

The chiral phase transition in (2+1)-flavor QCD

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- Critical behavior in the limit of vanishing light quark masses
- Finite size scaling and chiral limit
- The chiral PHASE TRANSITION temperature

The chiral **PHASE TRANSITION** temperature

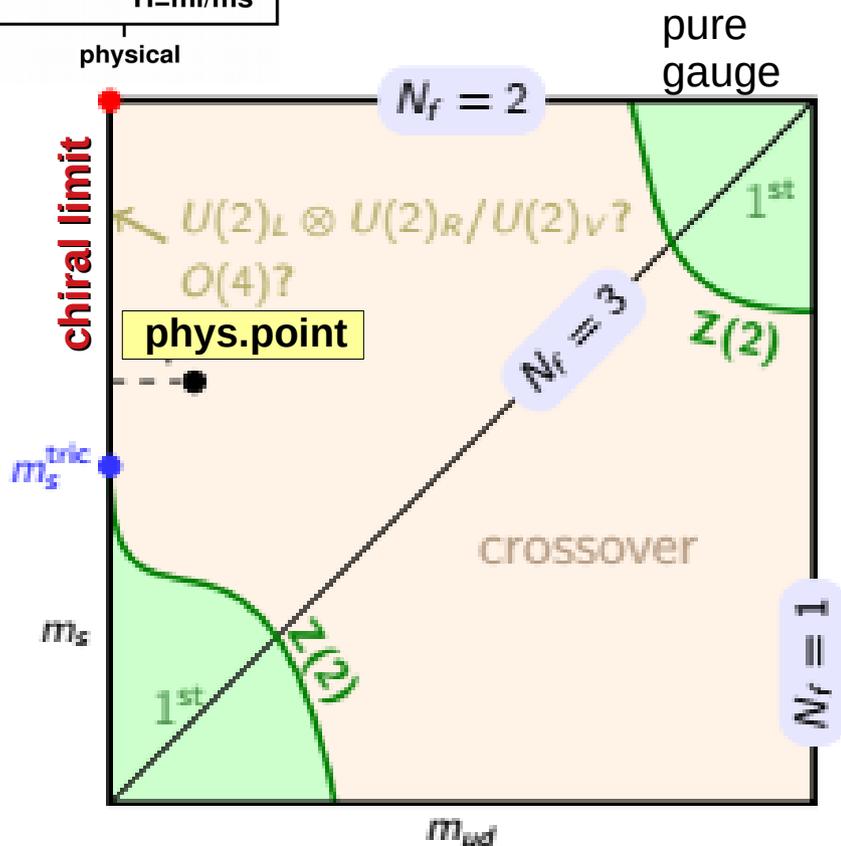
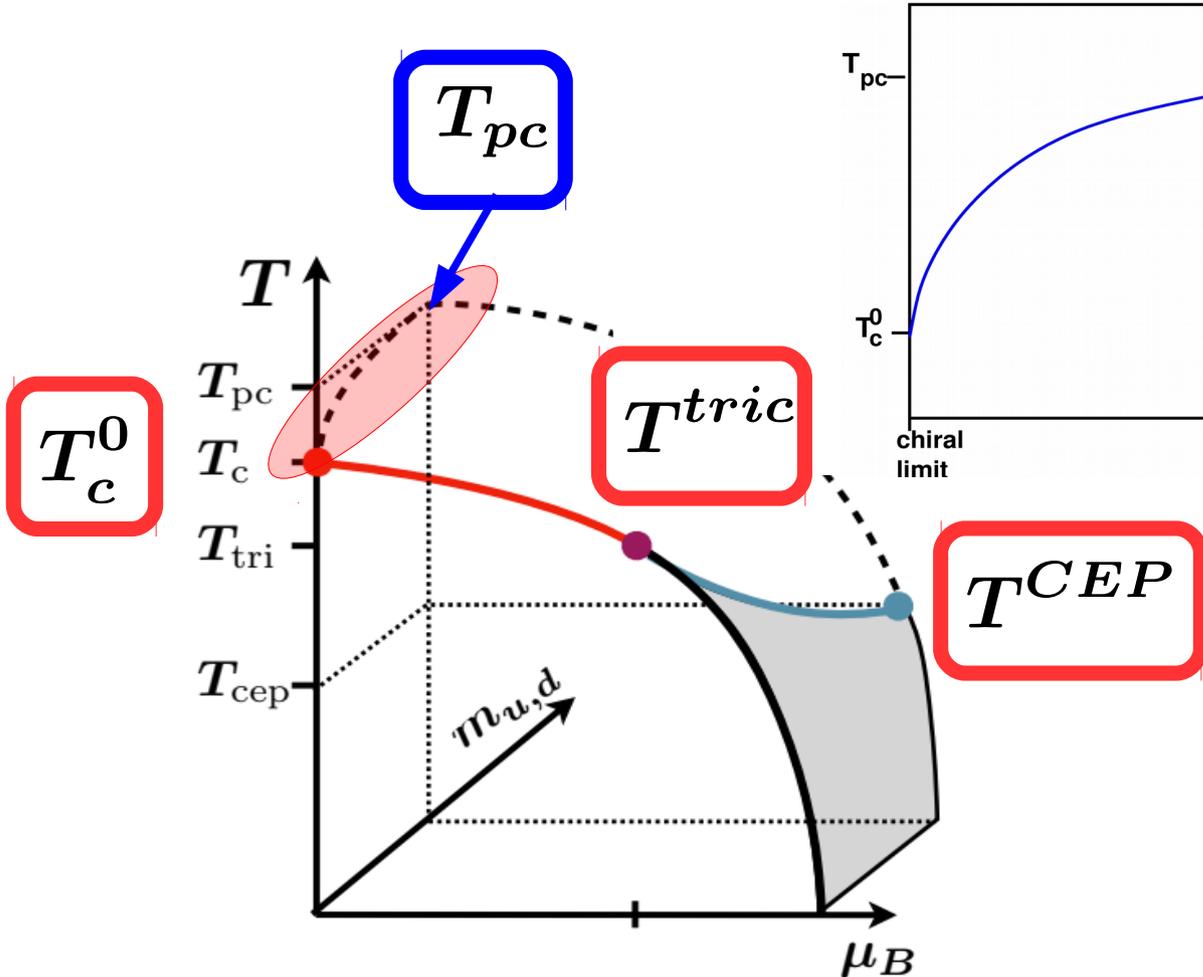
R. D. Pisarski, F. Wilczek,
[Remarks on the chiral phase transition in chromodynamics](#),
Phys. Rev. D 29 (1984) 338(R)

Abstract:

The phase transition restoring chiral symmetry at finite temperatures is considered in a linear σ model. For [three or more massless flavors](#), the perturbative ϵ expansion predicts the phase transition is of [first order](#). At high temperatures, the UA(1) symmetry will also be effectively restored.

- since 35 years it is understood that critical behavior in strong-interaction matter is due to **chiral symmetry restoration**
- the **phase transition temperature** in the chiral limit of QCD is one of the fundamental scales in strong-interaction physics
- **neither the order of the transition in 2 or (2+1)-flavor QCD nor the value of the transition temperature have been established so far**

Phases of strong-interaction matter



- determination of the critical temperatures
- and the order of the transition(s)

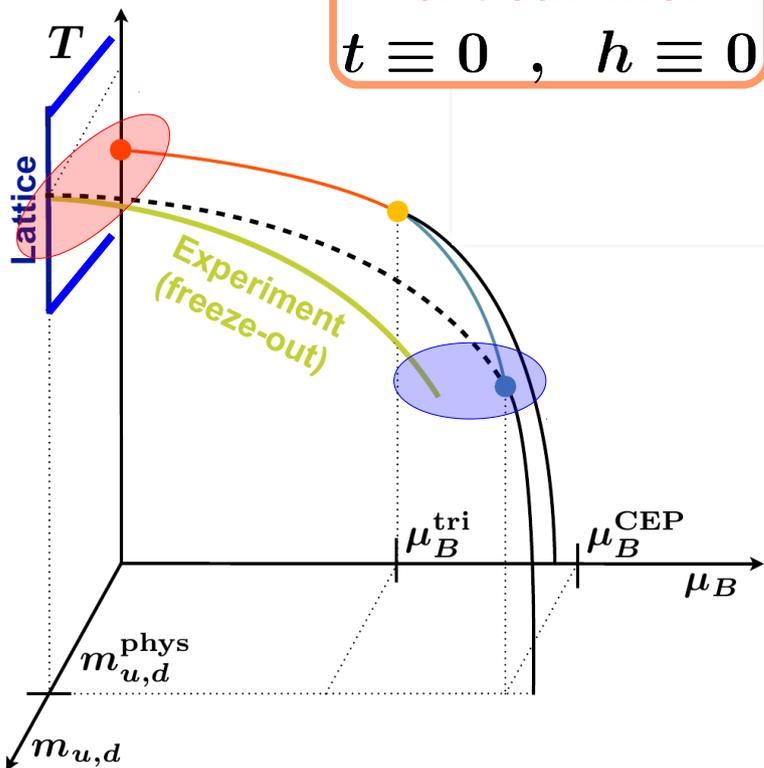
Critical behavior in QCD

- close to the chiral limit thermodynamics in the vicinity of the QCD transition(s) is controlled by a **universal scaling function**

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \vec{\mu}) = -h^{(2-\alpha)/\beta\delta} \overset{\text{singular}}{f_f(t/h^{1/\beta\delta})} - \overset{\text{regular}}{f_r(V, T, \vec{\mu})}$$

critical line:
 $t \equiv 0, h \equiv 0$

$$t \sim \frac{T - T_c}{T_c} + \kappa_2 \left(\frac{\mu_q}{T} \right)^2, \quad h \sim \frac{m_q}{T_c}$$



here only: $\mu_q \equiv 0$

question: Where is the chiral **PHASE TRANSITION** for $m_u = m_d = 0$ located? What is its influence on observables at the pseudo-critical temperature?

Scaling in the thermodynamic (infinite volume) limit

– approaching the chiral limit –

some definitions

$$z = \frac{t}{h^{1/\beta\delta}}$$

$$t \equiv \frac{1}{t_0} \frac{T - T_c}{T_c}$$

$$h = \frac{1}{h_0} H$$

$$H \equiv \frac{m_l}{m_s}$$

$$z_0 = h_0^{1/\beta\delta} / t_0$$

– order parameter M and its susceptibility

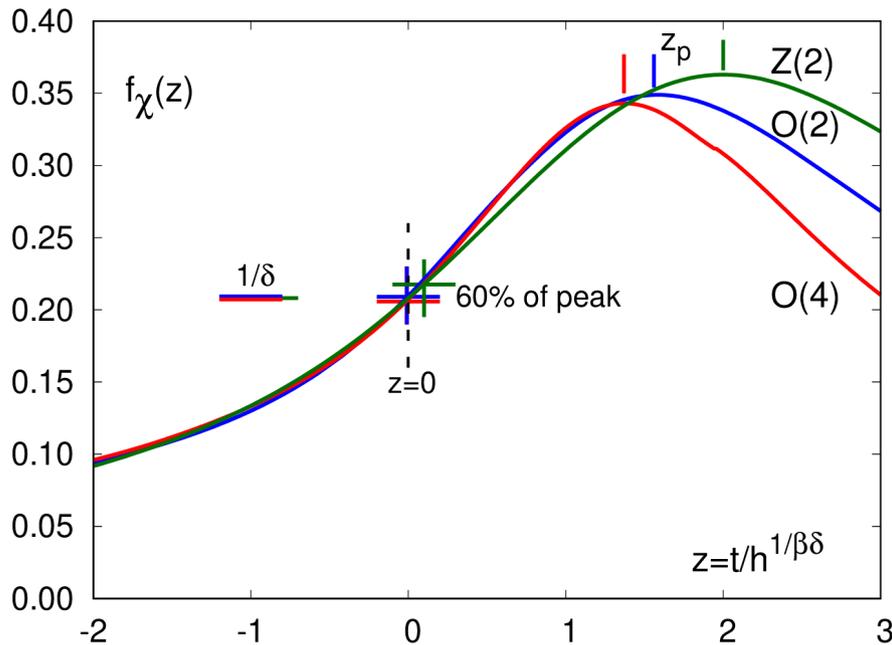
$$M = h^{1/\delta} f_G(z) + f_{sub}(T, H)$$

$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z) + \tilde{f}_{sub}(T, H)$$

for ANY fixed z:

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta} \right) + \text{sub leading}$$

↗ – corrections-to-scaling
↘ – regular terms

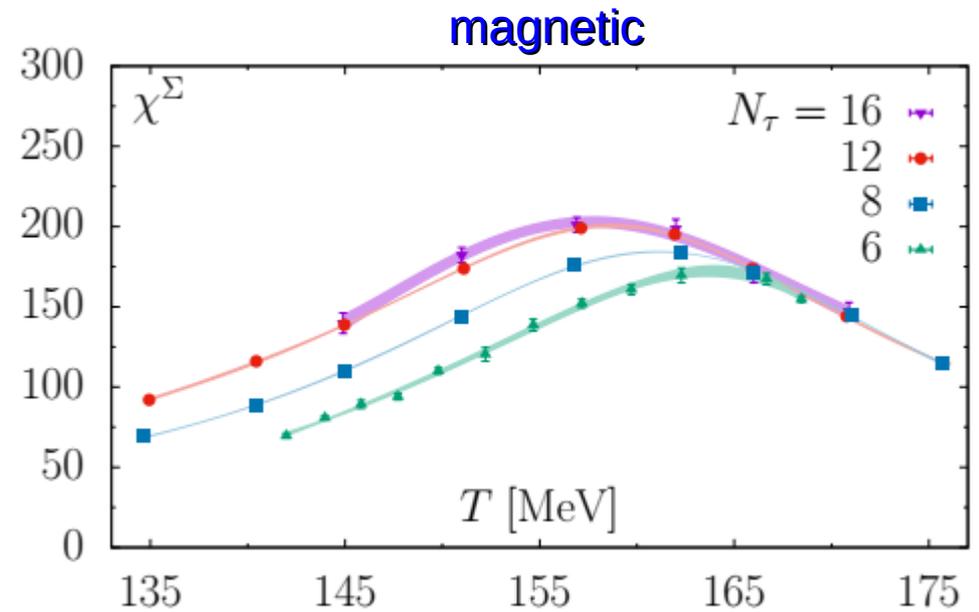
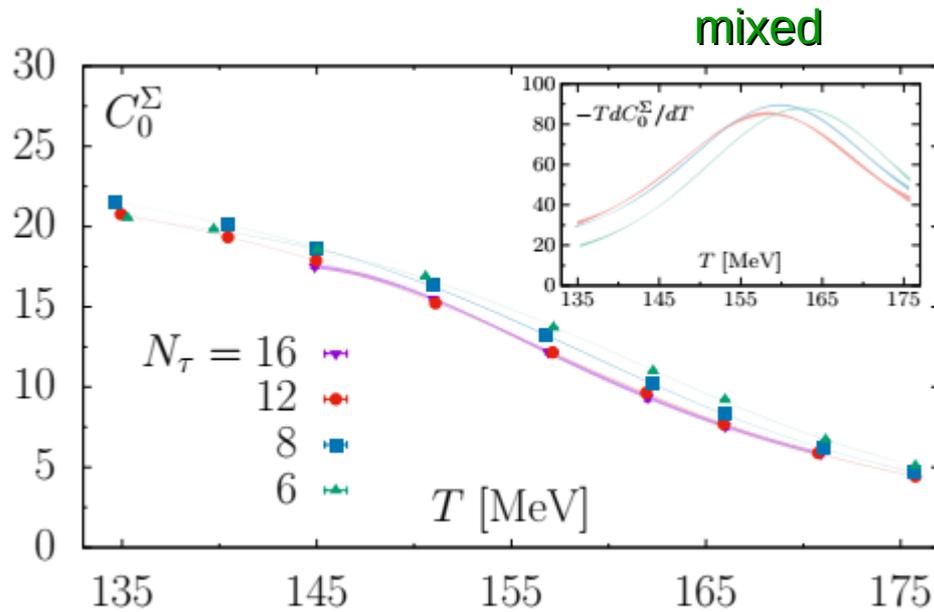


scaling functions $f_\chi(z)$ for some 3-d universality classes:

	δ	$1/\beta\delta$	z_p	z_{60}	z_δ
Z(2)	4.805	0.640	2.00(5)	0.10(1)	0
O(2)	4.780	0.599	1.58(4)	-0.005(9)	0
O(4)	4.824	0.545	1.37(3)	-0.013(7)	0

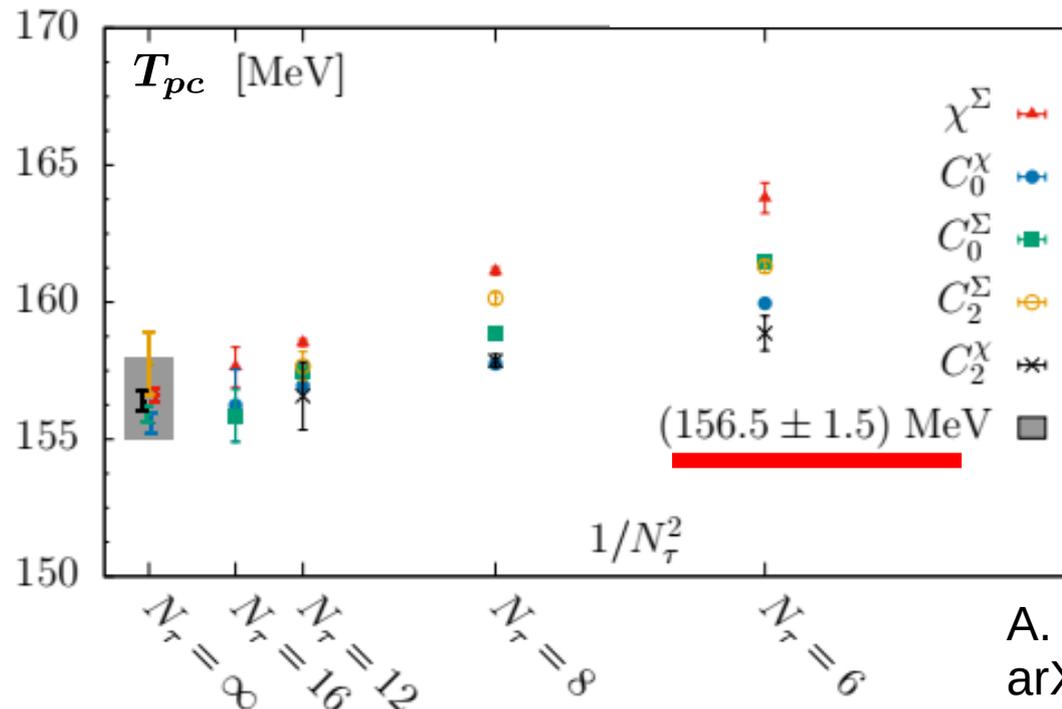
characteristic points on the scaling function $f_\chi(z)$

Pseudo-critical temperatures from chiral observables



physical
light & strange
quark masses;

continuum
extrapolated



$$\chi^\Sigma \Leftrightarrow \chi_M$$

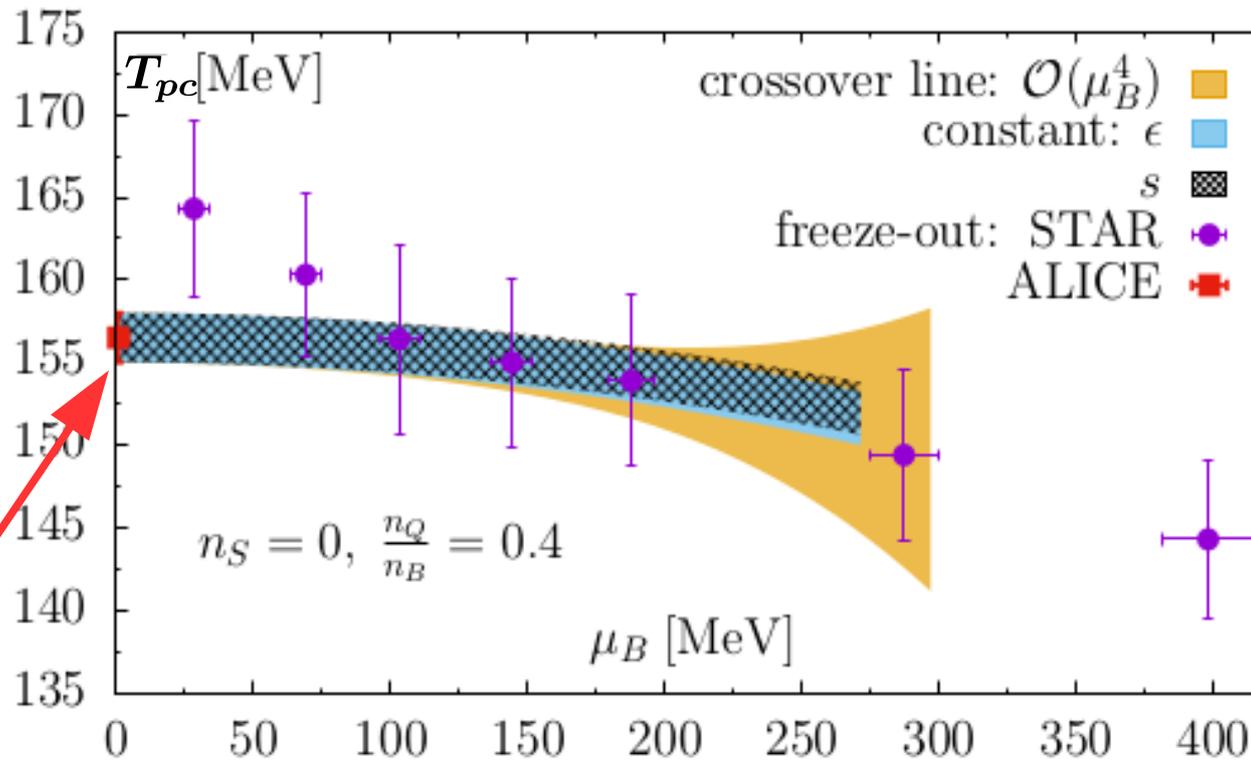
$$C_0^\Sigma \Leftrightarrow T(\partial M / \partial T)$$

A. Bazavov et al [HotQCD],
arXiv:1812.08235

Phases of strong-interaction matter

$$T_{pc}(\mu_B) = T_{pc} \left(1 + \kappa_2 \left(\frac{\mu_B}{T_c} \right)^2 + \dots \right)$$

phase diagram at physical values of the quark masses



A. Andronic et al.,
Nature 561 (2018)
321

$$T_{pc} = (156.5 \pm 1.5) \text{ MeV}$$

$$\kappa_2 = 0.012(4)$$

A. Bazavov et al. [HotQCD],
arXiv:1812.08235

Scaling in the thermodynamic (infinite volume) limit

– approaching the chiral limit –

some definitions

– order parameter M and its susceptibility

$$M = h^{1/\delta} f_G(z) + f_{sub}(T, H)$$

$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z) + \tilde{f}_{sub}(T, H)$$

for ANY fixed z :

$$T_{pc}(H) = T_c^0 \left(1 + \frac{z}{z_0} H^{1/\beta\delta} \right) + \text{sub leading}$$

$$z = \frac{t}{h^{1/\beta\delta}}$$

$$t \equiv \frac{1}{t_0} \frac{T - T_c}{T_c}$$

$$h = \frac{1}{h_0} H$$

$$H \equiv \frac{m_l}{m_s}$$

$$z_0 = h_0^{1/\beta\delta} / t_0$$

conventional steps to determine T_c^0

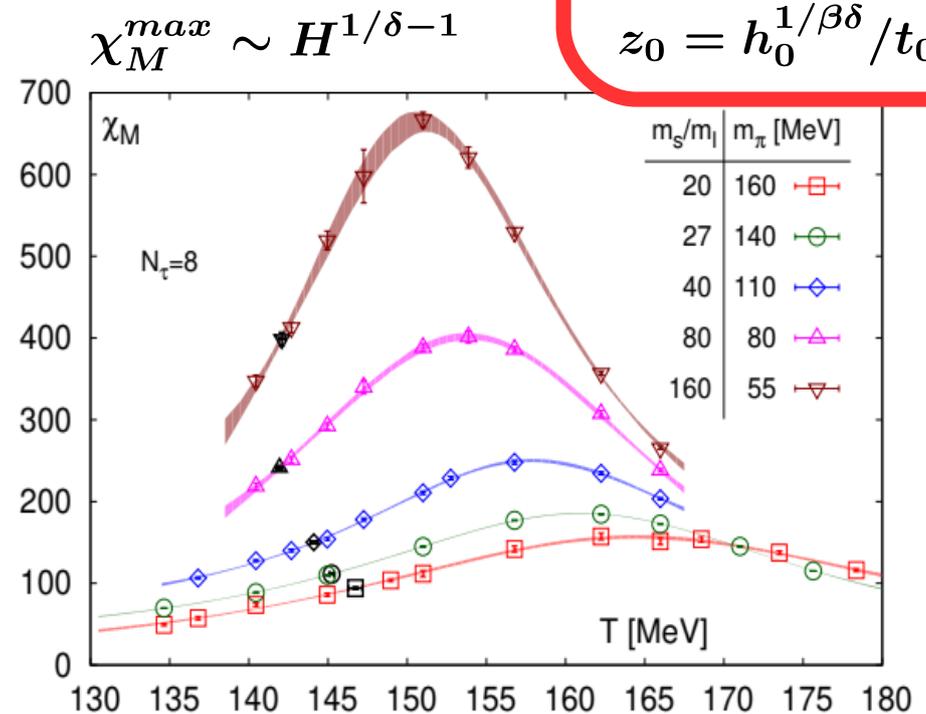
– choose a characteristic feature of χ_M

→ the maximum χ_M^{max}

– in the scaling regime this is located at z_p

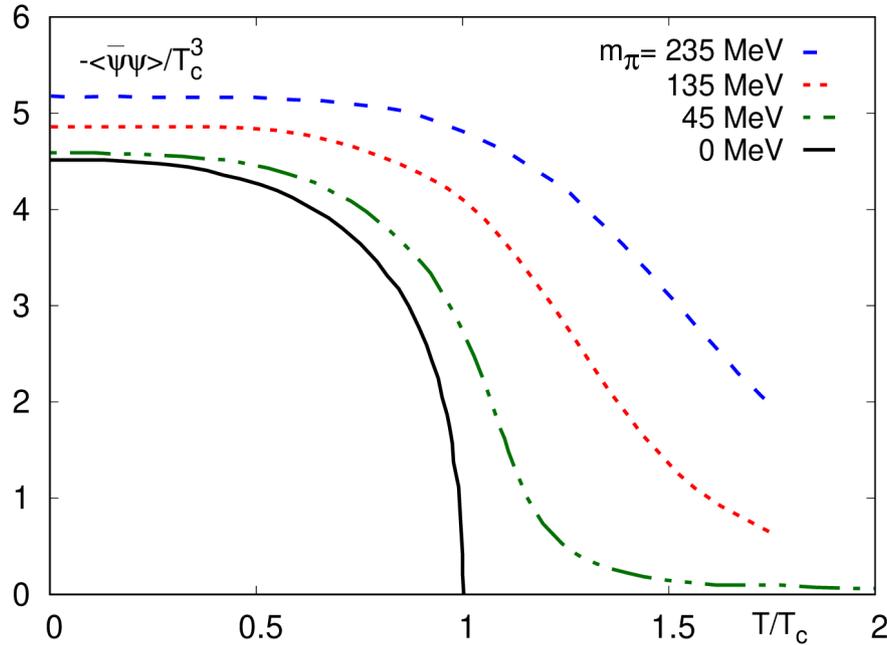
– using the scaling ansatz for $T_{pc}(H)$ allows to extract T_c^0

A. Lahiri et al, QM 2018, arXiv:1807.05727
H.T. Ding et al, arXiv:1903.04801



Chiral extrapolation in the Quark Meson Model

$$\Gamma_{\Lambda UV}[\phi] = \int d^4x \left\{ \bar{q}(\not{\partial} + gm_c)q + g\bar{q}(\sigma + i\vec{\tau} \cdot \vec{\pi}\gamma_5)q + \frac{1}{2}(\partial_\mu\phi)^2 + U_{\Lambda UV}(\phi) \right\}$$



$$\Delta T \equiv T_{pc}(m_\pi^{phys}) - T_c(0) \simeq 30 \text{ MeV}$$

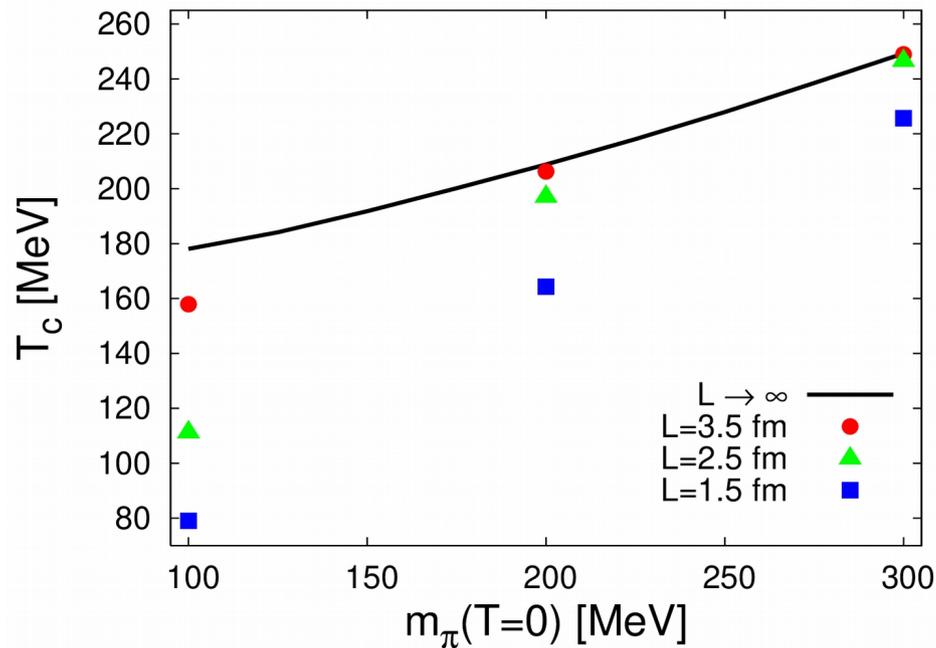
m_π [MeV]	0	45	135	230
$T_{pc}^{(1)}$ [MeV]	100.7	$\simeq 110$	$\simeq 130$	$\simeq 150$
$T_{pc}^{(2)}$ [MeV]	100.7	113	128	—

– strong pion mass dependence of $T_{pc}(m_\pi)$

$T_{pc}(m_\pi)$ almost linear in m_π ,
even for $m_\pi = m_\pi^{phys}$
trivial?
put O(4) in, get O(4) out?

J. Berges, D.U. Jungnickel, C. Wetterich,
Phys. Rev. D59 (1999) 034010

Chiral extrapolation and finite volume effects in the O(4) ϕ^4 model



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2 \phi^2 + \frac{\lambda}{4}\phi^4$$

$$\phi = (\phi_1, \dots, \phi_4)$$

$$\Delta T \equiv T_{pc}(m_\pi^{phys}) - T_c(0) \simeq 35 \text{ MeV}$$

L [fm]	$m_\pi^{(0)} = 100$ MeV	$m_\pi^{(0)} = 200$ MeV	$m_\pi^{(0)} = 300$ MeV
1.5	79.0 MeV	164.3 MeV	225.7 MeV
2.5	111.3 MeV	197.1 MeV	246.6 MeV
3.5	157.9 MeV	206.3 MeV	249.0 MeV
∞	178.1 MeV	208.3 MeV	249.3 MeV

– increasing volume dependence with decreasing pion mass

Finite size scaling functions of the 3-d, O(4) spin model

$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L)$$

$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L)$$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z, z_L)}{f_G(z, z_L)} + \text{sub leading}$$

$$\lim_{L \rightarrow \infty} \left(\frac{H\chi_M}{M} \right)_{z=0} = \frac{1}{\delta}$$

$$z_L = \frac{1}{Lh^{\nu_c}}$$

$$z = \frac{t}{h^{1/\beta\delta}}$$

$$\nu_c = \nu/\beta\delta$$

$$= (0.5 - 0.6)$$

volume dependence controlled by $z_L \sim 1/m_\pi^{2\nu_c} L$, $2\nu_c \simeq 1$

define $z_\delta(z_L)$ as the value z for given z_L at which $\left(\frac{H\chi_M}{M} \right)_{z_\delta(z_L)} = \frac{1}{\delta}$

$$T_\delta(H, L) = T_c^0 \left(1 + \frac{z_\delta(z_L)}{z_0} H^{1/\beta\delta} \right) + \text{sub leading}$$

$$z_\delta(0) = 0$$

$z_\delta \simeq 0 \Rightarrow$ weak H-dependence of T_δ even at finite H and/or L
 – almost perfect estimator for T_c in the limit $H \rightarrow 0$, $L \rightarrow \infty$

Finite size scaling functions of the 3-d, O(4) spin model

$$V \equiv L^3$$

$$\equiv (N_\sigma a)^3$$

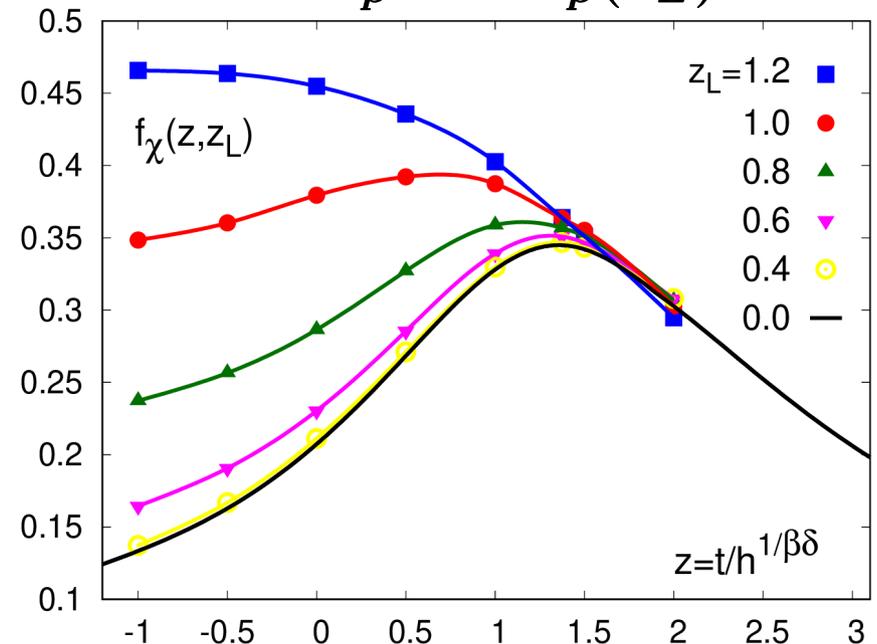
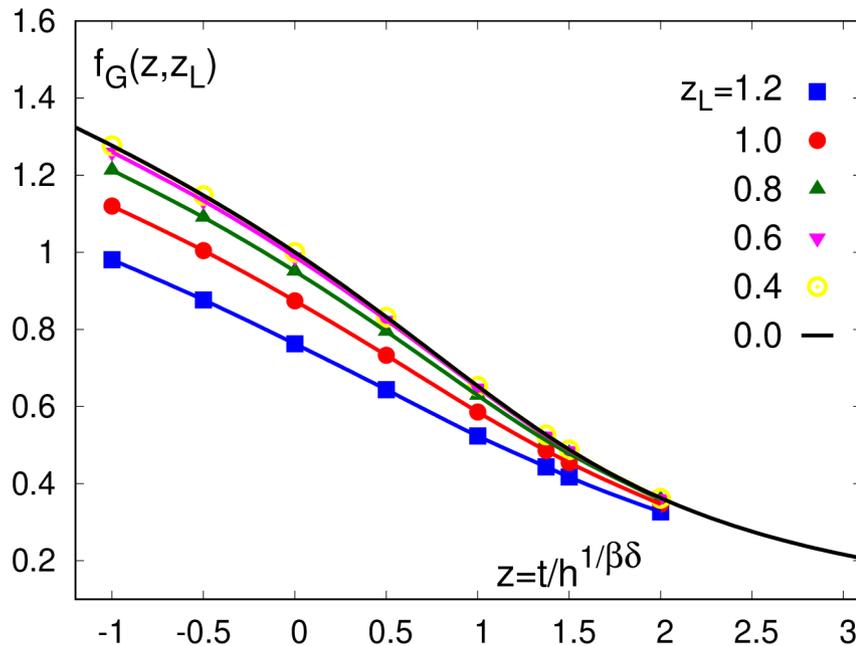
$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L)$$

$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L)$$

any "characteristic" z becomes a function of z_L :

$$T_{pc}(H, L) = T_c^0 \left(1 + \frac{z_p(z_L)}{z_0} H^{1/\beta\delta} \right) + \text{sub leading}$$

$$z_p \Rightarrow z_p(z_L)$$



J. Engels, FK, Phys. Rev. D90 (2014) 014501

Finite size scaling functions of the 3-d, O(4) spin model

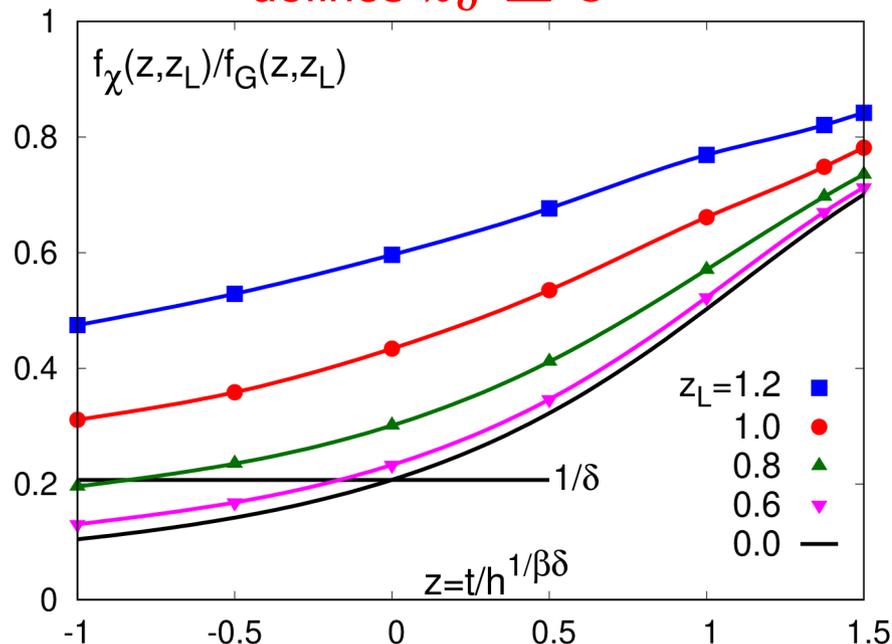
$$V \equiv L^3 \\ \equiv (N_\sigma a)^3$$

$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L) \\ \chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L)$$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z, z_L)}{f_G(z, z_L)} + \text{sub leading}$$

$$\lim_{L \rightarrow \infty} \left(\frac{H\chi_M}{M} \right)_{z=0} = \frac{1}{\delta}$$

defines $z_\delta \simeq 0$



Finite size scaling functions of the 3-d, O(4) spin model

$$V \equiv L^3$$

$$\equiv (N_\sigma a)^3$$

$$M = h^{1/\delta} f_G(z, z_L) + f_{sub}(T, H, L)$$

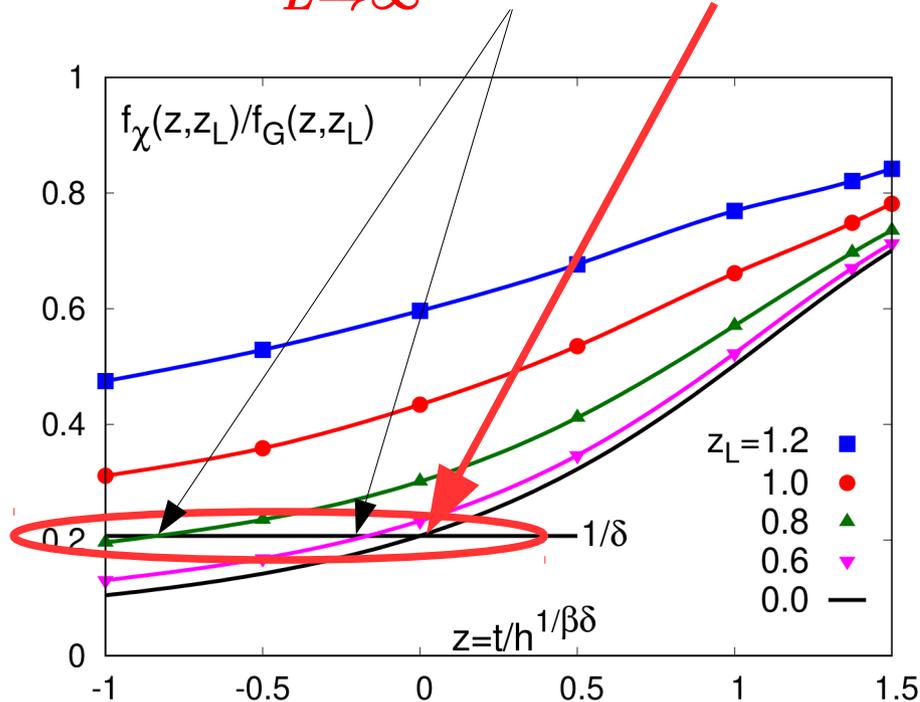
$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z, z_L) + \tilde{f}_{sub}(T, H, L)$$

$$\frac{H\chi_M}{M} = \frac{f_\chi(z, z_L)}{f_G(z, z_L)} + \text{sub leading}$$

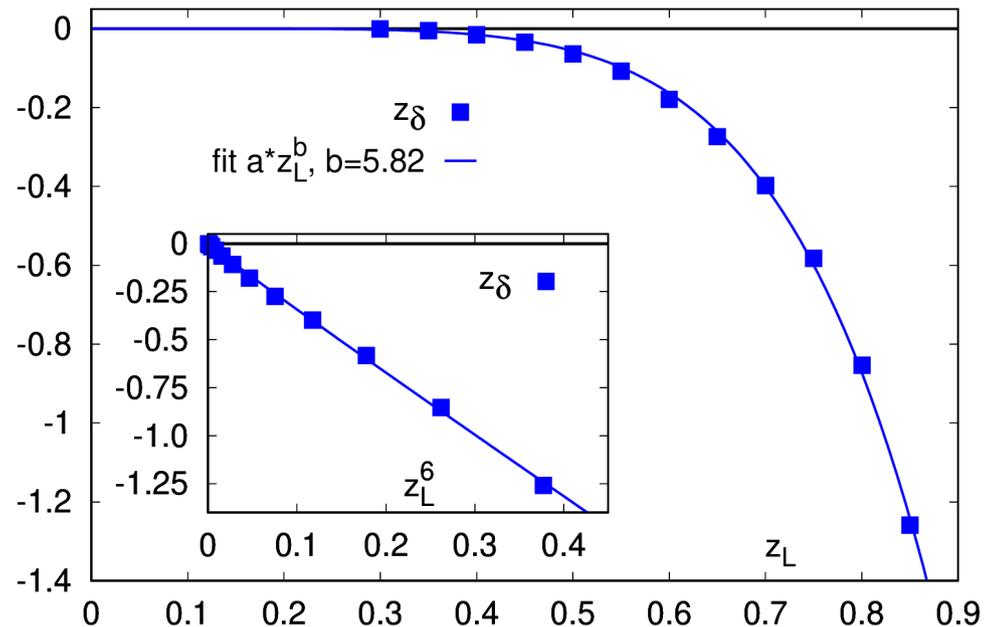
quark mass dependence arises only as a finite volume effect (+s.l.)

$$T_\delta(H, L) = T_c^0 \left(1 + \frac{z_\delta(z_L)}{z_0} H^{1/\beta\delta} \right)$$

$$\lim_{L \rightarrow \infty} T_\delta(L) = T_c^0$$

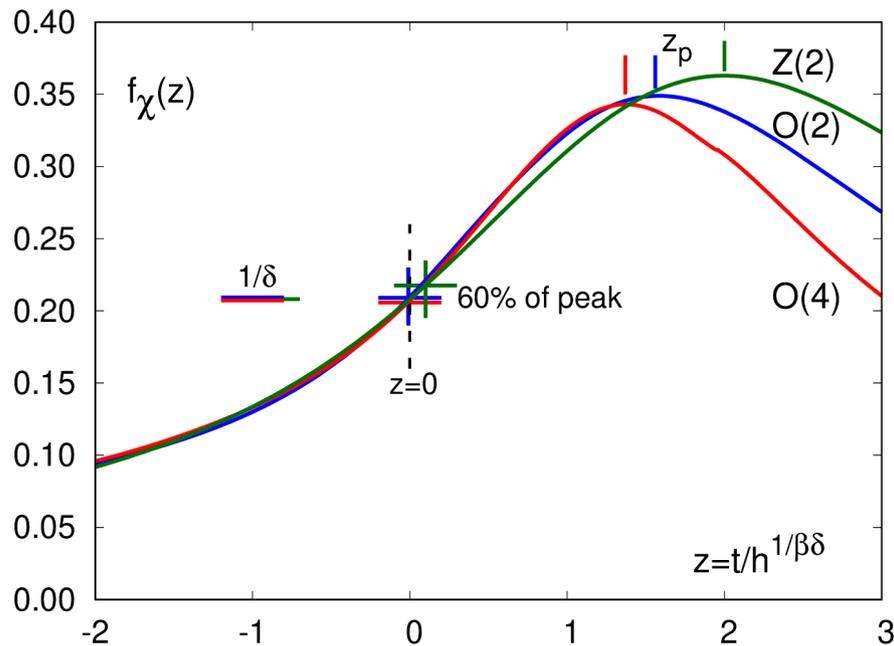


finite volume effects $\sim 1/V^2 \equiv 1/L^6$



Chiral PHASE TRANSITION in (2+1)-flavor QCD

A. Lahiri et al, QM 2018, arXiv:1807.05727
 H.T. Ding et al (HotQCD), arXiv:1903.04801



- physical strange quark mass
- vary light quark mass

$$55 \text{ MeV} \leq m_\pi \leq 160 \text{ MeV}$$

- use **new estimators** for pseudo-critical temperatures

$$T_\delta, T_{60}$$

- control finite volume effects

$$2 \leq m_\pi L \leq 5$$

- extrapolate to infinite volume limit and chiral limit

$$1/aT = 6, 8, 12$$

	δ	z_p	z_{60}	$f_G(z_p)$	$f_\chi(z_p)$	$f_\chi(0)/f_\chi(z_p)$
Z(2)	4.805	2.00(5)	0.10(1)	0.548(10)	0.3629(1)	0.573(1)
O(2)	4.780	1.58(4)	-0.005(9)	0.550(10)	0.3489(1)	0.600(1)
O(4)	4.824	1.37(3)	-0.013(7)	0.532(10)	0.3430(1)	0.604(1)

Chiral PHASE TRANSITION in (2+1)-flavor QCD

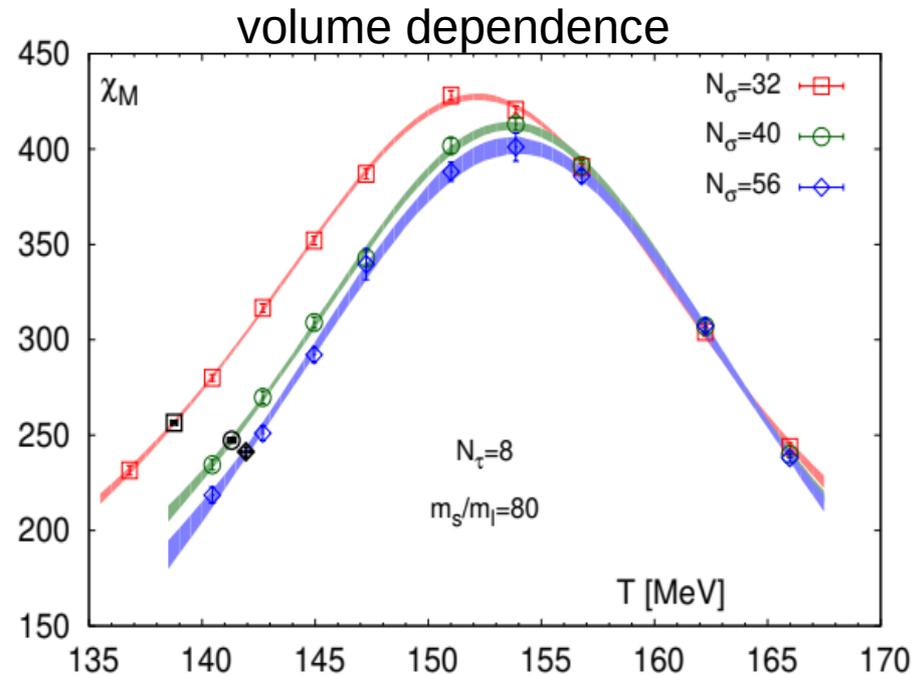
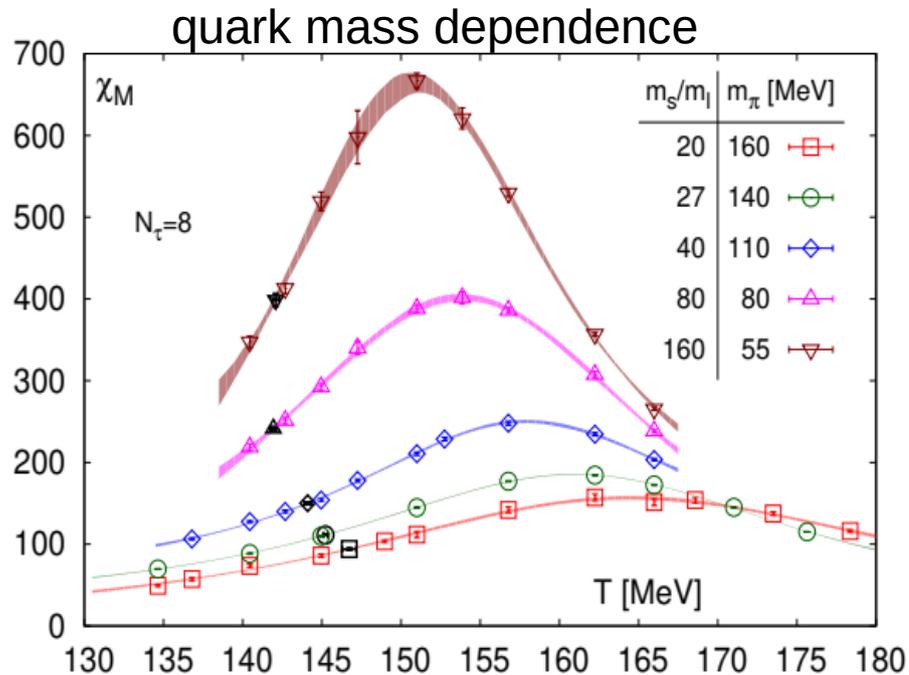
$$\langle \bar{\psi}\psi \rangle_f = \frac{T}{V} \frac{\partial \ln Z(T, V, m_u, m_d, m_s)}{\partial m_f}$$

$$\langle \bar{\psi}\psi \rangle_l = (\langle \bar{\psi}\psi \rangle_u + \langle \bar{\psi}\psi \rangle_d) / 2$$

renormalization group invariant order parameter: $M = 2 (m_s \langle \bar{\psi}\psi \rangle_l - m_l \langle \bar{\psi}\psi \rangle_s) / f_K^4$

chiral susceptibility: $\chi_M = m_s (\partial_u + \partial_d) M$

lattice sizes: $N_\sigma^3 \times N_\tau$, $4 \leq N_\sigma / N_\tau \leq 8$



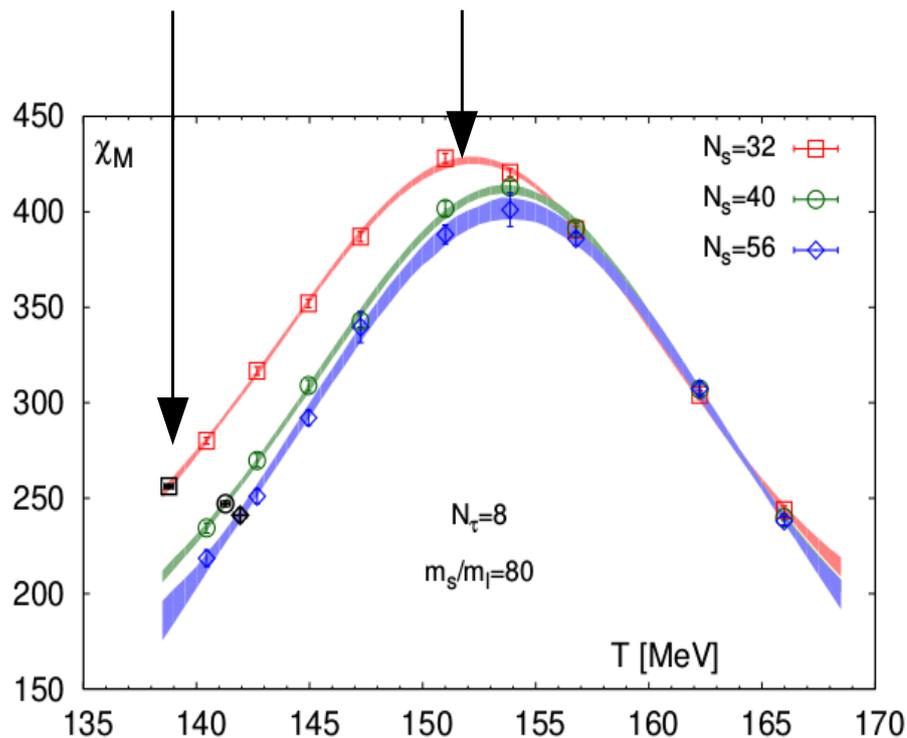
H.T. Ding et al, arXiv:1903.04801

Chiral PHASE TRANSITION in (2+1)-flavor QCD

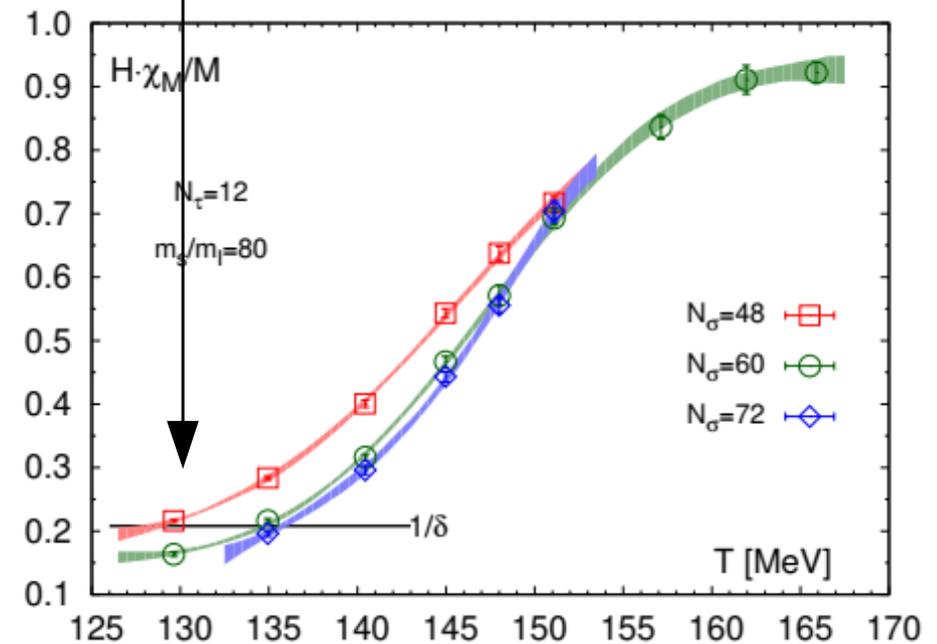
use two novel observables for the determination of the chiral PHASE TRANSITION TEMPERATURE, which in the infinite volume limit correspond to $z \simeq 0$, i.e. in the scaling regime they have almost no quark mass dependence

$$T_X(H, L) = T_c^0 \left(1 + \frac{z_X(z_L)}{z_0} H^{1/\beta\delta} \right) + \text{sub leading}, \quad X = \delta, 60$$

$$\chi_{M,60} = 0.6\chi^{max} \Rightarrow T_{60}$$



$$\frac{H\chi_M}{M} = \frac{1}{\delta} \Rightarrow T_\delta$$



H.T. Ding et al, arXiv:1903.04801

Finite size & and quark mass scaling

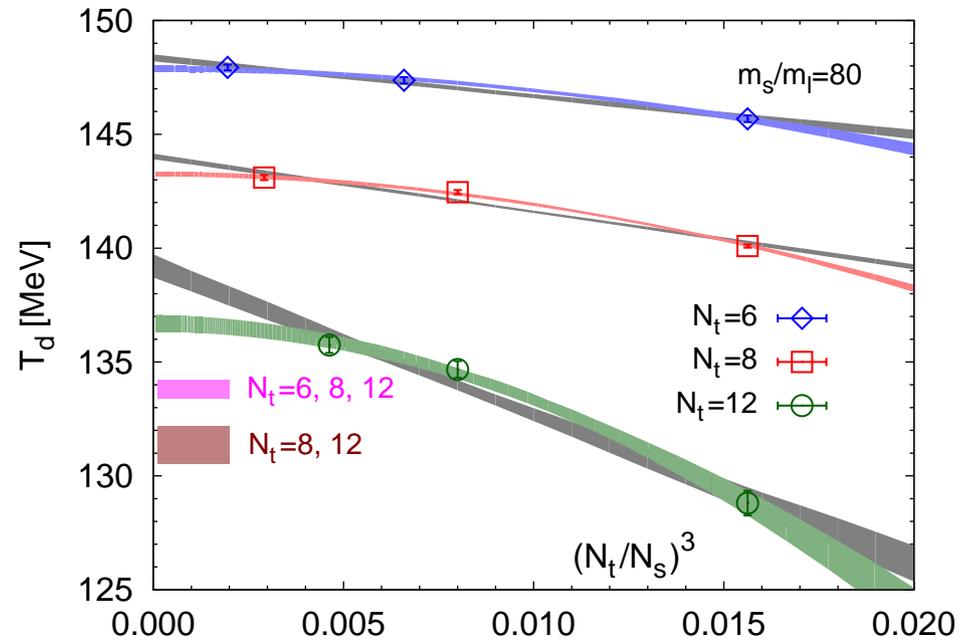
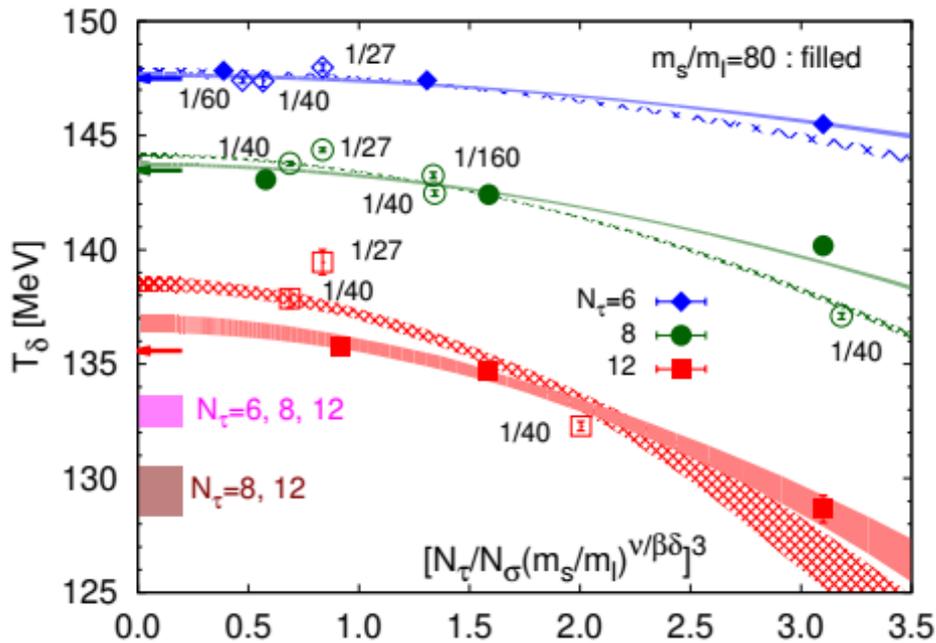
$$T_\delta(H, L) = T_c^0 \left(1 + \frac{z_\delta(z_L)}{z_0} H^{1/\beta\delta} \right) + \text{sub leading}$$

$$(T_\delta(H, L)/T_c^0 - 1) H^{-1/\beta\delta} - c_r H^{1-1/\delta} = \frac{z_\delta(z_L)}{z_0}$$

leading regular term

$$z_L = z_{L,0} \left(\frac{m_s}{m_l} \right)^{\nu_c} \frac{N_\tau}{N_\sigma}$$

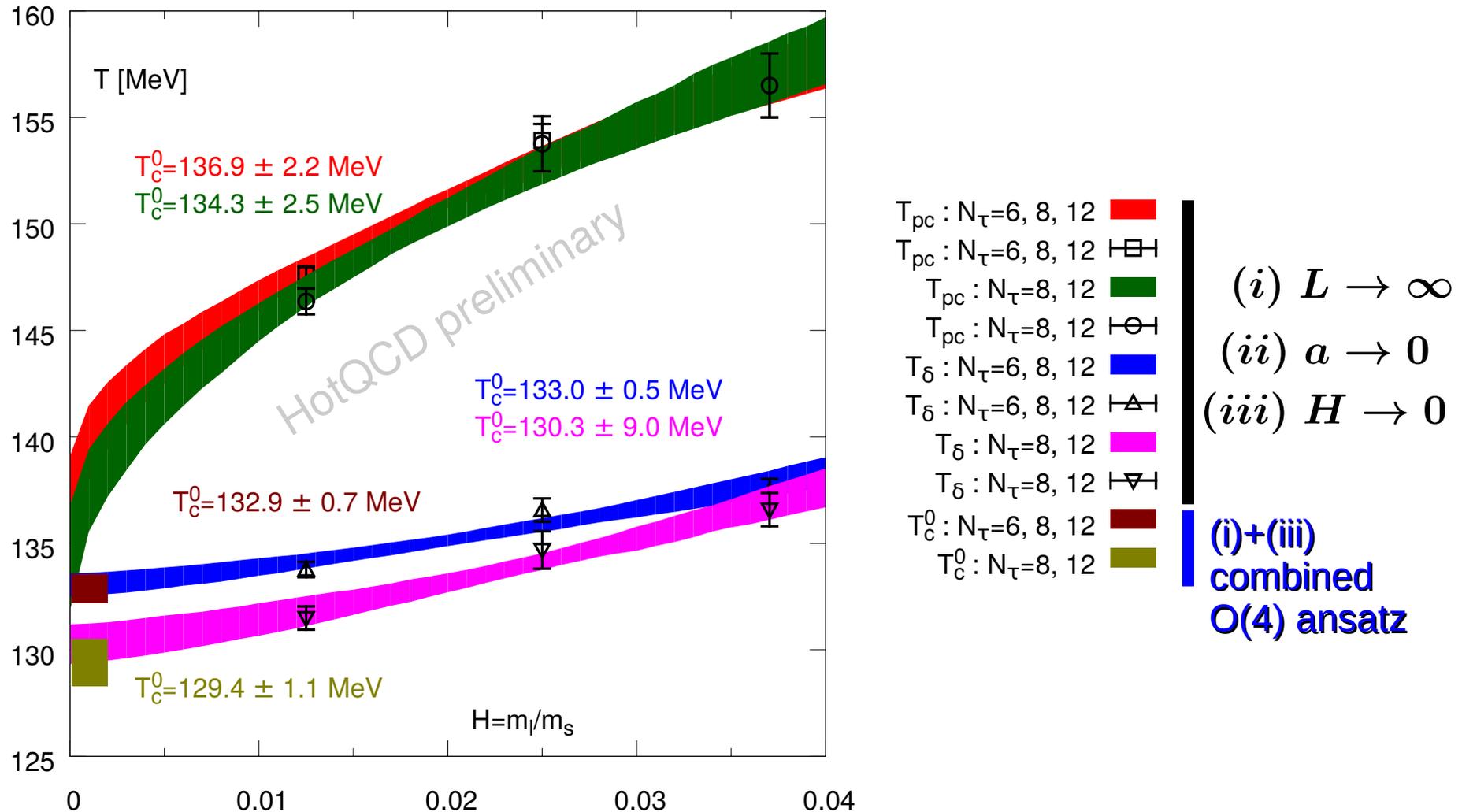
$$\nu_c = \nu/\beta\delta$$



$z_\delta(z_L)$ for O(4) from
J. Engels, FK, Phys. Rev. D90 (2014) 014501

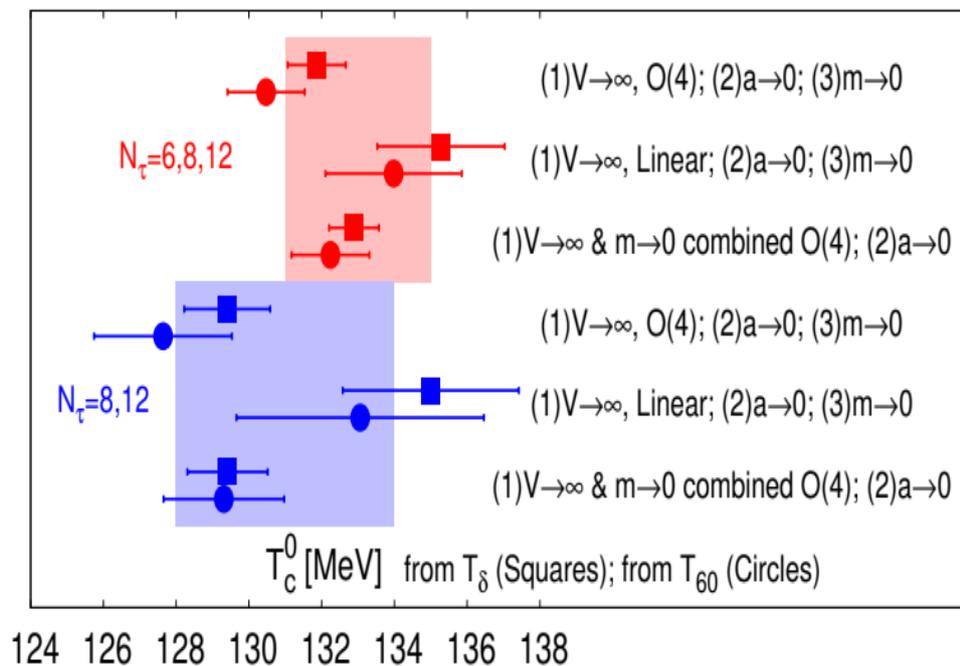
Finite size & and quark mass scaling

$$T_\delta(H, L) = T_c^0 \left(1 + \frac{z_\delta(z_L)}{z_0} H^{1/\beta\delta} \right) + \text{sub leading}$$



The chiral **PHASE TRANSITION** temperature

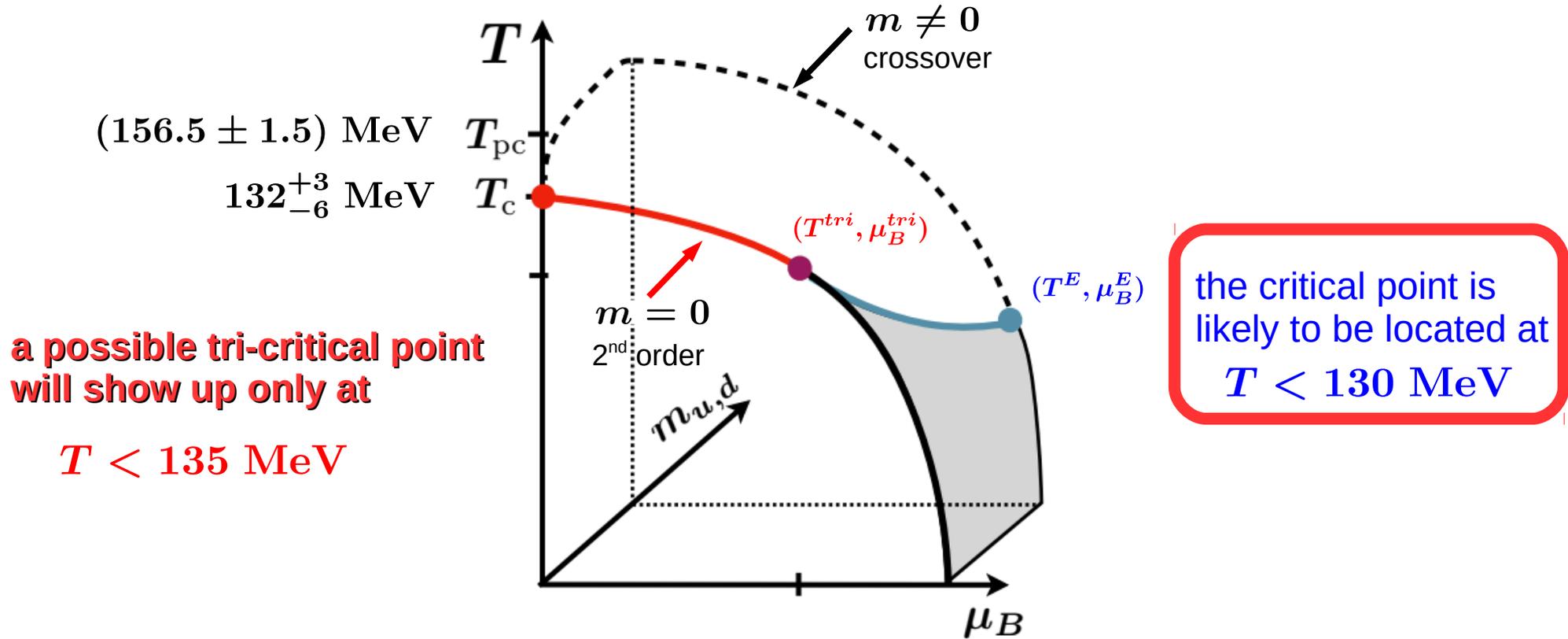
- using extrapolations linear in $1/V$ and m as well as $O(4)$ scaling ansatz
- extrapolations with and without data from coarsest lattice
- averaging results for T_δ and T_{60}



$$T_c = 132_{-6}^{+3} \text{ MeV}$$

H.T. Ding et al, arXiv:1903.04801

Crossover, chiral phase transition at $\mu_B = 0$ and the (tri)-critical point at $\mu_B > 0$



Random Matrix Model

QCD

NJL

A. Halasz, A.D. Jackson, R.E. Shrock, M.A. Stephanov, J.J., M. Verbaarschot, Phys. Rev. D58 (1998) 096007

M. Stephanov, Phys. Rev. D73 (2006) 094508

M. Buballa, S. Carignano, Phys. Lett. B791 (2019) 361

Conclusions

- no evidence for a 1st order transition in QCD for pion masses $m_\pi \geq 55$ MeV
- the chiral phase transition in QCD is likely to be 2nd order
- the chiral phase transition is (20-25) MeV smaller than the pseudo-critical temperature for physical values of the quark masses

$$T = 132_{-6}^{+3} \text{MeV}$$

- the chiral phase transition occurs at a pion density

$$n_\pi \simeq 0.12/\text{fm}^3$$

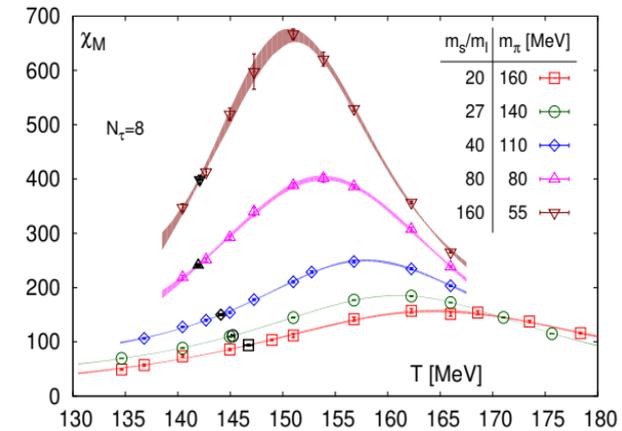
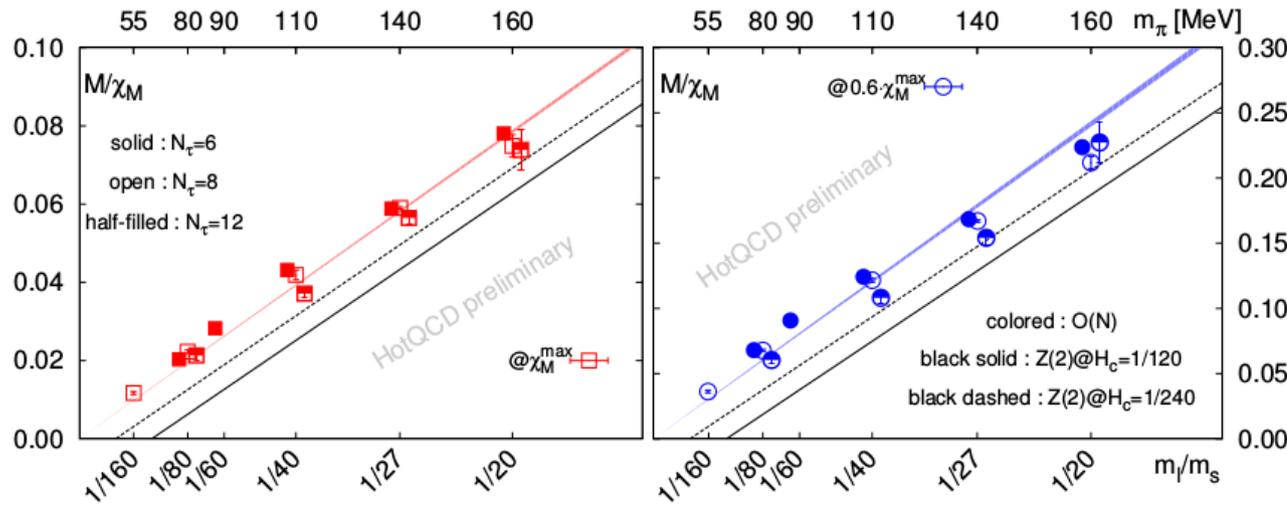
The chiral PHASE TRANSITION temperature – evidence for a 2nd order transition in the chiral limit–

in the thermodynamic limit: suppose there occurs a 1st order transition for $H < H_c$

$$M(T, H) \sim (H - H_c)^{1/\delta} f_G(z) \quad (M \text{ is "almost" an order parameter})$$

$$\chi(T, H) \sim (H - H_c)^{1/\delta-1} f_\chi(z) + \dots$$

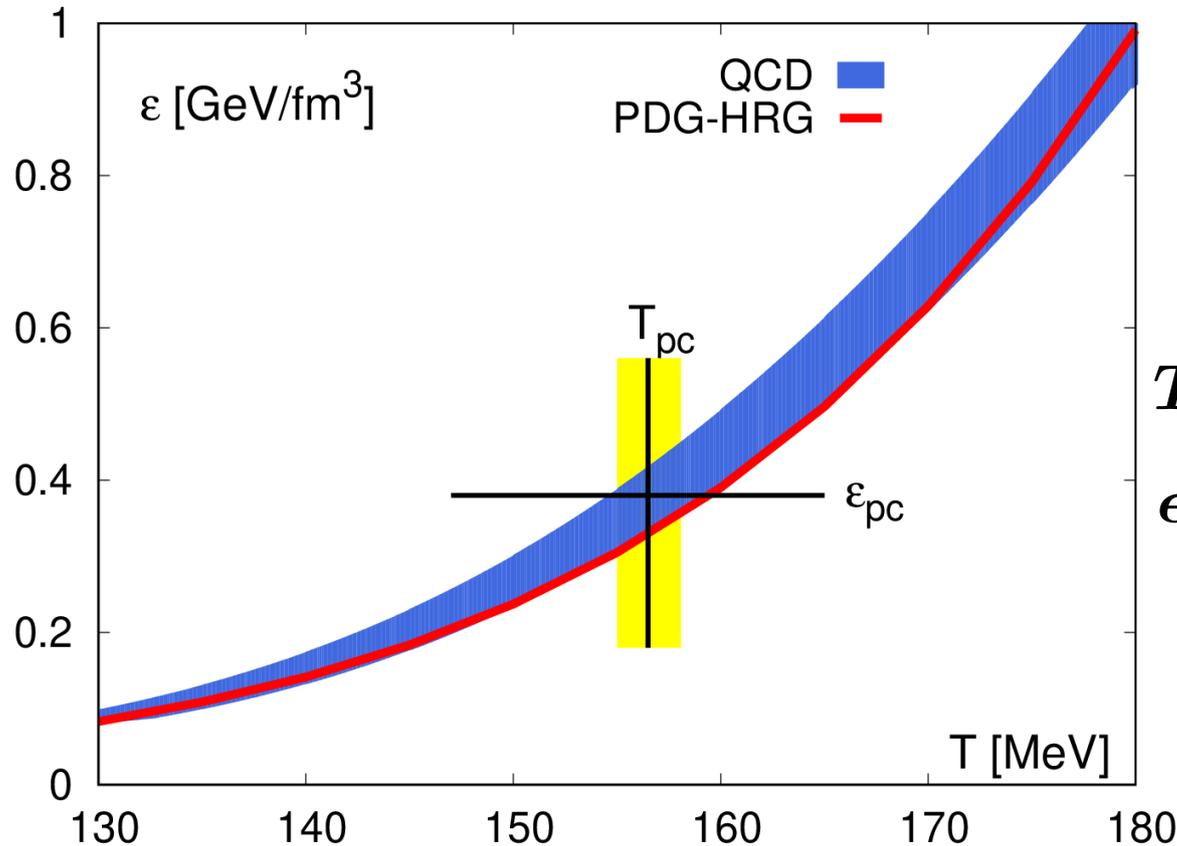
for ANY fixed z : $\frac{M}{\chi_M} \sim (H - H_c) \frac{f_G(z)}{f_\chi(z)} \Rightarrow$ bound on H_c



χ_M would diverge
already for non-zero
 H_c

Crossover transition parameters

PDG: Particle Data Group hadron spectrum



$$\mu_B/T = 0$$

physical quark masses

$$T_{pc} = (156.5 \pm 1.5) \text{ MeV}$$

$$\epsilon_{pc} = (0.42 \pm 0.06) \text{ GeV/fm}^3$$

compare with:

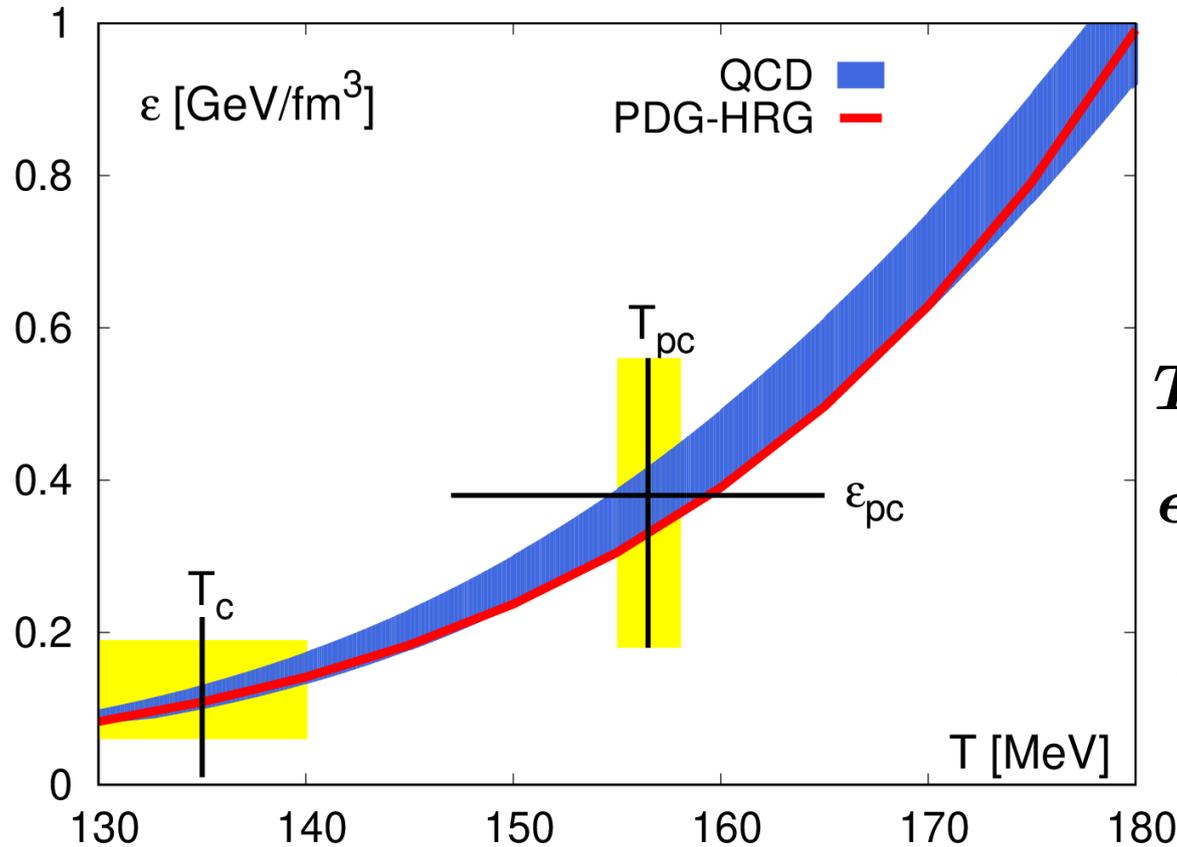
$$\epsilon^{\text{nucl. mat.}} \simeq 150 \text{ MeV/fm}^3$$

$$\epsilon^{\text{nucleon}} \simeq 450 \text{ MeV/fm}^3$$

A. Bazavov et al. (HotQCD),
Phys. Rev. D90 (2014) 094503
and arXiv:1812.08235

Crossover transition parameters – and chiral limit –

PDG: Particle Data Group hadron spectrum



$$\mu_B/T = 0$$

physical quark masses

$$T_{pc} = (156.5 \pm 1.5) \text{ MeV}$$

$$\epsilon_{pc} = (0.42 \pm 0.06) \text{ GeV/fm}^3$$

chiral limit

$$T_c = 132_{-6}^{+3} \text{ MeV}$$

$$\epsilon_c \simeq 0.15(5) \text{ GeV/fm}^3$$

compare with:

$$\epsilon^{\text{nucl. mat.}} \simeq 150 \text{ MeV/fm}^3$$

$$\epsilon^{\text{nucleon}} \simeq 450 \text{ MeV/fm}^3$$

A. Bazavov et al. (HotQCD),
Phys. Rev. D90 (2014) 094503
and arXiv:1812.08235

Transition parameters in the chiral limit

What drives the chiral transition?

- hadron resonance gas in the interval (132-156.5) MeV
- pion mass varies from 0 to its physical values

$T \simeq (130 - 156.5) \text{ MeV} :$

contributions to total energy density and pressure change by a factor 3
but, pion density stays roughly constant

