Quantum Gravity and Naturalness

KEK Theory Workshop 2019

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Plan of the talk

1. SM around Planck scale

2. Fine tunings by nature itself

3. Emergence of weak scale from Planck scale

Appendix A
Low energy effective theory of QG
Appendix B
Multiverse and naturalness

1. SM around Planck scale

Desert

SM is good to high energy scales.

Experimentally LHC

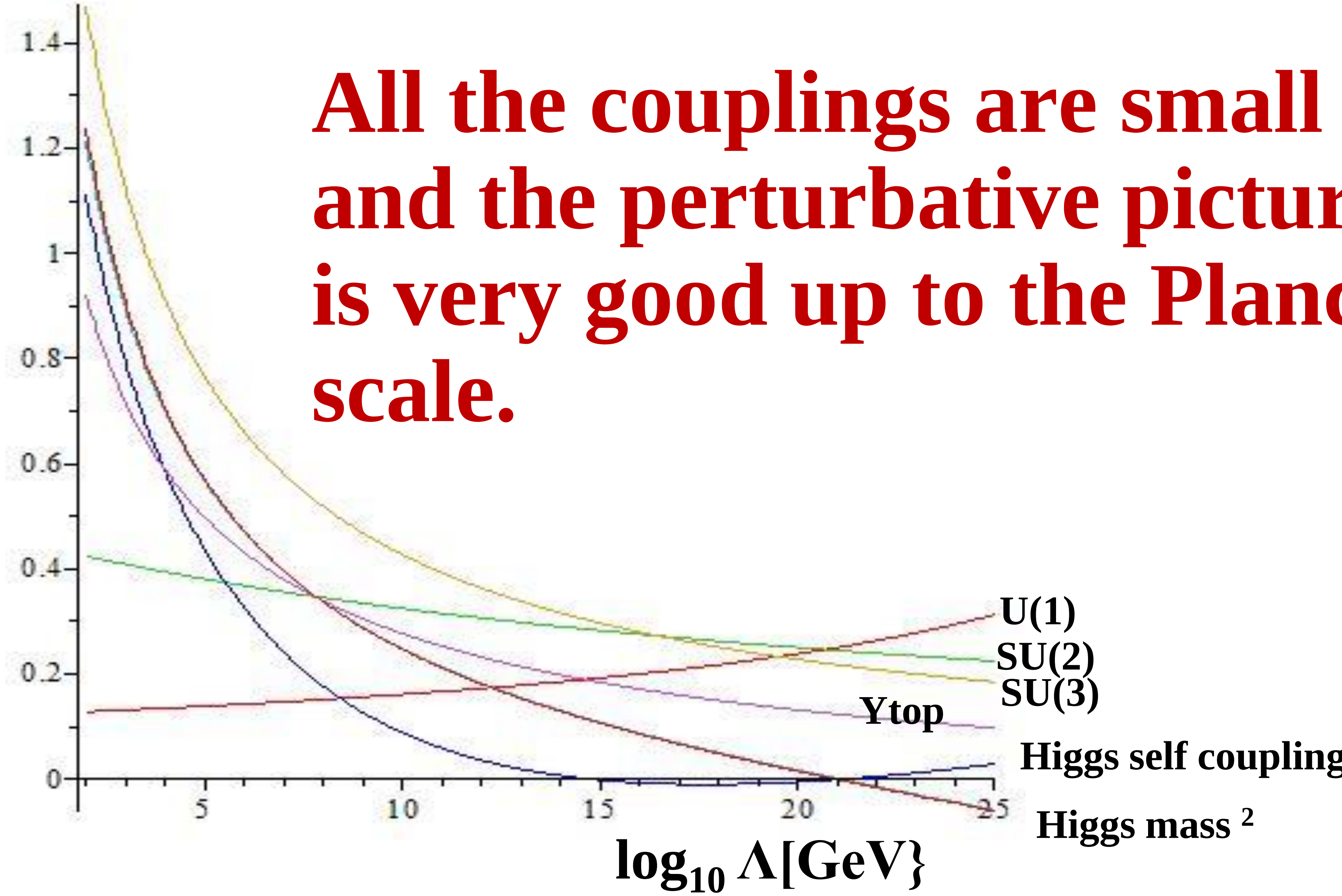
SM is good at least below a few TeV.

No signal for new particles or physics.

Especially no indication of low energy SUSY.

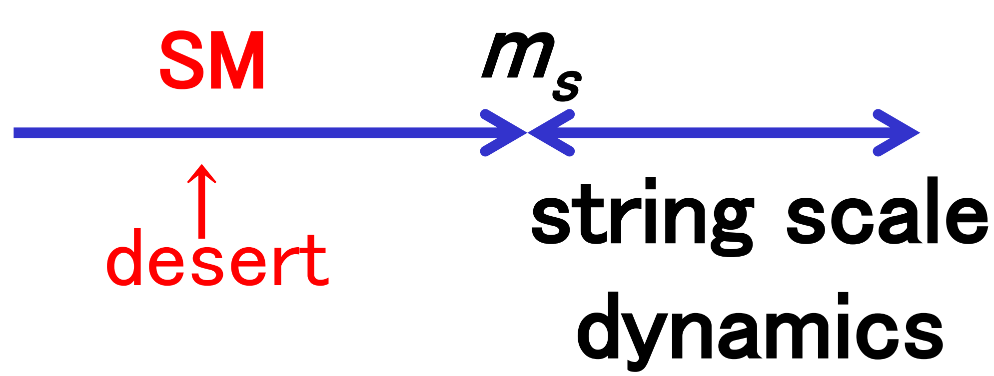
Theoretically UV region of SM by RG

No contradiction below Planck/string scale.



All the couplings are small and the perturbative picture is very good up to the Planck scale.

It is natural to imagine that SM is directly connected to the string scale dynamics without large modification.



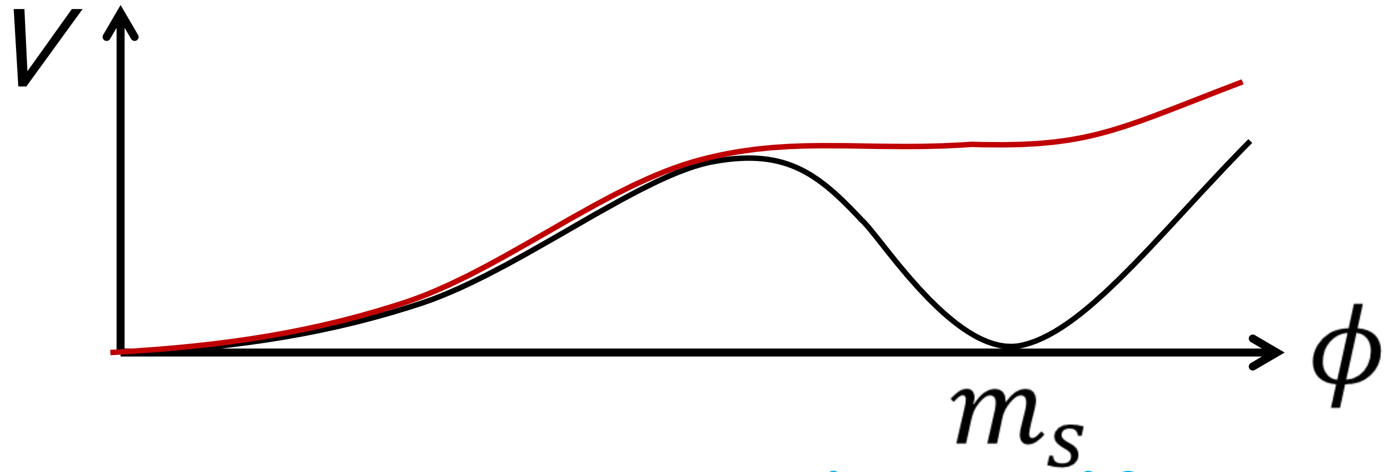
Triple coincidence

As we will see in the next 2 slides, RG analyses indicate

(1) The three quantities, $\lambda_B, \beta_\lambda(\lambda_B), m_B$

become zero around the string scale.

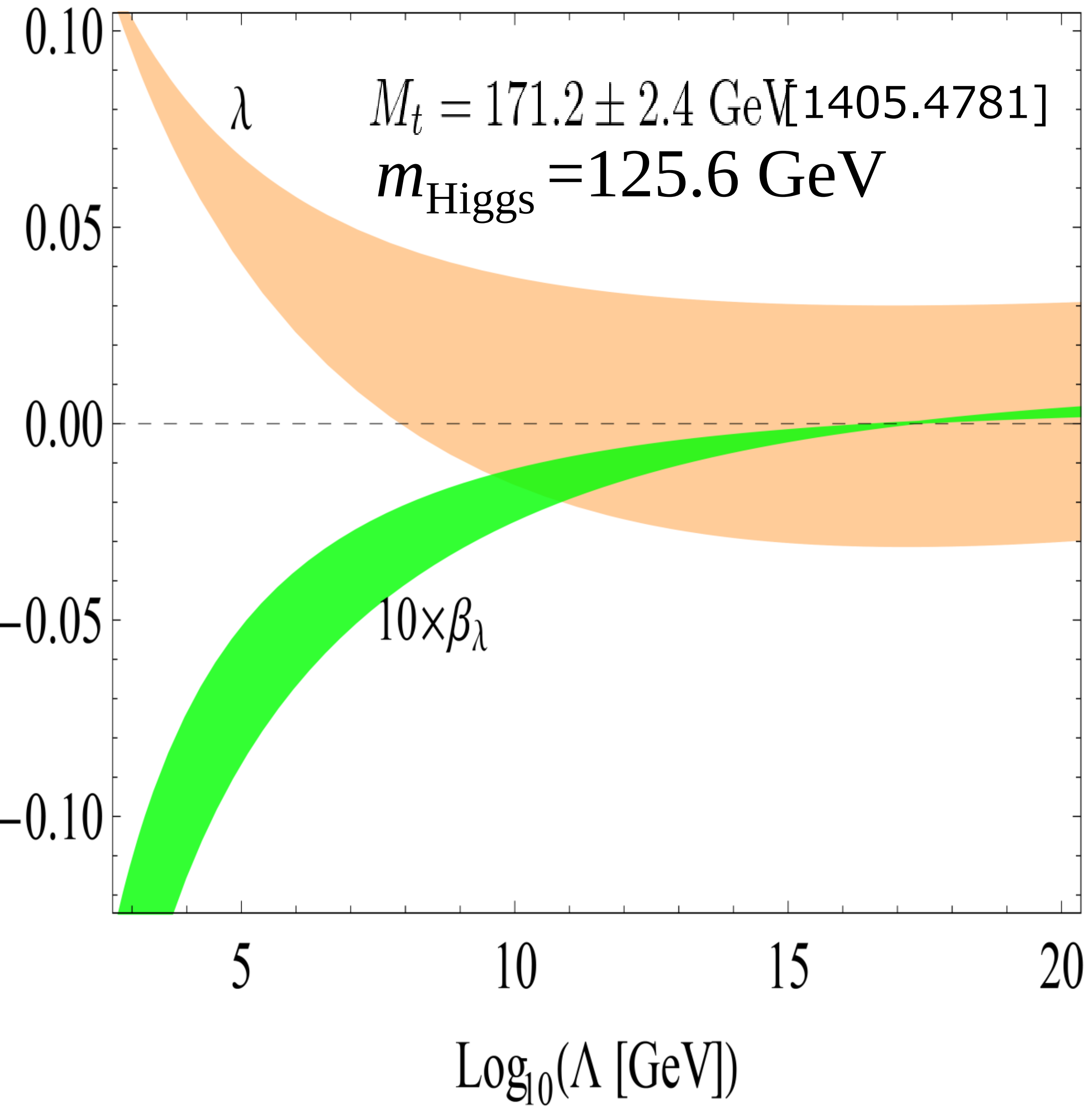
(2) The Higgs potential becomes flat (or zero) around the string scale.



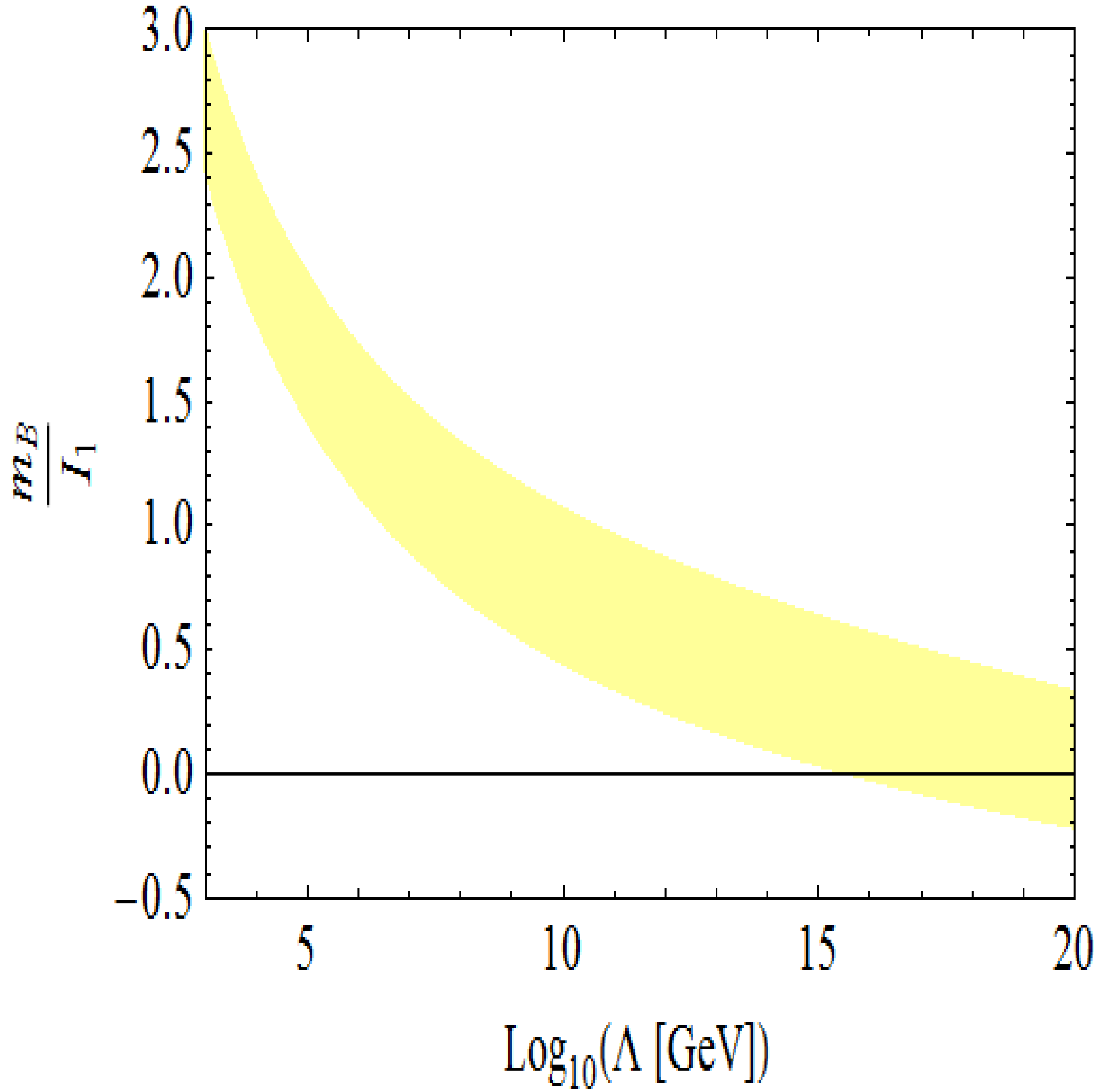
Froggatt and Nielsen '95.

Multiple Point Criticality Principle (MPP)

Higgs self coupling



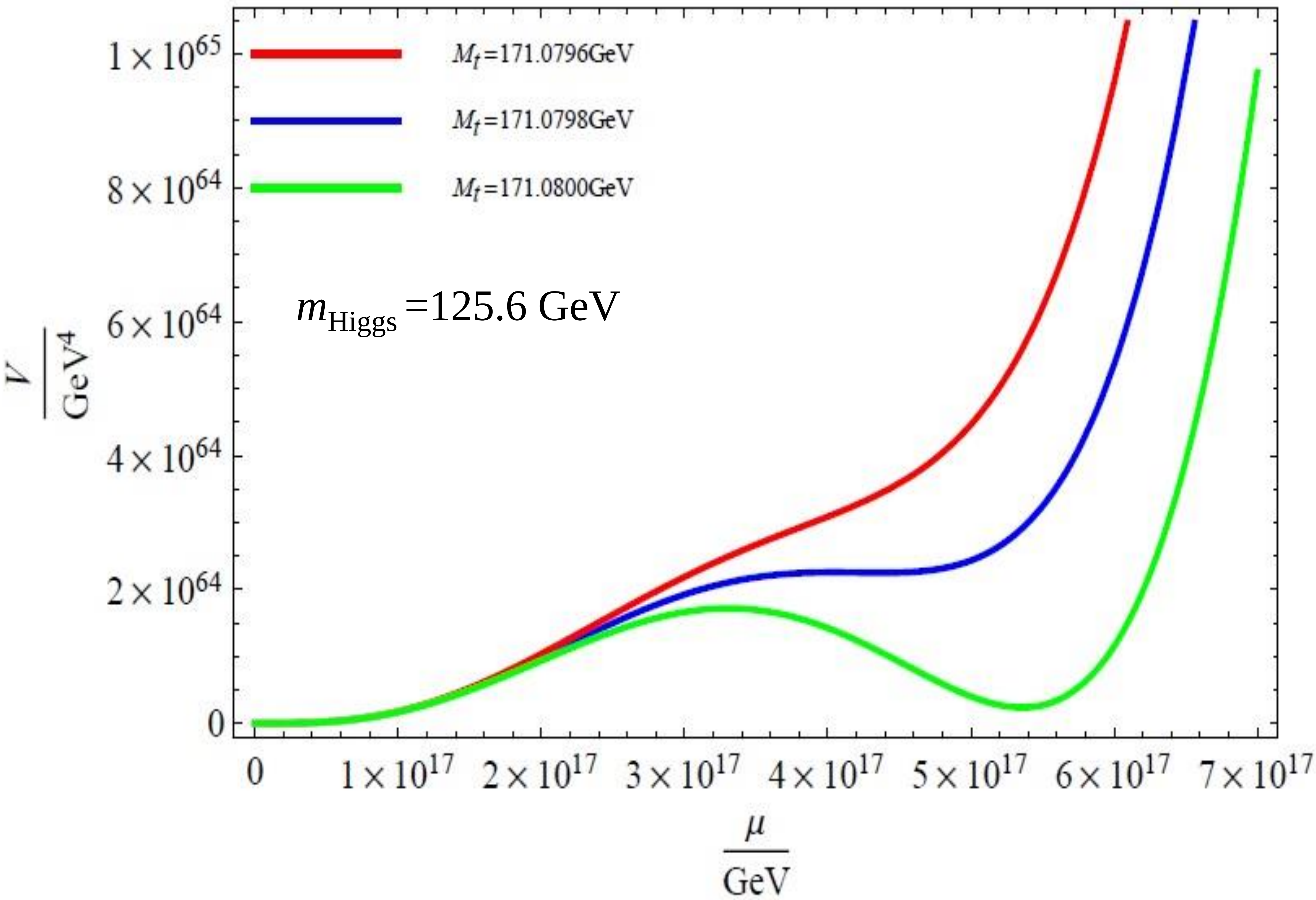
Bare mass



[Hamada, Oda, HK,1210.2538, 1308.6651]

Higgs potential

$$V(\varphi) = \frac{\lambda(\varphi)}{4}\varphi^4$$



Higgs inflation

Higgs potential may be flat around the string scale.

It suggests that the Higgs field can play the role of inflaton.

Here I will introduce two attempts.

(1) A toy model – Critical Higgs inflation

Hamada, Oda, Park and HK '14

Bezrukov,Shaposhnikov

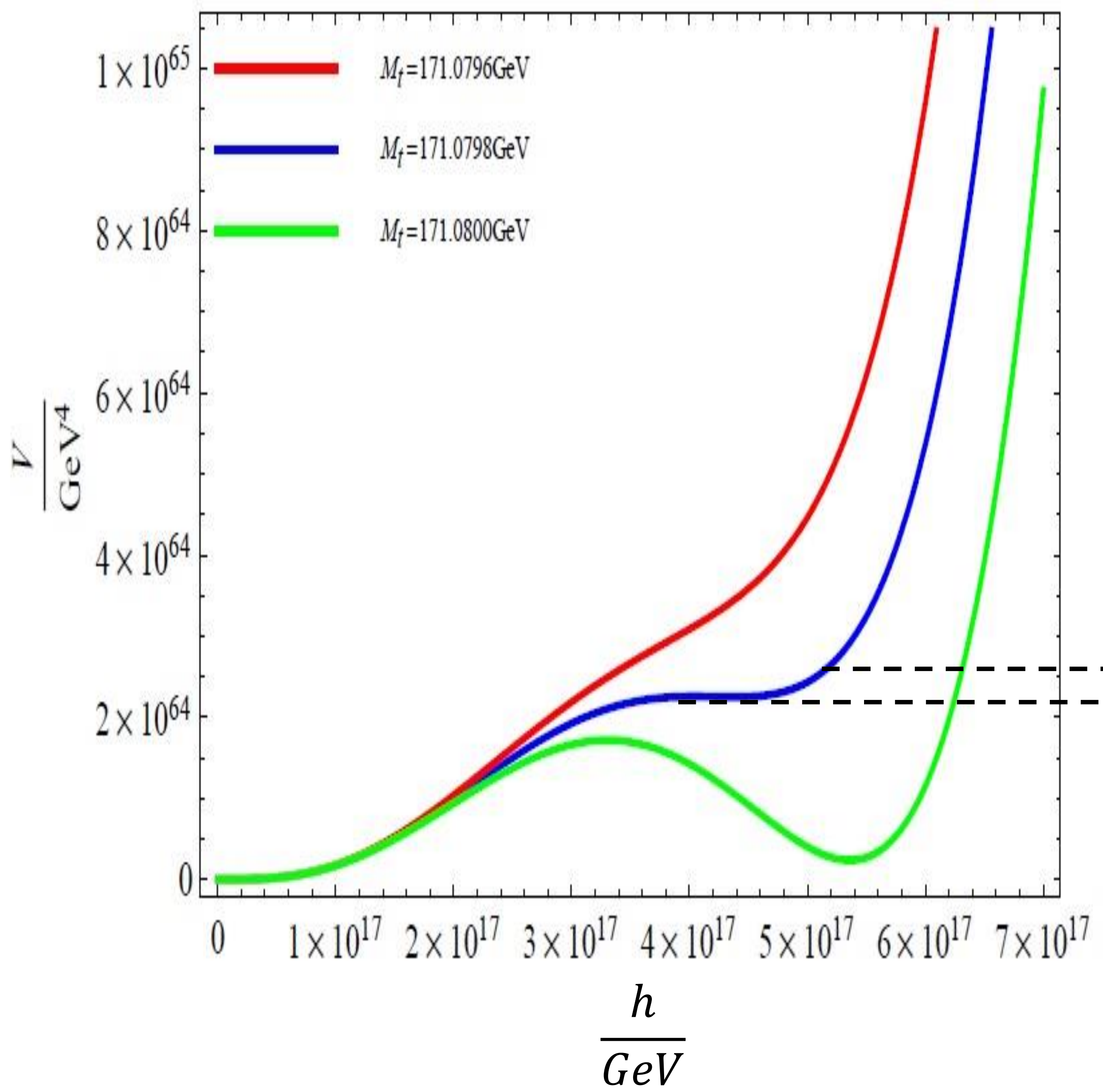
We assume

a) Nature does fine tunings so that the Higgs potential becomes flat around the string scale.

b) We can trust the Higgs potential including the string scale.

c) We introduce a non-minimal coupling $\xi R h^2$ of order $\xi \sim 10$.

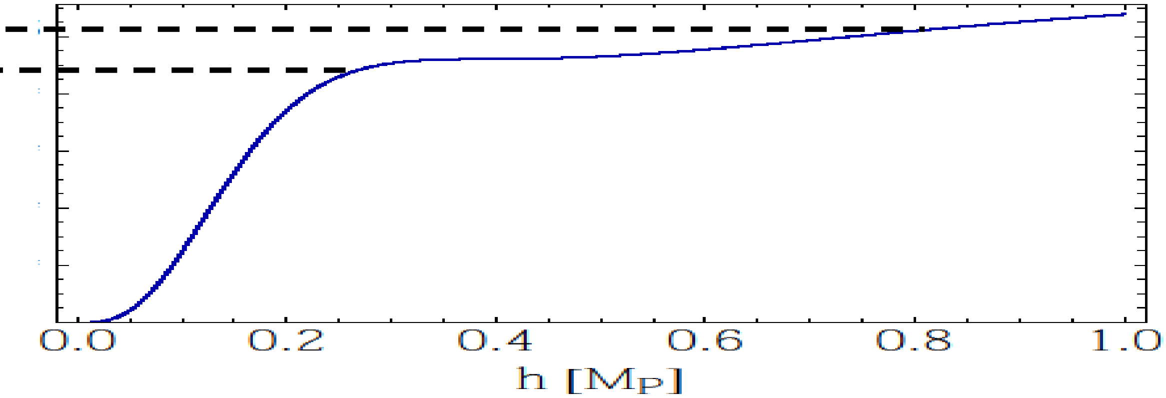
$\Rightarrow$ A realistic model can be constructed.



non-minimal coupling $\xi R h^2$

In the Einstein frame the effective potential becomes

$$V(\varphi_h), \quad \varphi_h = \frac{h}{\sqrt{1+\xi h^2/M_P{}^2}}.$$



We can make a realistic model of inflation.

ξ can be small as ~10.

(2) General bounds

Hamada, Nakanish, Oda and HK: arXiv: 1709.9035

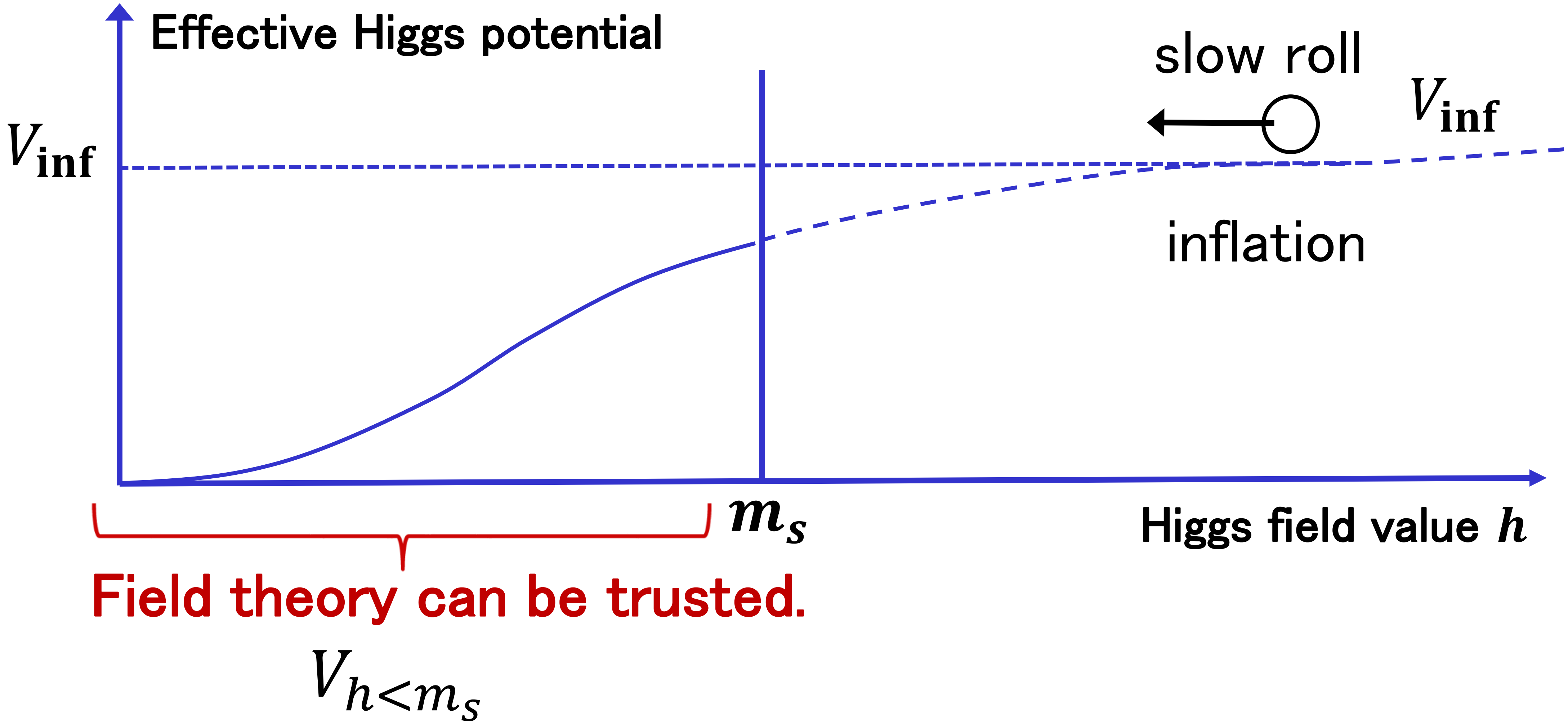
We trust the effective potential only below the string scale, and try to make bounds on the physical parameters.

We assume

a) Higgs field is the inflaton, and the inflation occurs beyond the string scale $m_s \sim 10^{17}\mathrm{GeV}$.

b) We can trust field theory below the string scale.

We then have a lower bound on the vacuum energy at the inflation.



Because h should roll down to 0, we have an equality

$$V_{\text{inf}} > V_{h<m_s}{}^{max} .$$

Because V_{inf} is proportional to the tensor perturbation as

$$A_t = 0.068\, V_{\text{inf}}/m_P^4 \quad (m_P = 2.4 \times 10^{18}\ \text{GeV}),$$

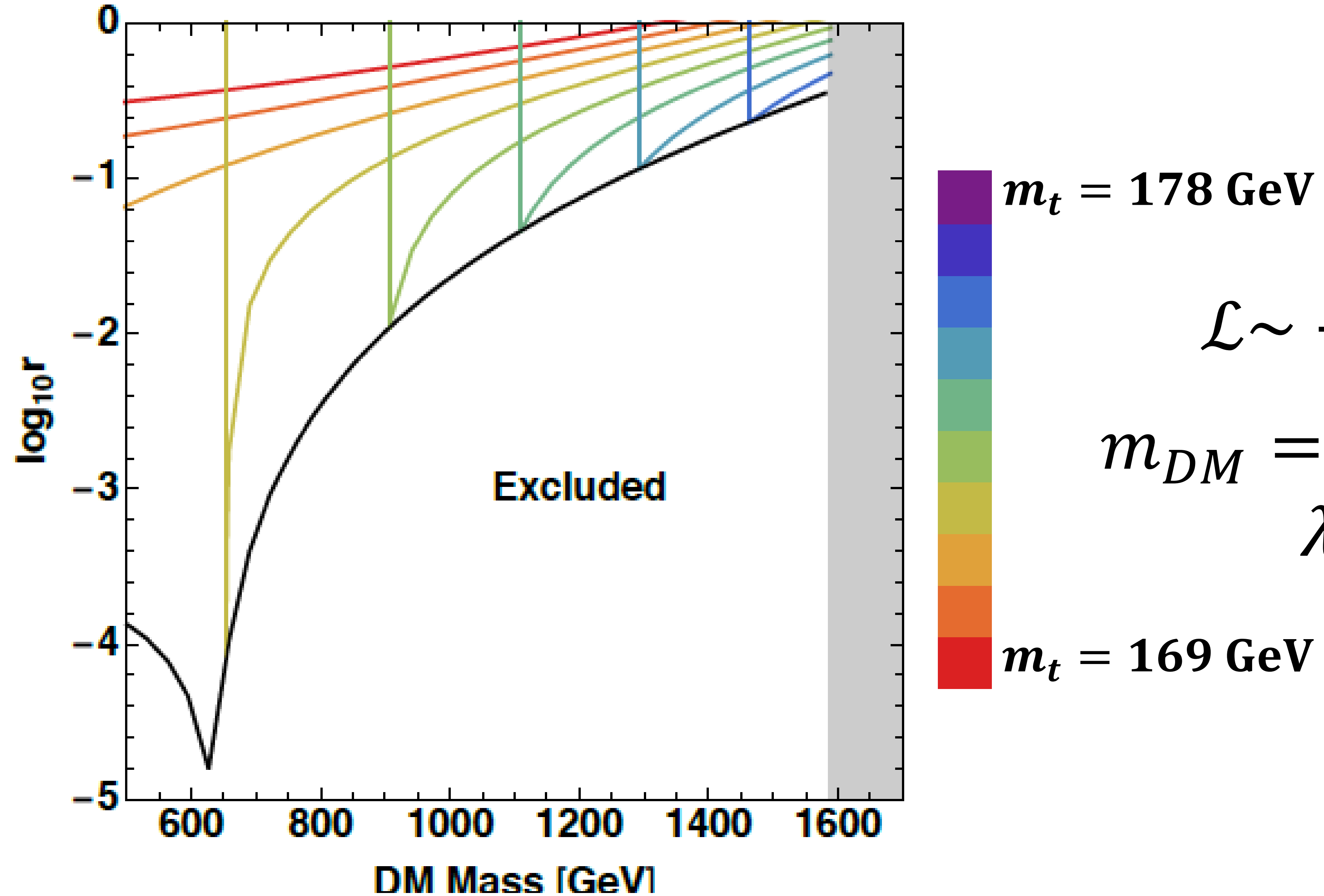
using the value $A_s = 2.2 \times 10^{-9}$, we have

$$r = \frac{A_t}{A_s} > V_{h<m_s}{}^{max} \ / \ \left(3.2 \times 10^{16}\text{GeV}\right)^4 .$$

This gives a rather strong constraint.

⇒ We can obtain bounds on possible modifications of SM.

(Ex.) SM + Higgs portal scalar dark matter



$$\mathcal{L} \sim -\frac{\kappa}{2} S^2 H^2$$

$$m_{DM} = \kappa \times 3.2 \text{ TeV}$$

$$\lambda_S = 0$$

Vertical line is from potential stability.

Upper region of “r” is allowed.

SM around Planck scale

- **Desert**

  SM is valid to the string scale at least theoretically.

  SM might be directly connected to string theory without large modification.

- **Marginal stability**

  Higgs field is near the stability bound.

- **Zero bare mass**

  The bare Higgs mass is close to zero at the string scale, which implies that Higgs is a massless state of string theory.

- **Flat potential and Higgs inflation**

  Higgs self coupling and its beta function become zero at the string scale.

  Higgs potential can be flat around the string scale, which suggests the Higgs inflation.

  Higgs inflation gives rather strong constraints on the modification of SM.

2. Fine tunings by nature itself

There are several attempts to extend the conventional framework of the local field theory in order to solve the fine tuning problem.

- asymptotic safety
  Weinberg, Shaposhnikov, …
- multiple point criticality principle
  Froggatt, Nielsen.
- classical conformality
  Bardeen
  Meissner, Nicolai,
  Foot, Kobakhidze, McDonald, Volkas
  Iso, Okada, Orikasa.
- baby universe and multi-local action
  Coleman
  Okada, Hamada, Kawana, Sakai, HK

They are related.

MPP of Froggatt and Nielsen

Imagine a system that is described by the path integral of not the canonical ensemble

$$\int [d\varphi] \exp\left(-S[\varphi]\right),$$

but the micro canonical ensemble

$$\int [d\varphi] \delta\left(S[\varphi] - C\right),$$

or an even more general ensemble (next slide)

$$\int [d\varphi]\, f\left(S_1[\varphi], S_2[\varphi], \cdots\right).$$

Still the system is equivalent to the ordinary field theory in the large space-time volume limit.

But the parameters of the corresponding field theory are automatically fixed such that the vacuum is at a (multiple) criticality point.

Integrating coupling constants

In fact we can show that the low energy effective theory of QG / string theory is given by the multi-local action:

Coleman '89
Tsuchiya-Asano-HK

$$S_{\text{eff}} = f\left(S_1, S_2, \cdots\right)$$
$$= \sum_i c_i S_i + \sum_{i\,j} c_{i\,j} S_i S_j + \sum_{i\,j\,k} c_{i\,j\,k} S_i S_j S_k + \cdots,$$
$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

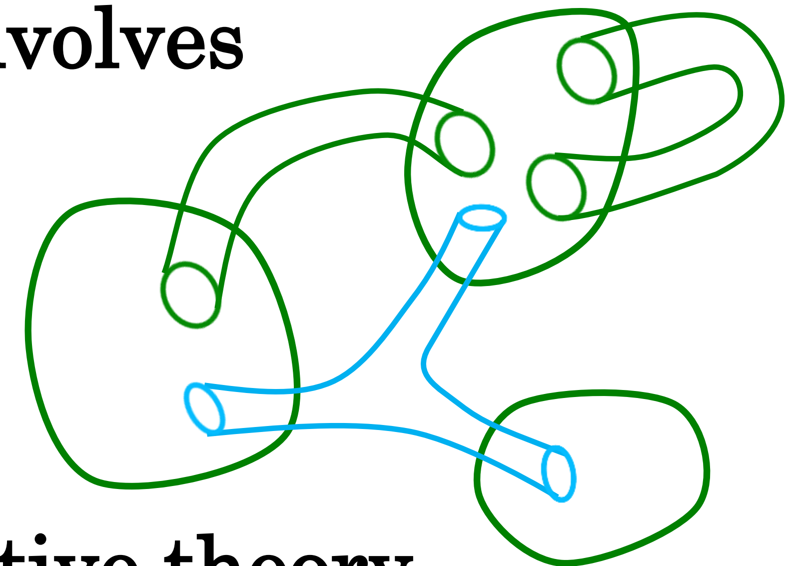
Here O_i are local scalar operators such as

$1\,,\, R\,,\, R_{\mu\nu}R^{\mu\nu}\,,\, F_{\mu\nu}\, F^{\mu\nu},\, \overline{\psi}\gamma^{\mu} D_{\mu}\psi\,,\, \cdots\,.$

Appendix
Low energy effective theory of quantum gravity

Coleman ('89)

Consider Euclidean path integral which involves the summation over topologies,



$$\sum_{\text{topology}} \int [dg] \exp(-S) \, .$$

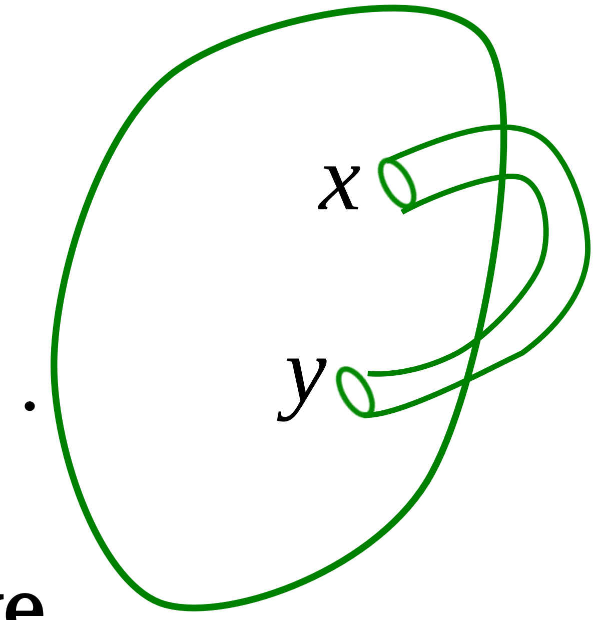
We consider the Wilsonian low energy effective theory after integrating out the short-distance configurations.

Among such configurations there should be a wormhole-like configuration in which a thin tube connects two points on the universe. Here, the two points may belong to either the same universe or different universes.

If we see such configuration from the side of the large universe(s), it looks like two small punctures.

But the effect of a small puncture is equivalent to an insertion of a local operator.

Therefore, after integrating out the metric of a wormhole, it contributes to the path integral as



$$\int [dg] \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y)\; \exp(-S)\,.$$

Summing over the number of wormholes, we have

$$\sum_{N=0}^{\infty} \frac{1}{n!} \left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y) \right)^n$$

$$= \exp\left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

Thus wormholes contribute to the path integral as

$$\int [dg] \exp\left(-S + \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)}\sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

bifurcated wormholes

⇒ cubic terms, quartic terms, …

Thus the effective action becomes a multi-local form

$$S_{\mathrm{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

End Appendix

Because S_{eff} is a function of S_i's , we can express $\exp(iS_{\text{eff}})$ by a Fourier transform as

$$\exp\left(iS_{eff}\left(S_1, S_2, \cdots\right)\right) = \int d\lambda\, w(\lambda_1, \lambda_2, \cdots) \exp\left(i \sum_i \lambda_i S_i\right),$$

where λ_i's are Fourier conjugate variables to S_i's, and w is a function of λ_i's .

Then the path integral for S_{eff} becomes

$$Z = \int [d\phi] \exp\left(iS_{\text{eff}}\right) = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i\right).$$

Because O_i are local operators,

$$\sum_i \lambda_i S_i = \int d^d x \sqrt{g(x)} \sum_i \lambda_i O_i(x)$$

is an ordinary local action where λ_i are regarded as coupling constants.

Therefore the system is the ordinary field theory, but we have to integrate over the coupling constants with some weight $w(\lambda)$.

Nature does fine tunings

We have seen

$$Z = \int [d\phi] \exp(iS_{\text{eff}}) = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right)$$

$$= \int d\lambda\, w(\lambda) Z(\lambda).$$

$$= Z(\lambda)$$

Ordinary field theory

But this theory is dangerous because the locality might be broken.

However, if a small region $\lambda \sim \lambda^{(0)}$ dominates the λ integral, it means that the coupling constants are fixed to $\lambda^{(0)}$, and the theory is equivalent to a local field theory.

Along this line, we can give some explanation to MPP.

Essence:

space−time volume

We can approximate $Z(\lambda) = \exp\left(-iVE_{vac}(\lambda)\right)$ **,**
because our universe has been cooled down for long time.

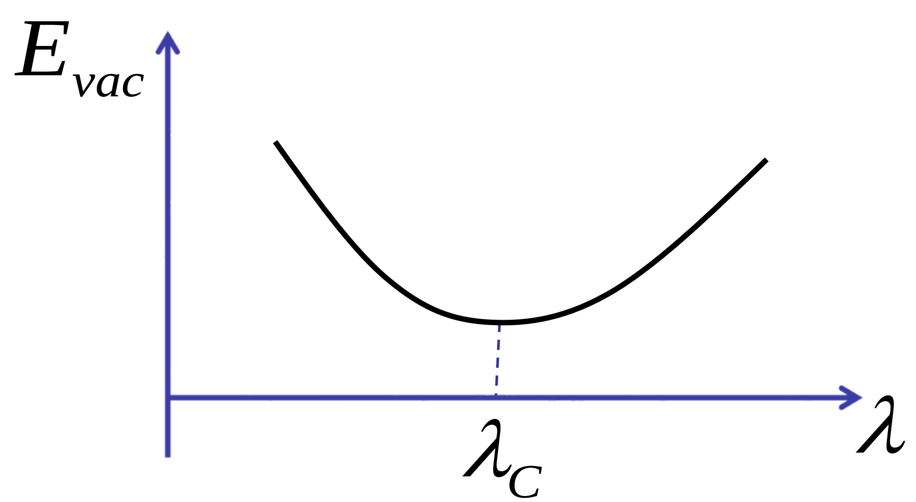
Some variations:

1) extremum

If $E_{vac}(\lambda)$ **is smooth and has an extremum at** λ_C **, the stationary point dominates and we have**

$$\exp\left(-iVE_{vac}(\lambda)\right) \sim \frac{\sqrt{2\pi}}{\sqrt{i\,V|E''(\lambda_c)|}}\,\delta(\lambda - \lambda_c) + O\left(\frac{1}{V}\right).$$

Thus λ **is fixed to** λ_C **in the limit** $V \to \infty$ **.**



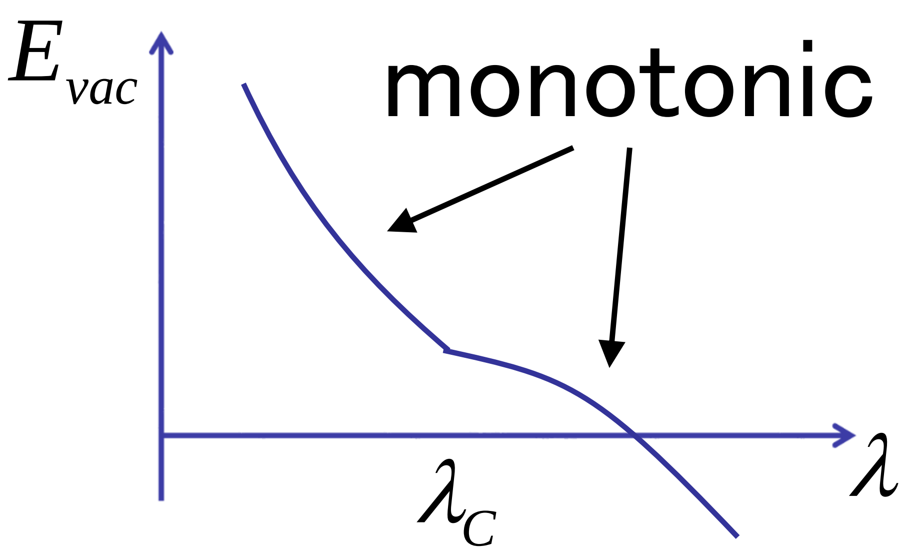
2) kink (need not be an extremum)

If $E_{vac}(\lambda)$ has a kink (first order phase transition), and monotonic in the both sides

$$Z(\lambda) = \exp\left(-iVE_{vac}(\lambda)\right)$$

$$\sim \frac{i}{V}\left(\frac{1}{E_{vac}'(\lambda_c+0)} - \frac{1}{E_{vac}'(\lambda_c-0)}\right)\delta(\lambda-\lambda_c) + O(\frac{1}{V^2})$$

Thus λ is fixed to λ_C in the limit $V \to \infty$. ⇒ original MPP



Proof

$$\int_a^b \mathrm{d}x \exp(iVf(x))\,\varphi(x)$$

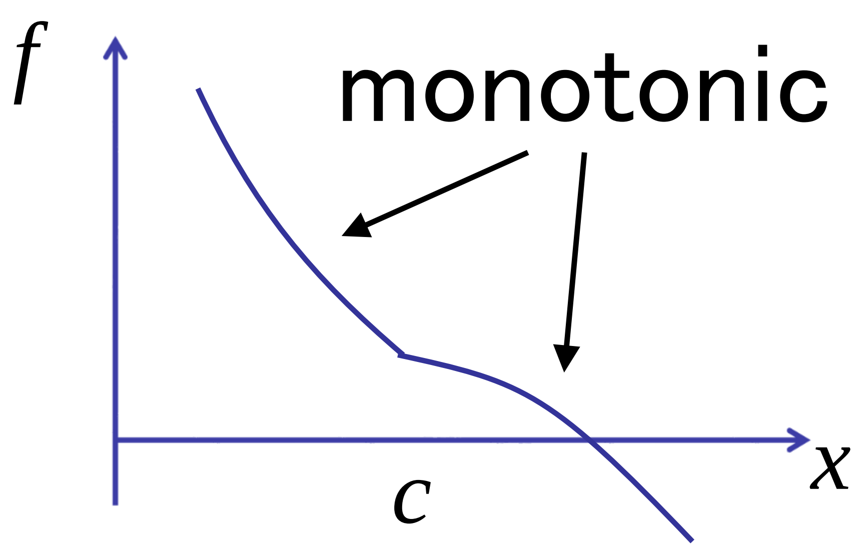
$$= \int_a^b \mathrm{d}x f'(x) \exp(iVf(x)) \frac{1}{f'(x)} \varphi(x)$$

$$= \left[\frac{1}{iV} \exp(iVf(x)) \frac{1}{f'(x)} \varphi(x)\right]_a^b + \mathrm{O}\left(\frac{1}{V^2}\right)$$

If the contributions from $\pm\infty$ is small,

$$\int_{-\infty}^{c} \mathrm{d}x \exp(iVf(x))\,\varphi(x) + \int_c^{\infty} \mathrm{d}x \exp(iVf(x))\,\varphi(x)$$

$$= \frac{1}{iV} \exp(iVf(x)) \left(\frac{1}{f'(c-0)} - \frac{1}{f'(c+0)}\right) \varphi(x) + \mathrm{O}\left(\frac{1}{V^2}\right)$$



Generalization

If we consider the time evolution of universe, the definition of $Z(\lambda)$ is not a priori clear.
For example, we need to specify the initial and final sates.

However, even if we do not know the precise form of $Z(\lambda)$, we expect that $Z(\lambda)$is determined by the late stage of the universe, because most of the space-time volume comes from the late stage.

From this we can make some predictions on λ's under some circumstances.
Here we consider two cases.

(1) Symmetry example θ_{QCD}

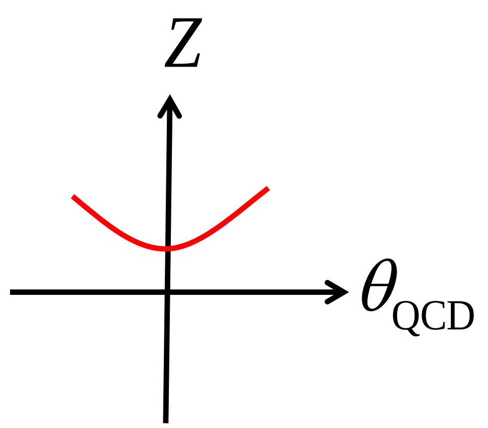
Nielsen, Ninomiya

1. It becomes important only after the QCD phase transition.
2. The masses and life-times of hadrons are invariant under

$$\theta_{QCD} \to -\theta_{QCD} \, .$$

⇒ We expect that Z is even in θ_{QCD}.

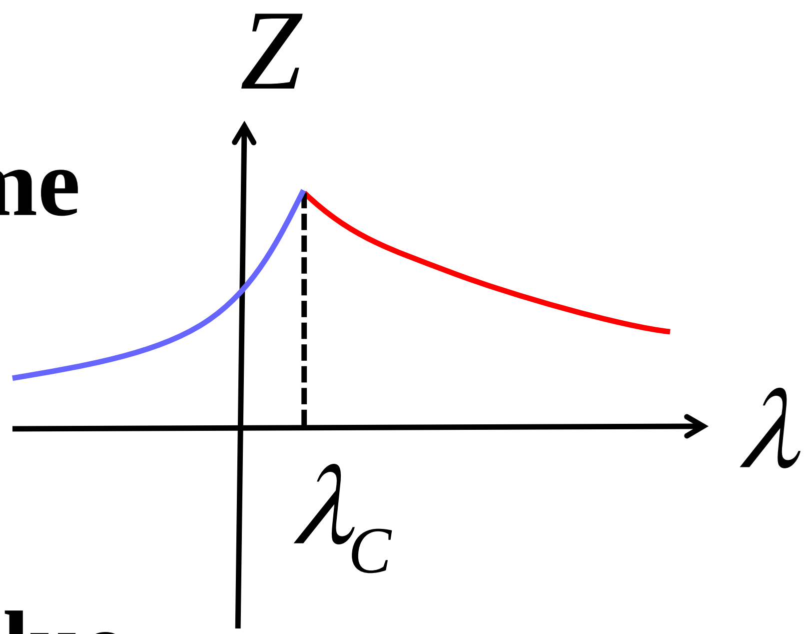
⇒ θ_{QCD} is tuned to 0 if Z behaves like



(2) Edge or drastic change

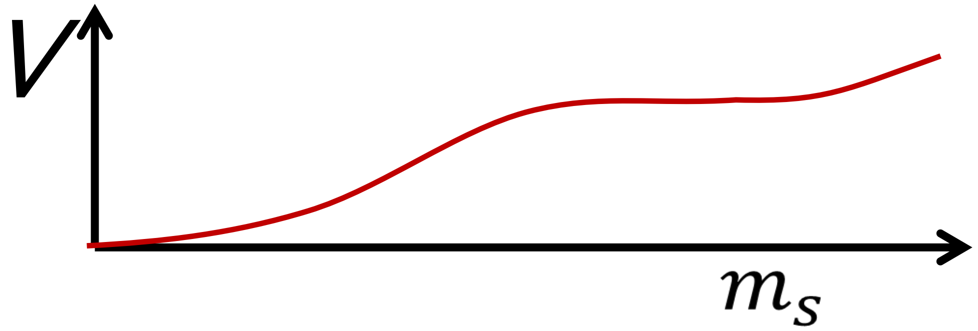
Conditions:

1. **Physics changes drastically at some value of the couplings.**
2. ***Z* is monotonic elsewhere.**

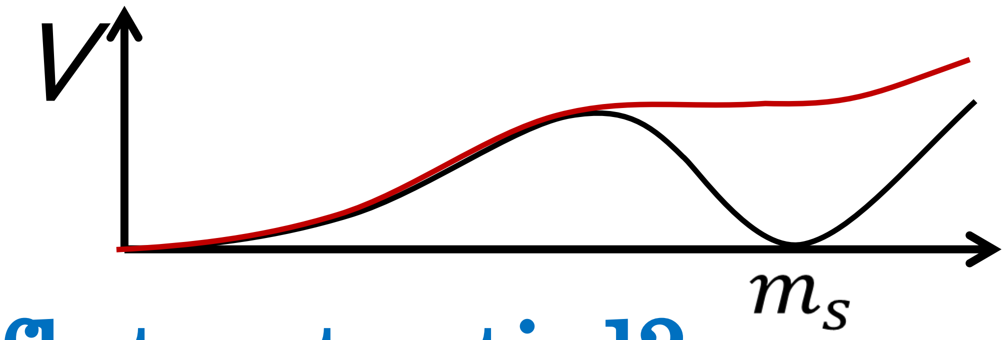


⇒ The couplings are tuned to the value, as we have seen for the case of kink.

Examples:
Cosmological constant,
Higgs inflation,
Classical conformality,
…



In this way we may introduce
the generalized MPP,
“ Coupling constants, which are relevant in low energy region, are tuned to values that significantly change the history of the universe when they are changed.”



Many open questions

- Degenerate vacuum or flat potential?
- Origin of the weak scale?
- Small cosmological constant?
- How many parameters are tuned?
  Too much big fix?

$\Rightarrow$ We need the precise form of $Z(\lambda)$.

$\Rightarrow$ We should investigate the wave function of multiverse.

Okada-HK

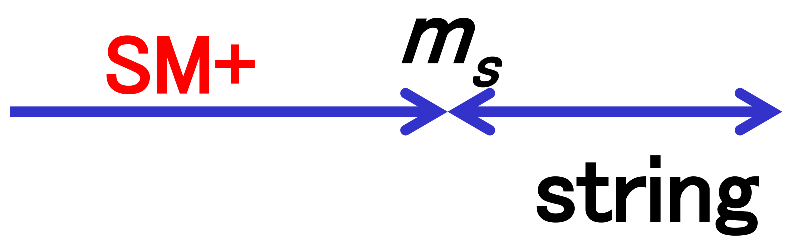
3. Emergence of weak scale from Planck scale

Based on a collaboration with J. Haruna, arXiv:1905.05656.

Weak scale as a non-perturbative effect

Basic assumptions:

(1) SM is directly connected to the string theory without large modification.



(2) The fundamental scale is only the Planck/string scale, which appears as the cut-off of the low energy effective field theory.

(3) Relevant operators (couplings with positive mass dimensions) are tuned by nature itself through the generalized MPP.

Question:

How does the weak scale appear?

Everybody's guess:

Weak scale should appear as a non-perturbative effect.

Then it is related to the Planck scale as

$$m_H = M_P \, e^{-\text{const.}/g_s}.$$

And the large hierarchy is naturally understood.

Problem:

Find a phenomenologically acceptable mechanism.

Various possibilities:

1. **QCD like dimensional transmutation.**

$$\Lambda_{\mathrm{QCD}} = \Lambda\, e^{-\mathrm{const.}/g_0^2}$$

Not compatible with weakly coupled Higgs.

2. **Coleman-Weinberg mechanism.**

 a) **Original idea is to explain SSB of SM from the massless Higgs.**
 Not acceptable. $m_H \ll v_H$

 b) **Additional gauge + complex scalar**
 Make a mass scale independently to the SM sector. Then transfer it to SM through VEV.
 Possible to make an acceptable model.

3. Even simpler (simplest) model.

Two real scalars. Adams, Tetradis, ⋯

$$\mathcal{L}_{\phi S} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial S)^2 - V$$

$$V = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

For a while we assume the $Z_2 \times Z_2$ invariance.

$$Z_2: \ \phi \to -\phi$$

$$Z_2: \ S \to -S$$

More important assumption is the classical conformality.

What is classical conformality?

Classical conformality

= “renormalized masses are 0”

This sounds nonsense for ordinary fields theorists: There is no quantum mechanical symmetry that guarantees masslessness of scalars except SUSY.

On the other hand, once we accept the existence of self-tuning mechanism, classical conformality is one of the natural choices.

More concretely, we can take the MPP:

“Coupling constants are fixed to critical points.”

As we have discussed, there are some variations in the meaning of “critical points”.

(1) Critical points for cosmological evolution

Critical values that the time evolution of universe changes drastically when they are changed.

Suppose that the universe starts from $\langle\phi\rangle = 0$.

$m^2 > 0 \Rightarrow \langle\phi\rangle = 0$ is (meta)stable.

$\Rightarrow$ Universe remains in that state for a while.

$m^2 < 0 \Rightarrow \langle\phi\rangle = 0$ is unstable.

$\Rightarrow$ Universe transitions quickly to another state.

So the time evolution of the universe changes drastically at the point $m^2 = 0$.

Thus "the classical conformality" is obtained.

(2) Critical points for the vacuum energy

Critical values that the phase of the vacuum changes when they are changed.

As we will see, there are various types of critical point for the two scalar system.

In any case, renormalized masses are fixed to some values, and we have similar predictions.

For a while, we concentrate on the case of classical conformality, and later we discuss the other criticalities.

3-1 SSB of the 2 scalar model

Two real scalar model

$$\mathcal{L}_{\phi S} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial S)^2 - \mathrm{V}$$

$$\mathrm{V} = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

Classical conformality:

$$\frac{\partial^2}{\partial\phi^2}V_{\mathrm{eff}}|_{\phi=S=0} = 0, \quad \frac{\partial^2}{\partial S^2}V_{\mathrm{eff}}|_{\phi=S=0} = 0.$$

Basic feature

For a large region of the parameter space, one of the fields has non-zero vacuum expectation value.

RG analysis

$$V = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

Beta functions:

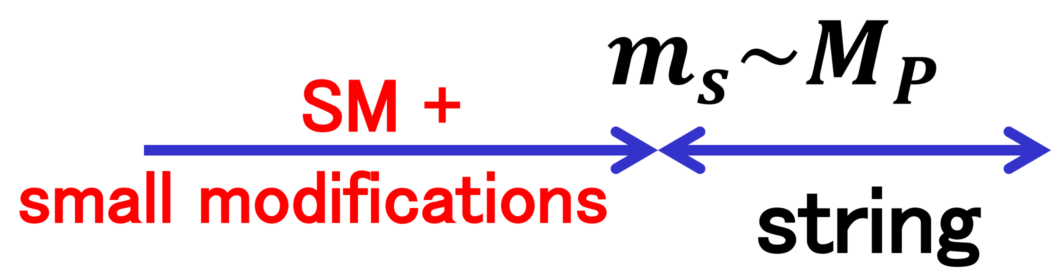
$$\beta_\rho = \frac{3}{16\pi^2}\left(\rho^2 + \kappa^2\right)$$

$$\beta_{\rho'} = \frac{3}{16\pi^2}\left(\rho'^2 + \kappa^2\right)$$

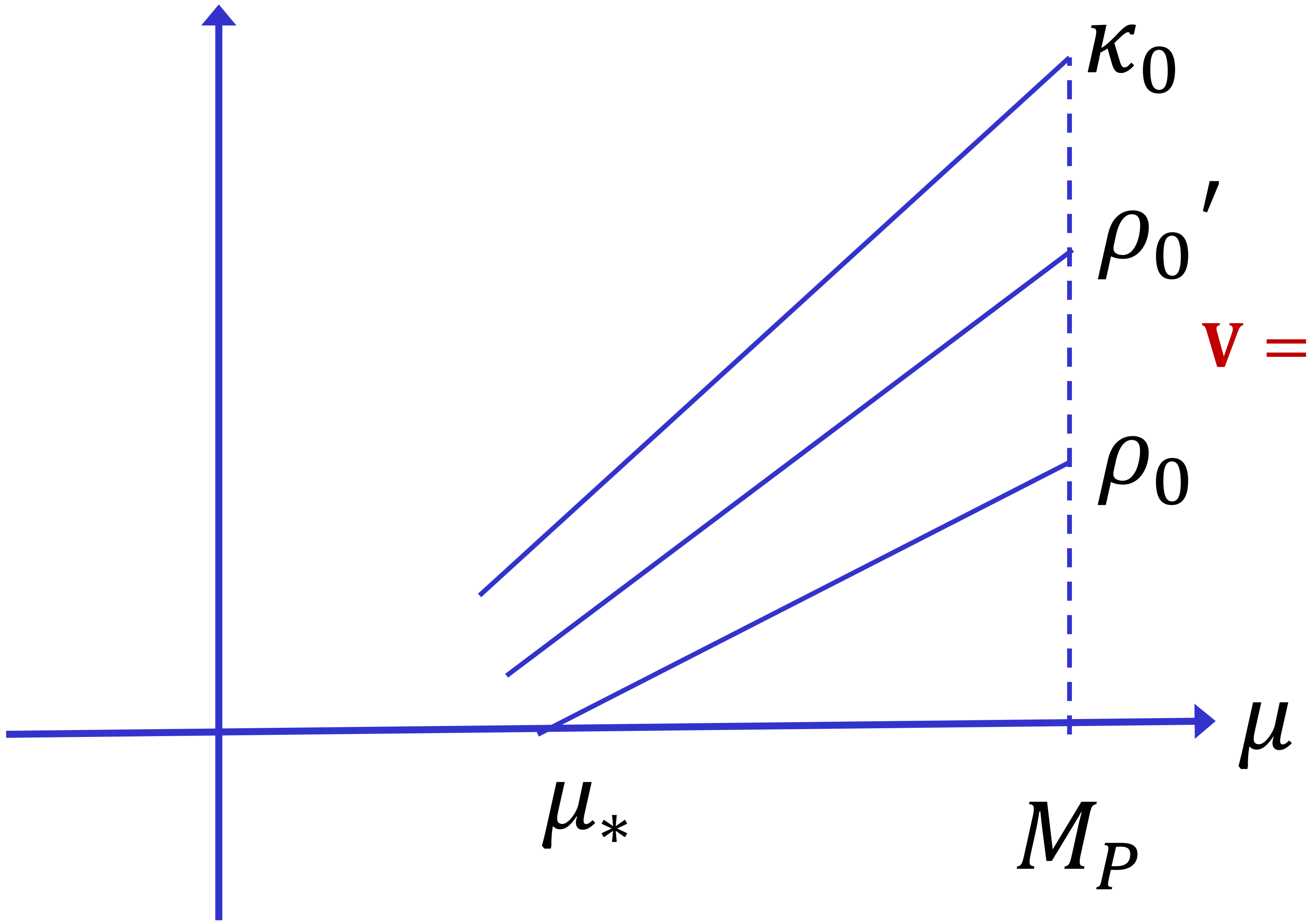
$$\beta_\kappa = \frac{1}{16\pi^2}\left(\rho\kappa + \rho'\kappa + 4\kappa^2\right)$$

Assumption: cut off at Planck scale,

$$\rho_0, \rho'_0, \kappa_0 > 0.$$



When we decrease the renormalization point, one of the couplings becomes zero.



$$V = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

We assume ρ becomes zero first at $\mu = \mu_*$.

This is possible if $\kappa_0 \gg \rho_0' > \rho_0$.

Then it is expected

$\langle\phi\rangle \neq 0 \Rightarrow S$ becomes massive through $\frac{\kappa}{4}\phi^2 S^2$

$$\Rightarrow \langle S\rangle = 0$$

One-loop effective potential

Effective potential for $\langle S\rangle = 0$:

$$V = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

$V_{\text{eff}}(\phi, S = 0)$

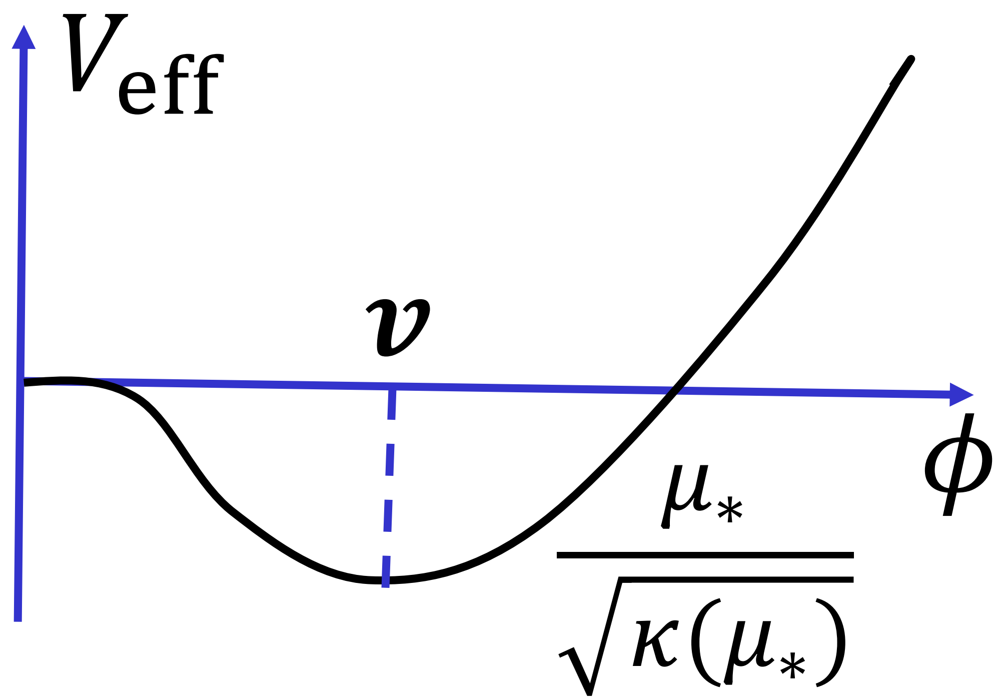
ϕ loop ↓ $\qquad$ S loop ↓

$$= \frac{\rho(\mu)}{4!}\phi^4 + \frac{\rho(\mu)^2}{256\pi^2}\phi^4 \log\left(\frac{\rho(\mu)\phi^2}{\mu^2}\right) + \frac{\kappa(\mu)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu)\phi^2}{\mu^2}\right)$$

If we take $\mu = \mu_*$, this becomes

$$= \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$

$$\Rightarrow \langle\phi\rangle = v = \text{const.}\frac{\mu_*}{\sqrt{\kappa(\mu_*)}}$$



A mass scale v emerges.

Relation between v and M_P

For simplicity we consider the case

$$\rho_0 < \rho_0' \ll \kappa_0 \ll 1.$$

$$\beta_\rho = \frac{3}{16\pi^2}\left(\rho^2 + \kappa^2\right)$$

$$\beta_\kappa = \frac{1}{16\pi^2}\left(\rho\kappa + \rho'\kappa + 4\kappa^2\right)$$

Then for $\mu_* \le \mu \le M_P$

$$\kappa(\mu) \sim \kappa_0, \;\; \beta_\rho \sim \frac{3\kappa_0^2}{16\pi^2}.$$

$$\Rightarrow \rho(\mu) \sim \rho_0 + \frac{3\kappa_0^2}{16\pi^2}\log\left(\frac{\mu}{M_P}\right)$$

Thus we have

$$\rho(\mu_*) = 0 \Rightarrow \mu_* \sim M_P \exp\left(-\frac{16\pi^2}{3}\frac{\rho_0}{\kappa_0^2}\right)$$

$$\Rightarrow v \sim M_P \frac{1}{\sqrt{\kappa_0}} \exp\left(-\frac{16\pi^2}{3}\frac{\rho_0}{\kappa_0^2}\right)$$

↑ Non-perturbative

Masses of the particles

mass of ϕ : $M_\phi^2 = \frac{d^2}{d\phi^2} V_{\rm eff} \Big|_{\phi=v} = \frac{\kappa(\mu_*)^2}{32\pi^2} v^2$

$$V_{\rm eff}(\phi, S=0) = \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$

mass of S: $M_S^2 = \frac{\kappa(\mu_*)}{2} v^2$

$$\mathrm{V} = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!} S^4$$

If $\kappa \ll 1$, the general pattern is

$$v \gg M_S \gg M_\phi \,.$$

For example,

$$\kappa(\mu_*) = 0.1 \Rightarrow v : M_S : M_\phi = 1 : 0.2 : \ 0.006 \,.$$

3-2 Coupling to SM

Incorporating two real scalar model into SM

Farzinnia-He-Ren, Sannino-Virkajarvi.
K.Ghorbani-H.Ghorbani, Jung-Lee-Nam.

Total action:

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\phi S} - \mathrm{V}_{\mathrm{H}\phi S}$$

$$\mathrm{V}_{\mathrm{H}\phi S} = -\frac{\eta}{2}\phi^2|\mathrm{H}|^2 + \frac{\eta'}{2}S^2|H|^2$$

We assume that $\mathcal{L}_{SM}$ does not contain the Higgs mass term because of the classical conformality.

In general, we should consider the mixing between Higgs and ϕ:

$$\tan(\theta) \sim \frac{v_H}{v}\,, \qquad v_H \sim 250\mathrm{GeV}, \quad v = \langle\phi\rangle.$$

If $v \gg v_H$, the mixing is small.

In that case, the Higgs potential is given by

$$\lambda|H|^4 - \frac{\eta}{2}\langle\phi\rangle^2|\mathrm{H}|^2,$$

which means

$$V_{H\phi S} = -\frac{\eta}{2}\phi^2|\mathrm{H}|^2 + \frac{\eta'}{2}S^2|H|^2$$

$$m_H^2 = \eta\, v^2,$$

and the rest is the same as SM:

$$\langle H\rangle = \frac{2m_H}{\sqrt{\lambda}}.$$

Weak scale is generated non-perturbatively from the Planck scale, as

$$M_P \to \langle\phi\rangle \to \langle H\rangle.$$

S as dark matter

Property of S

no vev: $\langle S \rangle = 0$

heavy but not too heavy: $M_S^2 = \frac{\kappa(\mu_*)}{2} v^2$

couples to Higgs: $\frac{\eta'}{2} S^2 |H|^2$

It is natural to regard S as the Higgs portal scalar dark matter.

Parameters of the model

Two scalar model has 3 parameters ρ, ρ', κ in addition to M_P.

$$V = \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

ρ is replaced by $\langle\phi\rangle = v$.

κ gives the ratios of M_ϕ, M_S, v .

ρ' is the self coupling of S.

$$V_{H\phi S} = -\frac{\eta}{2}\phi^2|H|^2 + \frac{\eta'}{2}S^2|H|^2$$

Coupling to the Higgs has 2 parameters.

η is determined by $m_H^2 = \eta\, v^2$.

η' gives coupling between Higgs and S .

$\Rightarrow$ **Only κ is new compared with the Higgs portal scalar dark matter scenario.**

Examples

(1) $\kappa = 0.1, M_S = 1\text{TeV}$

$\Rightarrow M_\phi = 25\text{GeV}, v = 4.5\text{TeV}, \theta = 0.057$

(2) $\kappa = 0.1, M_S = 6\text{TeV}$

$\Rightarrow M_\phi = 151\text{GeV}, v = 27\text{TeV}, \theta = 0.020$

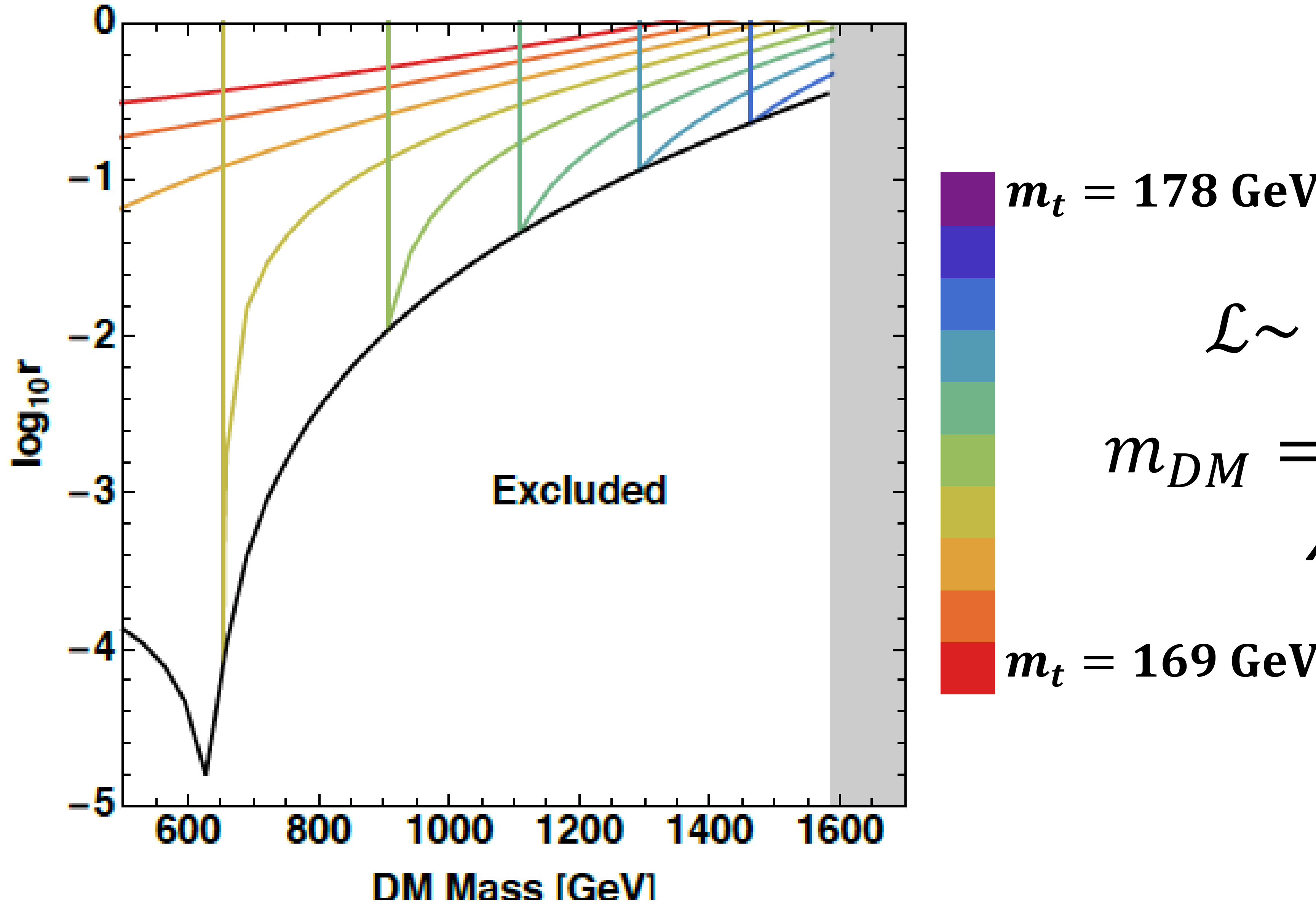
If we assume the Higgs inflation, (1) seems typical. (next slide)

It predicts a light scalar ~ 25GeV, which may be tested in near future experiments.

This gives a rather strong constraint.

⇒ We can obtain bounds on possible modifications of SM.

(Ex.) SM + Higgs portal scalar dark matter



$$\mathcal{L} \sim -\frac{\kappa}{2} S^2 H^2$$

$$m_{DM} = \kappa \times 3.2\ \mathrm{TeV}$$

$$\lambda_S = 0$$

Vertical line is from potential stability.

Upper region of “r” is allowed.

3-3 Other criticalities than classical conformality

Two real scalar model revisited

$$\mathcal{L}_{\phi S} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial S)^2 - \mathrm{V}$$

$$\mathrm{V} = \frac{m_0^2}{2}\phi^2 + \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

As we have discussed,

classical conformality is the assumption that the bare mass should be tuned so that the renormalized mass becomes zero:

$$\mathrm{m}^2 = 0.$$

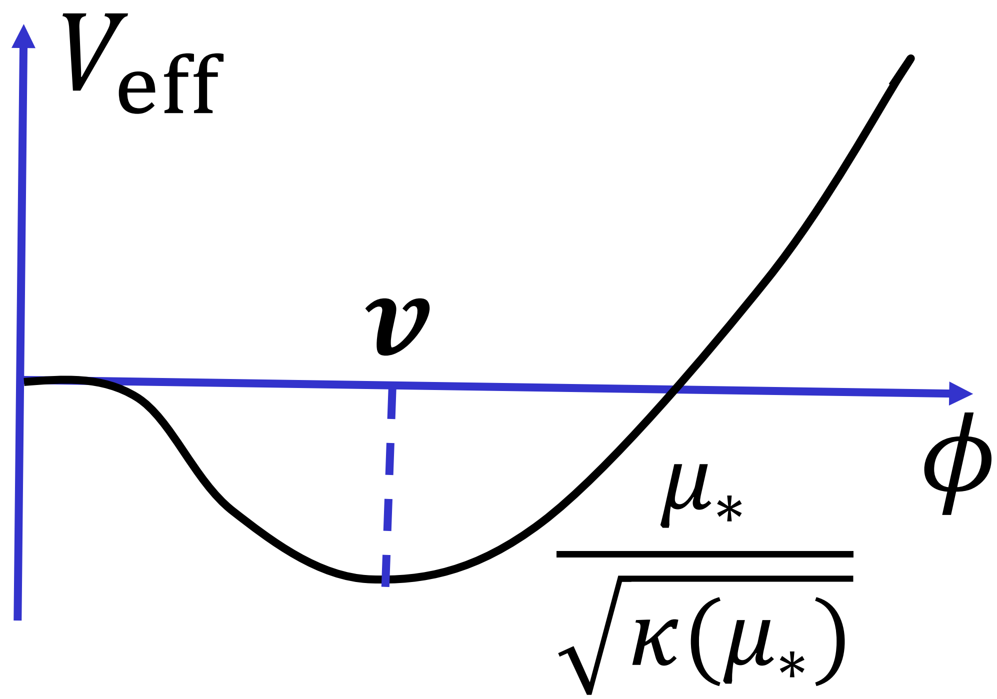
This is a critical point in that the time evolution of universe drastically changes at $\mathrm{m}^2 = 0$.

But there is another kind of critical point, that is, the 1-st order phase transition point.

$$V_{\text{eff}}(\phi, S=0) =$$

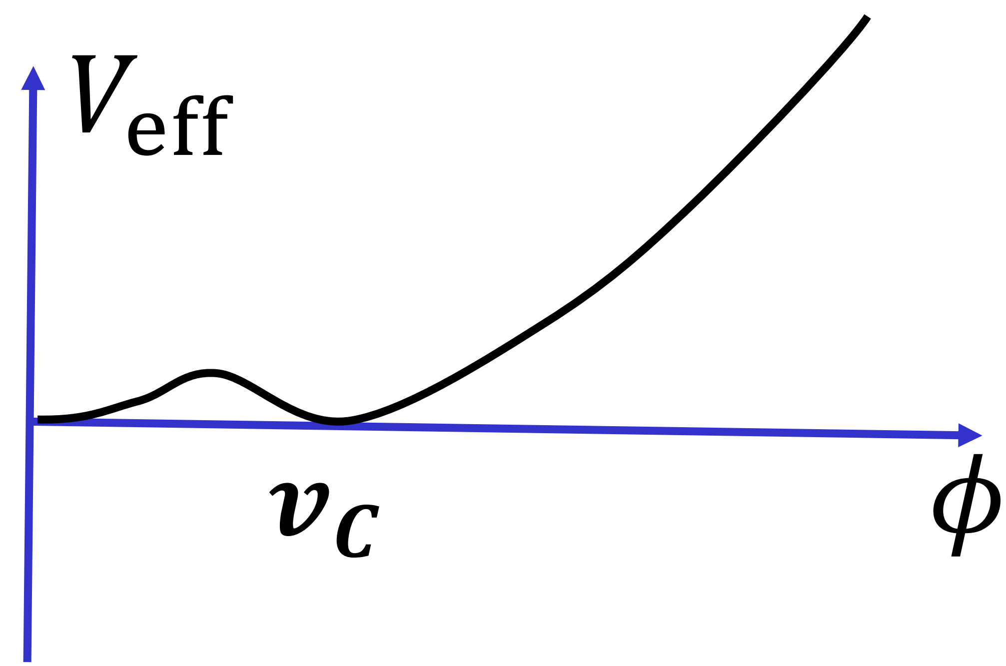
<u>**Classical conformality**</u>

$$\frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$



<u>**1-st order phase transition**</u>

$$\frac{m_C^2}{2}\phi^2 + \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$



$$v_C = v/\sqrt{e}$$

$$M'_\phi = M_\phi/\sqrt{2}$$

1-st order phase transition point is as plausible as classically conformality.

We can not tell which one is favored by nature unless we know the precise mechanism of MPP.

At any rate,

the generated mass scales v, M_ϕ, M_S change only by numerical factors.

Again we can say that the weak scale emerges from the Planck scale.

Further generalization

So far we have assumed $Z_2 \times Z_2$ symmetry.

Here we assume Z_2 only for S .

Then the action has 5 parameters with positive mass dimensions (relevant parameters):

$$\mathcal{L}_{\phi S} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial S)^2 - \mathrm{V}$$

$$\mathrm{V} = g\phi + \frac{m^2}{2}\phi^2 + \frac{h}{3!}\phi^3 + \frac{\sigma}{2}\phi S^2 + \frac{m'^2}{2}S^2 + \frac{\rho}{4!}\phi^4 + \frac{\kappa}{4}\phi^2 S^2 + \frac{\rho'}{4!}S^4$$

We can set $g = 0$ by shifting ϕ.

$\Rightarrow$ 4 relevant parameters.

Here we assume that all the relevant parameters are fixed by the generalized MPP.

The problem is to find tetra critical points in the space of 4 parameters m^2, h, σ, m'^2 .

$$V = \frac{m^2}{2}\phi^2 + \frac{h}{3!}\phi^3 + \frac{\sigma}{2}\phi S^2 + \frac{m'^2}{2}S^2 + \dots$$

Instead of seeking the general solutions, here we construct a special solution.

First we take the conditions $m^2 = m'^2 = 0$.

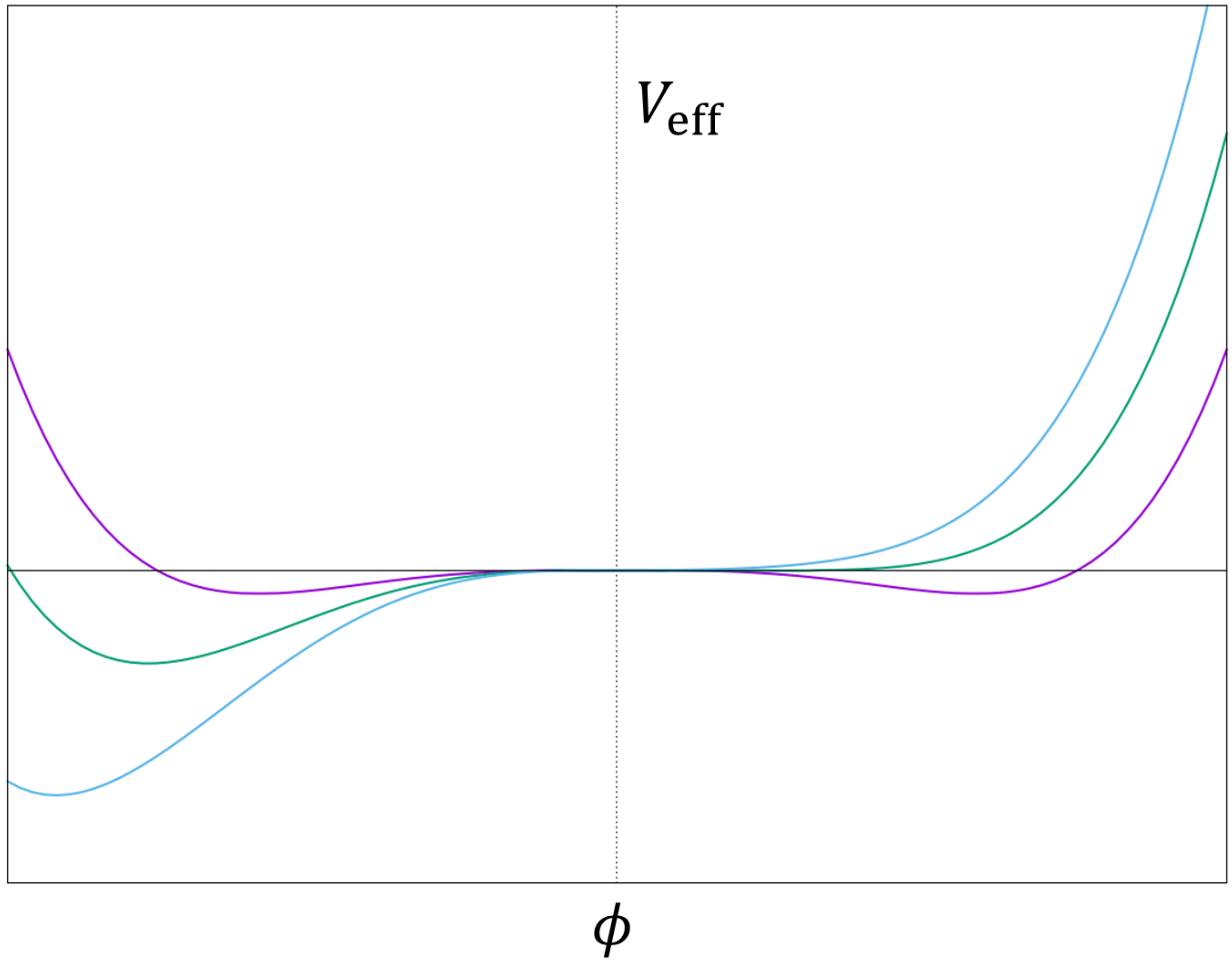
In fact these are criticality conditions because the behavior of $V_{eff} \sim \frac{m^2}{2}\phi^2 + \frac{m'^2}{2}S^2$ around $\phi = S = 0$ changes drastically depending on the signs of m^2 and m'^2 .

$$V = \frac{m^2}{2}\phi^2 + \frac{h}{3!}\phi^3 + \frac{\sigma}{2}\phi S^2 + \frac{m'^2}{2}S^2 + \dots$$

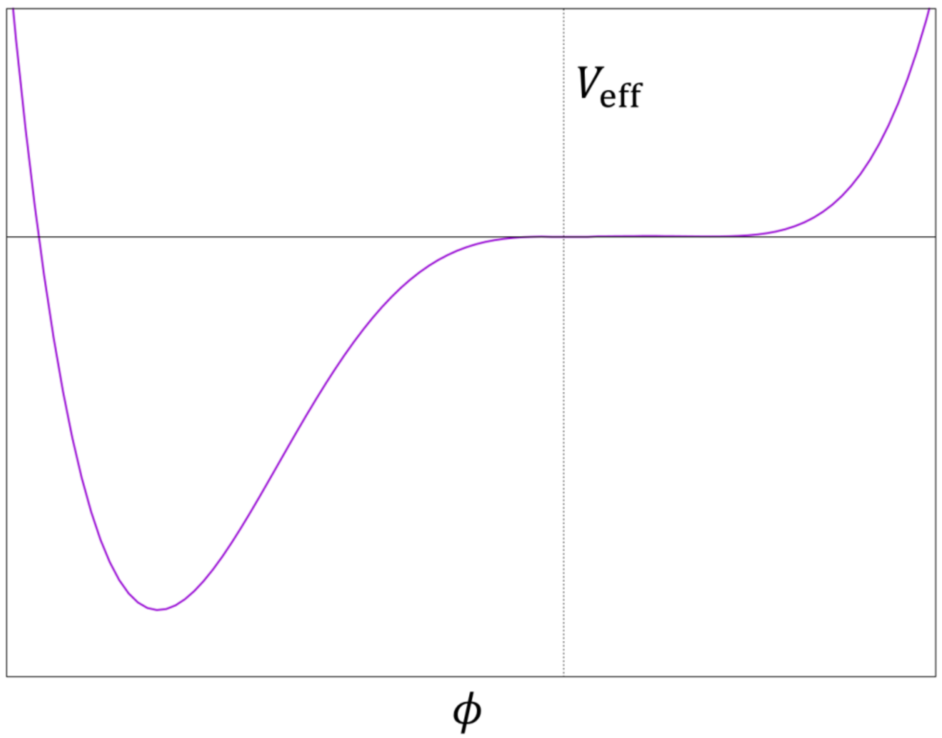
Then we take the condition $\sigma = 0$.
Again this is a criticality condition because the behavior of $V_{\text{eff}} \sim \frac{h}{3!}\phi^3 + \frac{\sigma}{2}\phi S^2$ changes drastically depending on the signs of σ .

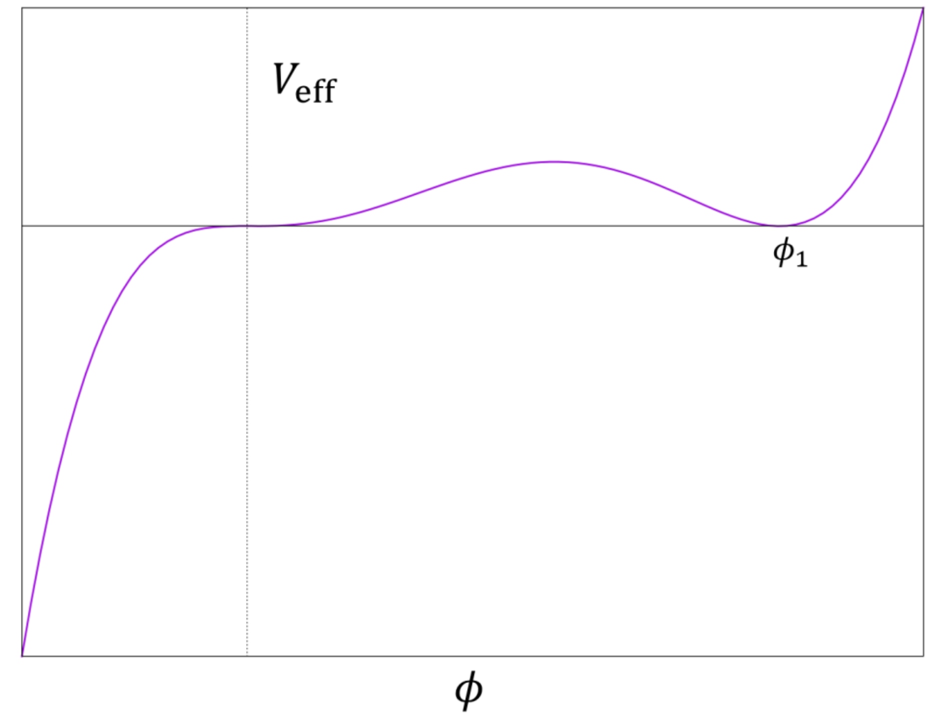
Then the only remaining parameter is h.
We determine it by the criticality condition of the effective potential as in the case of 1-st order phase transition:

$$V_{\text{eff}}(\phi, S = 0) = \frac{h}{6}\phi^3 + \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$

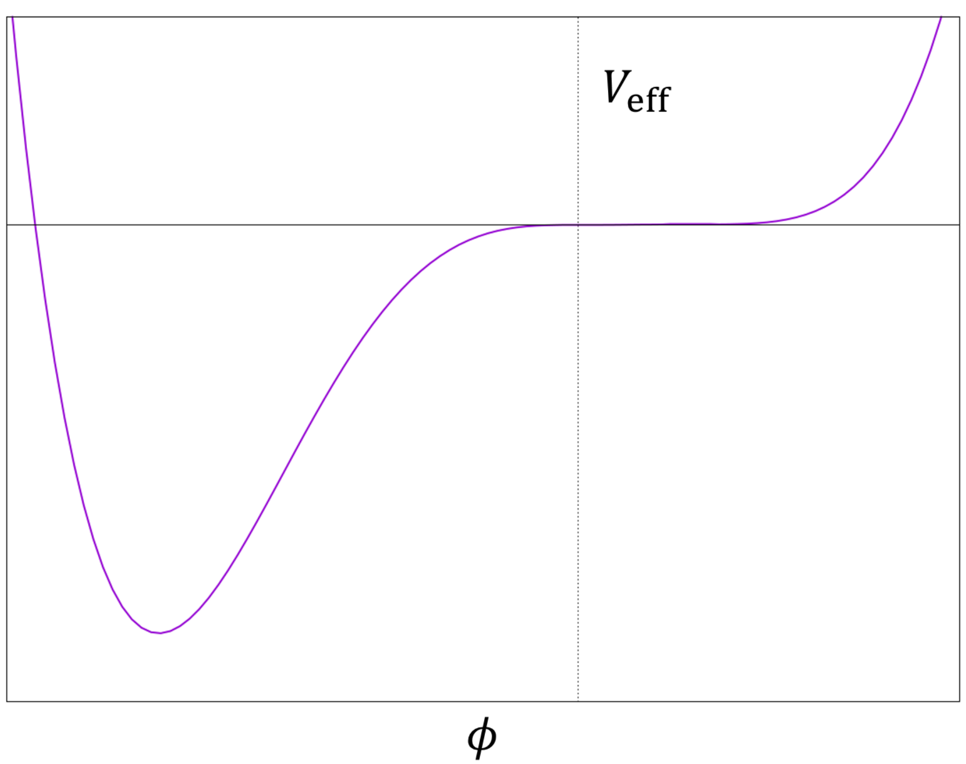


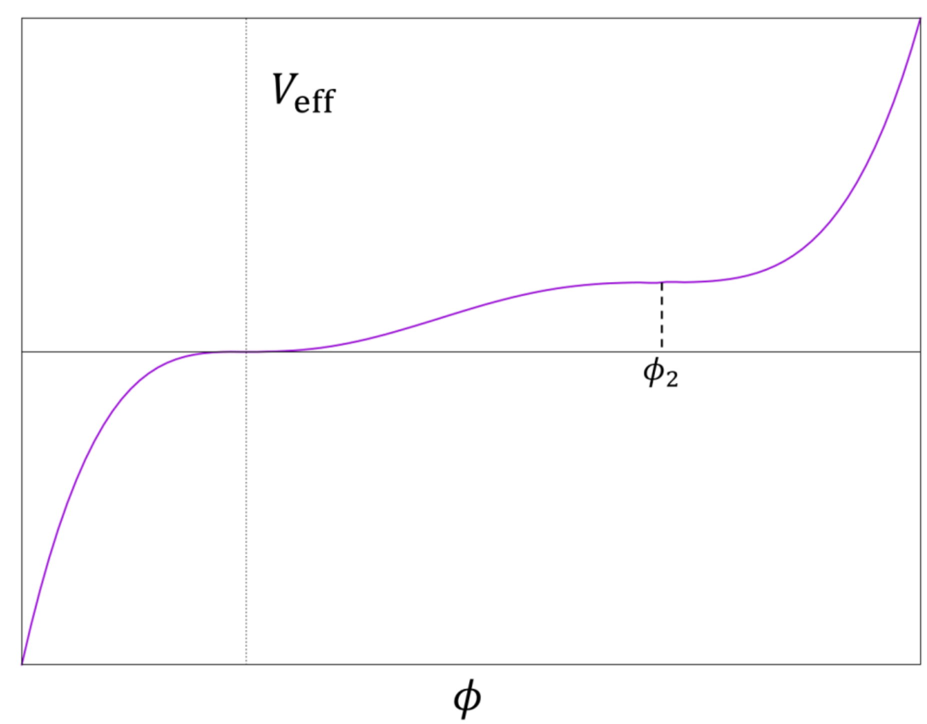
$$V_{\text{eff}}(\phi, S = 0) = \frac{h}{6}\phi^3 + \frac{\kappa(\mu_*)^2}{256\pi^2}\phi^4 \log\left(\frac{\kappa(\mu_*)\phi^2}{\mu_*^2}\right)$$





$$h_1 = 0.71\,\frac{\kappa^2 v}{32\pi^2}, \quad \phi_1 = 0.47v$$





$$h_2 = 0.74\,\frac{\kappa^2 v}{32\pi^2}, \quad \phi_2 = 0.37v$$

These do not have Z_2 symmetry for ϕ.
$\Rightarrow$ No cosmological domain wall problem.

Mass scales v, M_ϕ, M_S are similar to the previous ones.

Again the weak scale emerges from the Planck scale non-perturbatively.

Summary

In wide classes of quantum gravity or string theory, the low energy effective action has **the multi local form**:

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots .$$

The fine tuning problem might be solved by the dynamics of such action.

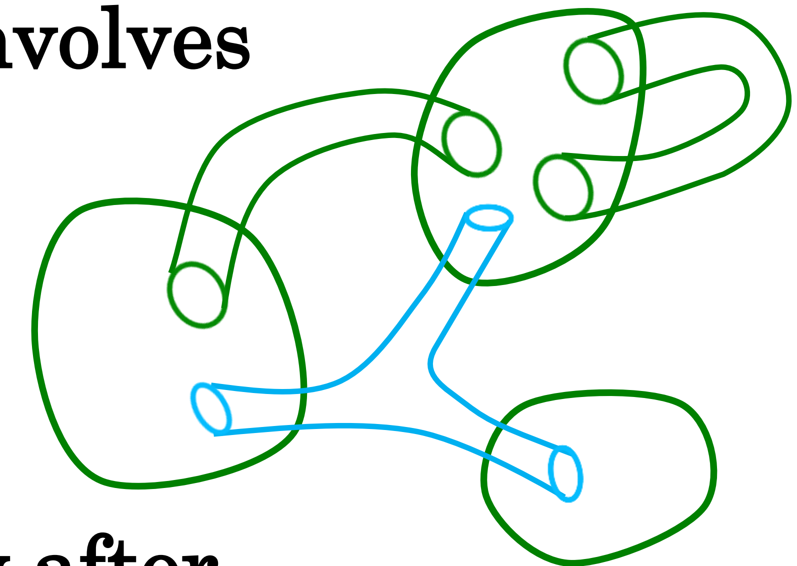
We need a good definition of the path integral for such action, but as an ad hoc assumption we can consider the generalized MPP and make non-trivial predictions.

It is interesting to consider a concrete simple mechanism by which the EW scale emerges from the Planck scale.

Appendix A
Low energy effective theory of quantum gravity/string theory

Coleman ('89)

Consider Euclidean path integral which involves the summation over topologies,



$$\sum_{\text{topology}} \int [dg] \exp(-S) \, .$$

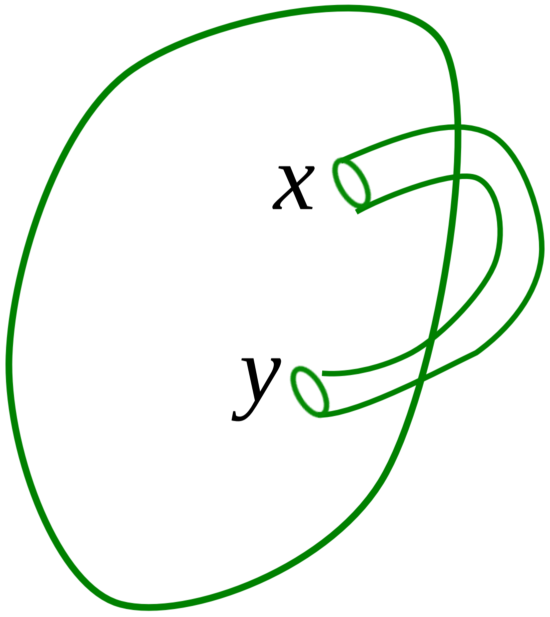
We consider the low energy effective theory after integrating out the short-distance configurations.

Among such configurations there should be a wormhole-like configuration in which a thin tube connects two points on the universe. Here, the two points may belong to either the same universe or different universes.

If we see such configuration from the side of the large universe(s), it looks like two small punctures.

But the effect of a small puncture is equivalent to an insertion of a local operator.

Therefore, a wormhole contributes to the path integral as



$$\int [dg] \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y)\, \exp(-S)\,.$$

Summing over the number of wormholes, we have

$$\sum_{N=0}^{\infty} \frac{1}{n!} \left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y) \right)^n$$

$$= \exp\left(\sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

Thus wormholes contribute to the path integral as

$$\int [dg] \exp\left(-S + \sum_{i,j} c_{ij} \int d^4x\, d^4y \sqrt{g(x)} \sqrt{g(y)}\, O^i(x)\, O^j(y) \right).$$

bifurcated wormholes

$\Rightarrow$ cubic terms, quartic terms, …

The effective action becomes a multi-local form

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

By introducing the Laplace transform

$$\exp\left(-S_{\text{eff}}\left(S_1, S_2, \cdots\right)\right) = \int d\lambda\, w\left(\lambda_1, \lambda_2, \cdots\right) \exp\left(-\sum_i \lambda_i S_i\right),$$

we can express the path integral as

$$Z = \int [d\phi] \exp\left(-S_{\text{eff}}\right) = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(-\sum_i \lambda_i S_i\right).$$

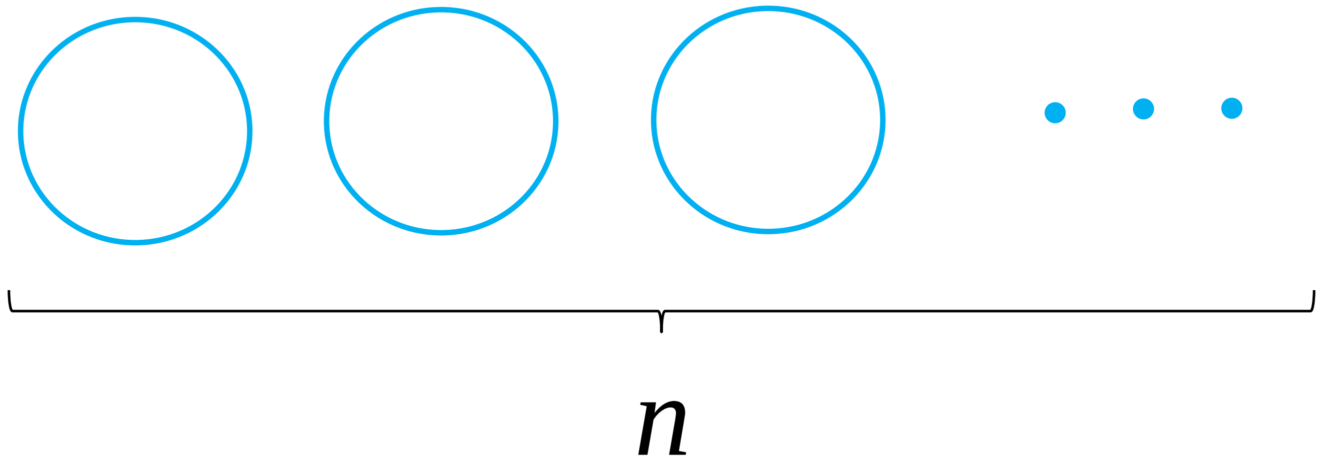
Coupling constants are not merely constant, but they should be integrated.

including multiverse

$$Z = \int d\lambda\, w(\lambda) \int [d\phi] \exp\left(-S(\lambda)\right)$$

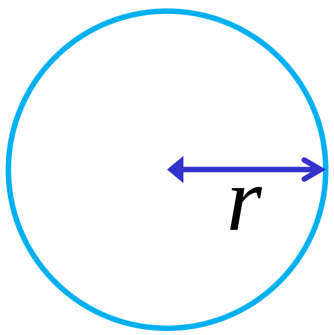
$$= \int d\lambda\, w(\lambda) \sum_{n=0}^{\infty} \frac{1}{n!} {Z_{\rm single}}^{n}$$

$$= \int d\lambda\, w(\lambda) \exp\left(Z_{\rm single}\right).$$

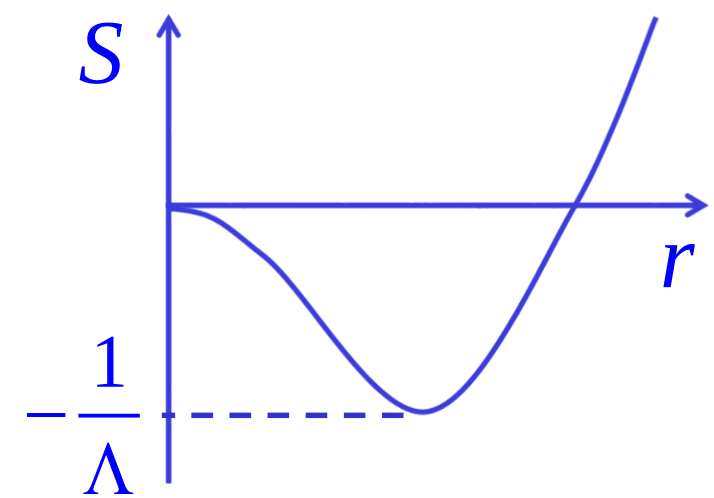


Coleman's "solution" to the cosmological constant problem

$$Z = \int d\Lambda\, w(\Lambda) \int [dg] \exp\left(-\int \sqrt{g} R - \Lambda \int \sqrt{g} \right).$$



S^4



$$\sim \int d\Lambda\, w(\Lambda) \int dr \exp\left(-\left(-r^2 + \Lambda r^4 \right) \right)$$

$$\sim \int d\lambda\, w(\Lambda) \begin{cases} \exp(1/\Lambda), & \Lambda > 0 \\ \text{no solution}, & \Lambda < 0 \end{cases}$$

$\Lambda \sim 0$ dominates irrespectively of $w(\Lambda)$.

<u>Difficulty</u>

Problem of the Wick rotation

WDW eq. $$H_{\text{total}}|\Psi\rangle = 0$$

$$H_{\text{total}} = H_{\text{universe}} + H_{\text{matter}} + H_{\text{graviton}} + \cdots$$

$$H_{\text{universe}} = -\left(\frac{1}{2}p_a{}^2 + \cdots\right)$$ **←wrong sign**

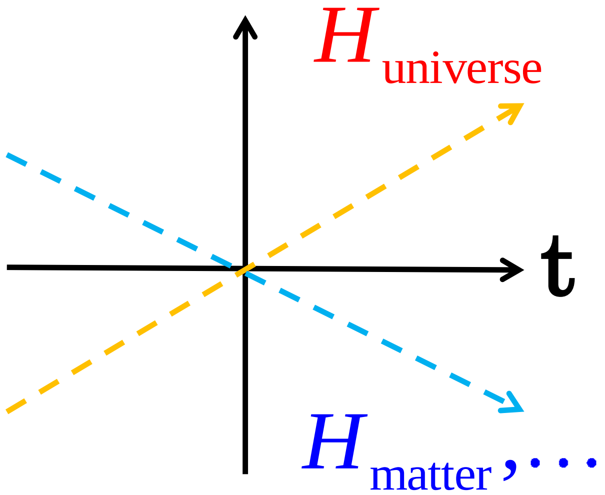
⇓ a : radius of the universe

"Ground state" does not exist.

Wick rotation is not well defined.

H_{matter} **is bounded from below.**

H_{universe} **is bounded from above.**



We expect that the physics with gravity should be expressed in Lorentzian signature, but the low energy effective theory is still given by the multi local action.

In fact we obtain the same effective Lagrangian in the IIB matrix model with Lorentzian signature.

Covariant derivatives as matrices

Y. Kimura,
M. Hanada and HK

The basic question :

In the large-N reduced model, a background of simultaneously diagonalizable matrices $A_\mu^{(0)} = P_\mu$ corresponds to the flat space, if the eigenvalues are uniformly distributed.

In other words, the background $A_\mu^{(0)} = i\partial_\mu$ represents the flat space.

How about curved space?

Is it possible to consider some background like

$$A_\mu^{(0)} = i\nabla_\mu \text{ ?}$$

Actually, there is a way to express the covariant derivatives on any *D*-dim manifold by *D* matrices.

More precisely, we consider

M: any *D*-dimensional manifold,

φ_α: a regular representation field on M.

Here the index α stands for the components of the regular representation of the Lorentz group $SO(D-1,1)$.

The crucial point is that for any representation r, its tensor product with the regular representation is decomposed into the direct sum of the regular representations:

$$V_r \otimes V_{reg} \simeq V_{reg} \oplus \cdots \oplus V_{reg}.$$

In particular the Clebsh-Gordan coefficients for the decomposition of the tensor product of the vector and the regular representaions

$$V_{vector} \otimes V_{reg} \simeq V_{reg} \oplus \cdots \oplus V_{reg}$$

are written as $C_{(a)\alpha}{}^{b,\beta}, \ (a = 1,..,D).$

Here *b* and β are the dual of the vector and the regular representation indices on the LHS. (a) indicates the a-th space of the regular reprezentation on the RHS, and α is its index.

Then for each a $(a = 1..D)$

$$\psi_\alpha = C_{(a)\alpha}{}^{b,\beta} \nabla_b \varphi_\beta$$

is a regular representation field on M.

In other words, if we define $\nabla_{(a)}$ by

$$\left(\nabla_{(a)}\varphi\right)_\alpha = C_{(a)\alpha}{}^{b,\beta} \nabla_b \varphi_\beta \ ,$$

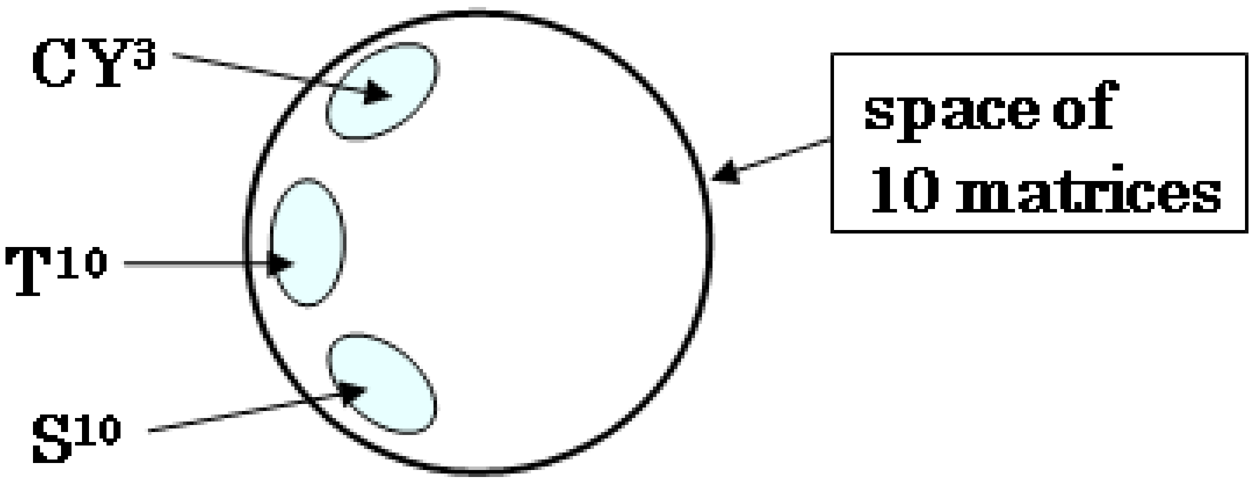
each $\nabla_{(a)}$ is an endomorphism on the space of the regular representation field on M.

Thus we have seen that the covariant derivatives on any D dimensional manifold can be expressed by D matrices.

Therefore any *D*-dimensional manifold *M* with $D \leq 10$ can be realized in the space of the IIB matrix model as

$$A_a{}^0 = \begin{cases} \nabla_{(a)}, & a = 1, \cdots, D \\ 0, & a = D+1, \ldots, 10 \end{cases},$$

where $\nabla_{(a)}$ is the covariant derivative on *M* multiplied by the C-G coefficients.



Low energy effective action of IIB matrix model

A. Tsuchiya, Y. Asano and HK

We have seen that any *D*-dim manifold is contained in the space of *D* matrices. Therefore IIB matrix model should contain the effects of the topology change of space-time.

As was pointed out by Coleman some years ago, such effects give significant corrections to the low energy effective action.

It is interesting to consider the low energy effective action of the IIB matrix model.

Actually we can show that if we integrate out the heavy states in the IIB matrix model, the remaining low energy effective action is not a local action but has a special form, which we call *the multi-local action*:

$$S_{\text{eff}} = \sum_i c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

Here O_i are local scalar operators such as

$$1, R, R_{\mu\nu}R^{\mu\nu}, F_{\mu\nu}F^{\mu\nu}, \overline{\psi}\gamma^{\mu}D_{\mu}\psi, \cdots.$$

S_i are parts of the conventional local actions.
The point is that S_{eff} is a function of S_i's.

This is essentially the consequence of the well-known fact that the effective action of a matrix model contains multi trace operators.

More precisely, we first decompose the matrices A_a into the background $A^0{}_a$ and the fluctuation ϕ :

$$A_a = A^0{}_a + \phi_a .$$

Here we assume that the background $A^0{}_a$ contains only the low energy modes, and ϕ contains the rest. We also assume that this decomposition can be done in a SU(N) invariant manner.

Then we integrate over ϕ to obtain the low energy effective action.

Substituting the decomposition into the action of the IIB matrix model, and dropping the linear terms in ϕ, we obtain

$$S = \frac{1}{4} Tr\Big(\left[A_a^0, A_b^0 \right]^2$$

$$+ 2\left[A_a^0, \phi_b \right]^2 + \left[A_a^0, A_b^0 \right]\left[\phi_a, \phi_b \right] - 2\left[A_a^0, \phi_b \right]\left[A_b^0, \phi_a \right]$$

$$+ 4\left[A_a^0, \phi_b \right]\left[\phi_a, \phi_b \right] + \left[\phi_a, \phi_b \right]^2 + \text{fermion} \Big).$$

In principle, the 0-th order term $S_0 = \frac{1}{4} Tr\left(\left[A_{(a)}^0, A_{(b)}^0 \right]^2 \right)$ **can be evaluated with some UV regularization, which should give a local action.**

The one-loop contribution is obtained by the Gaussian integral of the quadratic part.

Then the result is given by a double trace operator as usual:

$$W = \sum K^{abc\cdots,pqr\cdots} Tr\left(A_a^0 A_b^0 A_c^0 \cdots\right) Tr\left(A_p^0 A_q^0 A_r^0 \cdots\right)$$

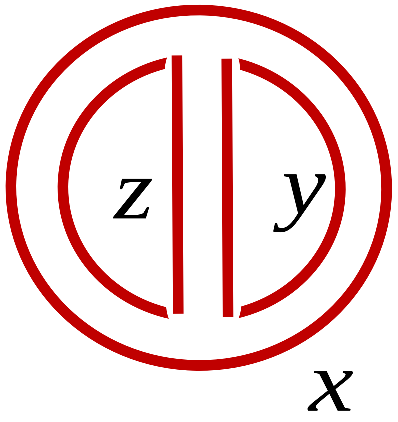
The crucial assumption here is that both of the diffeomorphism and the local Lorentz invariance are realized as a part of the SU(*N*) symmetry.

Then each trace should give a local action that is invariant under the diffeomorphisms and the local Lorentz transformations:

$$S_{\text{eff}}^{\text{1-loop}} = \sum_{ij} \frac{1}{2} c_{ij} S_i S_j, \quad S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

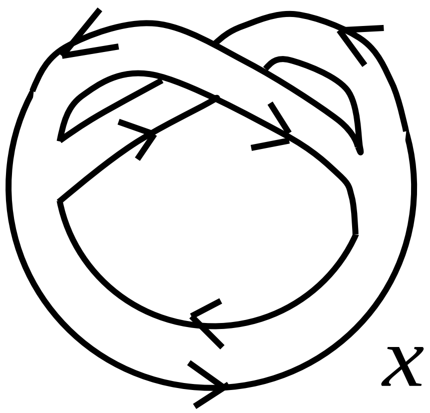
Similar analyses can be applied for higher loops.

In the two loop order, from the planar diagrams we have a cubic form of local actions



$$S_{\text{eff}}^{\text{2-loop Planar}} = \sum_{i,j,k} \frac{1}{6} c_{ijk} S_i S_j S_k ,$$

while non-planar diagrams give a local action



$$S_{\text{eff}}^{\text{2-loop NP}} = \sum_{i} c_i' S_i .$$

We have seen that

the low energy effective theory of the IIB matrix model is given by *the multi-local action*:

$$S_{\mathrm{eff}} = \sum_{i} c_i S_i + \sum_{ij} c_{ij} S_i S_j + \sum_{ijk} c_{ijk} S_i S_j S_k + \cdots,$$

$$S_i = \int d^D x \sqrt{g(x)} O_i(x).$$

This reminds us of the theory of baby universes by Coleman.

Appendix B
Multiverse and naturalness

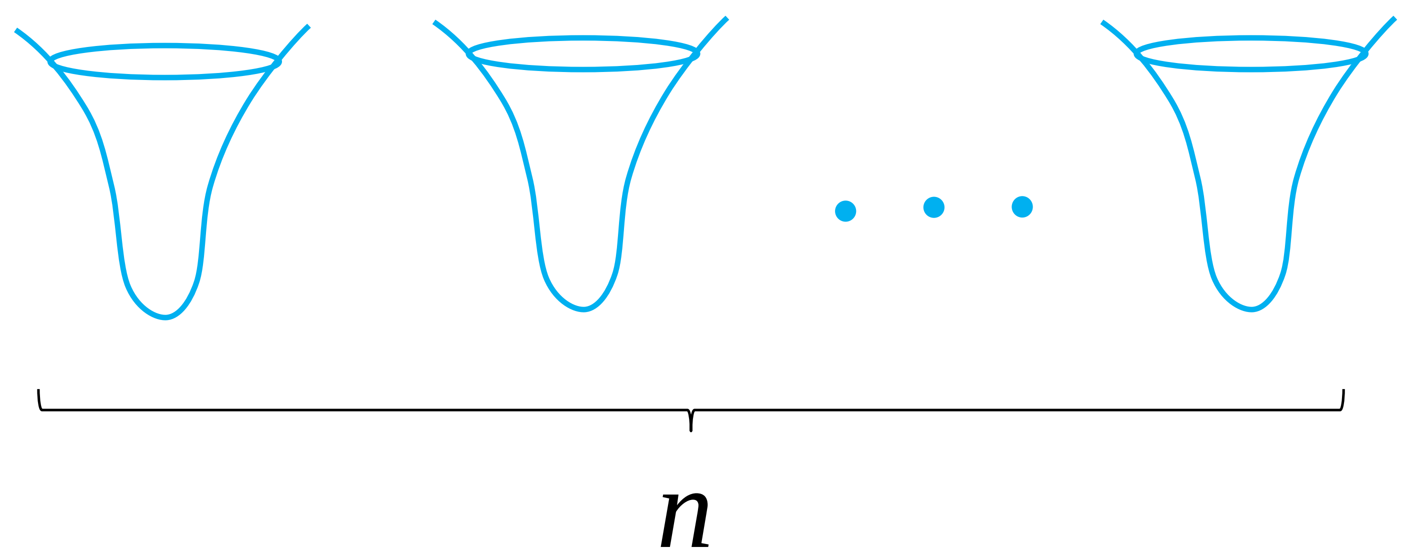
3-1. Partition function of the multiverse

It is natural to apply this action to the multiverse.

$$Z = \int [d\phi] \exp(i S_{\text{eff}}) = \int d\lambda w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right).$$

$$\int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right) = \sum_{n=0}^{\infty} \frac{1}{n!} Z_1{}^n$$

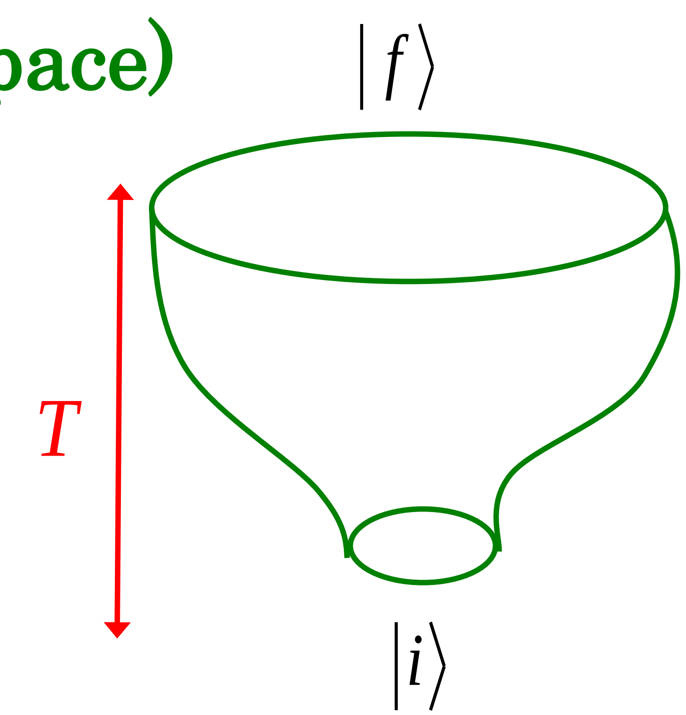
$$Z_1 = \int [d\phi]_{\text{single universe}} \exp\left(i \sum_i \lambda_i S_i \right)$$



$$= \int d\lambda w(\lambda) \exp(Z_1(\lambda)).$$

Path integral for a universe

If the initial and final states are given, the path integral is evaluated as usual: (mini superspace)



$$Z_1(\lambda) = \int [d\phi] \exp(i S)$$

$$= \langle f | \int [da\,dp\,dN] \exp\left(i \int_0^1 dt \left(p\dot{a} - N H_\lambda \right) \right) | i \rangle$$

$$= \langle f | \int_{-\infty}^{\infty} dT \exp\left(-iT\hat{H}_\lambda \right) | i \rangle$$

$$= \langle f | \delta\left(\hat{H}_\lambda \right) | i \rangle$$

$$= \langle f | \phi_{E=0} \rangle \langle \phi_{E=0} | i \rangle$$

$$\langle \phi_E | \phi_{E'} \rangle = \delta\left(E - E' \right)$$

$$\hat{H}_\lambda = -\frac{1}{2} \frac{1}{\sqrt{a}} p^2 \frac{1}{\sqrt{a}} - a^3 U(a)$$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{matt}}{a^3} - \frac{C_{rad}}{a^4}$$

a : radius of the universe

Question:

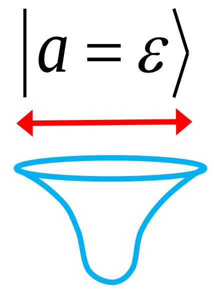
Is there a natural choice for them?

Initial state

For the initial state, we assume that the universe emerges with a small size ε.

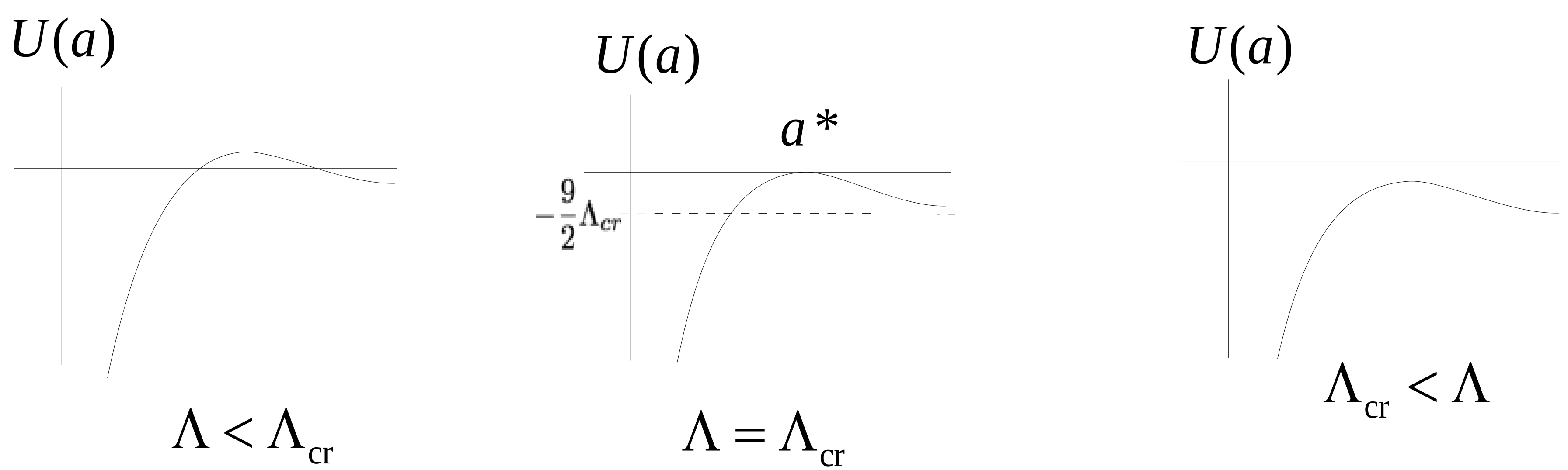
$$\left|i\right\rangle = \mu \left|a = \varepsilon\right\rangle \otimes \left|matter \cdots\right\rangle,$$

μ : probability amplitude of a universe emerging.



Evolution of the universe

S^3 topology



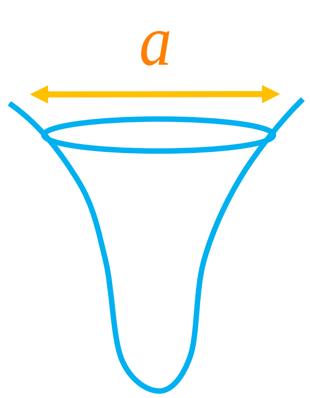
$\Lambda \sim$ curvature

$\sim$ energy density

WKB solution

$$\phi_{E=0}(a,\lambda) \sim \frac{1}{\sqrt{a^{-1} p(a,\lambda)}} \sin\left(\int_0^z da' p(a',\lambda) + \alpha \right)$$

$$S = \int d^4x \sqrt{-g}\left(R - \Lambda + matter\right)$$



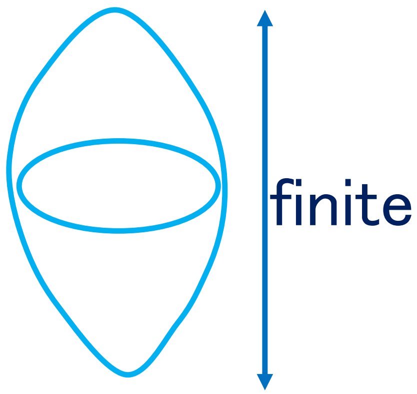
with $p(a,\lambda) = \sqrt{-2a^4 U(a)}.$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{matt}}{a^3} - \frac{C_{rad}}{a^4}$$

For the final state, we have two possibilities.

Final state: case 1

$$\Lambda < \Lambda_{\mathrm{cr}}$$



The universe is closed.
We assume the final state is

$$|f\rangle = \mu' |a = \varepsilon\rangle \otimes |matter \cdots\rangle .$$

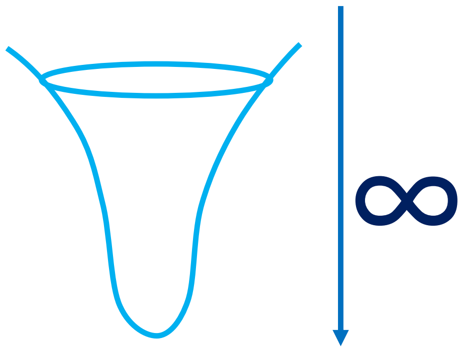
The path integral

$$Z_1(\lambda) = \langle f | \delta(\hat{H}_\lambda) | i \rangle$$

$$\sim const \left|\phi_{E=0}(\varepsilon)\right|^2 .$$

Final state: case 2

$\Lambda > \Lambda_{\mathrm{cr}}$



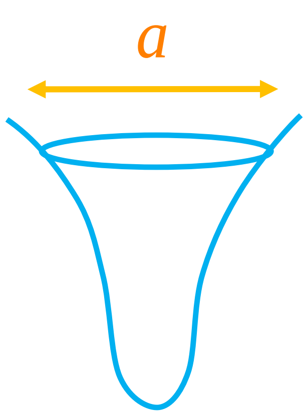
The universe is open.

It is not clear how to define the path integral for the universe:

$$Z_1(\lambda) = \int [d\phi] \exp(i S) \,.$$

As an ad hoc assumption **we consider**

$$| f \rangle = \lim_{a_{IR} \to \infty} c\sqrt{a_{IR}} \, |a_{IR}\rangle \otimes |matter \cdots \rangle .$$



Then the partition function becomes

$$Z_1(\lambda) = \mu\, c \sqrt{a_{IR}}\, \phi_{E=0}(a_{IR})\, \phi^*{}_{E=0}(\varepsilon)$$

$$\sim \mu\, c \sqrt{a_{IR}}\, \frac{1}{\sqrt{a_{IR}\sqrt[4]{\Lambda}}} \sin\left(a_{IR}{}^3 \sqrt{\Lambda} + \alpha' \right) \phi^*{}_{E=0}(\varepsilon)$$

$$\sim \mu\, c \frac{1}{\sqrt[4]{\Lambda}} \sin\left(a_{IR}^3 \sqrt{\Lambda} + \alpha' \right) \phi^*{}_{E=0}(\varepsilon).$$

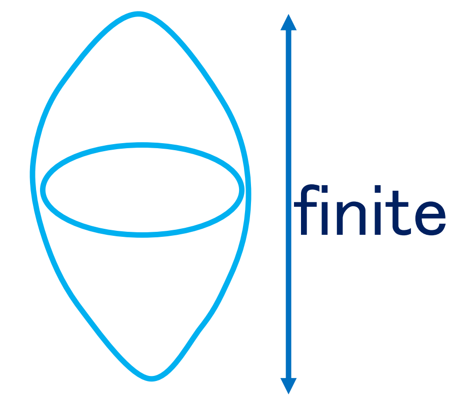
$$\phi_{E=0}(a) \sim \frac{1}{\sqrt{a^{-1}\, p(a)}} \sin\left(\int_0^z da' p(a') + \alpha \right)$$

$$p(a,\lambda) = \sqrt{-2a^4 U(a)} \qquad U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{\mathrm{mat}}}{a^3} - \frac{C_{\mathrm{rad}}}{a^4}$$

The result does not depend on a_{IR} except for the phase which come from the classical action.

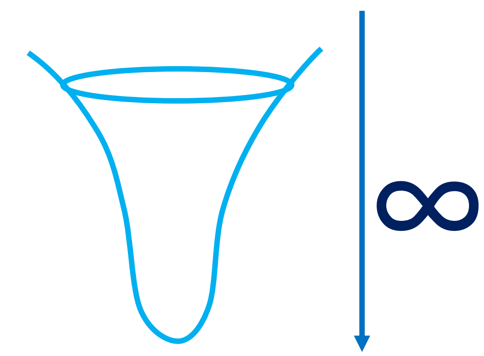
Thus we have

the path integral for a universe $Z_1(\lambda)$



for $\Lambda < \Lambda_{cr}$ $\quad$ *const* of order 1,

for $\Lambda > \Lambda_{cr}$ $\quad$ $const \frac{1}{\sqrt[4]{\Lambda}} \sin\left(a_{IR}{}^{3} \cdot \sqrt{\Lambda} + \alpha' \right).$



Then the λ **integration for the multiverse**

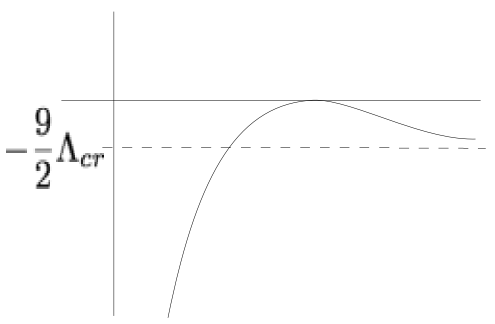
$$Z = \int d\lambda w(\lambda) \exp\left(Z_1(\lambda)\right).$$

has a large peak at $\Lambda(\lambda) \sim \Lambda_{cr}$**, which means that the cosmological constant at the late stages of the universe almost vanishes.**

3-2. Maximum Entropy Principle

Maximum entropy principle

For simplicity we assume the S^3 topology of the space and that all matters decay to radiation at the late stages.



$\Lambda = \Lambda_{\rm cr}$

$$U(a) = \frac{1}{a^2} - \Lambda - \frac{C_{\rm rad}}{a^4}$$

$$\Lambda_{\rm cr} \sim 1/C_{\rm rad}$$

Then the multiverse partition function is given by

$$Z = \int d\lambda\, w(\lambda) \exp\left(Z_1(\lambda)\right)$$

$$\sim \exp\left(\mathrm{const}\,\frac{1}{\sqrt[4]{\Lambda_{cr}}}\right) \sim \exp\left(\mathrm{const}\sqrt[4]{C_{\rm rad}(\lambda)}\right).$$

Maximum entropy principle (MEP)

The low energy couplings are determined in such a way that the entropy at the late stages of the universe is maximized.

There are many ways to obtain MPP:

Suppose that we pic up a universe randomly from the multiverse. Then the most probable universe is expected to be the one that has the maximum entropy.

Okada and HK '11

Flatness of the Higgs potential

We may understand the flatness of the Higgs potential as a consequence of MEP.

If we accept the inflation scenario in which universe pops out from nothing and then inflates, most of the entropy of the universe is generated at the stage of reheating just after the inflation stops. Therefore the potential of the inflaton should be tuned in such a way that inflation occurs.

Furthermore, if the Higgs field plays the role of inflaton, the above analysis asserts that the SM parameters are tuned such that the Higgs potential becomes flat at high energy scale.

3-3. Probabilistic interpretation of multiverse wave function

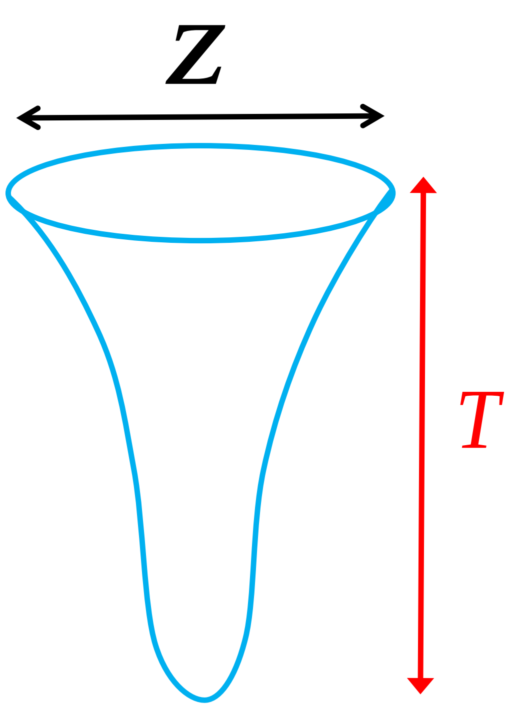
Probabilistic interpretation (1)

postulate $\psi(z) = \mu\, \phi_{E=0}(z)$

$$|\psi(z)|^2 dz \propto \text{probability of finding a universe of size } z$$

meaning of this measure

$$\int dz \left|\phi_{E=0}(z)\right|^2 \sim \int dz \frac{1}{z\,p(z)} \qquad \phi_{E=0}(z) \sim \frac{1}{\sqrt{z\,p(z)}} \exp\left(i \int^z dz'\, p(z')\right)$$



$$= \int dz \frac{1}{\dot{z}} = \int dT \qquad H = z\left(-\frac{1}{2}p^2 + \cdots\right) \rightarrow \dot{z} = \frac{\partial H}{\partial p} = -zp$$

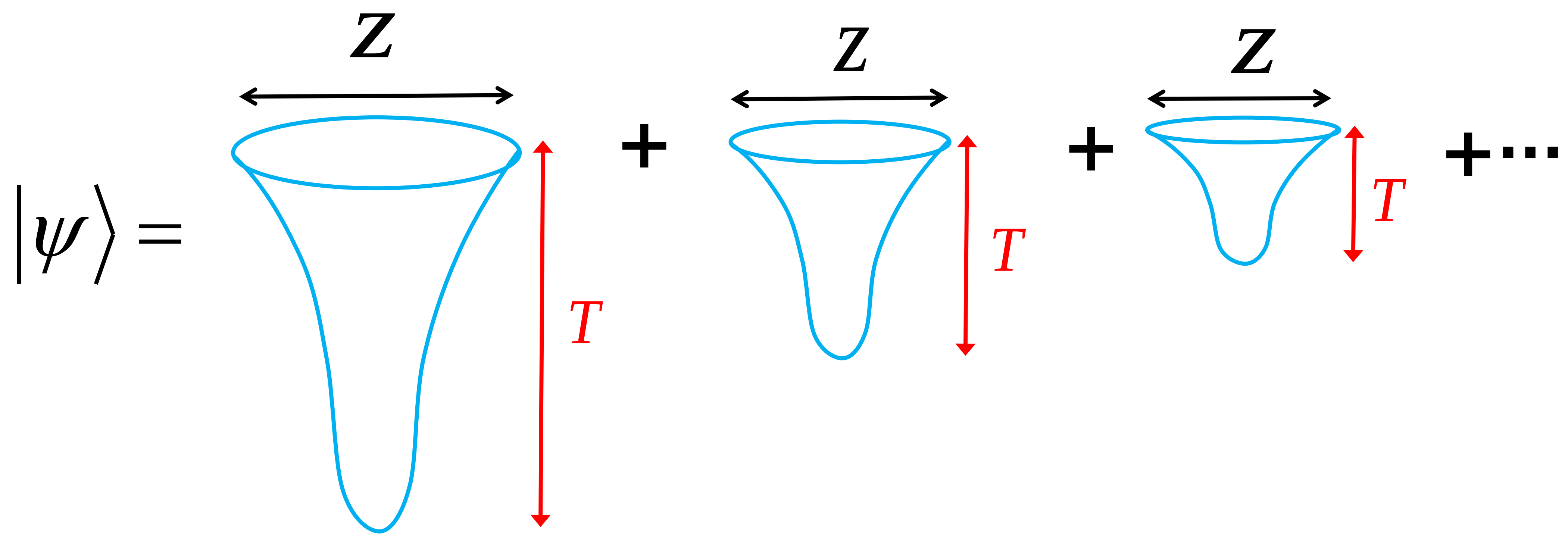
T : age of the universe
the time that has passed
after the universe is created

$$\Rightarrow \quad |\psi(z)|^2 dz \sim |\mu|^2 dT$$

$|\mu|^2 =$ probability of a universe emerging in unit time

Probabilistic interpretation (2)

$|\psi\rangle$ **is a superposition of the universe with various age,**

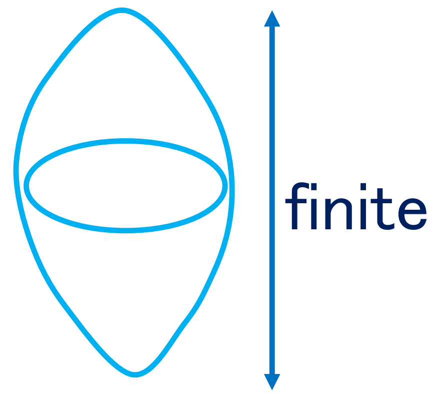


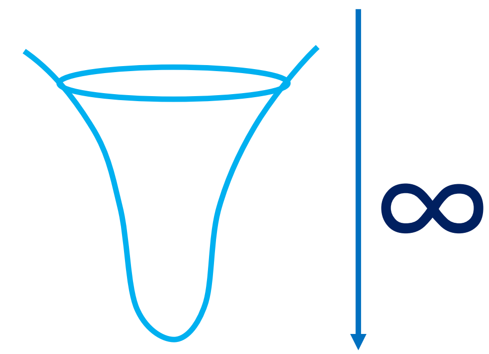
$|\psi(z)|^2 dz \sim |\mu|^2 dT$ **gives the probability of finding a universe of age** $T \sim T + dT$ **.**

Lifetime of the universe

$$\int \left|\psi(z)\right|^2 dz \sim \left|\mu\right|^2 \int dT = \left|\mu\right|^2 \times (\text{life time of the universe})$$

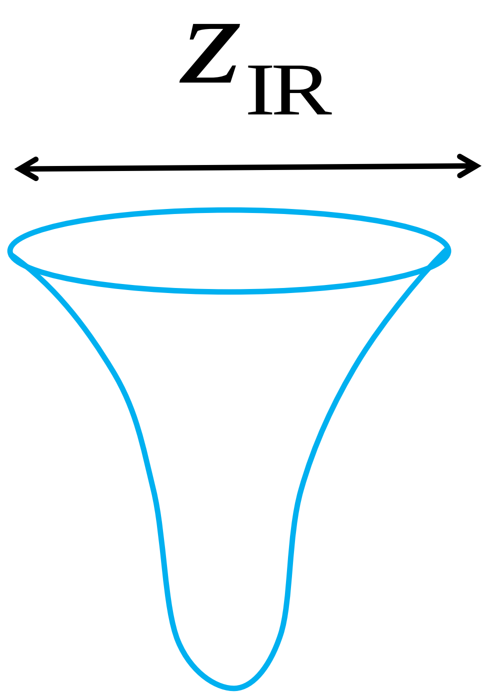
dimensionless





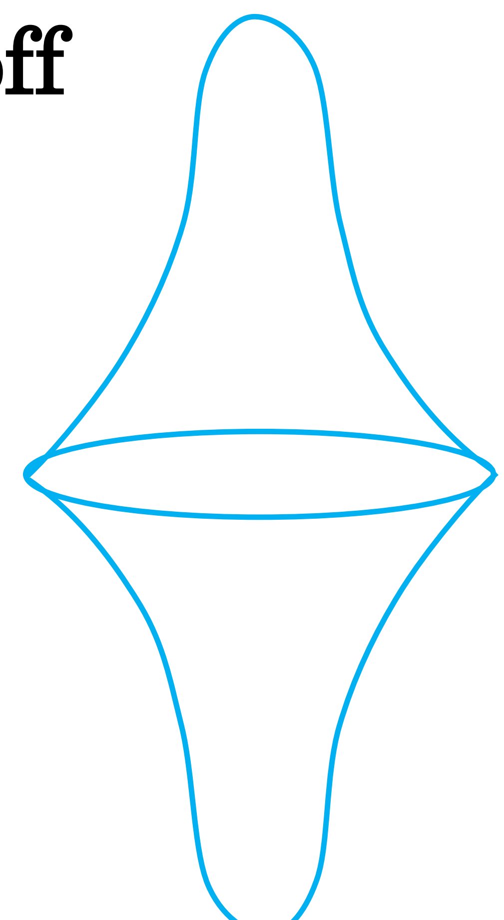
infrared cutoff

We introduce an infrared cutoff for the size of universes.



ceases to exist

or

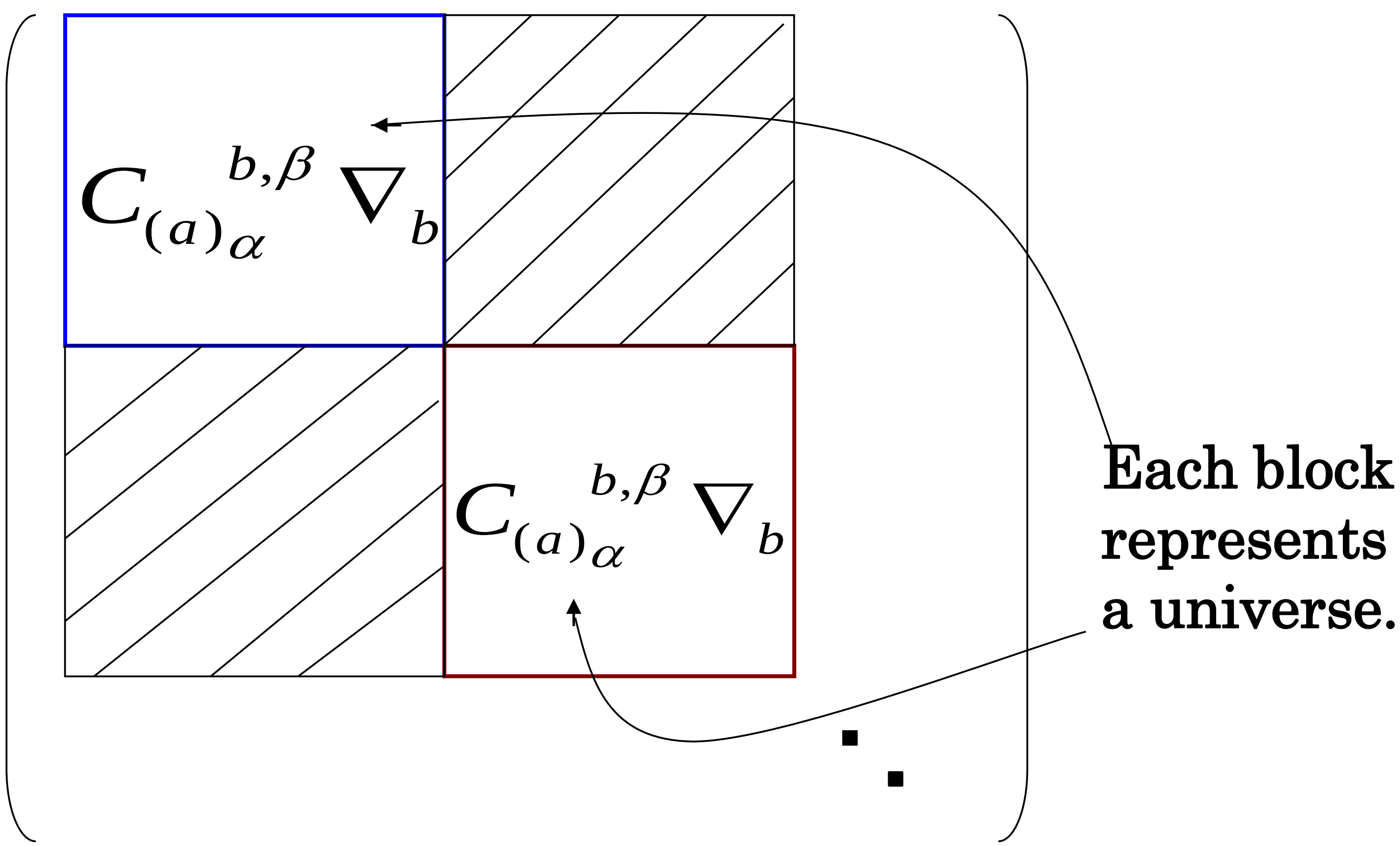


bounces back

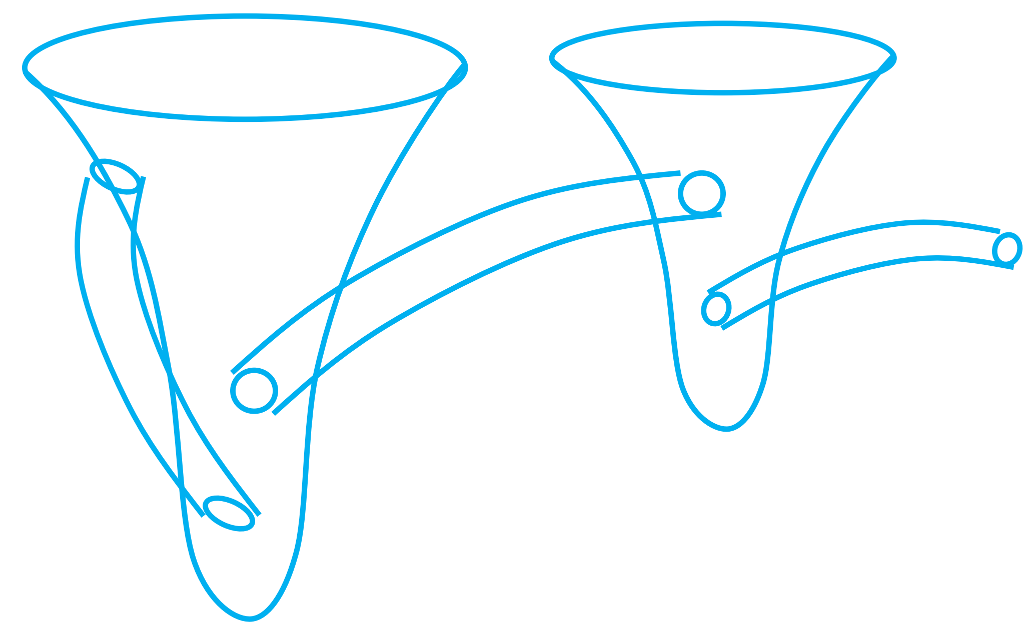
Wave Function of the multiverse (1)

Okada, HK

Multiverse appears naturally in quantum gravity / string theory.



matrix model



quantum gravity

Wave Function of the multiverse (2)

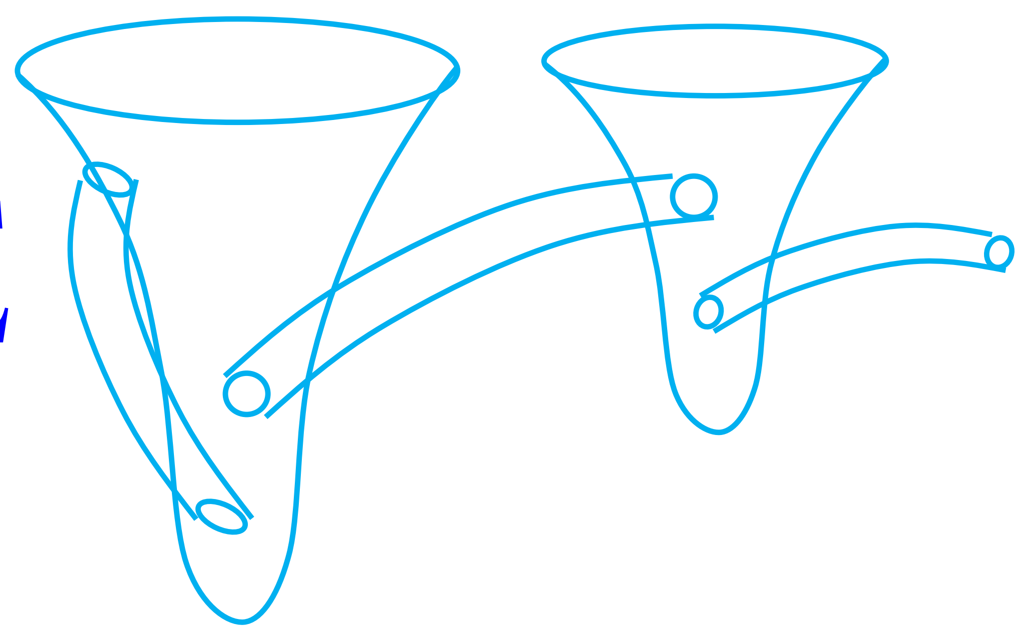
The multiverse sate is a superposition of N-verses.

$$\left|\Psi_{\text{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_{N=0}^{\infty} \left|\Psi_N, \lambda\right\rangle \otimes \left|\lambda\right\rangle \quad \leftarrow Z = \int d\lambda w(\lambda) \int [d\phi] \exp\left(i \sum_i \lambda_i S_i \right)$$

$$\Psi_N\left(z_1, \cdots, z_N, \lambda\right) = \psi\left(z_1, \lambda\right) \cdots \psi\left(z_N, \lambda\right)$$

$$\left|\psi, \lambda\right\rangle \otimes \left|\psi, \lambda\right\rangle \otimes \cdots$$

$$\left|\Psi_{\text{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_N$$



Wave Function of the multiverse (3)

Probabilistic interpretation

$$\left|\Psi_{\text{multi}}\right\rangle = \int d\lambda\, w(\lambda) \sum_{N=0}^{\infty} \left|\Psi_N, \lambda\right\rangle \otimes \left|\lambda\right\rangle$$

$$\Psi_N\left(z_1, \cdots, z_N, \lambda\right) = \psi\left(z_1, \lambda\right) \cdots \psi\left(z_N, \lambda\right)$$

$\left|\Psi_N\left(z_1, \cdots, z_N, \lambda\right)\right|^2 dz_1 \cdots dz_N \left|w(\lambda)\right|^2 d\lambda$ **represents**

the probability of finding N universes with size

$$z_1 \sim z_1 + dz_1,\ \cdots,\ z_N \sim z_N + dz_N$$

and finding the coupling constants in

$$\lambda \sim \lambda + d\lambda.$$

Probability distribution of λ

$$P(\lambda) = \sum_{N=0}^{\infty} \int \frac{dz_1 \cdots dz_N}{N!} \left| \Psi_N (z_1, \cdots, z_N, \lambda) \right|^2 \left| w(\lambda) \right|^2$$

$$= \exp\left(\int dz \left| \psi(z, \lambda) \right|^2 \right) \left| w(\lambda) \right|^2 \quad \leftarrow \Psi_N (z_1, \cdots, z_N, \lambda) = \psi(z_1, \lambda) \cdots \psi(z_N, \lambda)$$

$$= \exp\left(\left| \mu \tau(\lambda) \right|^2 \right) \left| w(\lambda) \right|^2 \quad \leftarrow \left| \psi \right\rangle = \mu \left| \phi_{E=0} \right\rangle$$

$$\tau(\lambda) = \int dz \left| \phi_{E=0} (z, \lambda) \right|^2 \sim \text{(life time of the universe)}$$

$\tau(\lambda)$ can be very large.

⬇

λ is chosen in such a way that $\tau(\lambda)$ is maximized, irrespectively of $w(\lambda)$.

If we accept the probabilistic interpretation of the multiverse wave function, the coupling constants are chosen in such a way that the lifetime of the universe becomes maximum.

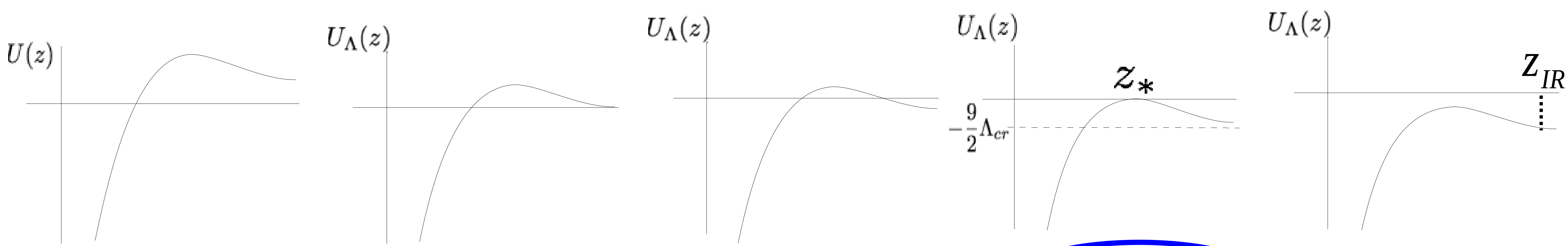
Cosmological constant

What value of Λ maximizes $\int dz \left| \mu \phi_{E=0}(z,\lambda) \right|^2$?

WKB sol $\phi_{E=0}(z,\lambda) \sim \dfrac{1}{\sqrt{z\, p(z,\lambda)}}$ with $p(z,\lambda) = \sqrt{-2U(z)}$. S^3 topology

$$U(z) = \frac{1}{z^{2/3}} - \Lambda - \frac{C_{matt}}{z} - \frac{C_{rad}}{z^{4/3}}$$

assuming all matters decay to radiation



$\Lambda < 0$ | $\Lambda = 0$ | $0 < \Lambda < \Lambda_{cr}$ | $\Lambda = \Lambda_{cr}$ | $\Lambda_{cr} < \Lambda$

The cosmological constant in the far future is predicted to be very small.

Λ ～curvature～energy density

$\Lambda_{cr} \sim 1/C_{rad}$ (extremely small)

The other couplings (Big Fix)

$$P(\lambda) = \exp\left(\left|\mu\tau(\lambda)\right|^2\right)\left|w(\lambda)\right|^2 \qquad \leftarrow \tau(\lambda) = \int dz \left|\phi_{E=0}(z,\lambda)\right|^2$$

The exponent is divergent, and regulated by the IR cutoff :

$$\int dz \left|\phi_{E=0}(z,\lambda)\right|^2 \sim \int_0^{z_{IR}} \frac{1}{z\sqrt{\Lambda_{\text{cr}}}} \sim \sqrt{C_{\text{rad}}}\log z_{IR}\,. \qquad \leftarrow \Lambda_{\text{cr}} \sim 1/C_{\text{rad}}$$

assuming all matters decay to radiation

MEP

λ are determined in such a way that $C_{\text{rad}}(\lambda)$ is maximized.
Again we have MEP.