a composite vector boson field $\mathscr{X}_\mu(x)$ from $\mathscr{A}_\mu(x)$ and $\hat{\phi}(x)$ as

$$g\mathscr{X}_\mu(x):=i\left[\hat{\phi}(x),\mathscr{D}_\mu[\mathscr{A}]\hat{\phi}(x)\right],\quad(7)$$

which transforms in the adjoint way under the gauge transformation $U(x)\in SU(2)$:

$$\mathscr{X}_\mu(x)\to\mathscr{X}'(x)=U(x)\mathscr{X}_\mu(x)U^\dagger(x).\quad(8)$$

Moreover, the kinetic term of the scalar field is identical to the mass term of the vector field $\mathscr{X}_\mu(x)$:

$${\rm tr}\left[\left(\mathscr{D}_\mu[\mathscr{A}]\phi\right)\left(\mathscr{D}_\mu[\mathscr{A}]\phi\right)\right]=M^2_{\mathscr{X}}{\rm tr}\left(\mathscr{X}_\mu\mathscr{X}_\mu\right),\quad M_{\mathscr{X}}:=gv,\quad(9)$$

as long as the constraint (5) holds. It is clear that by observing (8), the obtained mass term of $\mathscr{X}_\mu$ is gauge-invariant. Therefore, $\mathscr{X}_\mu(x)$ can become massive without breaking the original gauge symmetry. It should be emphasized that we do not choose a specific vacuum of $\phi(x)$ and hence no spontaneous symmetry breaking occurs.

By using the definition of the massive vector field $\mathscr{X}_\mu$, the original gauge field $\mathscr{A}_\mu$ is separated into two pieces [4]: $\mathscr{A}_\mu=\mathscr{V}_\mu+\mathscr{X}_\mu$, where $\mathscr{V}_\mu$ can be written in terms of $\mathscr{A}_\mu$ and $\hat{\phi}$ as

$$g\mathscr{V}_\mu(x)=gc_\mu(x)\hat{\phi}(x)-i\left[\hat{\phi}(x),\partial_\mu\hat{\phi}(x)\right],\quad(10)$$

where $c_\mu(x):=2{\rm tr}\left(\mathscr{A}_\mu(x)\hat{\phi}(x)\right)$.

Then, we regard a set of field variables $\{c_\mu(x),\mathscr{X}_\mu(x),\hat{\phi}(x)\}$ as begin obtained from $\{\mathscr{A}_\mu(x),\hat{\phi}(x)\}$ based on a change of variables, and identify $c_\mu(x)$, $\mathscr{X}_\mu(x)$, and $\hat{\phi}(x)$ with the fundamental field variables for describing the massive Yang–Mills theory anew, which means that we should perform the quantization with respect to the variables $\{c_\mu(x),\mathscr{X}_\mu(x),\hat{\phi}(x)\}$ appearing in the path-integral measure. However, the degrees of freedom carried by $\hat{\phi}(x)$ is extra if we wish to obtain the (pure) Yang–Mills theory from the "complementary" gauge-scalar model. These two d.o.f.s are eliminated by imposing the two constraints that we call the reduction condition. We choose, e.g.,

$$\chi(x):=\left[\hat{\phi}(x),\mathscr{D}_\mu[\mathscr{A}]\mathscr{D}_\mu[\mathscr{A}]\hat{\phi}(x)\right]=0.\quad(11)$$

The reduction condition indeed eliminates the two extra d.o.f.s introduced by $\hat{\phi}(x)$, since ${\rm tr}\left(\chi(x)\hat{\phi}(x)\right)=0$.

Following the Faddeev–Popov procedure, we insert unity into the functional integral to incorporate the reduction condition:

$$1=\int\mathcal{D}\chi^\theta\,\delta\left(\chi^\theta\right)=\int\mathcal{D}\theta\,\delta\left(\chi^\theta\right)\Delta_{\rm red},\quad(12)$$

where $\chi^\theta:=\chi[\mathscr{A},\phi^\theta]$ is the reduction condition gauge-rotated by $\theta=\theta^A(x)T_A$, and $\Delta_{\rm red}$ is the associated Faddeev–Popov determinant. Then we obtain

$$\begin{aligned}Z&=\int\mathcal{D}\hat{\phi}\mathcal{D}\mathscr{A}\,\delta\left(\chi\right)\Delta_{\rm red}\exp\left\{-S_{\rm YM}[\mathscr{A}]-S_{\rm kin}[\mathscr{A},\phi]\right\}\\&=\int\mathcal{D}\hat{\phi}\mathcal{D}c\mathcal{D}\mathscr{X}\,\delta\left(\tilde{\chi}\right)\tilde{\Delta}_{\rm red}\exp\left\{-S_{\rm YM}[\mathscr{V}+\mathscr{X}]-S_{\rm m}[\mathscr{X}]\right\}.\end{aligned}\quad(13)$$

Therefore, we obtain the massive Yang–Mills theory that keeps the original gauge symmetry

$$S_{\rm mYM}=\int_0^{T^{-1}}d\tau\int d^3\boldsymbol{x}\,{\rm tr}\left[\frac{1}{2}\mathscr{F}_{\mu\nu}\mathscr{F}_{\mu\nu}+M^2_{\mathscr{X}}\mathscr{X}_\mu\mathscr{X}_\mu\right].\quad(14)$$

It should be remarked that the solutions of the field equations of the gauge-scalar model

$$\mathscr{D}_\mu[\mathscr{A}]\mathscr{F}_{\mu\nu}+igv^2\left[\hat{\phi},\mathscr{D}_\nu[\mathscr{A}]\hat{\phi}\right]=0,\quad(15)$$

$$\mathscr{D}_\mu[\mathscr{A}]\mathscr{D}_\mu[\mathscr{A}]\hat{\phi}-2{\rm tr}\left(\hat{\phi}\mathscr{D}_\mu[\mathscr{A}]\mathscr{D}_\mu[\mathscr{A}]\hat{\phi}\right)\hat{\phi}=0,\quad(16)$$

satisfy the reduction condition (11) automatically. (But the converse is not true.) From this fact, we find that the solutions of the coupled field equations (15) and (16) can play the very important role of the configurations satisfying the reduction condition (11) in a massive Yang–Mills theory through the path integral (13).

Construction of the Yang–Mills dyon

We adopt the Julia–Zee ansatz with a unit magnetic charge [5] for the Euclidean space

$$g\mathscr{A}_j(x)=\epsilon^{jAk}T_A\frac{x^k}{r}\frac{1-\tilde{f}(r)}{r},\quad(17)$$

$$g\mathscr{A}_4(x)=T_A\frac{x^A}{r}\tilde{a}(r),\quad\hat{\phi}(x)=T_A\frac{x^A}{r}\tilde{h}(r),\quad(18)$$

where Roman indices run from 1 to 3 and r is the radius of $\mathbb{R}^3$, i.e., $r=\sqrt{x_jx_j}$. Notice that this ansatz is "static", i.e., τ-independent. The field equations (15) and (16) are written in terms of the profile functions $\tilde{f}(r)$, $\tilde{a}(r)$, and $\tilde{h}(r)$ as

$$\tilde{a}''(r)+\frac{2}{r}\tilde{a}'(r)-\frac{2}{r^2}\tilde{a}(r)\tilde{f}^2(r)=0,\quad(19)$$

$$\tilde{f}''(r)-\frac{1}{r^2}\left(\tilde{f}^3(r)-\tilde{f}(r)\right)-\left(\tilde{a}^2(r)+g^2v^2\tilde{h}^2(r)\right)\tilde{f}(r)=0,\quad(20)$$

$$\left(\tilde{h}^2(r)-1\right)\left(\tilde{h}''(r)+\frac{2}{r}\tilde{h}'(r)-\frac{2}{r^2}\tilde{h}(r)\tilde{f}^2(r)\right)=0.\quad(21)$$

The radially fixing constraint (5) is also written in terms of $\tilde{h}(r)$ as

$$\tilde{h}^2(r)-1=0,\quad(22)$$

which yields

$$\tilde{h}(r)=\pm1.\quad(23)$$

Thus, the equation (21) is automatically satisfied by the constraint (5). By substituting $\tilde{h}(r)=\pm1$ into the equation (20), we obtain

$$\tilde{f}''(r)-\frac{1}{r^2}\left(\tilde{f}^3(r)-\tilde{f}(r)\right)-\left(\tilde{a}^2(r)+g^2v^2\right)\tilde{f}(r)=0.\quad(24)$$

For a numerical calculation, we introduce the dimensionless variable $\rho=gvr$ and functions $\tilde{a}(r)=gva(\rho)$, $\tilde{f}(r)=f(\rho)$, then the remaining two equations become

$$a''(\rho)+\frac{2}{\rho}a'(\rho)-\frac{2}{\rho^2}a(\rho)f^2(\rho)=0,\quad(25)$$

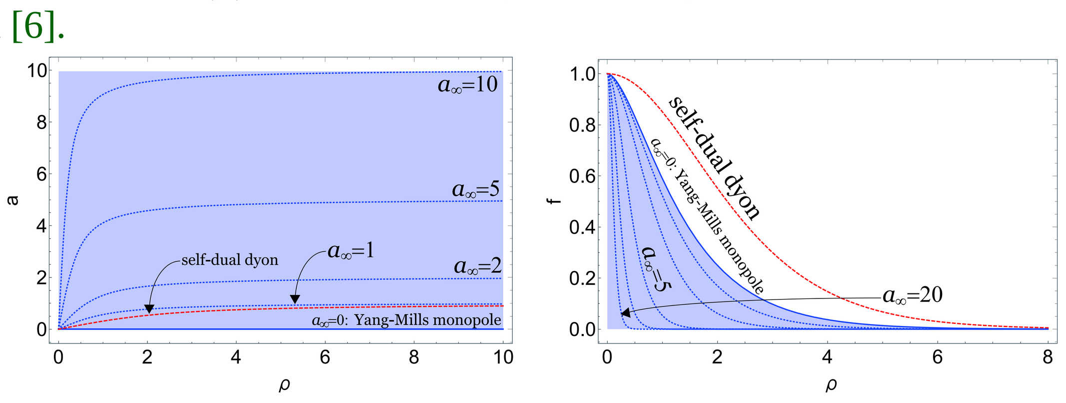
$$f''(\rho)-\frac{1}{\rho^2}\left(f^3(\rho)-f(\rho)\right)-\left(a^2(\rho)+1\right)f(\rho)=0.\quad(26)$$

Therefore, we solve the field equations (25) and (26) under the boundary conditions

$$a(0)=0,\quad a(\infty)=a_\infty,\quad(27)$$

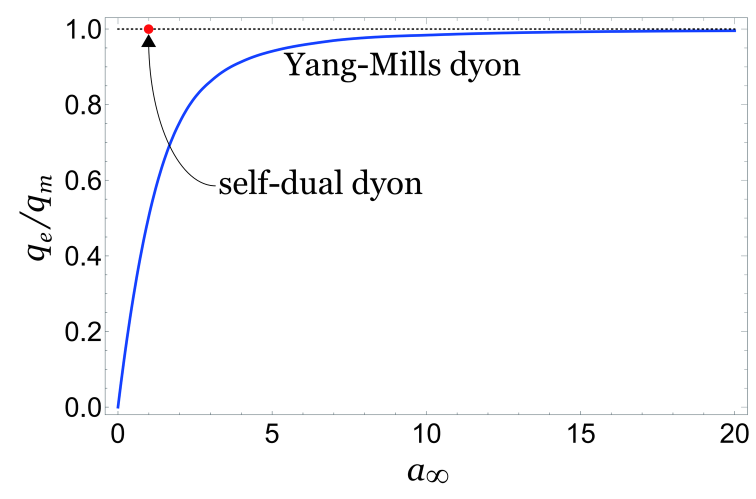
$$f(0)=1,\quad f(\infty)=0,\quad(28)$$

the dyon solution is obtained. It should be noticed that there is no condition to specify the asymptotic value a_∞ of $a(\rho)$. Notice that if $a(\rho)$ is a solution of equations (25) and (26), then $-a(\rho)$ is also a solution of them. Therefore, we can restrict $a_\infty\geq0$ without loosing the generality. The solution $a(\rho)\equiv0$ is the Yang–Mills magnetic monopole obtained in [6].



The Yang–Mills dyon cannot become self-dual for a finite asymptotic value a_∞ of $a(\rho)$, the upper bound of the electric charge q_e is obtained numerically as

$$\left|\frac{q_e}{q_m}\right|\lesssim1.\quad(29)$$



Asymptotic holonomy of the Yang–Mills dyon

We define the Polyakov loop operator $L(\boldsymbol{x})$ as

$$L(\boldsymbol{x}):=\frac{1}{{\rm tr}(\mathbf{1})}{\rm tr}\mathscr{P}\exp\left[ig\int_0^{T^{-1}}d\tau\,\mathscr{A}_4(\boldsymbol{x},\tau)\right],\quad(30)$$

where $\mathscr{P}$ denotes the path-ordering prescription. The asymptotic holonomy $\mathcal{P}_\infty$ is defined by the Polyakov loop operator at the spatial infinity

$$\mathcal{P}_\infty:=\lim_{|\boldsymbol{x}|\to\infty}L(\boldsymbol{x}).\quad(31)$$

By performing the gauge transformation to the unitary (or stringy) gauge $\hat{\phi}^A(x)=\delta^{A3}$, so that the "time" component $\mathscr{A}_4(x)$ of the gauge field becomes diagonal

$$g\mathscr{A}_4(x)\equiv g\mathscr{A}^A_4(x)T_A=\tilde{a}(r)\frac{\sigma_3}{2}=gva(\rho)\frac{\sigma_3}{2},\quad(32)$$

the asymptotic holonomy can be calculated as

$$\begin{aligned}\mathcal{P}_\infty&=\lim_{|\boldsymbol{x}|\to\infty}\frac{1}{2}{\rm tr}\exp\left[i\int_0^{T^{-1}}d\tau\,\tilde{a}(r)\frac{\sigma_3}{2}\right]\\&=\lim_{r\to\infty}\frac{1}{2}{\rm tr}\exp\left[\frac{i\tilde{a}(r)}{2T}\sigma_3\right]=\lim_{r\to\infty}\cos\frac{\tilde{a}(r)}{2T}=\cos\left[\frac{gva_\infty}{2T}\right],\end{aligned}\quad(33)$$

where we have used $\tilde{a}(\infty)=gva(\infty)=gva_\infty$.

As seen from the above figure, in the Yang–Mills dyon the asymptotic value a_∞ of the profile function $a(r)$ can be expressed as a function of the ratio of the charges q_e/q_m. This means that since we have fixed the magnetic charge q_m to the unit $q_m=4\pi/g$, the asymptotic holonomy $\mathcal{P}_\infty$ depends on the electric charge q_e through a_∞:

$$\mathcal{P}_\infty=\cos\left[\frac{gv}{2T}q_e(a_\infty)\right].\quad(34)$$

The vanishing electric charge $q_e\to0$ that is nothing but the Yang–Mills monopole means $a_\infty\to0$, which yields the trivial holonomy $\mathcal{P}_\infty\to1$. Conversely, the asymptotic holonomy becomes nontrivial as long as the Yang–Mills dyon has a nonzero electric charge. It should be compared with the (anti-)self-dual dyons, which are the constituents of the KvBLL calorons. The electric charge of the (anti-)self-dual dyons cannot be changed continuously, since it is fixed by definition to $q_e=\pm q_m$.

Conclusion and discussion

We obtained the novel dyon solution in the $SU(2)$ gauge-adjoint scalar model whose radial degree of freedom is fixed. This dyon solution in $S^1\times\mathbb{R}^3$ space becomes the field configuration of the $SU(2)$ Yang–Mills theory with a gauge-invariant mass term through the path integral derived by the gauge-independent description of the Brout–Englert–Higgs mechanism. We observed that the Yang–Mills dyon canoot acquire the electric charge equal to the magnetic charge. This is caused by a gauge-invariant mass term. In the contexts of the KvBLL calorons, the constituent (anti-)self-dual dyon has the electric charge equal to the magnetic charge (up to its sign) by definition, however, there do not exist such a self-dual object in our theory due to the mass term.

We also found that the Yang–Mills dyon in $S^1\times\mathbb{R}^3$ space has a nontrivial holonomy. This implies that our (non-self-dual) dyon with a nontrivial holonomy $\mathcal{P}_\infty$ can be used to explain the confinement/deconfinement phase transition in the Yang–Mills theory at finite temperature based on the dual superconductivity picture, instead of using the traditional KvBLL calorons or the self-dual dyons.

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