Partial Deconfinement

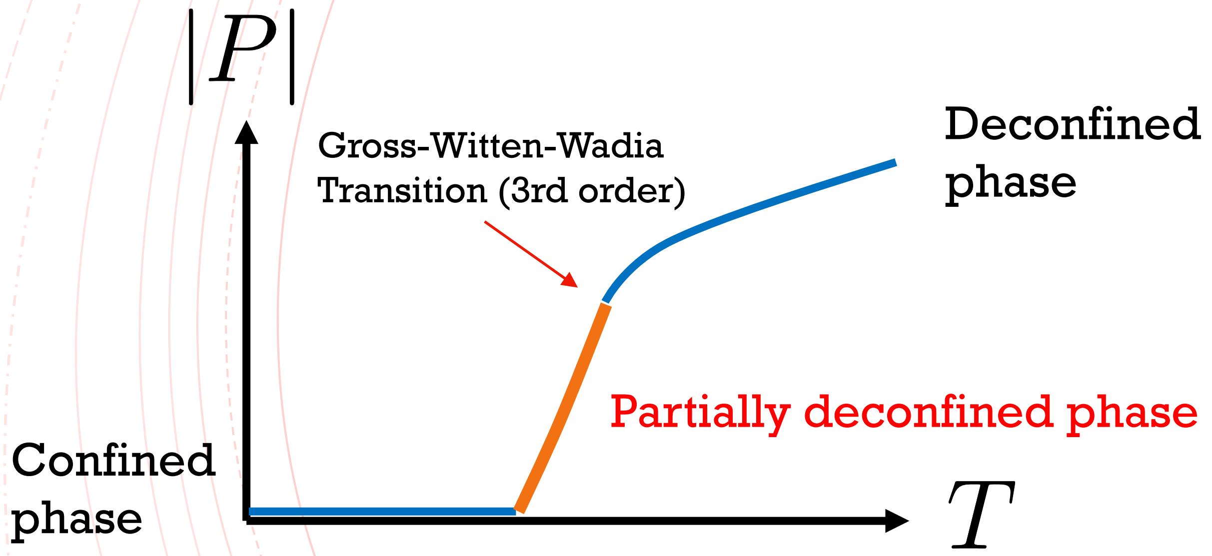
Goro Ishiki (University of Tsukuba)

Based on Hanada-Ishiki-Watanabe,
JHEP 1903 (2019) 145

See also Watanabe's poster
& Enrico Rinaldi's poster

Partial Deconfinement

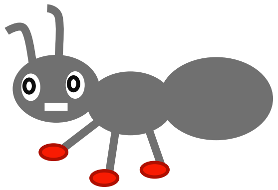
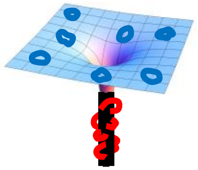
- We mainly consider large-N SU(N) gauge theories with adjoint matters at finite temperature. (with the center symmetry)
- The Polyakov loop is a good order parameter for confine/deconfine transition.
- In some gauge theories, there exists "Partially deconfined phase. (PDP)"



D. O. F for SU(M) (M<N) subgroup are deconfined and the rest are confined

- PDP can be characterized by the Polyakov loop phases.
- PDP shears a common structure with the Gross-Witten-Wadia transition.

Contents of my talk

1. Collective motion of ants 
2. Black holes in string theory 
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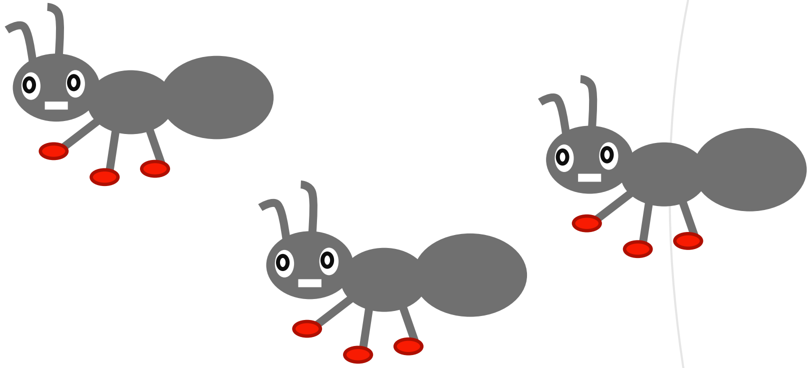
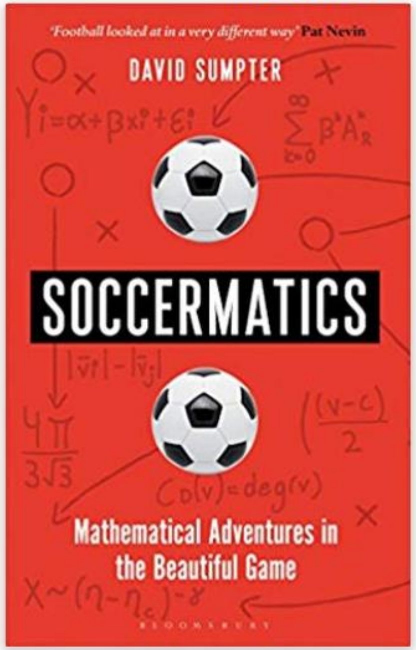


1. Collective motion of ants

◆ One day, Hanada-san was reading a book about strategy of soccer

洋書 › Professional & Technical › Professional Science

なか見!検索



Soccermatics: Mathematical Adventures in the Beautiful Game (Bloomsbury Sigma) (英語) ハードカバー – 2016/5/31

David Sumpter (著)

カスタマーレビューを書きませんか？

› その他（4）の形式およびエディションを表示する

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◆ In the reference list, he found another interesting article about ants

Phase transition between disordered and ordered foraging in Pharaoh's ants

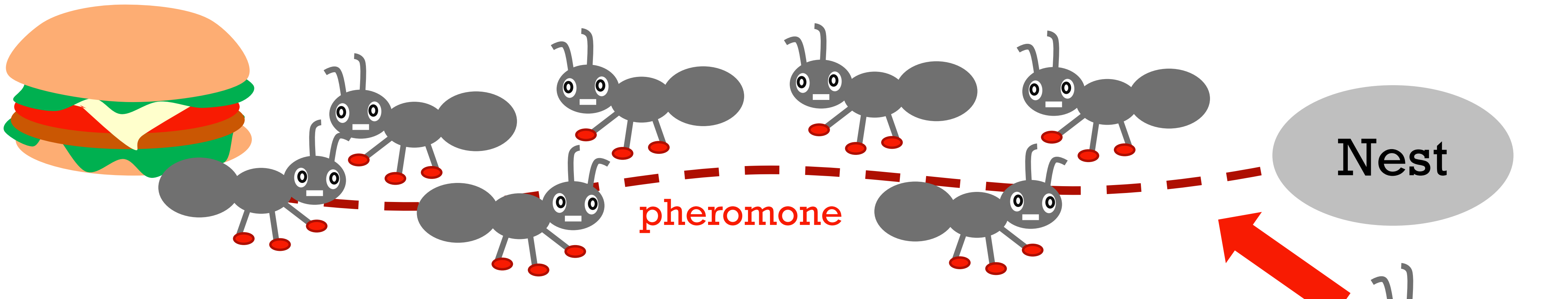
Madeleine Beekman*†, David J. T. Sumpter‡, and Francis L. W. Ratnieks*

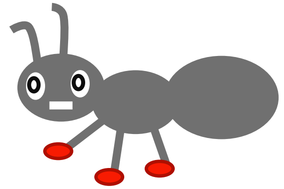
*Laboratory of Apiculture and Social Insects, Department of Animal and Plant Sciences, Sheffield University, Sheffield S10 2TN, United Kingdom; and
‡Centre for Mathematical Biology, Mathematical Institute, Oxford University, 24-29 St. Giles, Oxford OX1 3LB, United Kingdom

Communicated by I. Prigogine, Free University of Brussels, Brussels, Belgium, June 7, 2001 (received for review August 12, 2000)

PNAS Proceedings of the National Academy of Sciences of the United States of America

- When ants find food, they form a trail to the nest.





- Ants on the trail produce pheromone so that other ants can also find the trail.
- In this article, a phenomenological equation describing this behavior is proposed.
- They also made an experiment using real ants and show some agreement with their model.

◆ The more ants on the trail ⇒ The more pheromone ⇒ The more ants joining the trail Positive feedback

◆ A phenomenological equation for the number of ants in a single trail:

$$\frac{dN_{\text{trail}}}{dt} = \text{\#(ants joining the trail) - \#(ants leaving the trail)}$$

$$= (\alpha + pN_{\text{trail}})(N - N_{\text{trail}}) - \frac{sN_{\text{trail}}}{s + N_{\text{trail}}}$$

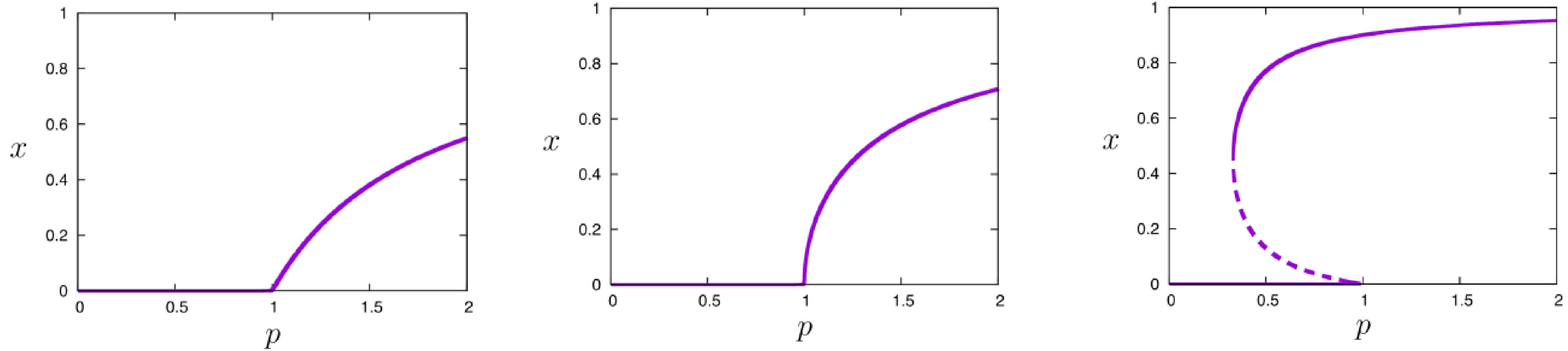
Positive feedback by pheromone

- N : The total number of ants
- N_{trail} : The Number of ants on the trail
- α : A parameter for the positive feedback term
- p : A parameter of the strength of pheromone
- s : A parameter of the leaving rate (larger s ⇒ more ants leaving the trail)

◆ The "thermodynamic limit" of the ant equation: $\alpha \sim N^0, p \sim N^0, s \sim N^1 \quad \tilde{s} = s/N$

◆ Solving the equilibrium condition $\frac{dN_{\text{trail}}}{dt} = 0$, we can find the solutions: $\left(x = \frac{N_{\text{trail}}}{N} \right)$

$\tilde{s} = 5$ (Many ants leave the trail) | $\tilde{s} = 1$ | $\tilde{s} = 0.1$ (Less ants leave the trail)

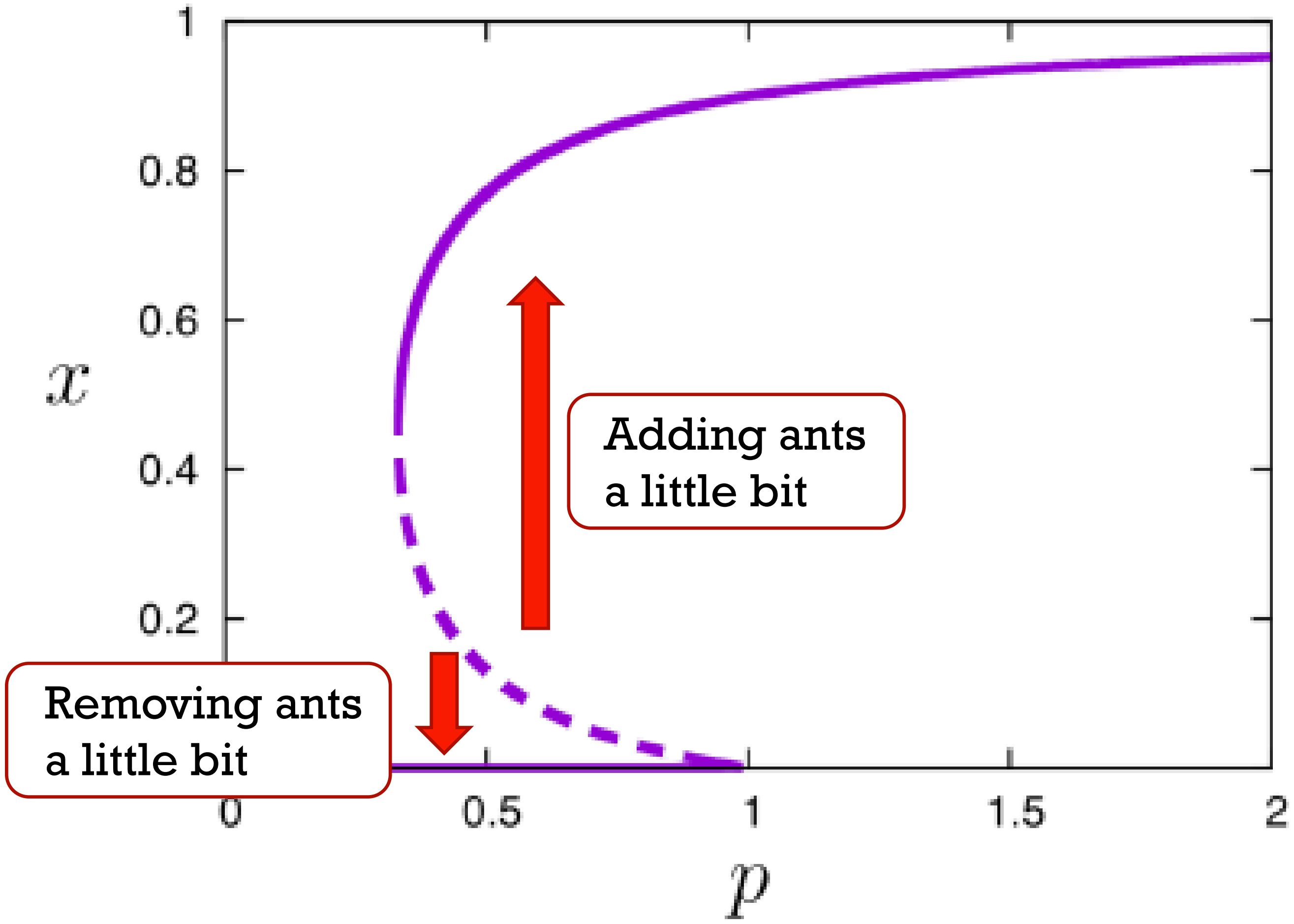


◆ When the pheromone become stronger, there is a sudden phase transition

◆ There are three possible patterns depending on a value of the parameter $\tilde{s}$

$$\tilde{s} = 0.1$$ (Less ants leave the trail)

- In this case, we have "a first order phase transition".

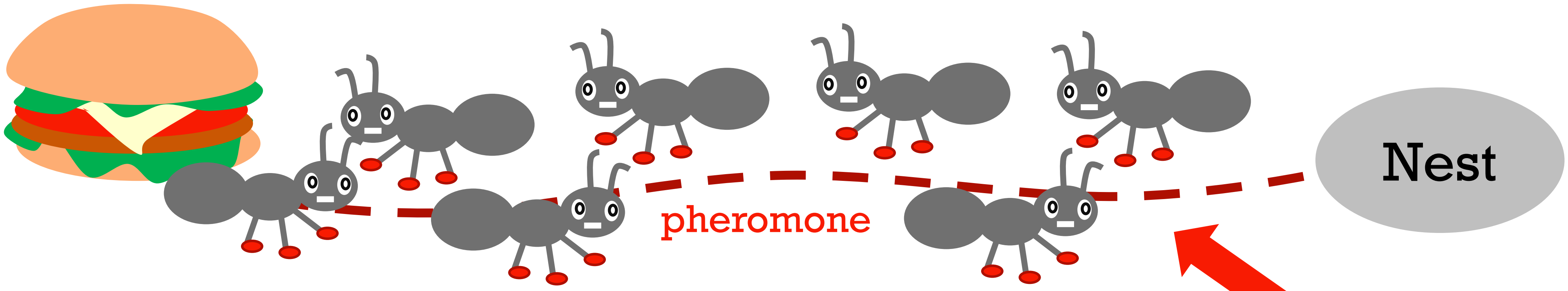


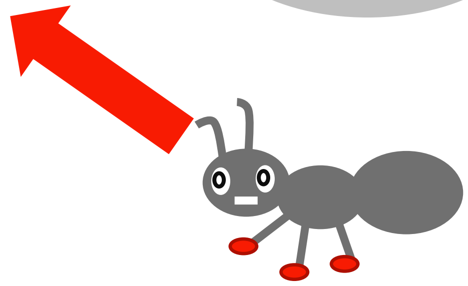
- The dotted line is an unstable solution.
  Under perturbations, it flows to the solid lines.

- On the dotted line, by adding some ants to the trail by hand, more and more ants will join the trail. Then, the system goes to the upper solid line

- Similarly, by removing some ants from the trail, the system goes to the lower solid line

Existence of three saddles at intermediate p comes from a subtle balance between the attractive/repulsive forces of ants

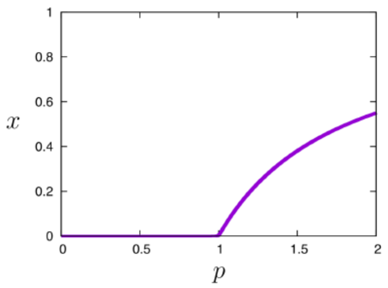




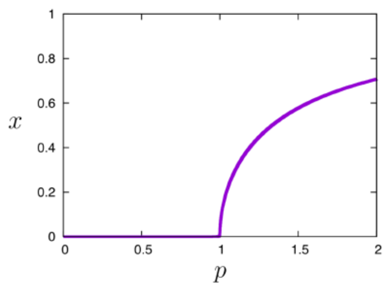
Summary so far

- The ant equation: a many-body system with a positive feedback
- There are three kinds of behaviors

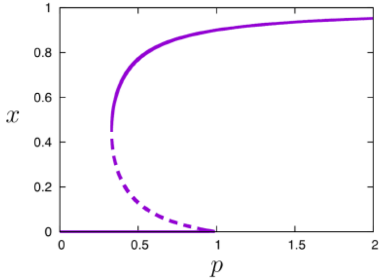
$\tilde{s} = 5$



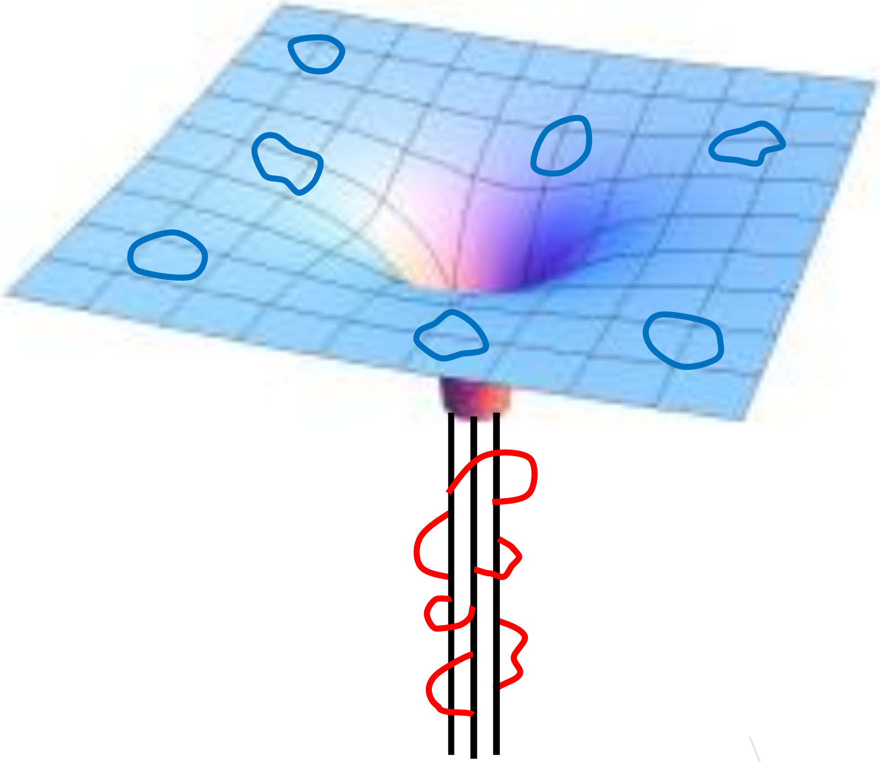
$\tilde{s} = 1$



$\tilde{s} = 0.1$



- In particular, there is "a first order transition" when the positive feedback is very strong



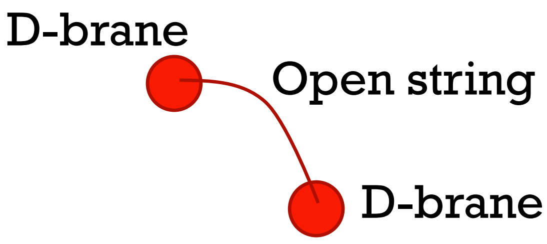
2. Black holes in string theory

- Reading the paper of ants, Hanada-san got excited.
  "Oh, this is very similar to systems with D-branes in string theory!"

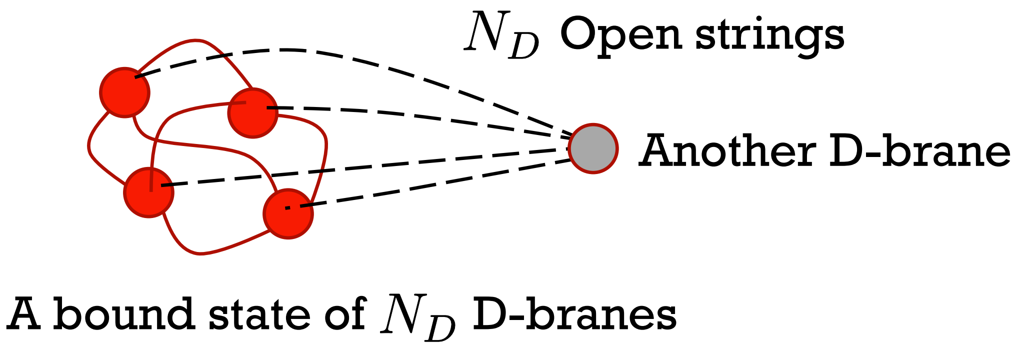
- D-branes are fundamental objects in string theory. [Polchinski]

  - D0-branes ⇔ particles
  - D1-branes ⇔ strings
  - D2-branes ⇔ membranes
  - D3-branes ⇔ 1+3 dim objects, and so on

- D-branes interact with each other through open strings

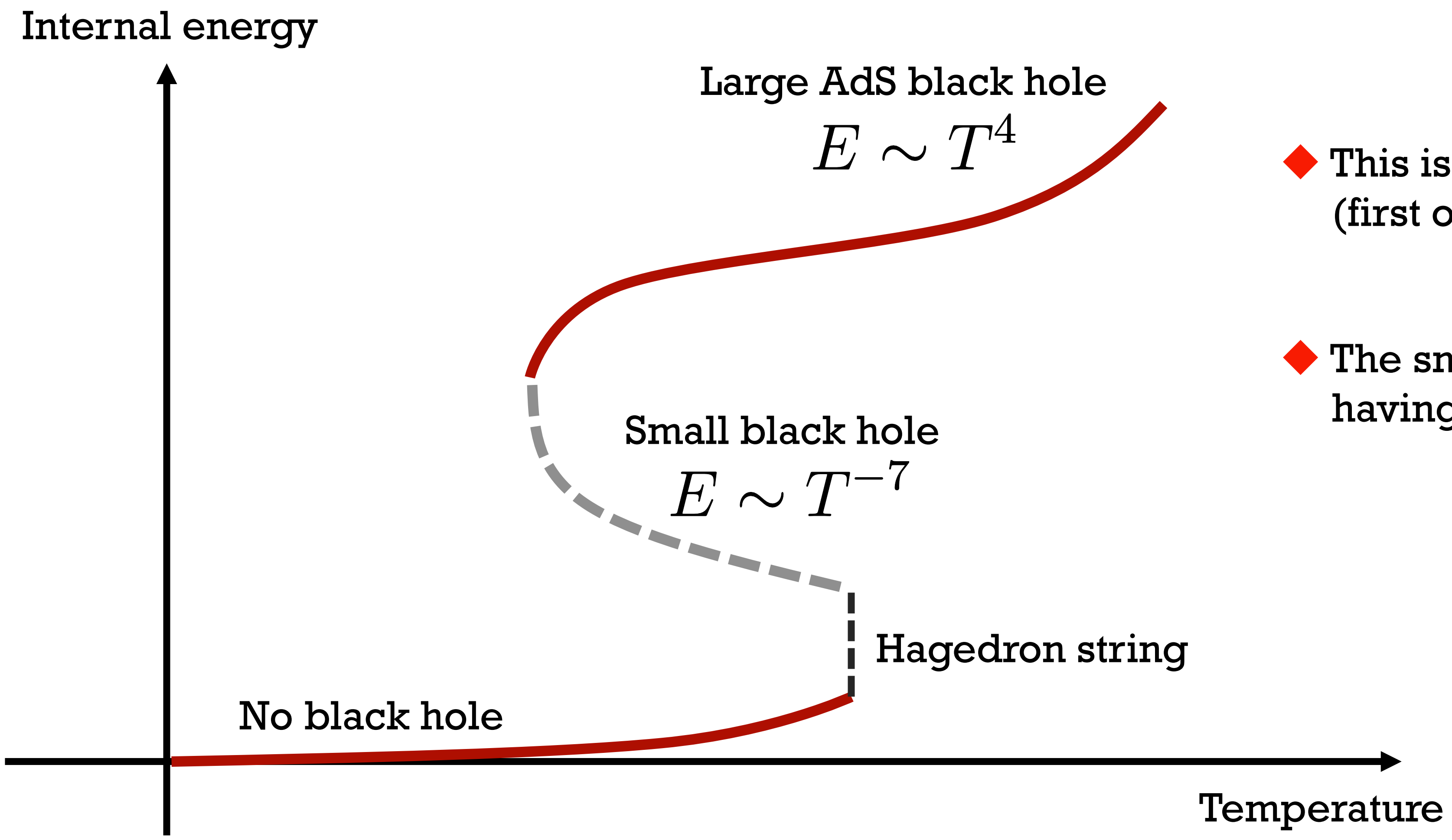


- D-branes have an interaction with a positive feedback



The larger N_D is , the more strongly the bound state attracts the other D-brane, because there are N_D open strings in between.

- When the energy of D-branes is very high, they form a black hole, because string theory contains gravity.
- The following diagram is known for the type IIB superstring theory on AdS5xS5 at finite temperature



- This is the Hawking-Page transition (first order)
- The small black hole is unstable having a negative specific heat

Very similar to ants!

- Positive feedback
- First order transition
- Unstable saddle

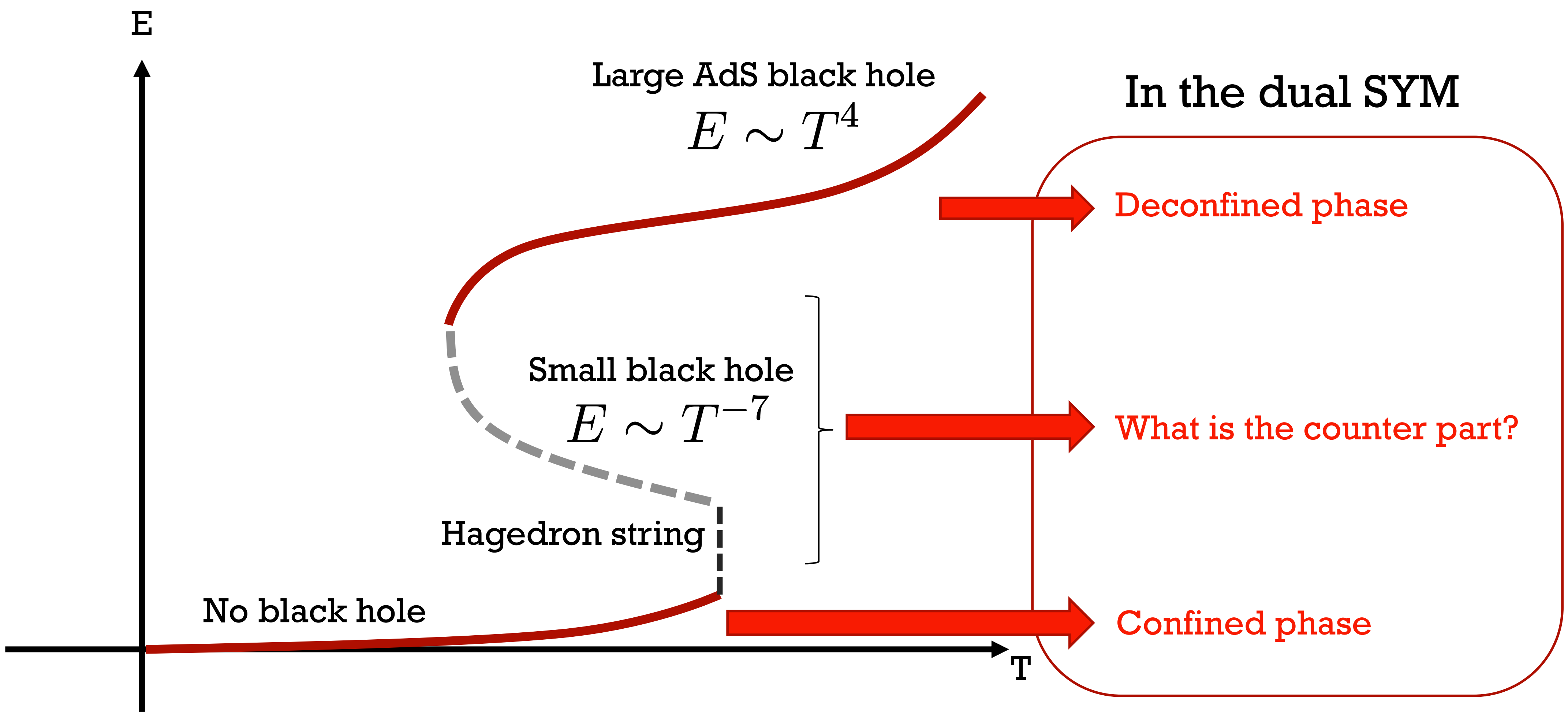
- The gauge/gravity correspondence [Maldacena]

- Then, the similarity between string theory and ants will extend to gauge theories

"Gauge theories should behave like ants!"

- In SU(N) gauge theories with adjoint matters, fields can be represented as $N \times N$ matrices.
  Each matrix element couples to all the other elements, and this structure is the same as the D-branes.
  Thus, the gauge theories will also have the positive feedback structure.

◆ The Hawking-Page transition ⇔ the confine/deconfine transition [Witten]



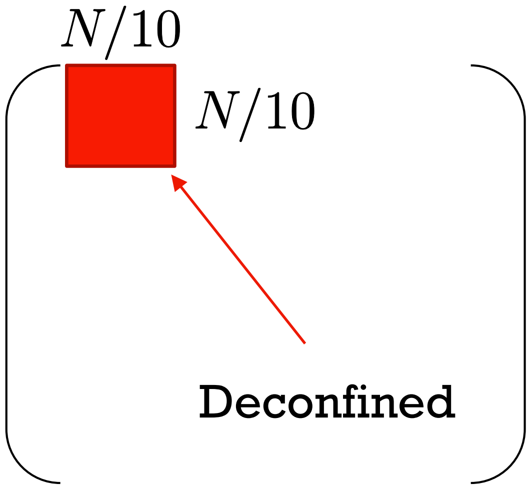
◆ The unstable saddle is in between the confined and deconfined phases.
Then, it is natural to conjecture that this saddle is "a partially deconfined state" in the gauge theory. [Hanada-Maltz, Berenstein]

◆ Partial deconfinement in large-N gauge theories?

$$E \sim \#(\text{DOF}) \sim \begin{cases} N^2 & \text{for deconfined phase} \\ N^0 & \text{for confined phase} \end{cases}$$

What about the intermediate energy region such as $E \sim N^2/100$?

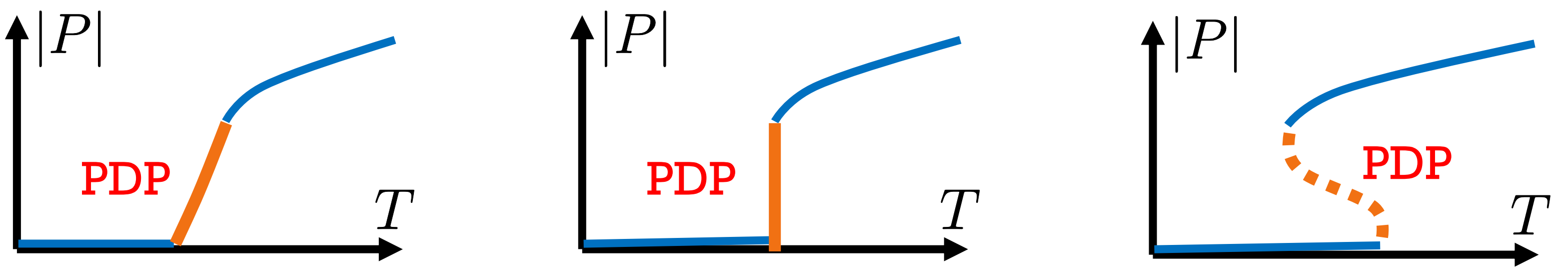
This case will correspond to states with $\#(\text{DOF}) \sim N^2/100$



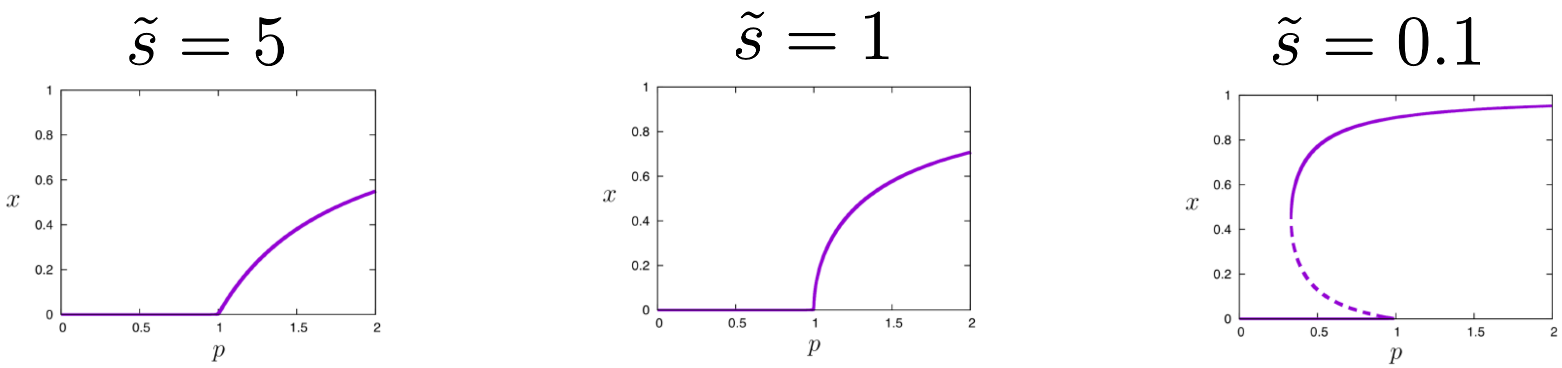
There will be the partial deconfinement in the intermediate energy region.

- In the next (last) section, I will introduce the partially deconfined phase (PDP) in gauge theories.

- I will show that (1) PDP indeed exists for some gauge theories
  (2) PDP can be either stable or unstable depending on a theory (or a parameter)



- This is in a nice analogy with the ant model:



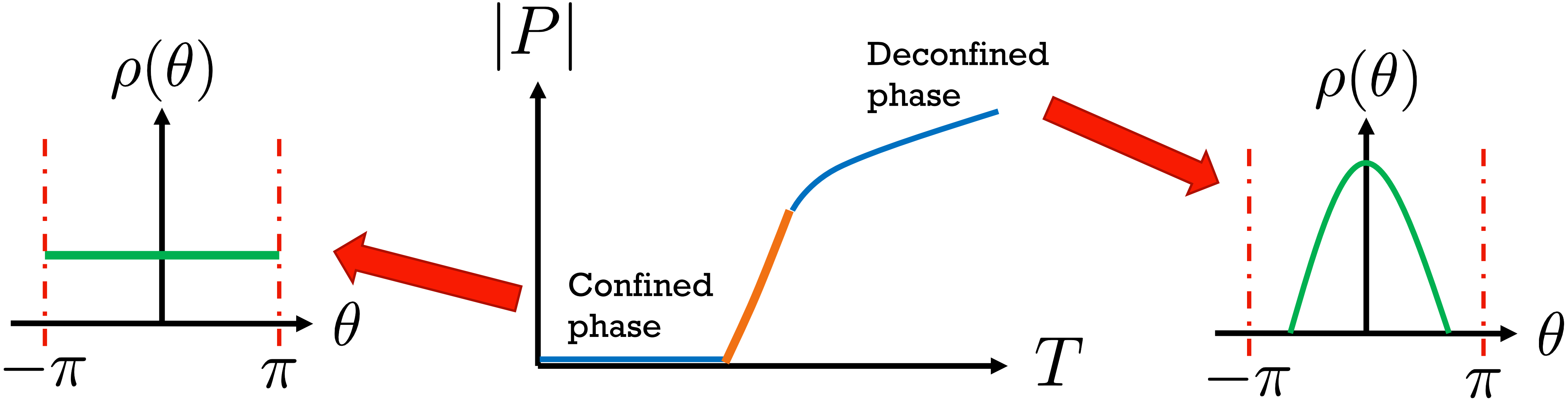
3. Partial deconfinement

Confine/deconfine transition

- We mainly consider large-N SU(N) gauge theories with adjoint matters at finite temperature. (center sym)
- Polyakov loop

$$P = \frac{1}{N}\mathrm{Tr}\mathcal{P}e^{i\int A_0} = \frac{1}{N}\sum_{i=1}^{N} e^{i\theta_i} = \int_0^{2\pi} d\theta e^{i\theta}\rho(\theta) \qquad \left(\rho(\theta) = \frac{1}{N}\sum_{i=1}^{N}\delta(\theta-\theta_i) \ \text{phase distribution}\right)$$

- Polyakov loop is a good order parameter for the confine/deconfine transition

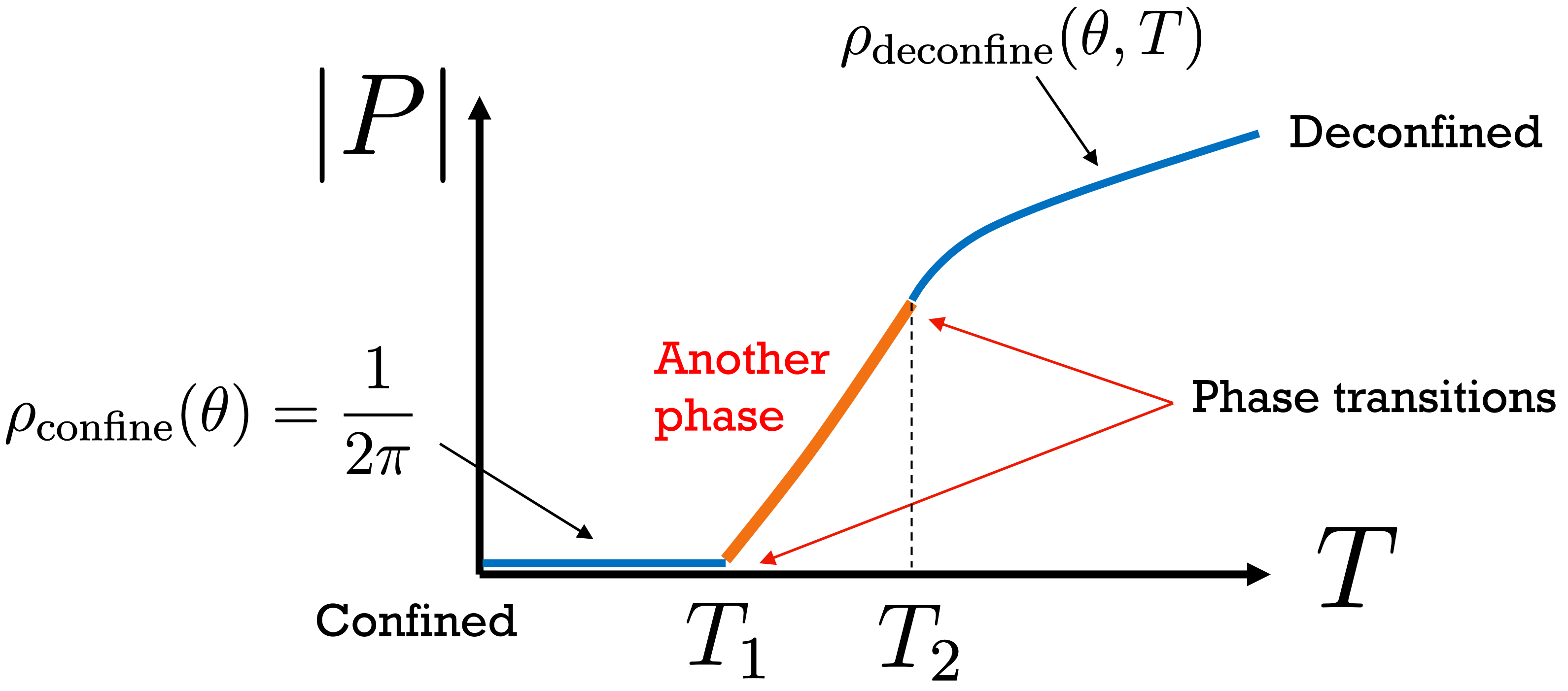


Flat in confined phase

Nonuniform in deconfined phase

◆ Partially deconfined phase (PDP)

Suppose that there exists another phase in between the confined and deconfined phases.



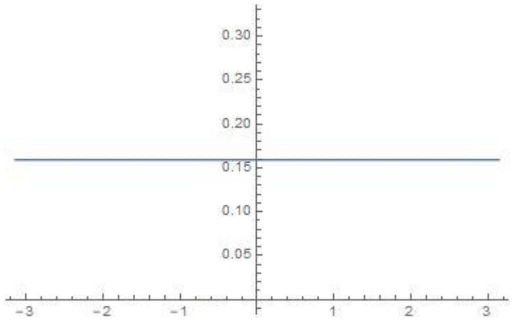
It is natural to call it PDP, if the phase distribution takes the following form

$$\rho(\theta, T) = \frac{N-M}{N}\rho_{\rm confine}(\theta) + \frac{M}{N}\rho_{\rm deconfine}(\theta, T_2) \qquad (0 < M < N)$$

i.e. D. O. F for SU(M) subgroup are already deconfined but the rest is still confined

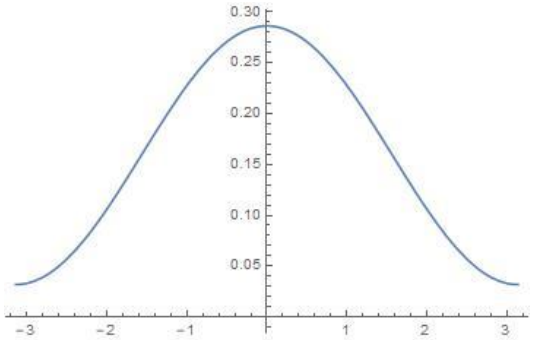
◆ Phase distributions in various phases

Confined phase



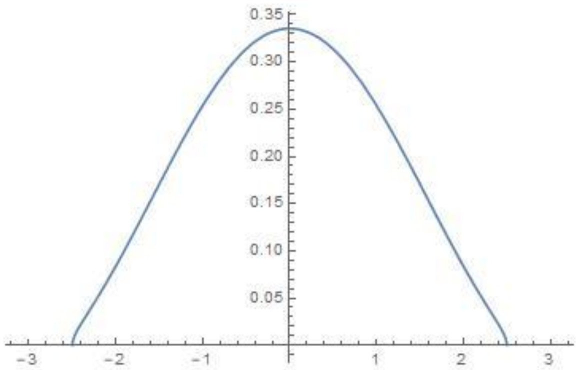
Uniform

PDP



Non-uniform
Gapless

Deconfined phase



Non-uniform
Gapped

$$\rho(\theta, T) = \frac{N-M}{N}\rho_{\text{confine}}(\theta) + \frac{M}{N}\rho_{\text{deconfine}}(\theta, T_2)$$

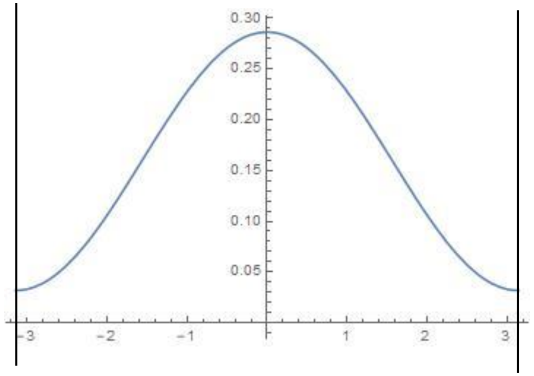
◆ Relation to the Gross-Witten-Wadia (GWW) transition

GWW transition : a third order phase transition in 2D lattice pure YM.

In all the examples we considered, the phase transition from PDP to the deconfined phase has the same structure as the GWW transition

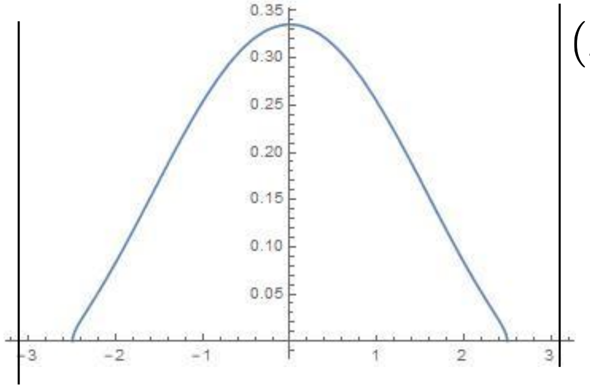
$$\rho_{\mathrm{PDP}}(\theta) = \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right)$$

$$\rho_{\mathrm{deconfine}}(\theta) = \frac{2}{\pi\kappa}\cos\frac{\theta}{2}\sqrt{\frac{\kappa}{2} - \sin^2\frac{\theta}{2}}$$



$(\kappa = 2.5)$

PDP



$(\kappa = 1.8)$

Deconfined phase

These functional form and the order of transition coincide with those of GWW.

It is suggested that GWW's structure is a universal feature of PDP.

◆ Example 1: N=4 SYM on $S^1 \times S^3$ [Sundborg, Aharony et. al.]

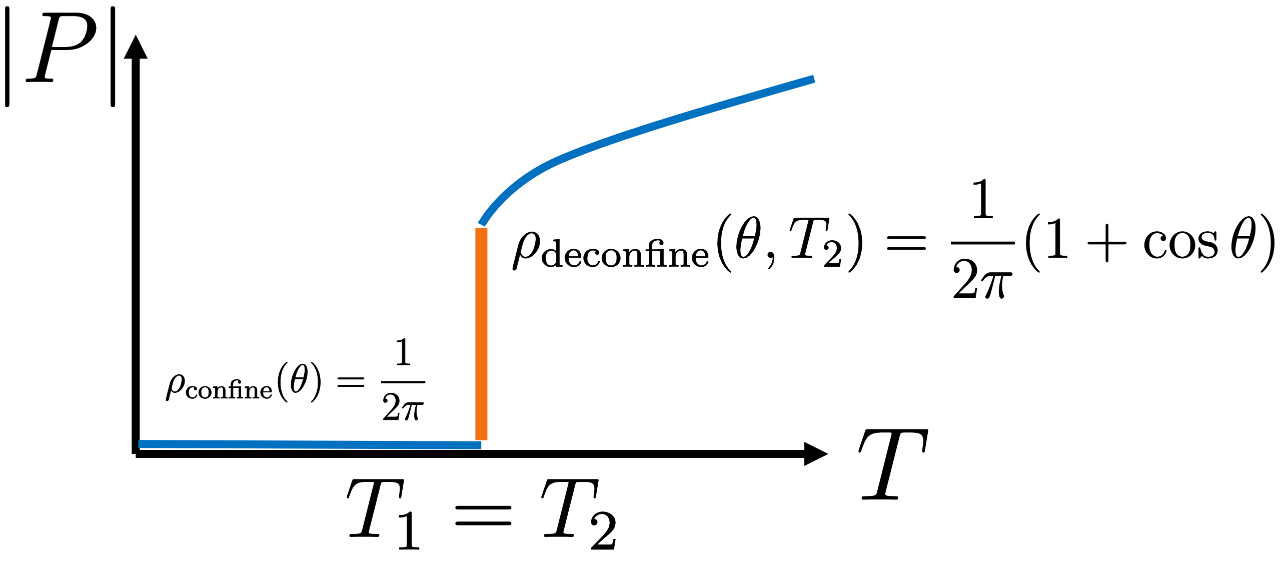
<u>Weak coupling limit (1-loop approx)</u>

On the vertical orange line, we have

$$\rho(\theta) = \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right)$$

where κ parametrizes the orange line and

$$\begin{cases} \kappa = 2 & \text{at the upper edge} \\ \kappa = \infty & \text{at the lower edge} \end{cases}$$



This satisfy the condition for PDP:

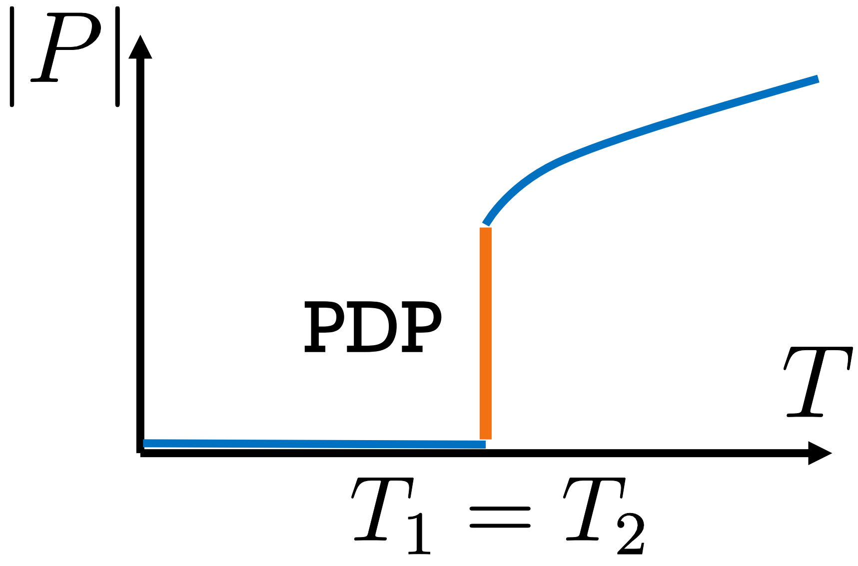
$$\rho(\theta) = \left(1 - \frac{2}{\kappa}\right)\frac{1}{2\pi} + \frac{2}{\kappa}\frac{1}{2\pi}(1 + \cos\theta)$$

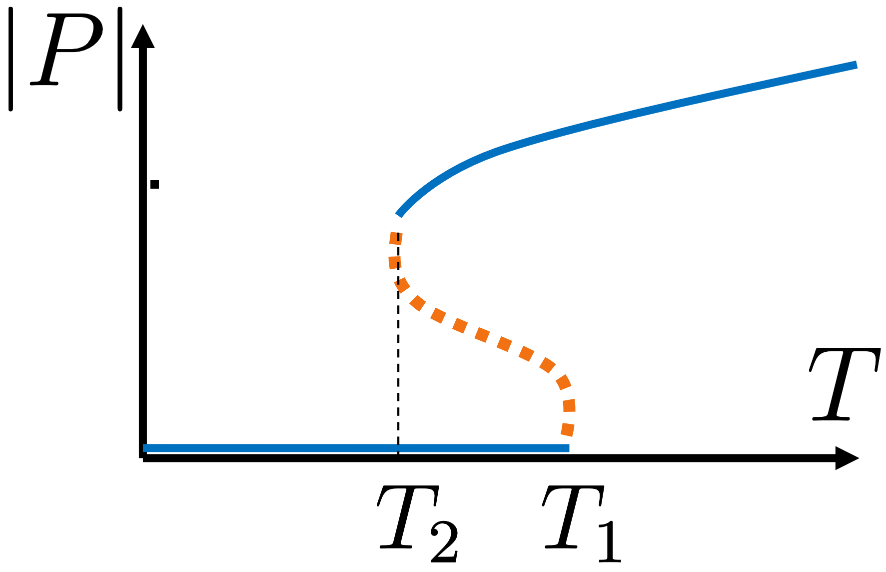
$$= \left(1 - \frac{2}{\kappa}\right)\rho_{\text{confine}}(\theta) + \frac{2}{\kappa}\rho_{\text{deconfine}}(\theta, T_2)$$

Identification

$$\frac{2}{\kappa} = \frac{M}{N}$$

◆ From weak to strong coupling



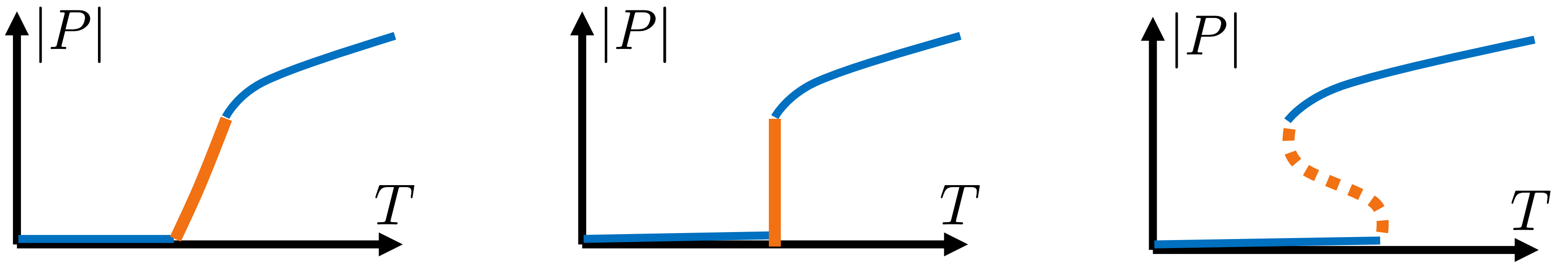


In N=4 SYM at finite coupling, PDP seems to be always unstable.
(⇔ small BH is always unstable in string theory)

◆ Example 2: Other gauge theories on $S^1 \times S^3$ with adjoint matters

Weak coupling analysis [Aharony et. al.]

Depending on matter contents, there are three patterns



For the left and center cases, the condition for PDP is satisfied

$$\rho(\theta, T) = \frac{N-M}{N}\rho_{\text{confine}}(\theta) + \frac{M}{N}\rho_{\text{deconfine}}(\theta, T_2)$$

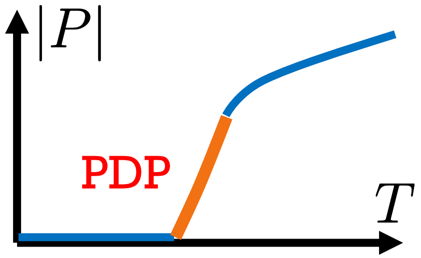
◆ Example 3 : Matrix quantum mechanics

$$S = N\int dt \mathrm{Tr}\left(\frac{1}{2}(DX_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{2}X_i^2 - \frac{\mu^2}{8}X_a^2 - i\mu\epsilon^{ijk}X_iX_jX_k\right)$$

$$\begin{cases} X_I(t) : \ N\times N \ \text{Hermitian matrices} & (I,J=1,2,\cdots,9,\ i,j,k=1,2,3,\ a=4,5,\cdots,9) \\ DX_I = \partial_t X_I - i[A_0, X_I] \end{cases}$$

◆ This model has the following phase structure:

$\mu = 0$

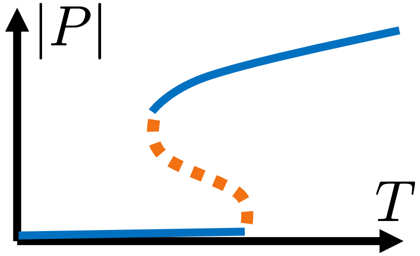


[Kawahara-Nishimura-Takeuchi, 2007]

PDP condition satisfied

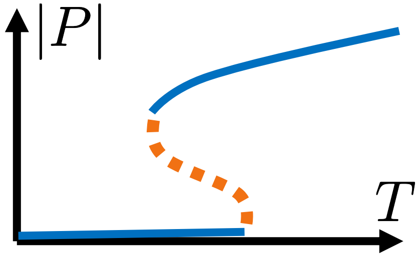
Modified
More data
Larger N

$\mu = 0$



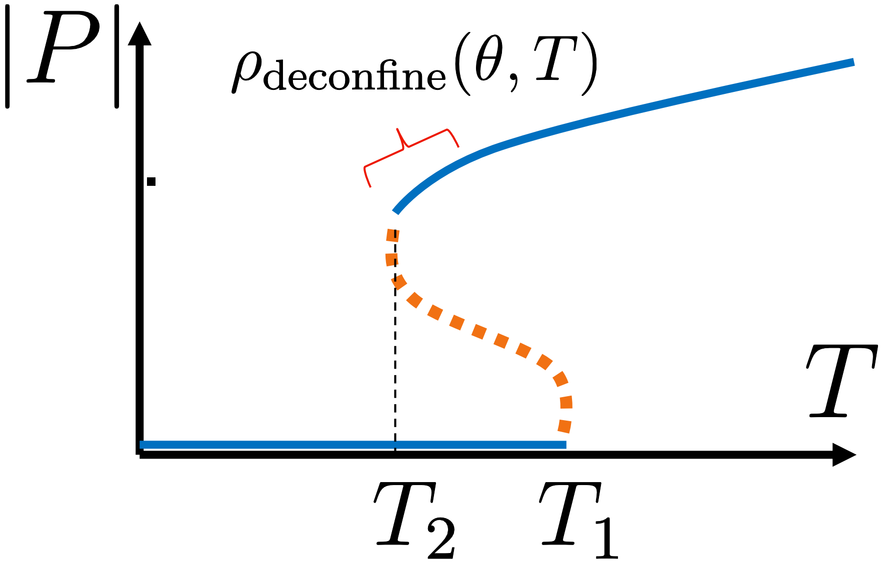
[see Enrico's poster]

large μ

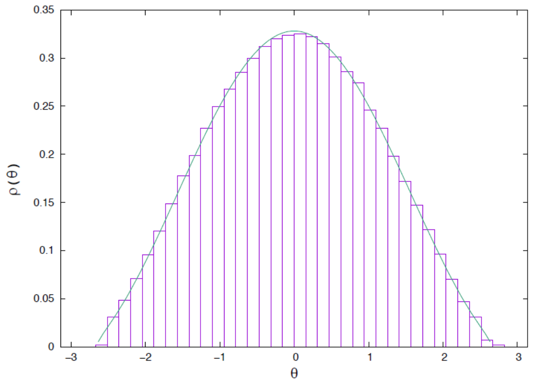


We studied this case numerically

◆ We numerically computed $\rho_{\text{deconfine}}(\theta, T)$ near $T = T_2$



◆ We find that $\rho_{\text{deconfine}}(\theta, T)$ can be fitted well by the GWW form.



$$\mu = 5, N = 128, L = 16, T = 1.54$$

Fitting function (GWW form)

$$\rho(\theta) = \frac{2}{\pi\kappa}\cos\frac{\theta}{2}\sqrt{\frac{\kappa}{2} - \sin^2\frac{\theta}{2}} \quad \text{with } \kappa = 1.88$$

Suggesting the universality of GWW and existence of unstable PDP saddle

Conclusion

Ants behave like gauge theories and vise versa

- We found a qualitative similarity between ants/strings/gauge theories. Positive feedbacks, phase transitions, emergence of unstable saddles etc.
- We characterized PDP for gauge theories in terms of Polyakov loop phases
- PDP can be either stable/unstable depending on a theory or a parameter
- We found some examples where PDP indeed exists. They all have GWW's structure.

Outlook

- Partial deconfinement may be a generic feature for theories with positive feedback
- How universal is PDP? It should also be studied for finite N, fundamental matters, and so on
- It would be interesting if PDP exists for QCD (even in an approximate sense)
  cf. QCD at finite density: Cross over at $\mu = 0$ and 1^{st} order at $\mu \neq 0$
- We would like to study the correspondence between PDP and small Schwarzschild black holes.