Correlation functions with gradient flow from lattice QCD

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CRC-TR 211 Strong-interaction matter under extreme conditions

Introduction

The determination of spectral and transport properties from lattice QCD require the calculation of certain Euclidean-time correlation functions and a spectral reconstruction as the correlators are related to the spectral function through an integral equation,

$$G(\tau) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\rho(\omega)K(\omega,\tau)\,, \qquad K(\omega,\tau) = \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)}\,.$$

For the spectral reconstruction a fine temporal resolution of the correlator and small statistical errors are required. Making contact to perturbation theory in the UV and to use perturbative prior information for this reconstruction requires to continuum extrapolate the correlators.

The goal of this study, still in the quenched approximation, is to understand the effect of the gradient flow, as a noise reduction technique that can also be applied in full QCD studies, on correlation functions and to learn how the continuum limit needs to be performed.

Color-electric field correlator

Heavy Quark Effective Theory (HQET) in the large quark mass limit for a single quark in medium leads to a (pure gluonic) “color-electric correlator” [1,2],

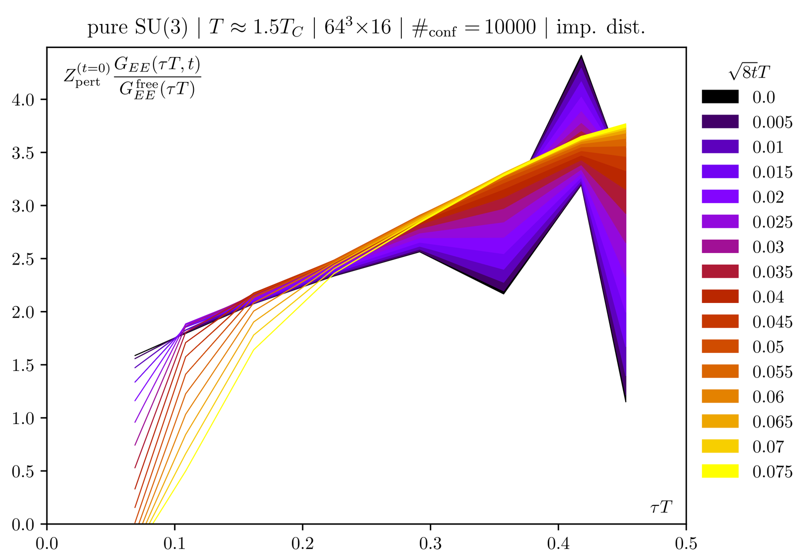
$$G_{\rm E}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3}\frac{\left\langle \text{Re Tr}\left[U(\beta;\tau)\,gE_i(\tau,\mathbf{0})\,U(\tau;0)\,gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \text{Re Tr}\left[U(\beta;0)\right]\right\rangle}\,, \beta \equiv \frac{1}{T}$$

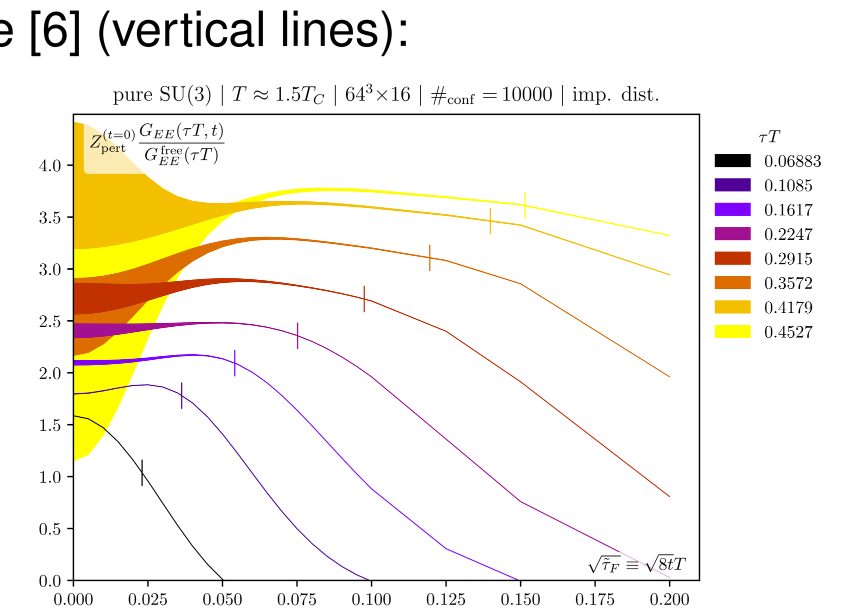
which is related to the heavy quark momentum diffusion coefficient κ through a Kubo formula,

$$\kappa \equiv \lim_{\omega\to 0}\frac{2T\rho_{\rm E}(\omega)}{\omega}\,,$$

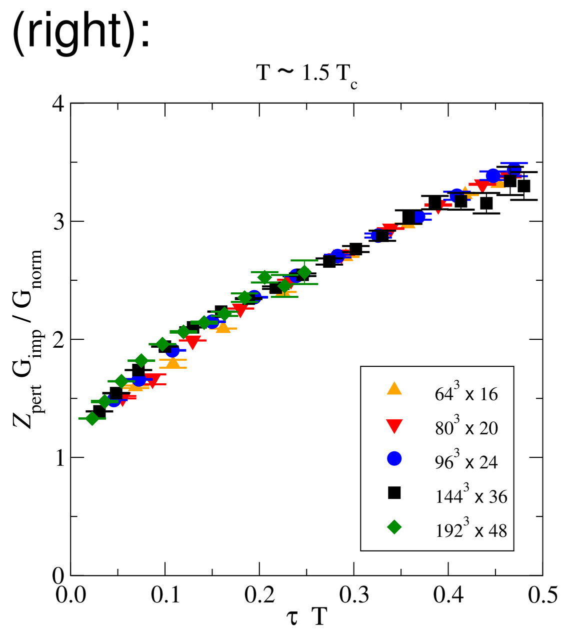
also relates to the thermal quarkonium mass shift [3]. To obtain a good signal for the correlation function, in a previous study in the quenched approximation we have used mulit-level and link-integration noise reduction techniques to obtain reliable results that were then extrapolated to the continuum [4]. The noise reduction techniques used in the quenched approximation are not applicable in full QCD calculations. The gradient flow method [5] is also known to reduce UV fluctuations and is a noise-reduction technique that can also be applied to full QCD. This method smoothens the gauge fields which leads to a reduction of UV fluctuations but also changes the short distance part of the correlators.

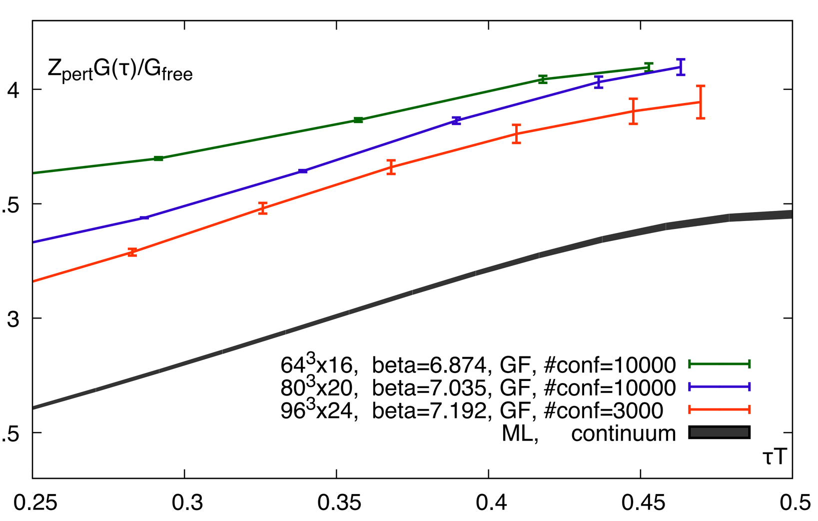
The flow time dependence of the correlators together with a perturbative estimate of the maximal allowed flow time [6] (vertical lines):





Comparison of the results of [3] using multi-level (left and black line in the right figure) to the correlators obtained after a gradient flow of $\sqrt{8t}\simeq 0.075N_\tau$ (right):





The next step in this project is to perform the continuum and small flow time extrapolation to find agreement in both methods.

Correlators of topological charge density

The spectral function determined from correlation functions of the topological charge density is related to the sphaleron rate,

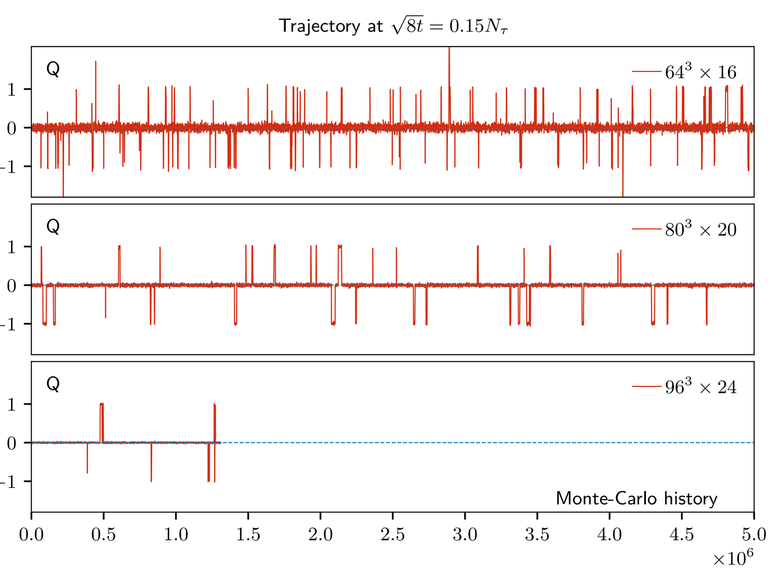
$$\Gamma_{\text{sphal}} = -\lim_{\omega\to 0}\frac{2T\rho_q(\omega,0)}{\omega}\,,$$

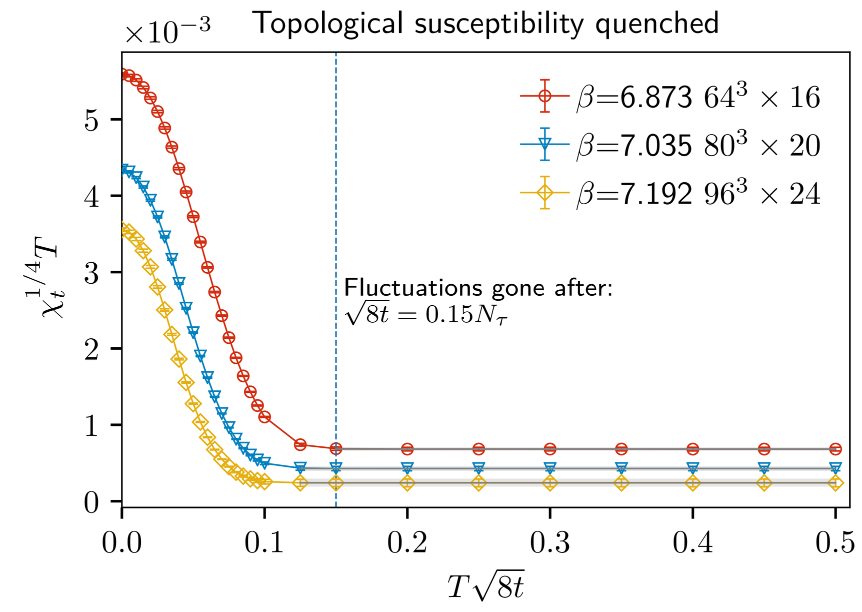
which defines the diffusion constant for topological number and is related to the rate of equilibration of axial light quark number in QCD. The topological charge density can be calculated on the lattice using the gluonic definition,

$$q(x) = \frac{g^2}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\text{tr}\left\{F_{\mu\nu}(x)F_{\rho\sigma}(x)\right\}\,,$$

once the gauge fields are sufficiently smooth, e.g. after smearing the gauge fields using the gradient flow method. This provides a proper gluonic definition of the topological charge and topological susceptibility,

$$Q = \int d^4x\, q(x) \quad , \quad \chi_Q = \lim_{V\to\infty}\frac{\langle Q^2\rangle}{V} = \int d^4x\,\langle q(x)q(0)\rangle.$$

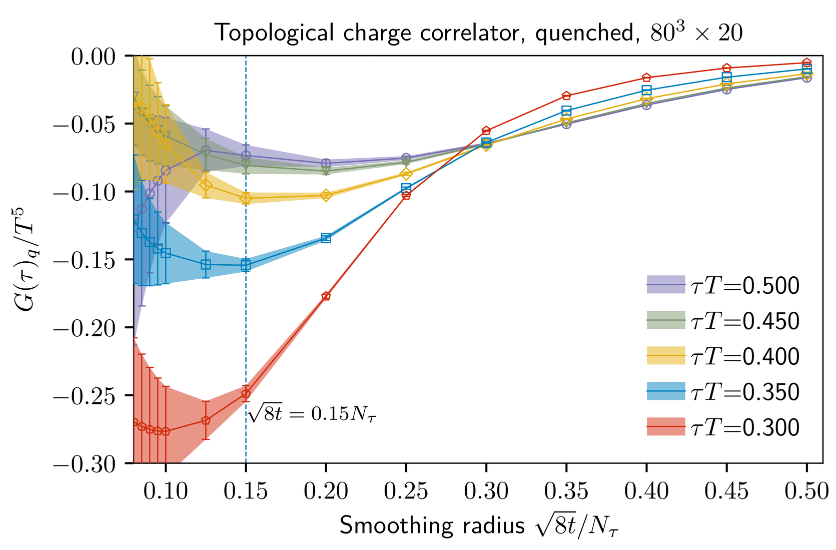


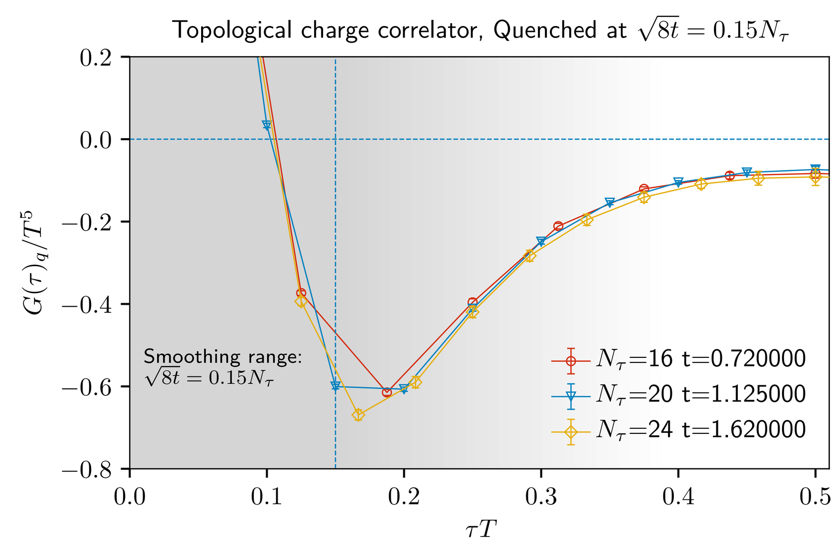


In addition to providing a proper definition of the topological charge, the gradient flow also reduces UV fluctuations in correlation functions of the topological charge density, i.e. is used as a noise reduction technique for the correlator of topological charge density,

$$G_q(\tau,\mathbf{p}) = \int d^3x\, e^{i\mathbf{px}}\langle q(\mathbf{x},\tau)q(\mathbf{0},0)\rangle.$$

The flow time dependence of the correlator at different distances, shown in the left plot, shows an effective reduction of the errors with increasing t. At least for the larger distances it reaches a plateau after a flow of around $\sqrt{8t}\simeq 0.15N_\tau$, before the signal will be destroyed by too much flow. In the right plot we show the result at a fixed flow time for three lattices spacings. The shaded gray area is the region which is affected by flow, while the largest distance part is not influenced by the gradient flow:





The results show only small cut-off effects and using an additional even finer lattice will allow for a controlled continuum extrapolation at fixed physical flow time and a subsequent small flow time limit in the future, providing continuum correlation functions required for a spectral reconstruction in the future. Both studies are currently being extended to full QCD calculations.

References

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