Extracting equation of state from neutron star observation using machine learning

Yuki Fujimoto[1], **Kenji Fukushima**[1,2], **and Koichi Murase**[3]

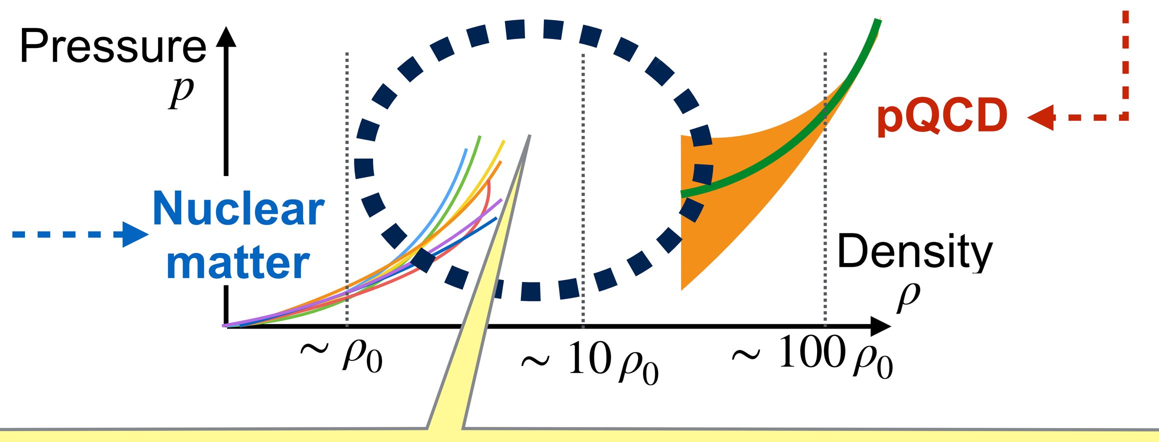
[1]Department of Physics, The University of Tokyo, [2]Institute for Physics of Intelligence (IPI), The University of Tokyo, [3]Department of Physics, Sophia University

References: [1] **YF**, K. Fukushima & K. Murase, Phys. Rev. D **98**, 023019 (2018); [2] **YF**, K. Fukushima & K. Murase, arXiv:1903.03400 [nucl-th].

1. Introduction: dense matter equation of state (EoS)

- **Equation of State** (EoS): static relation between pressure p and density ρ
- **The current status of the EoS:**
  - At lower density Nuclear calculation, well-constrained around ρ_0 … but reliability above ρ_0 decline with growing density
  - At higher density Perturbative QCD calculation is feasible … but plagued with large uncertainty

$\rho_0 \sim 0.16$ fm^{-3}; normal nuclear density

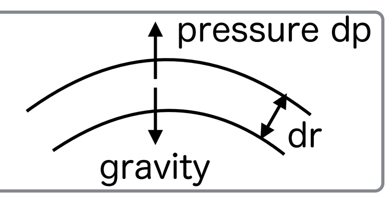


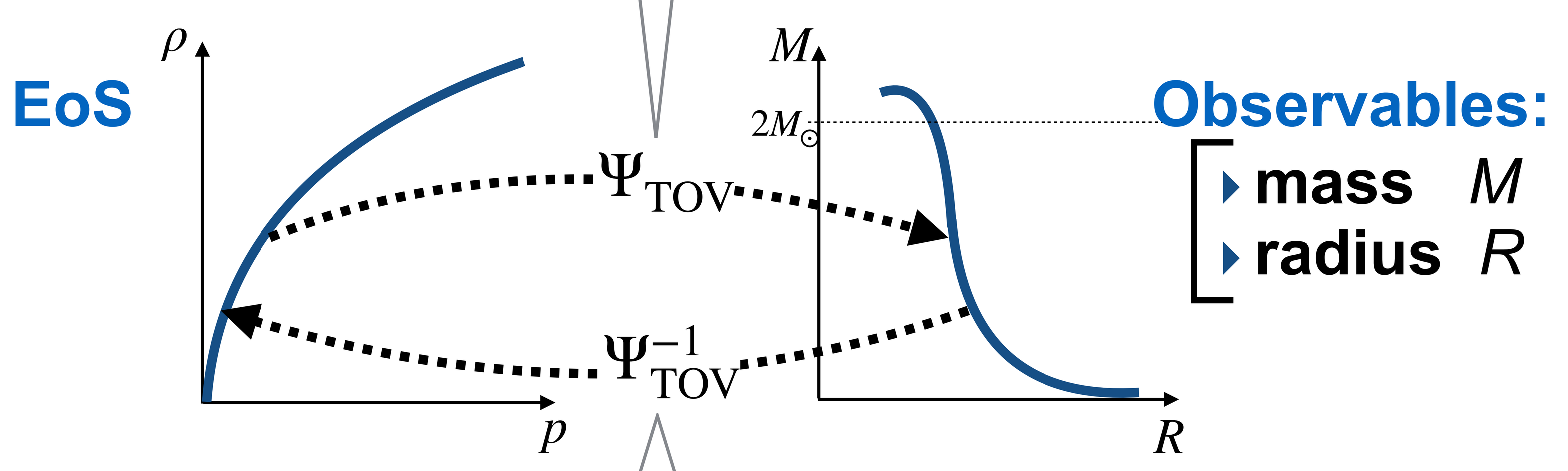
We extract this lacking information of the EoS from neutron star observables using machine learning

2. Neutron star phenomenology

- **TOV equation** connects EoS and neutron star observables:

Tolman-Oppenheimer-Volkoff (TOV) equation
General relativistic equation for hydrostatic equilibrium



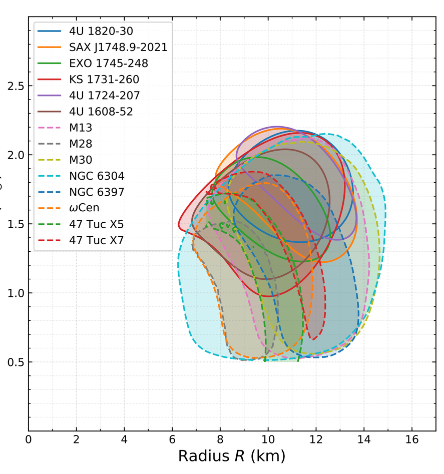


If M-R curve is given, there is **one-to-one correspondence** Lindblom (1992)

- **Observables**:
  X-ray measurement of neutron star masses and radii
- 14 of them are distributed on the website

Contour plot (68 %) for each stars →

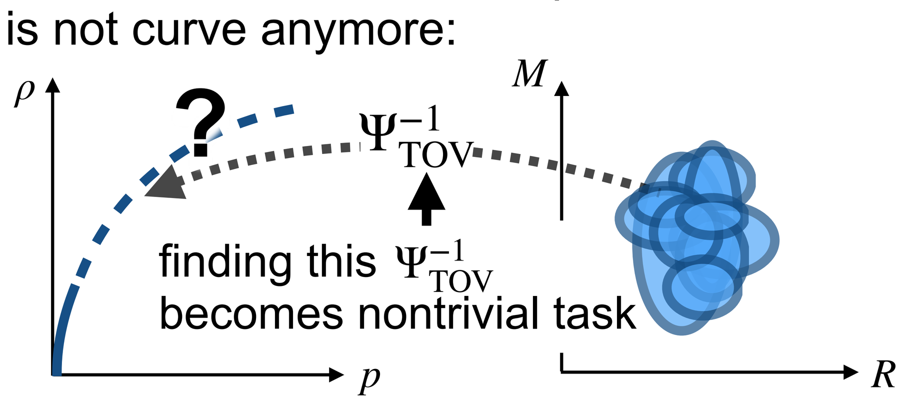
Özel et al. (2016)



Using the one-to-one mapping between M-R and EoS, estimate the EoS from the real observables

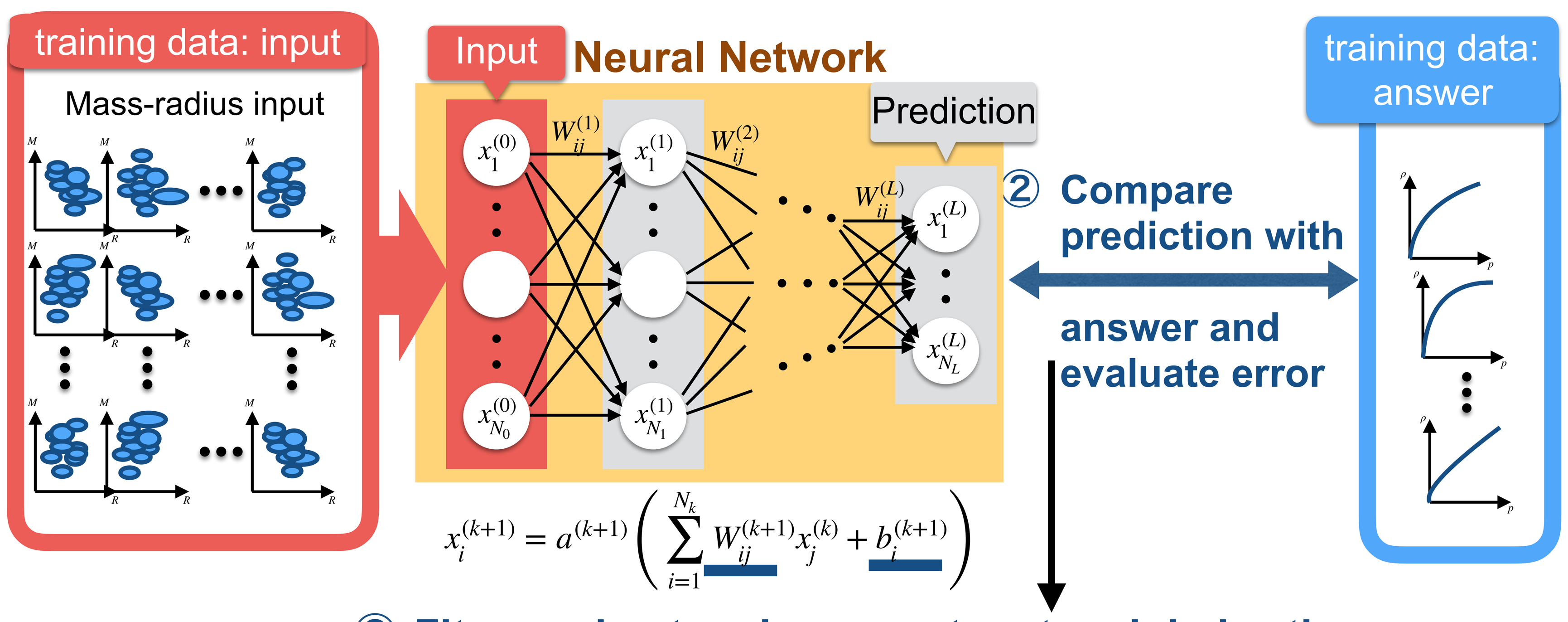
3. Machine learning method for EoS estimation

- In reality, however, above one-to-one correspondence is not exact because the M-R is not curve anymore:



- To get "EoS predictor," express this mapping in terms of deep neural network

① **Prepare training data (input and answer)**

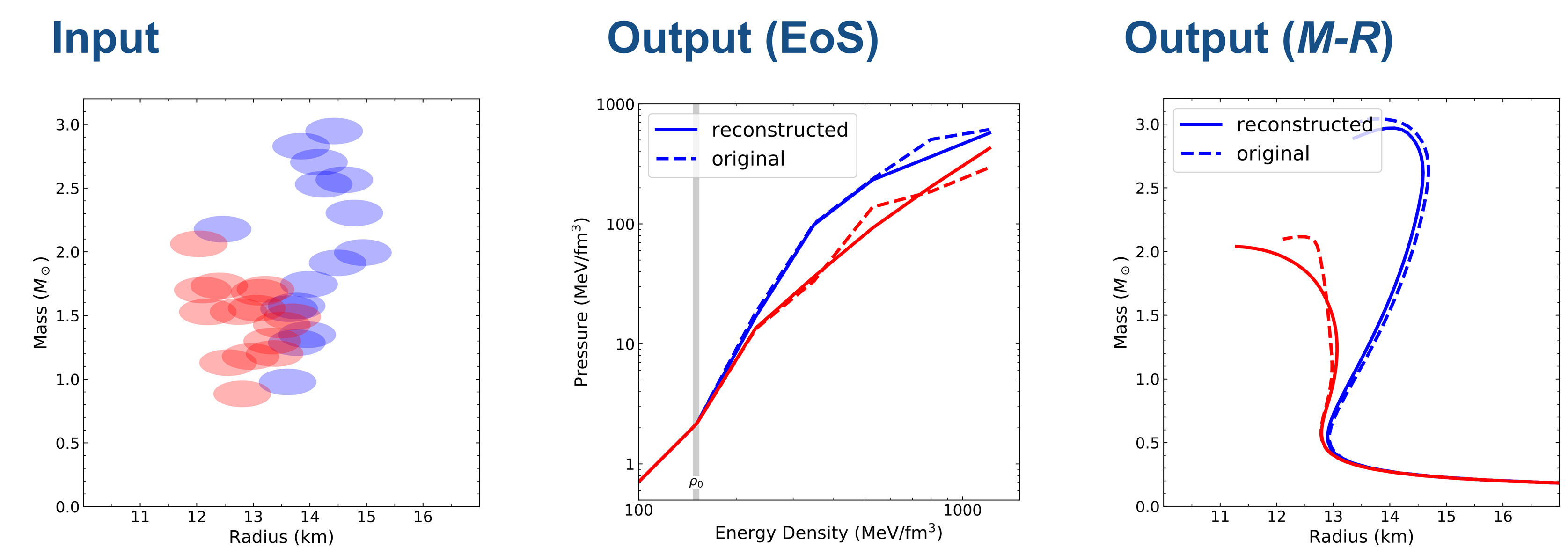


③ **Fit neural network parameters to minimize the error**

- Use bootstrapping method for uncertainty estimation

4. Mock data analysis

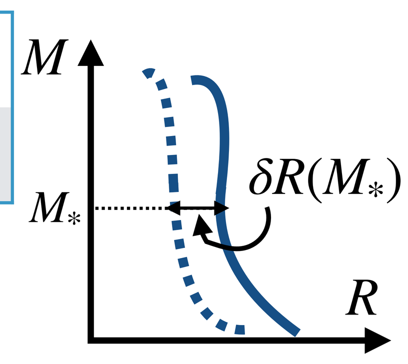
- **EoS reconstruction accuracy: typical examples**



- **Reconstruction accuracy in M-R space**

Mass ($M_\odot$)	0.6	0.8	1.0	1.2	1.4	1.6	1.8
σ_R (km)	0.16	0.12	0.10	0.099	0.11	0.11	0.12

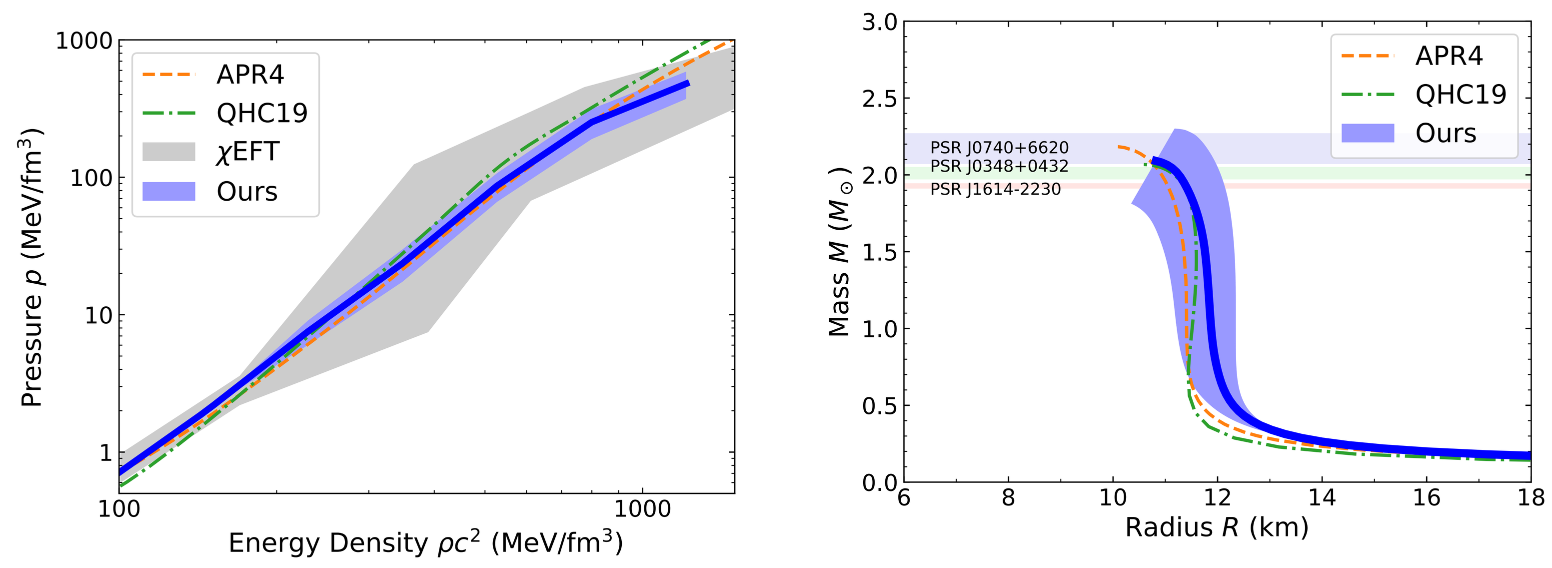
$\sigma_R = \mathrm{RMS}[\delta R(M)]$



- Reconstruction accuracy: ~0.1 km
  surpassing the current observational uncertainties!

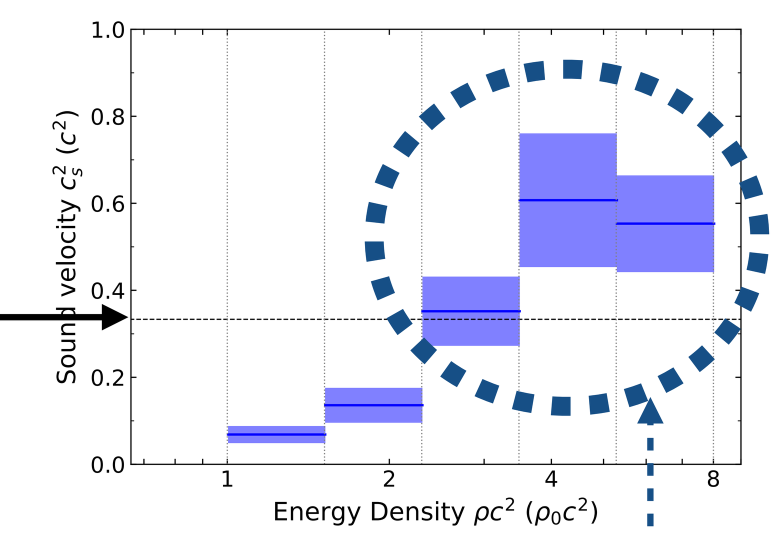
5. Results and discussion

- **Inferred result**



- **Sound velocity**
  - **Sound velocity:** $c_s^2 = \dfrac{\partial p}{\partial \rho}$
  - Upper limit for ultra-relativistic particle:
    $c_s^2 = 1/3$ **(Conformal limit)**
  - Sound velocity exceeds conformal limit!

Bedaque-Steiner (2015)
Tews-Carlson-Gandolfi-Reddy (2018)



- **Consistency check with independent result**
  - **Chiral effective theory:** shown above in grey region, consistent Hebeler et al. (2013)
  - **Nuclear theory:** EoS based on realistic potential such as APR, consistent
  - **Quark-hadron continuity:** three-window EoS (QHC19), consistent Baym et al. (2019)
  - **GW170817:** calculated tidal deformability with our result, consistent
    $\Lambda_{\rm ours}(1.4\,M_\odot) = 320^{+120}_{-110}$, $\Lambda_{\rm GW170817}(1.4\,M_\odot) = 190^{+390}_{-120}$ Annala et al. (2017) Abbott et al. (2018)
- **Relation with Bayesian inference of EoS**
  We want to find $\Psi^{-1}_{\rm TOV}$ such that $\boldsymbol{\theta} = \Psi^{-1}_{\rm TOV}(\mathscr{D})$ ($\boldsymbol{\theta}$: EoS parameter, $\mathscr{D}$: observation)
  - **Bayesian (e.g. Maximum a posterior estimator)**
    $\Psi^{-1}_{\rm MAP} = \mathrm{argmax}_{\boldsymbol{\theta}} \Pr(\boldsymbol{\theta}\,|\,\mathscr{D}) = \mathrm{argmax}_{\boldsymbol{\theta}} \Pr(\boldsymbol{\theta})\Pr(\mathscr{D}\,|\,\boldsymbol{\theta})$
  - **Neural network**
    find Ψ^{-1} through minimizing: $\langle \ell[\Psi^{-1}] \rangle = \int d\boldsymbol{\theta} d\mathscr{D}\, \Pr(\boldsymbol{\theta})\Pr(\mathscr{D}\,|\,\boldsymbol{\theta})\,\boxed{\ell(\boldsymbol{\theta}, \Psi^{-1}(\mathscr{D}))}$
    approximated estimate of loss function → Bayesian

6. Conclusions and future prospects

Conclusions
- We established the method to estimate EoS using machine learning
- Put significant constraint on EoS based the real neutron star observations
- Result seems to be consistent with independent study

Future prospects
- Rigorous treatment of the uncertainty; bootstrapping is known to be optimistic
- Study the bias effect of other contributions

Contact: fujimoto@nt.phys.s.u-tokyo.ac.jp