Correlations and probability distributions in high-energy nuclear collisions

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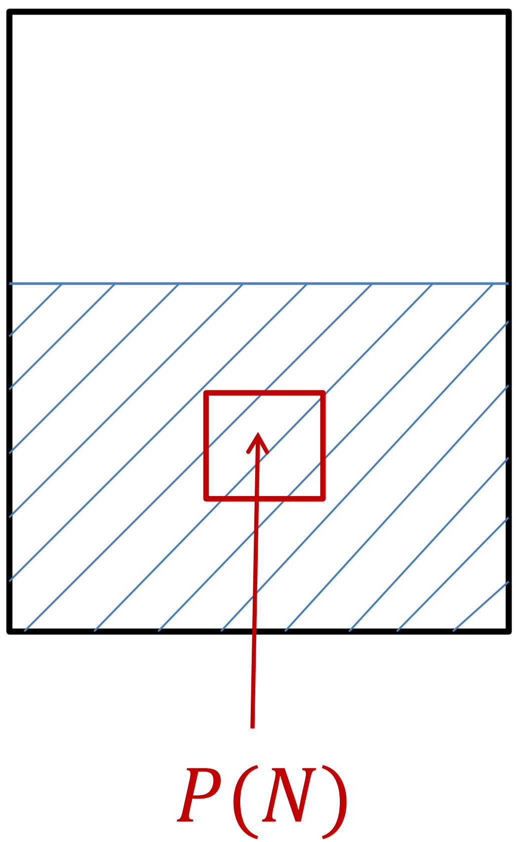
AB, V.Koch, N.Strodthoff, PRC 95 (2017) 054906
AB, V.Koch, V.Skokov, EPJC 77 (2017) 288
AB, V.Koch, D.Oliinychenko, J.Steinheimer, PRC 98 (2018) 054901
AB, V.Koch, 1811.04456

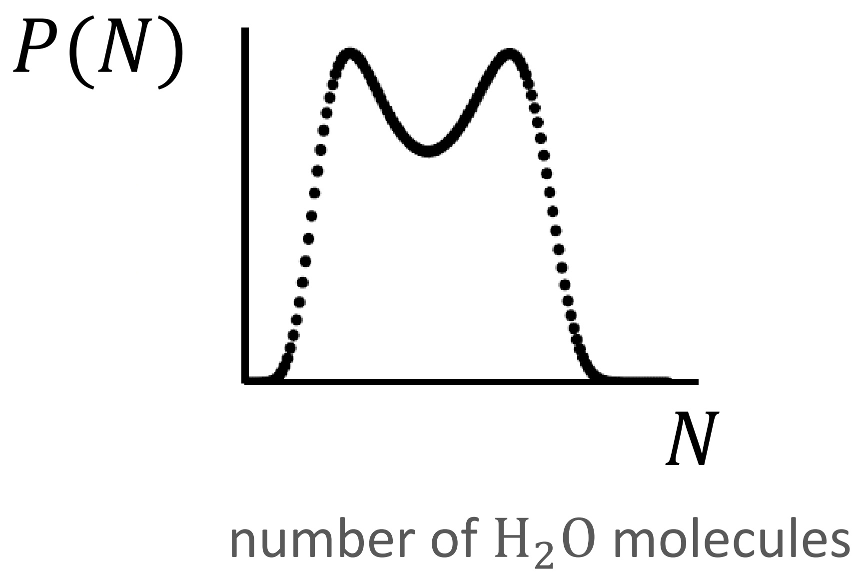
Outline

- introduction
- factorial cumulants, cumulants
- STAR data
- long-range correlations
- bimodal distribution
- statistics friendly distributions
- summary

Consider water vapour transition

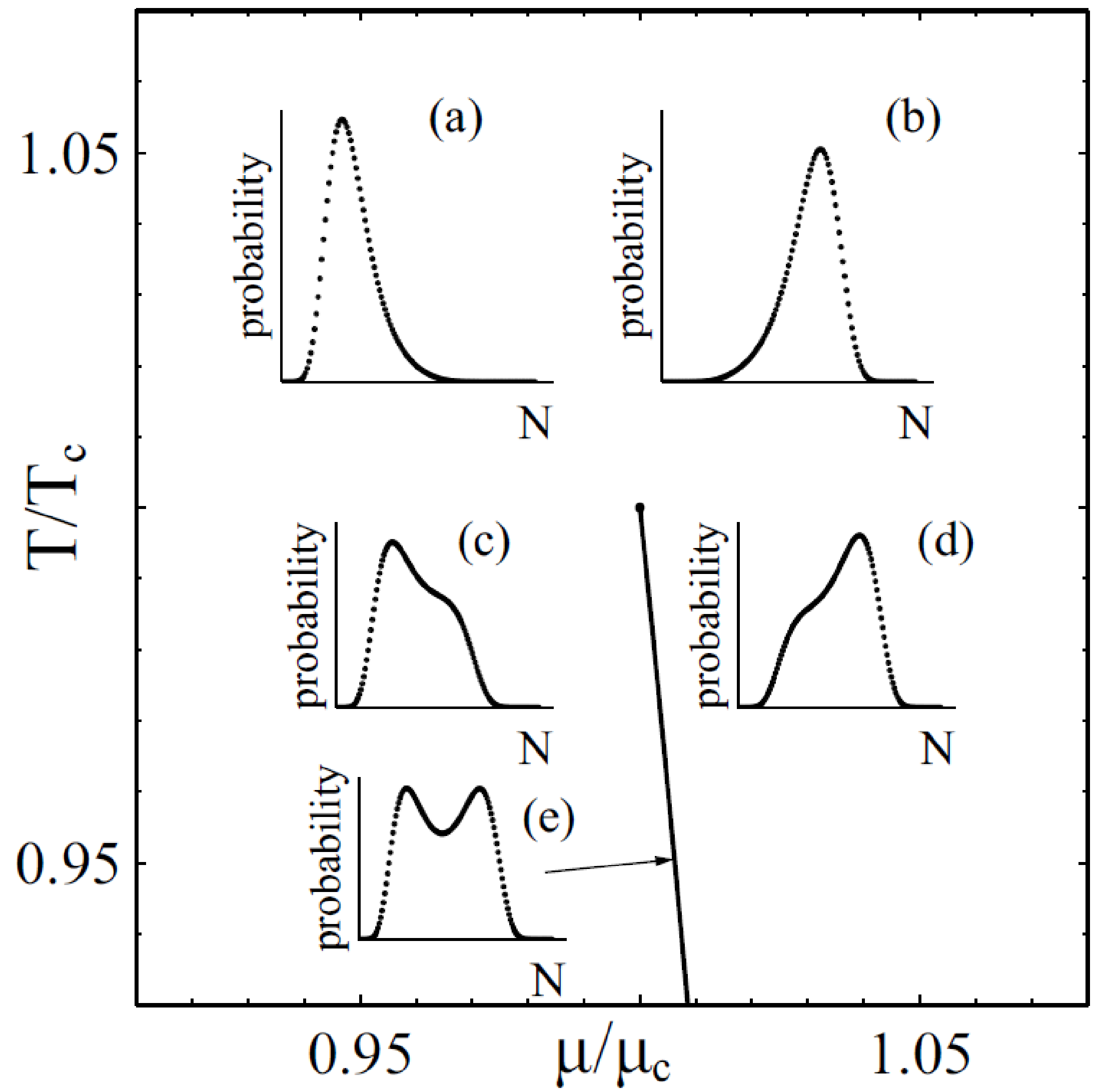


right at the phase transition



In QCD we use, e.g., baryon density.

A finite volume van der Waals model



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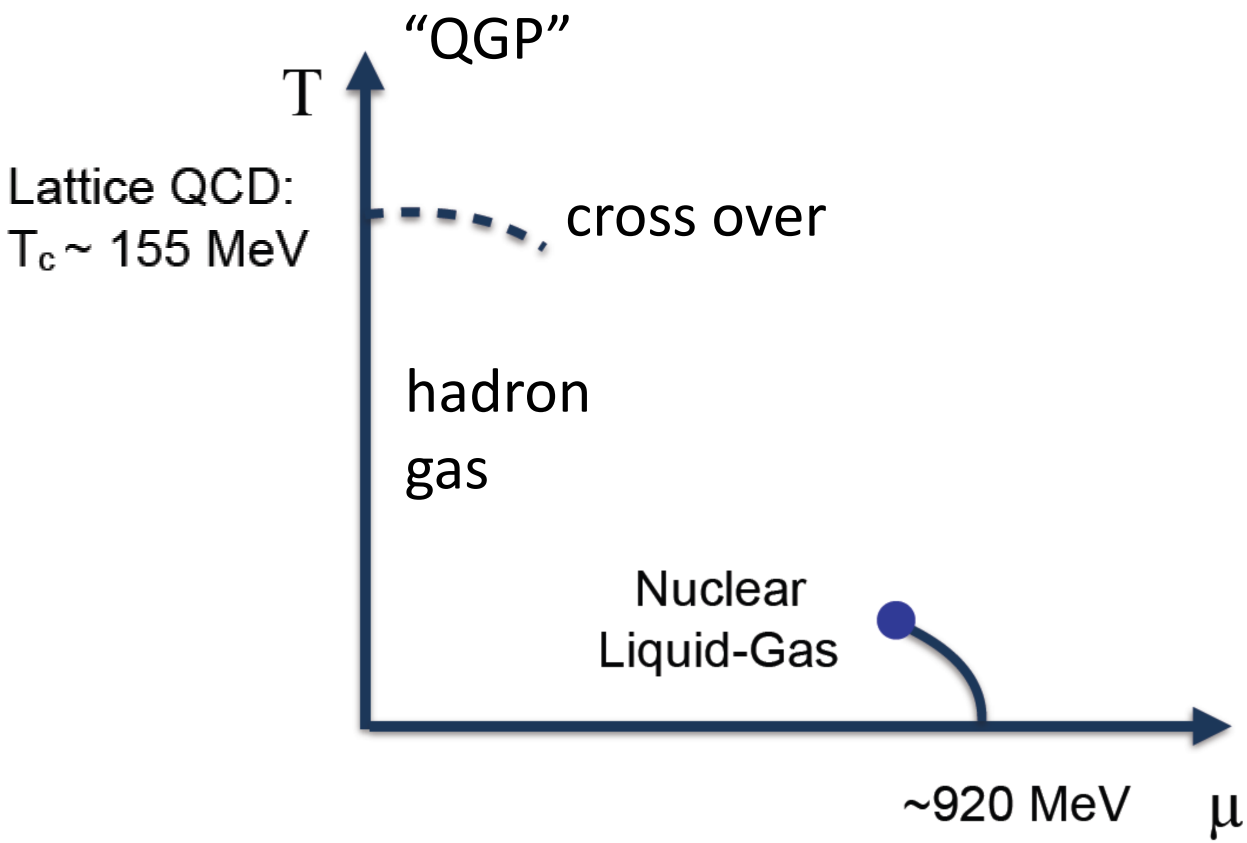
The (known) QCD phase diagram



The rest is everybody's guess.

On the experimental side all we can do is to measure various fluctuation observables and hope to see some nontrivial energy or/and system-size dependence

see, e.g.,
Stephanov, Rajagopal, Shuryak, PRL (1998)
Stephanov, PRL (2009)
Skokov, Friman, Redlich, PRC (2011)

There are some intriguing results:

STAR, HADES

Higher order cumulants

Proton v_1 (STAR)

HBT radii (STAR)

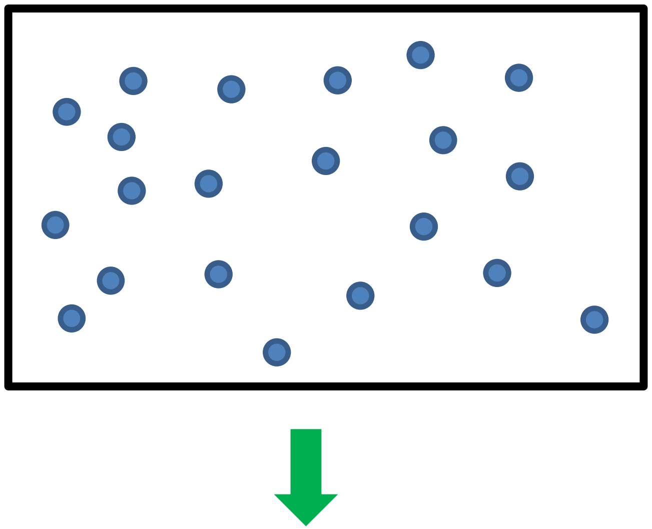
NA 49, 61/SHINE

Intermittency in the transverse momentum phase space

Scaled variance

Strongly intensive variables

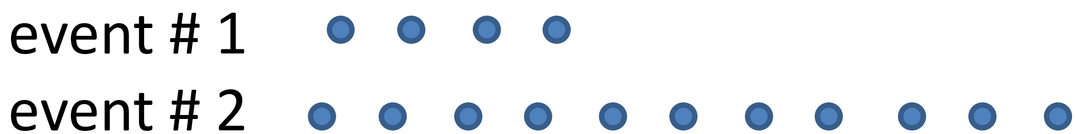
Poisson distribution



$$N = 10^{10}$$

$$p = 10^{-9}$$

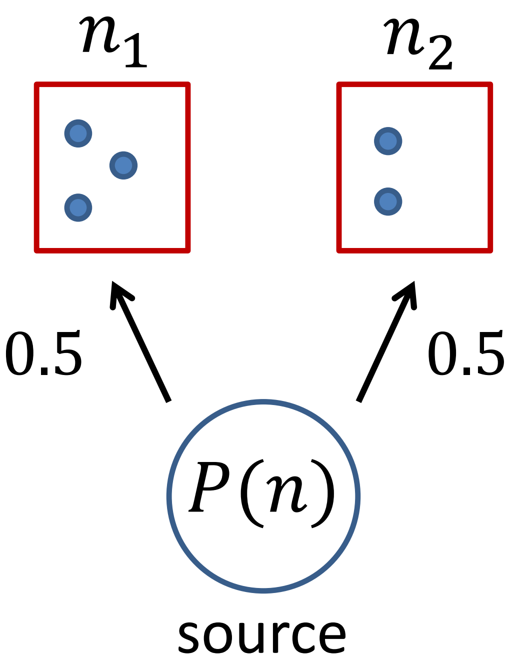
$$\langle n \rangle = Np = 10$$



$$P(n) = \text{Poisson} \quad \text{if} \quad N \to \infty,\ p \to 0,\ \ Np = \langle n \rangle$$

Such source (multiplicity distribution) is characterized by
All **factorial cumulants $\boldsymbol{C_n = 0}$**, $n = 2,3,\ldots$ (“no correlations”)

In what sense "no correlations"?



$$P(n_1, n_2) \stackrel{?}{=} P(n_1)P(n_2)$$

It is true for $P(n)$ = Poisson only

fixed N

finite N

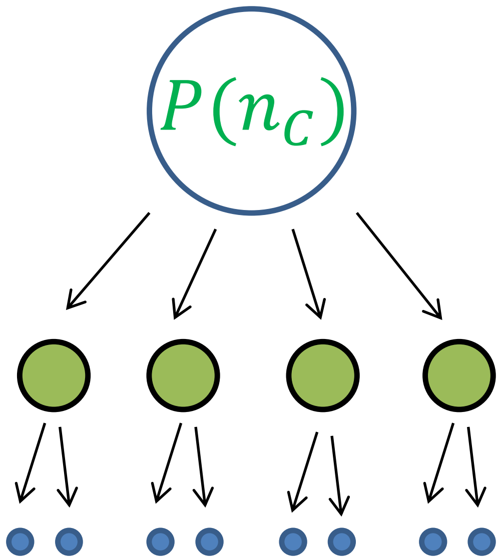
resonances

volume fluctuation

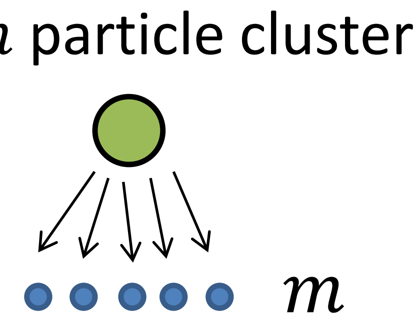
$$P(n_1, n_2) = P(n)\frac{n!}{n_1!\, n_2!}\left(\frac{1}{2}\right)^{n_1}\left(\frac{1}{2}\right)^{n_2}$$

$$n = n_1 + n_2$$

Multiparticle correlations - **factorial cumulants**



Poisson



$$\boldsymbol{C_2} \neq 0$$

$$\boldsymbol{C_k} = 0, k > 2$$

$$\boldsymbol{C_{2,3,\ldots,m}} \neq 0$$

$$\boldsymbol{C_k} = 0, k > m$$

factorial cumulants

$$\boldsymbol{C_k} = \frac{d^k}{dz^k} \ln\left(\sum_n P(n) z^n\right)\Big|_{z=1}$$

Two-particle correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

Integrating both sides over some bin in rapidity

$$\langle n(n-1)\rangle = \langle n\rangle^2 + \boldsymbol{C_2}$$

$$\langle n(n-1)\rangle = \int \rho_2(y_1, y_2)dy_1dy_2$$

$$\langle n\rangle = \int \rho(y)dy$$

factorial cumulant (integrated correlation function)

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2)dy_1dy_2$$

Genuine three-particle correlation

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3) + \rho(y_1)\boldsymbol{C_2}(y_2, y_3) + \cdots$$

three possibilities

$$+ \boldsymbol{C_3}(y_1, y_2, y_3)$$

Integrating both sides

$$\langle n(n-1)(n-2)\rangle = \langle n\rangle^3 + 3\langle n\rangle \boldsymbol{C_2} + \boldsymbol{C_3}$$

factorial cumulant (integrated correlation function)

$$\boldsymbol{C_3} = \int \boldsymbol{C_3}(y_1, y_2, y_3) dy_1 dy_2 dy_3$$

and analogously for higher-order correlation functions

Factorial cumulants vs cumulants

factorial cumulant

$$\boldsymbol{C_i} = \frac{d^i}{dz^i} \ln\left(\sum\nolimits_n P(n) z^n\right)\Big|_{z=1}$$

cumulant

$$K_{\boldsymbol{i}} = \frac{d^i}{dt^i} \ln\left(\sum\nolimits_n P(n) e^{tn}\right)\Big|_{t=0}$$

cumulants naturally appear in statistical physics

$$\ln(Z) = \ln\left(\sum_i e^{-\beta(E_i - \mu N_i)}\right)$$

Poisson:

$C_i = 0$

$K_i = \langle n \rangle$

We have

$$K_2 = \langle N \rangle + \boldsymbol{C_2}$$

$$K_3 = \langle N \rangle + 3\boldsymbol{C_2} + \boldsymbol{C_3}$$

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulants mix integ. correlation functions of different orders

$$K_5 = \langle N \rangle + 15C_2 + 25C_3 + 10C_4 + C_5$$

$$K_6 = \langle N \rangle + 31C_2 + 90C_3 + 65C_4 + 15C_5 + C_6$$

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2)dy_1 dy_2$$

factorial cumulant

See, e.g.,

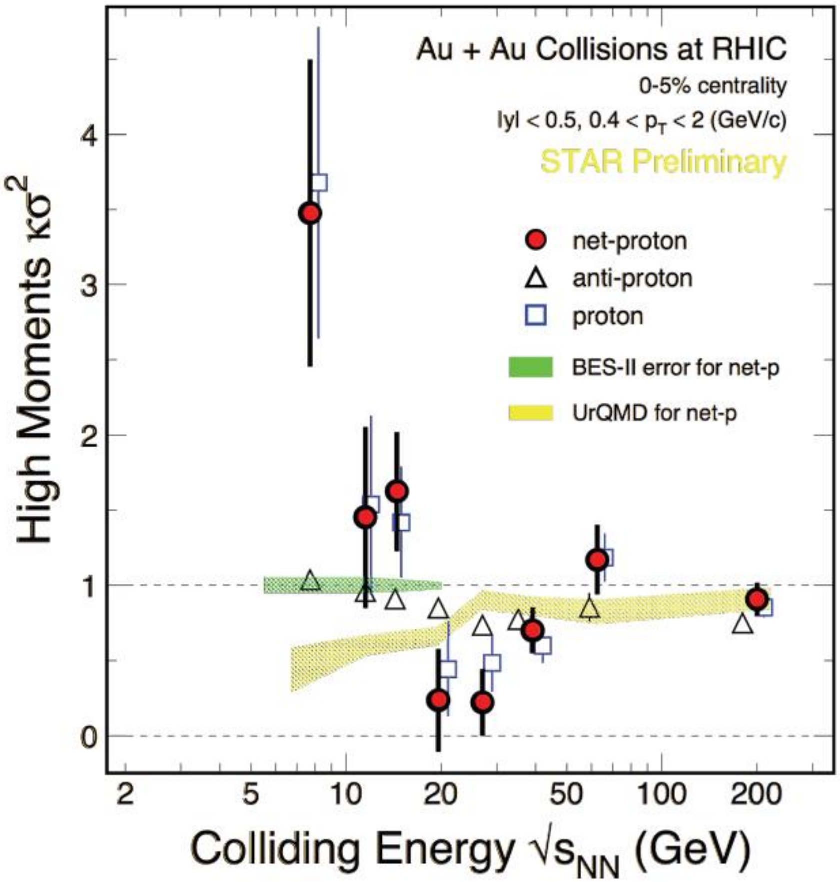
B. Ling, M. Stephanov, PRC 93 (2016) 034915

AB, V. Koch, N. Strodthoff , PRC 95 (2017) 054906

AB, V. Koch, D. Oliinychenko, J. Steinheimer, PRC 98 (2018) 054901

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



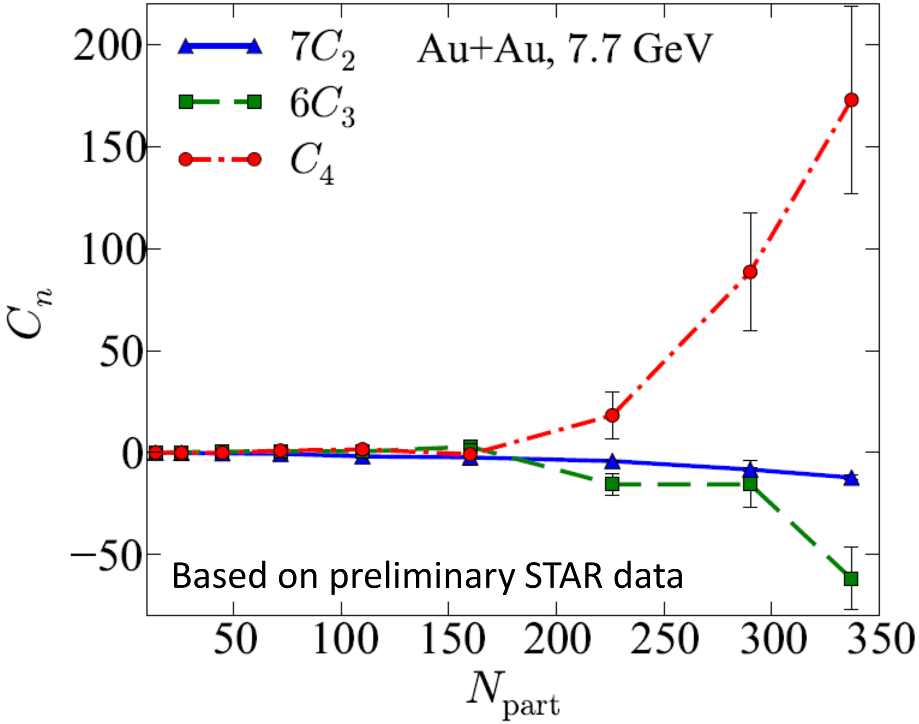
my notation

K_4/K_2

Is proton signal at 7.7 GeV large?
Is physics changing between 7 and 19 GeV?

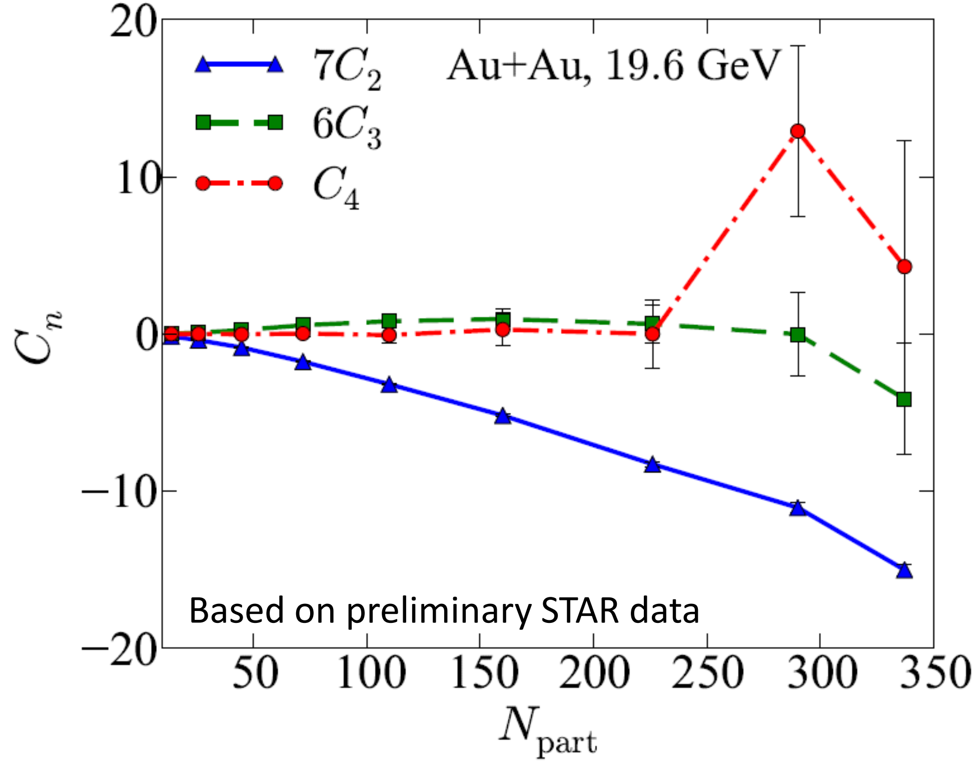
Using preliminary STAR data we obtain C_n

central signal at $\mathbf{7.7}$ GeV is driven by large 4-particle correlations



$C_4(7.7) \sim \mathbf{170}$

central signal at $\mathbf{19.6}$ GeV is driven by 2-particle correlations



C_4 and $6C_3$ cancelation in most central coll.

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

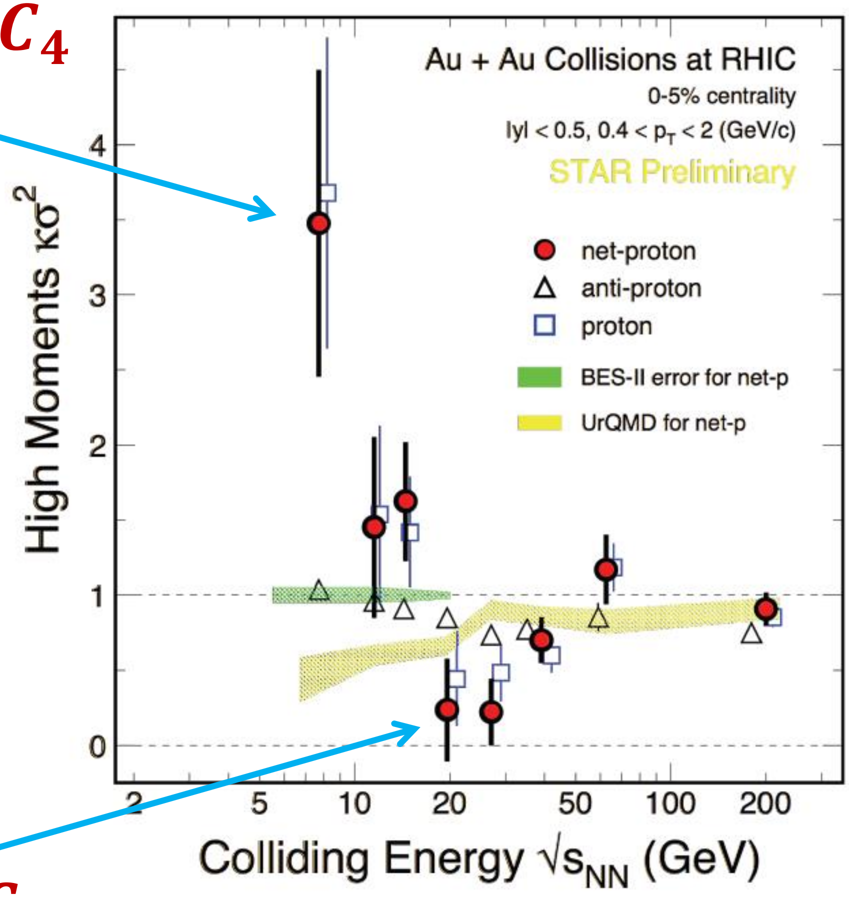
STAR preliminary
X.Luo, N.Xu, 1701.02105

here we see $\boldsymbol{C_4}$

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$



and here $\boldsymbol{C_2}$

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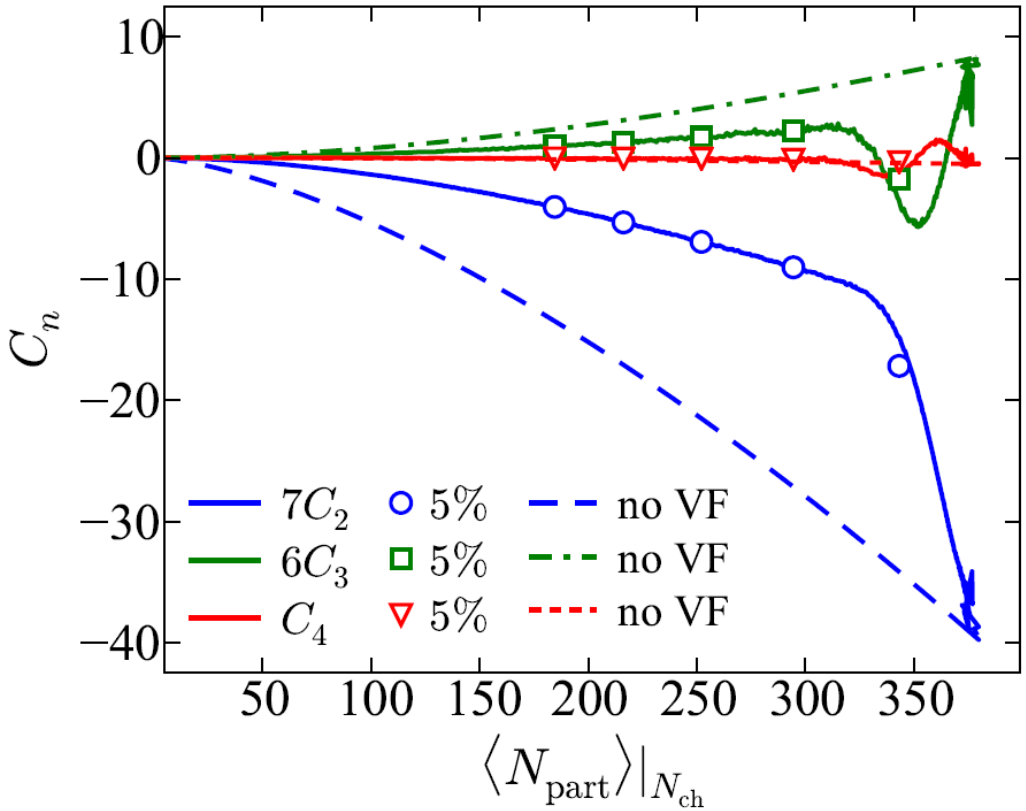
Baryon conservation + volume fluctuation (minimal model)

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV



STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, EPJC 77 (2017) 288

See also, P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 960 (2017) 114

Volume fluctuation + baryon conservation seems to be important for C_2 but irrelevant for C_3 and C_4 (7.7 GeV).

C_4 observed by STAR is larger by almost **three orders of magnitude** than the minimal model.

To explain C_4 we need a strong source of multi-proton correlations.

Proton clusters?

Rapidity independent correlations

$$R_n(y_1, \dots, y_n) = \frac{C_n(y_1, \dots, y_n)}{\rho(y_1) \cdots \rho(y_n)}$$

if $R_n(y_1, \dots, y_n) = const = R_n^0$

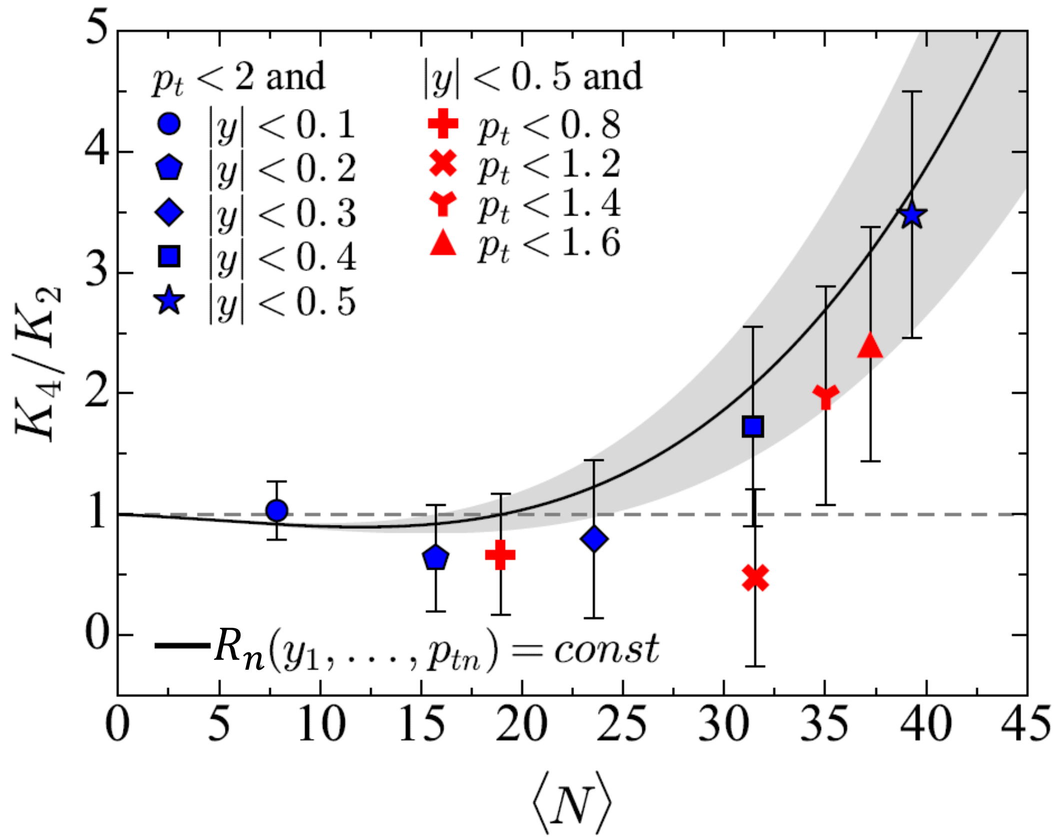
$$C_n = R_n^0 \int \rho(y_1) \cdots \rho(y_n) dy_1 \cdots dy_n = R_n^0 \langle N \rangle^n \sim (\Delta y)^n$$

$C_n \sim \langle N \rangle^n \sim (\Delta y)^n$

Constant correlation

$$R_n(y_1, p_{t1}, \ldots, y_n, p_{tn}) = const$$

physics independent on rapidity and transverse momentum



$\sqrt{s} = 7.7$ GeV

Acceptance: the missing link between models and data

AB, V. Koch, PRC 96 (2017) 054905

Cumulant ratios strongly depend on acceptance in rapidity (as actually expected) and in transverse momentum.

Comparison with models which do not have experimental acceptance is questionable (should be done with extra caution).

For small enough $\langle N \rangle$ things look like Poisson but this is actually a bit misleading.

Perhaps the best thing to measure is $\frac{C_n}{\langle N \rangle^n}$

Can we describe the STAR data at 7.7 GeV with simple multiplicity distributions?

Bimodal distribution

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

Poisson, binomial, etc..	Poisson, binomial, etc.

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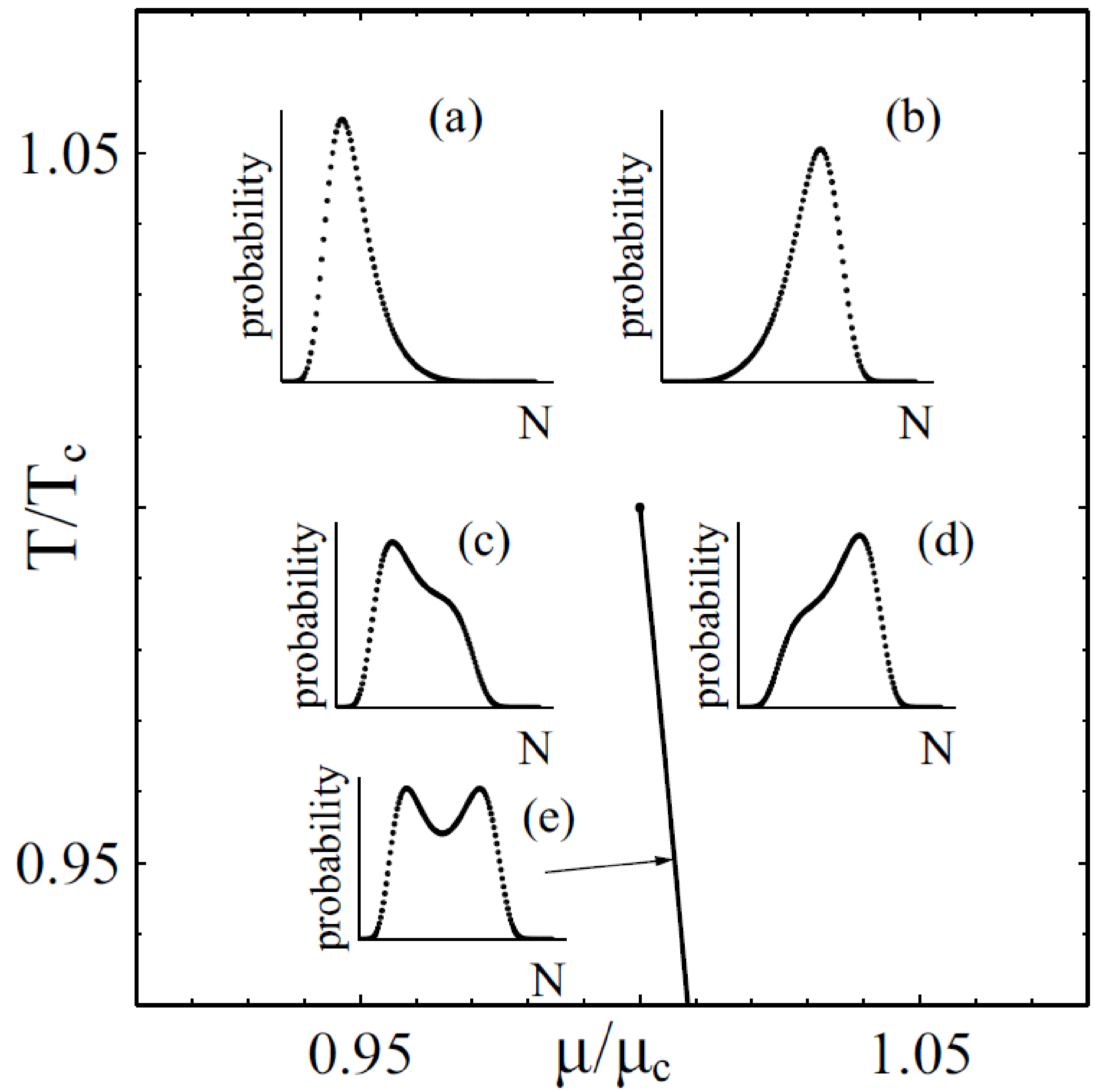
A finite volume van der Waals model



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$$C_2 = \alpha(1-\alpha)\overline{N}^2 \approx \alpha\overline{N}^2,$$
$$C_3 = -\alpha(1-\alpha)(1-2\alpha)\overline{N}^3 \approx -\alpha\overline{N}^3,$$
$$C_4 = \alpha(1-\alpha)(1-6\alpha+6\alpha^2)\overline{N}^4 \approx \alpha\overline{N}^4,$$
$$\overline{N} = \langle N_{(a)} \rangle - \langle N_{(b)} \rangle$$

$$\frac{C_6}{C_5} \approx \frac{C_5}{C_4} \approx \frac{C_4}{C_3} = -17 \pm 6$$

parameter-free prediction at 7.7 GeV ($\alpha \ll 1$)

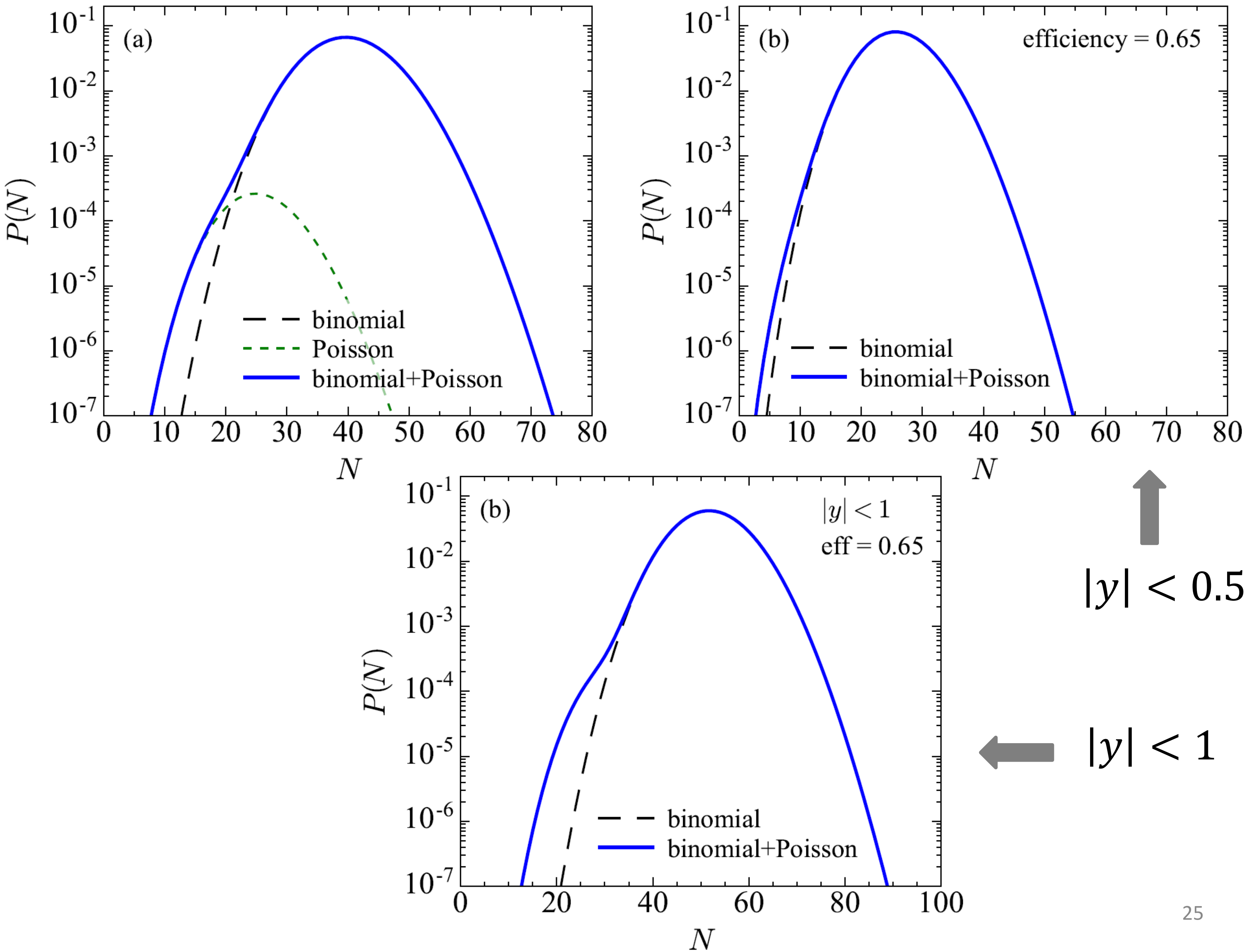
$$C_5 \approx -2645,$$
$$C_6 \approx 40900,$$

assuming $C_4 = 170$

We can describe the data with $\alpha \approx 0.0033$

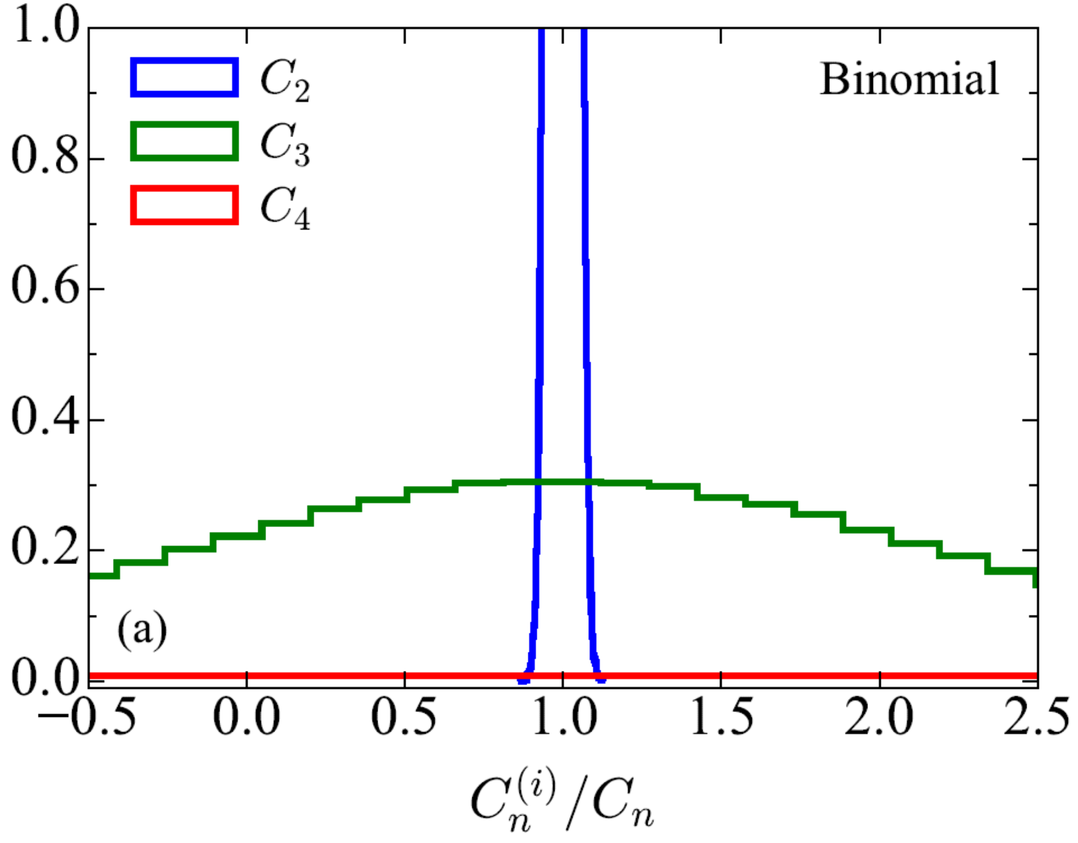
$$\langle N_{(a)} \rangle \approx 40,\ \langle N_{(b)} \rangle \approx 25$$

Now we can plot $P(N)$


(a)
binomial
Poisson
binomial+Poisson
P(N)
N
(b)
efficiency = 0.65
binomial
binomial+Poisson
P(N)
N
(b)
|y| < 1
eff = 0.65
binomial
binomial+Poisson
P(N)
N
|y| < 0.5
|y| < 1

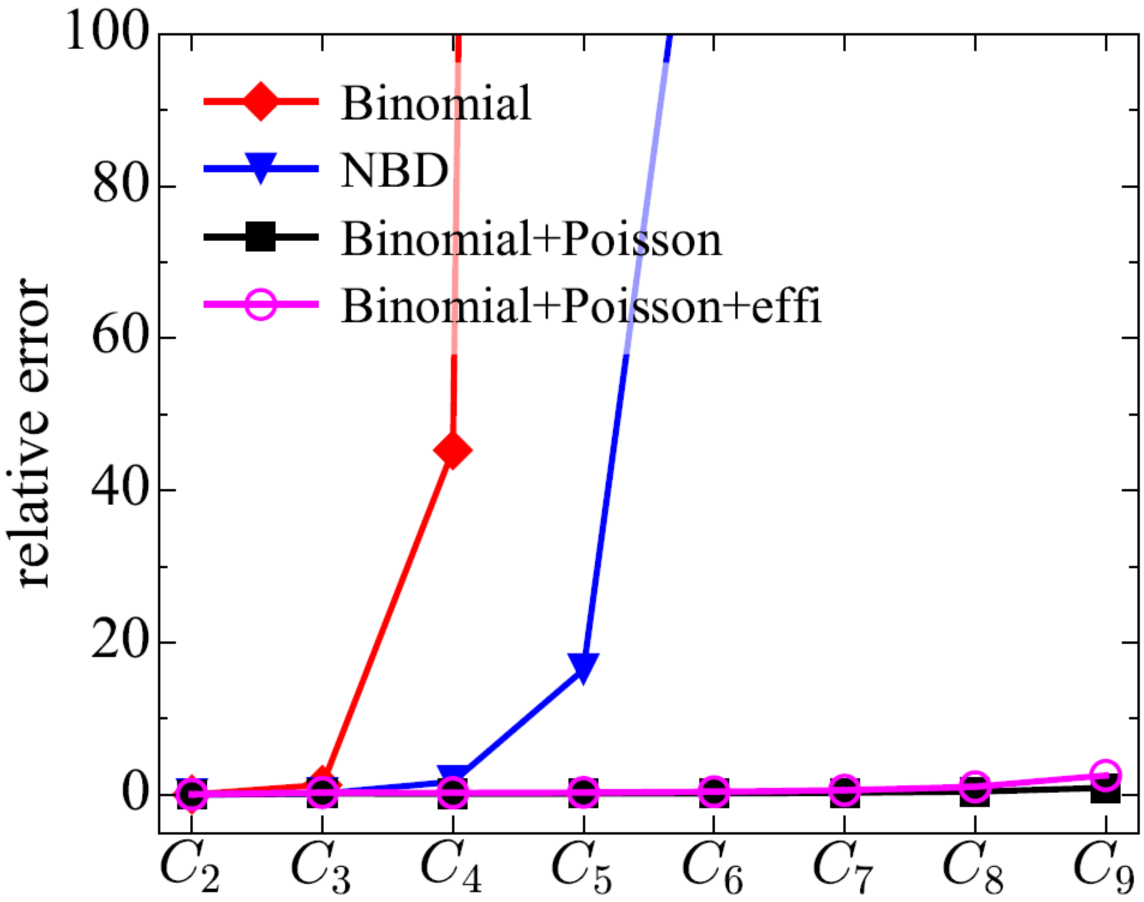
Can we verify the model (C_5, C_6, …) with 144393 events available at STAR?

Statistics hungry distributions (SHD): binomial, Poisson, NBD,…



Histogram of $C_n^{(i)}/C_n$ based on 144393 events.

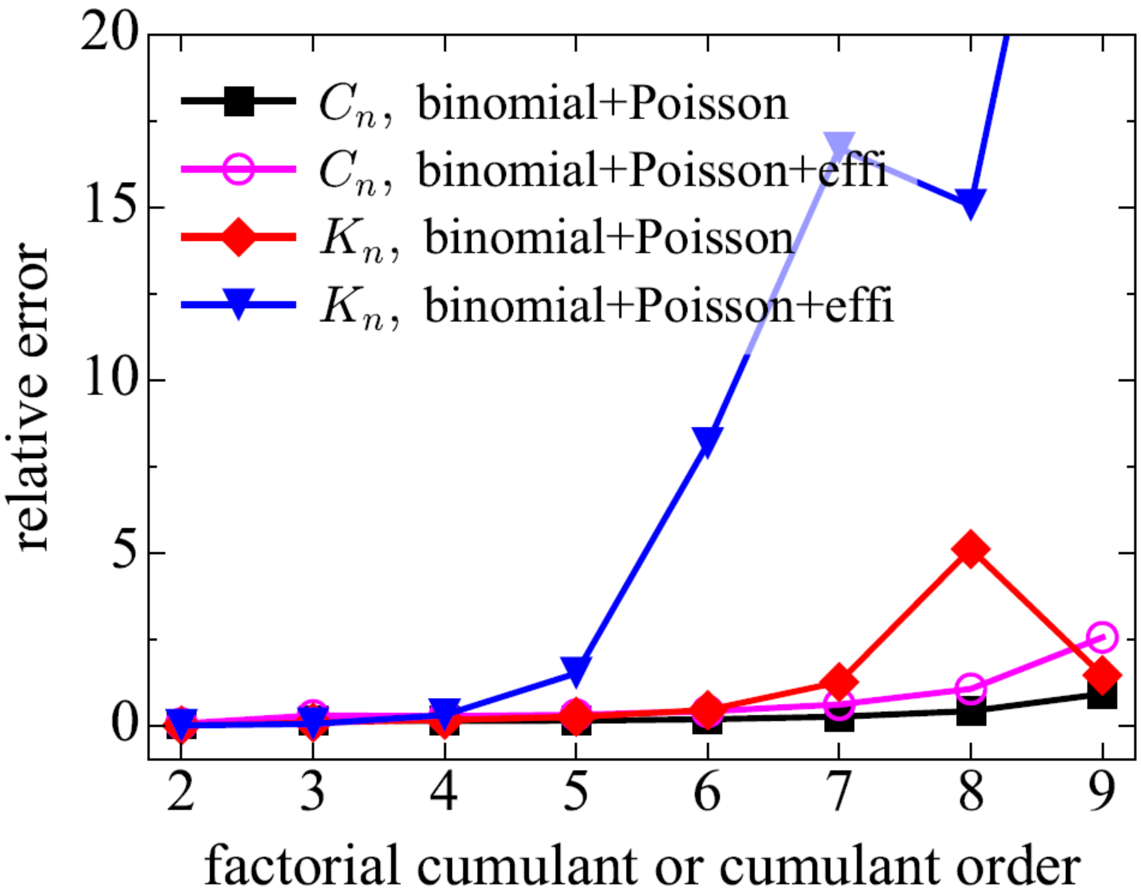
Are there **statistics friendly distributions** (SFD)? Yes.



based on 144393 events

AB, V.Koch, 1811.04456

Regular cumulants are not as friendly



All of this can be understood

AB, V.Koch, 1811.04456

Using preliminary STAR data we have (factorial cumulants)

$$C_5 \sim -2650$$

$$C_6 \sim +40900$$

$$C_7 \sim -615000$$

$$C_8 \sim +8520000$$

For Poisson $C_n = 0$

More details in 1811.04456

Worth measuring by STAR

Conclusions

Four-proton factorial cumulant (int. correlation function) at 7.7 GeV is surprisingly large. Three orders of magnitude larger than the minimal model.

Volume fluctuation and baryon conservation seem to be irrelevant for C_3 and C_4. C_2 (and K_2) is likely contaminated by background.

Rapidity and transverse momentum independent correlations.

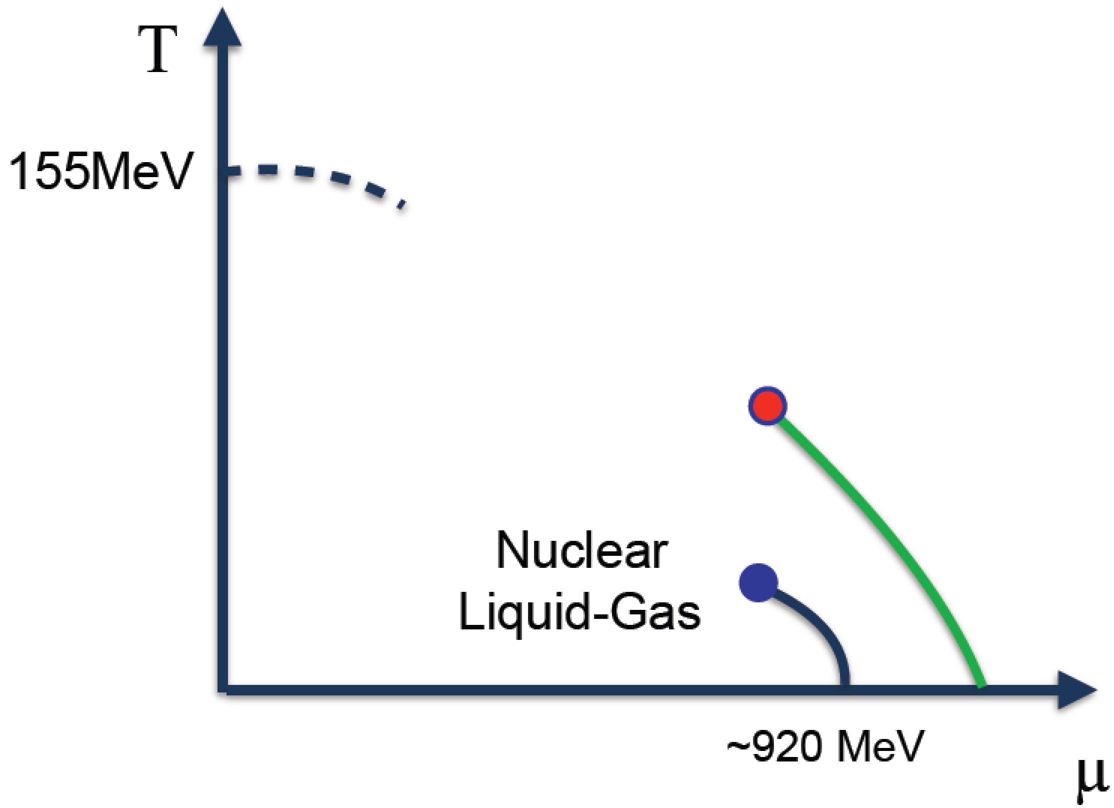
Proton clusters?

Bimodal $P(N)$?

Statistics friendly distributions close to the phase transition?

Backup

Usual expectation based on various effective models



Critical point: everybody's guess

M. Stephanov, hep-lat/0701002

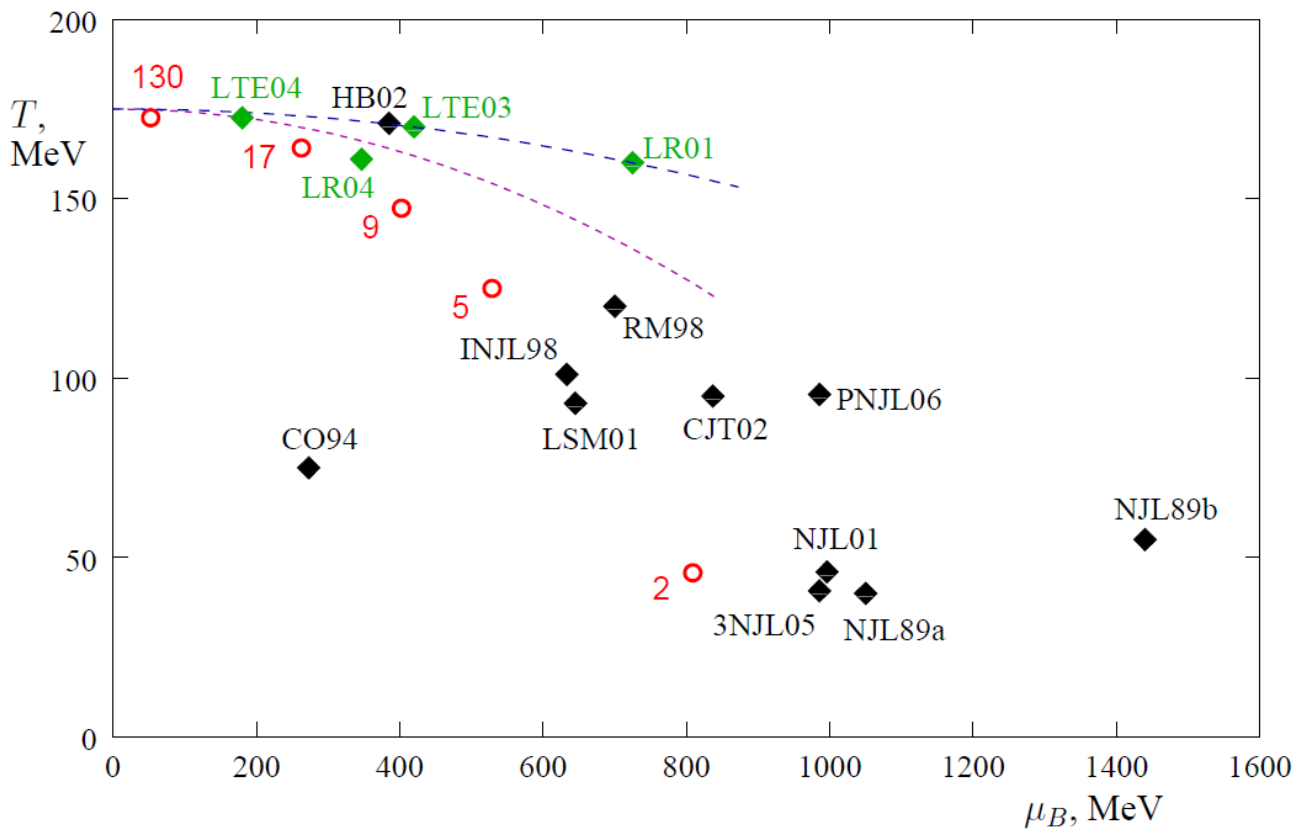
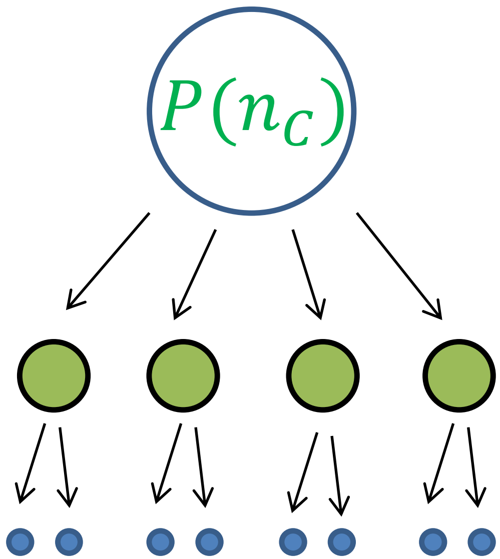


Figure 4: Comparison of predictions for the location of the QCD critical point on the phase diagram. Black points are model predictions: NJLa89, NJLb89 – [12], CO94 – [13, 14], INJL98 – [15], RM98 – [16], LSM01, NJL01 – [17], HB02 – [18], CJT02 – [19], 3NJL05 – [20], PNJL06 – [21]. Green points are lattice predictions: LR01, LR04 – [22], LTE03 – [23], LTE04 – [24]. The two dashed lines are parabolas with slopes corresponding to lattice predictions of the slope $dT/d\mu_B^2$ of the transition line at $\mu_B = 0$ [23, 25]. The red circles are locations of the freezeout points for heavy ion collisions at corresponding center of mass energies per nucleon (indicated by labels in GeV) – Section 5.

Suppose we have a system with two-particle clusters only



Poisson

In this case all information is contained in $\langle n \rangle$ and K_2.
No point to measure $K_{3,4,\ldots}$

$$\boldsymbol{C_2} = 2\langle n_C \rangle \qquad \boldsymbol{C_{3,4,\ldots}} = 0$$

$$\boxed{K_i = 2^i \langle n_C \rangle}$$

and, e.g,: $\dfrac{K_4}{K_2} = 4, \ \dfrac{K_6}{K_2} = 16$

looks nontrivial but no new information

Let's put the STAR numbers in perspective.

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Suppose that we have **clusters** (distributed according to Poisson) decaying always to 4 protons

$$\boldsymbol{C_k} = \langle N_{\rm cl} \rangle \cdot 4!/(4-k)!$$

↑ mean number of clusters

$$\boldsymbol{C_4} = \langle N_{\rm cl} \rangle \cdot 24$$

for 5-proton clusters:

$$\boldsymbol{C_k} = \langle N_{\rm cl} \rangle \cdot 5!/(5-k)!$$

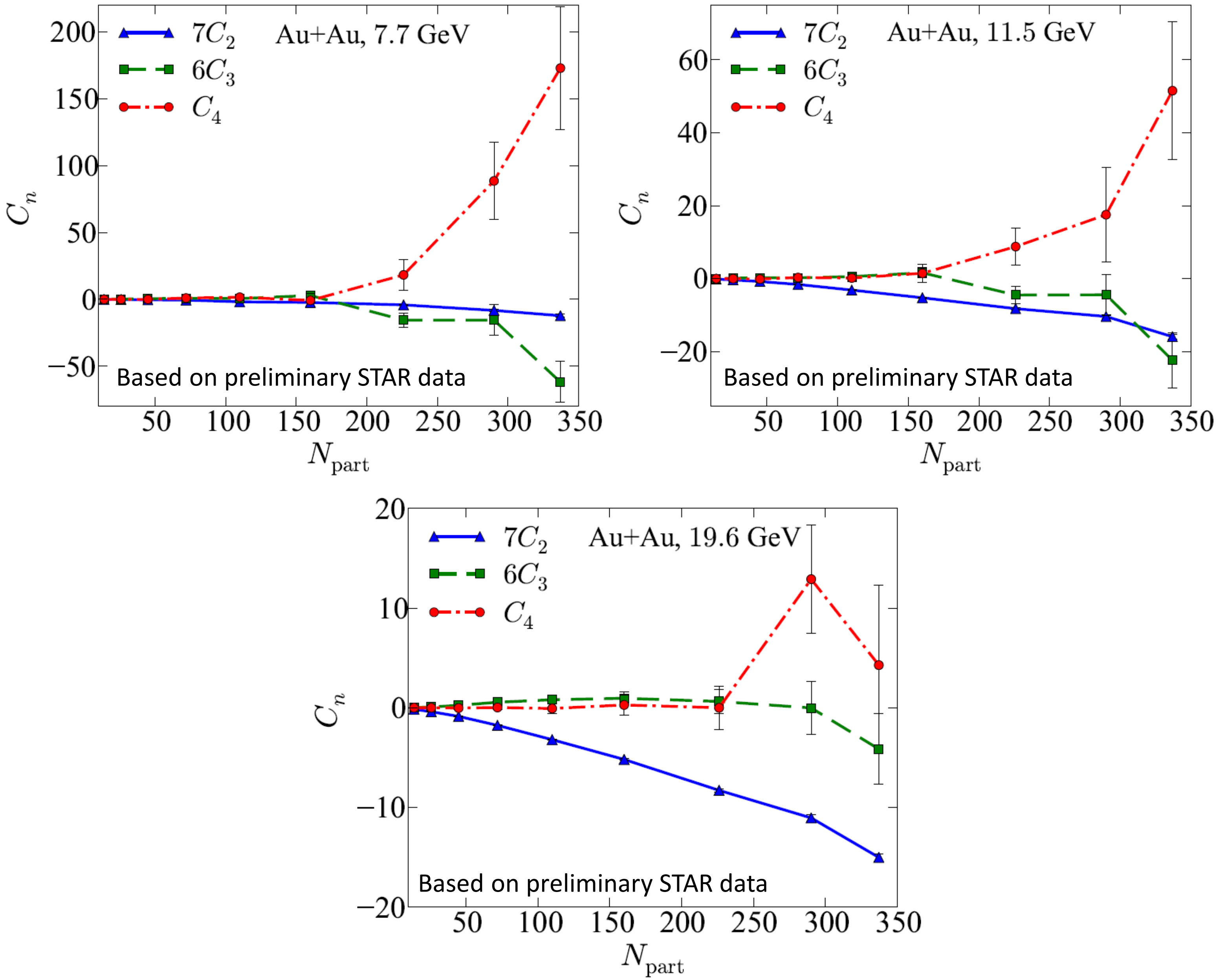
$$\boldsymbol{C_4} = \langle N_{\rm cl} \rangle \cdot 120$$

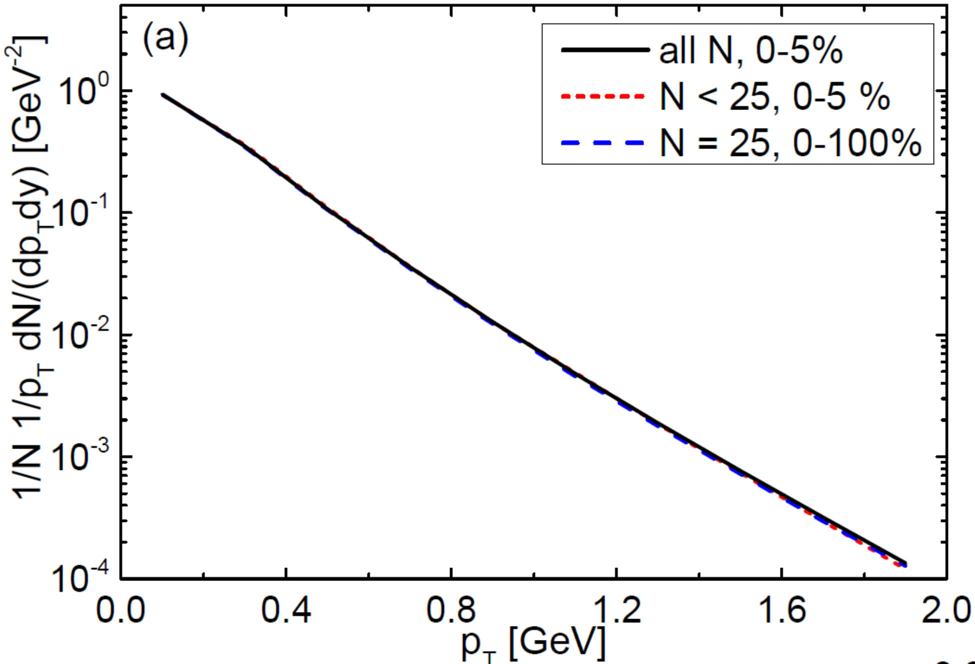
and $\langle N_{\rm cl} \rangle \sim 1$

To obtain $\boldsymbol{C_4} \approx 170$ we need $\langle N_{\rm cl} \rangle \sim 7$, it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

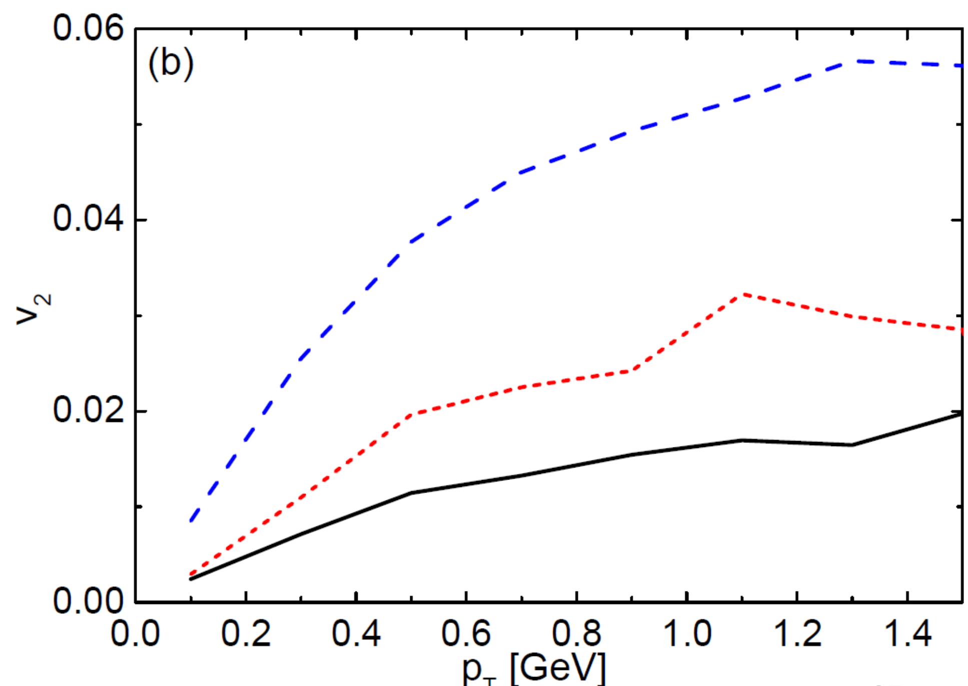
Comparison of 7.7, 11.5 and 19.6 GeV





Cuts in the number of protons for central collisions

UrQMD



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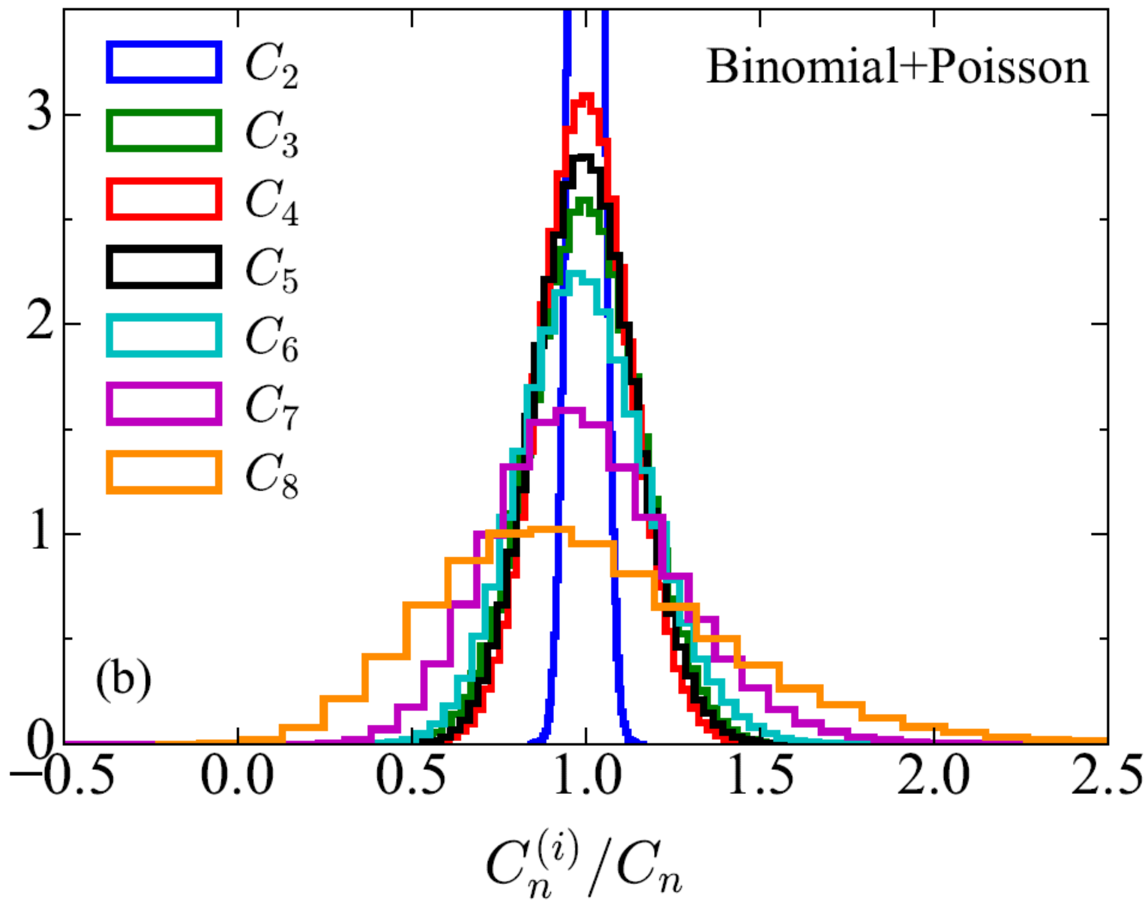
Histogram of $C_n^{(i)}/C_n$ based on 144393 events.



It survives when hit with efficiency of 0.65