Selecting $\mu \to e$ conversion targets to distinguish lepton flavour-changing operators

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S. Davidson, Y. Kuno and M.Y., PLB 790 (2019) 380

$\mu - e$ conversion $\quad \mu^- + N(A,Z) \to e^- + N(A,Z)$

One of the most hopeful mode to discover Charged Lepton Flavor Violation (CLFV)

Limit
$$\mathrm{BR}(\mu^- + \mathrm{Au} \to e^- + \mathrm{Au}) < 7 \times 10^{-13}$$
$$\mathrm{BR}(\mu^- + \mathrm{Ti} \to e^- + \mathrm{Ti}) < 4.3 \times 10^{-12}$$

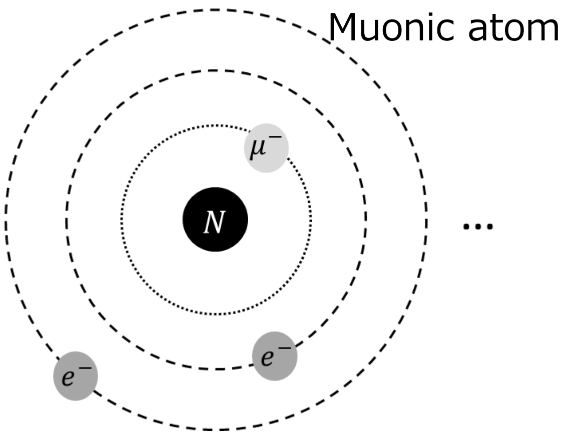
SINDRUM-II

Has been searched for with **Au**, **Ti**, **S**, **Cu**

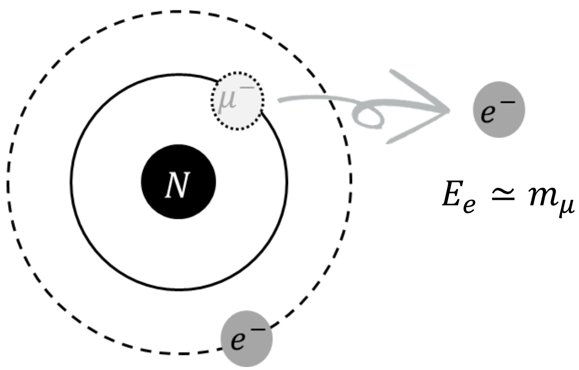
Will be searched for with **Al** in near future

COMET, Mu2e

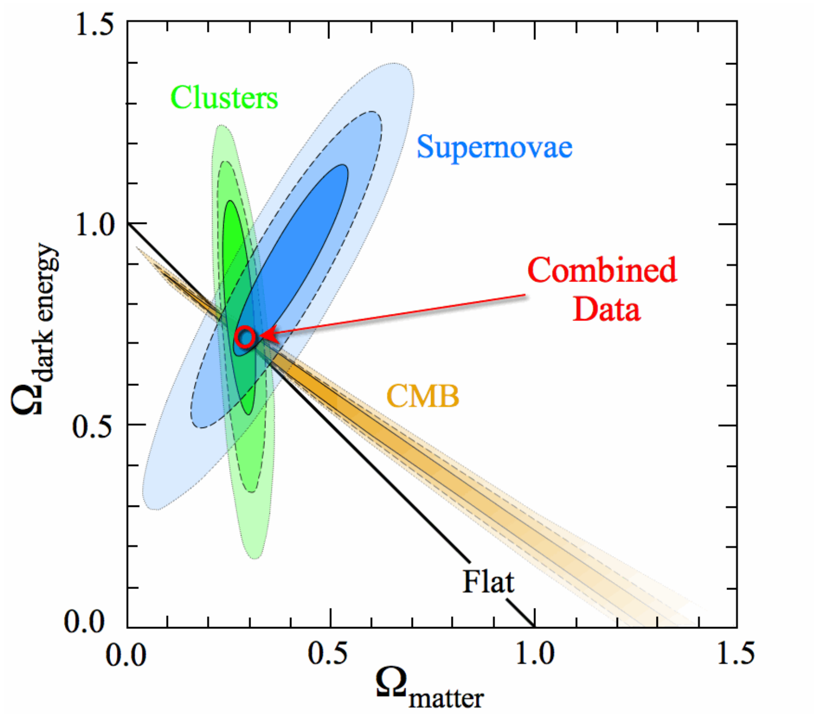
How large an amount of information of CLFV can be obtained from experiments?

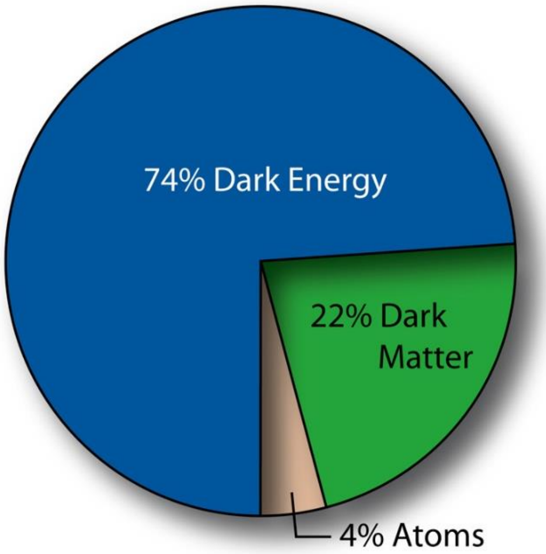


If charged lepton flavor is violated…



Sufficient number of information to get unknown



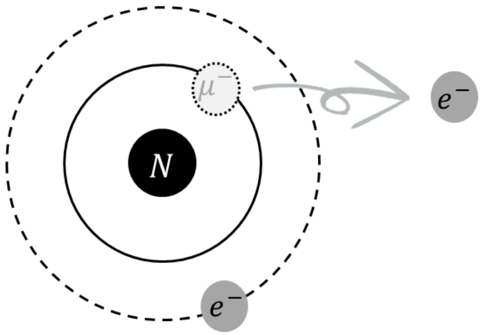


Example of ideal case : cosmological parameters

(# of observational data) > (#of unknown)

Is it sufficient to get flavor structure?

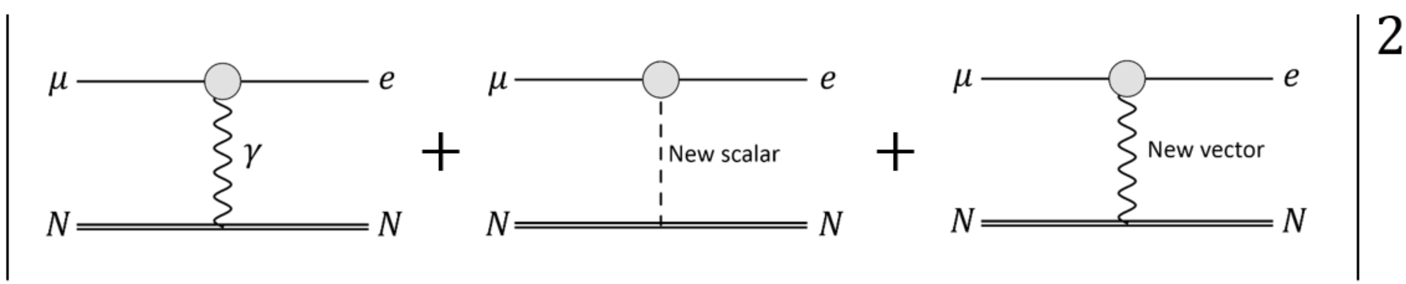
$\mu^- - e^-$ conversion



Challenging to obtain the comprehensive information!!

- Coherent contributions of 22 independent CLFV operators
- Uncertainties in theoretical calculations

Reaction rate $\propto$



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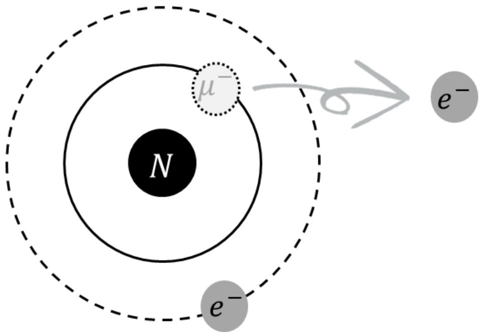
$\mu^- - e^-$ conversion



Challenging to obtain the comprehensive information!!

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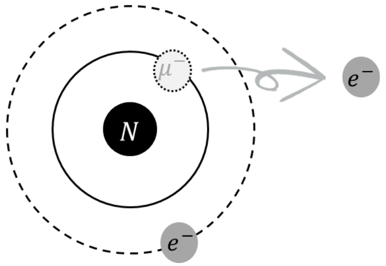
Aim of work

- ☐ **Potential of $\mu - e$ conversion for discriminating CLFV scenarios**
- ☐ **Which is remarkable target in future?**
- ☐ **How many CLFV operators could be constrained?**

Reaction rate R. Kitano, M. Koike, Y. Okada, PRD (2002)



Interactions@experimental scale

$$\mathcal{L}_{\mu A\to eA}(\Lambda_{expt}) = -\frac{4G_F}{\sqrt{2}}\sum_{N=p,n}\Big[m_\mu\left(C_{DL}\overline{e_R}\sigma^{\alpha\beta}\mu_L F_{\alpha\beta} + C_{DR}\overline{e_L}\sigma^{\alpha\beta}\mu_R F_{\alpha\beta}\right)$$
$$+\left(\widetilde{C}_{SL}^{(NN)}\overline{e}P_L\mu + \widetilde{C}_{SR}^{(NN)}\overline{e}P_R\mu\right)\overline{N}N$$
$$+\left(\widetilde{C}_{VL}^{(NN)}\overline{e}\gamma^\alpha P_L\mu + \widetilde{C}_{VR}^{(NN)}\overline{e}\gamma^\alpha P_R\mu\right)\overline{N}\gamma_\alpha N + h.c.\Big]$$

Branching ratio

V, S, D : overlap of wave functions of μ, e, and nucleon

$$\mathrm{BR}_{SI} = \frac{32G_F^2 m_\mu^5}{\Gamma_{cap}}\left[\left|\widetilde{C}_{V,R}^{pp}V^{(p)} + \widetilde{C}_{S,L}^{pp'}S^{(p)} + \widetilde{C}_{V,R}^{nn}V^{(n)} + \widetilde{C}_{S,L}^{nn'}S^{(n)} + C_{D,L}\frac{D}{4}\right|^2 + \{L \leftrightarrow R\}\right]$$

Each type of CLFV are coherently added in the amplitude $\mathcal{M}$

Challenging to extract each contribution from experimental results ($\propto |\mathcal{M}|^2$)

Geometric and intuitive measure of targets ability

<u>Branching ratio</u>

$$\mathrm{BR}_{SI} = \frac{32 G_F^2 m_\mu^5}{\Gamma_{cap}} \left[\left| \widetilde{C}^{pp}_{V,R} V^{(p)} + \widetilde{C}^{pp\prime}_{S,L} S^{(p)} + \widetilde{C}^{nn}_{V,R} V^{(n)} + \widetilde{C}^{nn\prime}_{S,L} S^{(n)} + C_{D,L} \frac{D}{4} \right|^2 + \{L \leftrightarrow R\} \right]$$

Target vector

elements: overlap of wave functions

$$\vec{v}_Z = \left(\frac{D_Z}{4}, V_Z^{(p)}, S_Z^{(p)}, V_Z^{(n)}, S_Z^{(n)} \right)$$

Coefficient vector

elements: coefficient of CLFV operators

$$\vec{C}_L = (\widetilde{C}_{D,R}, \widetilde{C}^{pp}_{V,L}, \widetilde{C}^{pp}_{S,R}, \widetilde{C}^{nn}_{V,L}, \widetilde{C}^{nn}_{S,R})$$

$$\mathrm{BR}_{SI} = B_Z \left[\left| \hat{v}_Z \cdot \vec{C}_L \right|^2 + \{L \leftrightarrow R\} \right] \qquad \left(B_Z = \frac{32 G_F^2 m_\mu^5 |\vec{v}_Z|^2}{\Gamma_{cap}(Z)} \right)$$

If target vectors are misaligned, they could discriminate the coefficients

of targets to discriminate coefficient vectors

◆ Neglect dipole coefficients

$\mathrm{BR}(\mu \to e\gamma) \lesssim 10^{-14}$ and $\mathrm{BR}(\mu \to 3e) \lesssim 10^{-16}$ are better sensitivity than COMET/Mu2e

◆ Neglect operators of e_R final state

Interference of e_L and e_R final states is negligible, and an experimental bound simultaneously sets bounds on both ope.

Geometric and intuitive measure of targets ability

Branching ratio

$$\mathrm{BR}_{SI} = \frac{32 G_F^2 m_\mu^5}{\Gamma_{cap}} \left[\left| \tilde{C}^{pp}_{V,R} V^{(p)} + \tilde{C}^{pp'}_{S,L} S^{(p)} + \tilde{C}^{nn}_{V,R} V^{(n)} + \tilde{C}^{nn'}_{S,L} S^{(n)} + C_{D,L} \frac{D}{4} \right|^2 + \{L \leftrightarrow R\} \right]$$

Target vector
elements: overlap of wave functions

$$\vec{v}_Z = \left(\frac{D_Z}{4}, V_Z^{(p)}, S_Z^{(p)}, V_Z^{(n)}, S_Z^{(n)} \right)$$

Coefficient vector
elements: coefficient of CLFV operators

$$\vec{C}_L = (\tilde{C}_{D,R}, \tilde{C}^{pp}_{V,L}, \tilde{C}^{pp}_{S,R}, \tilde{C}^{nn}_{V,L}, \tilde{C}^{nn}_{S,R})$$

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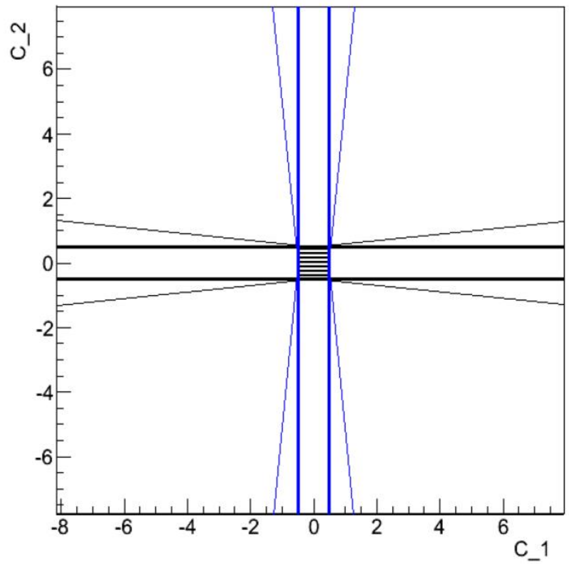
If target vectors are misaligned, they could discriminate the coefficients

# of remaining coefficients = 4 @ experimental scale

Search for $\mu - e$ conversion on sufficiently different 4 targets

Relations among observables, parameter, and uncertainty



Calculations with an uncertainty for 2 observables

obs. 1 $\propto |c_1(1 \pm \epsilon) + \epsilon c_2|^2$

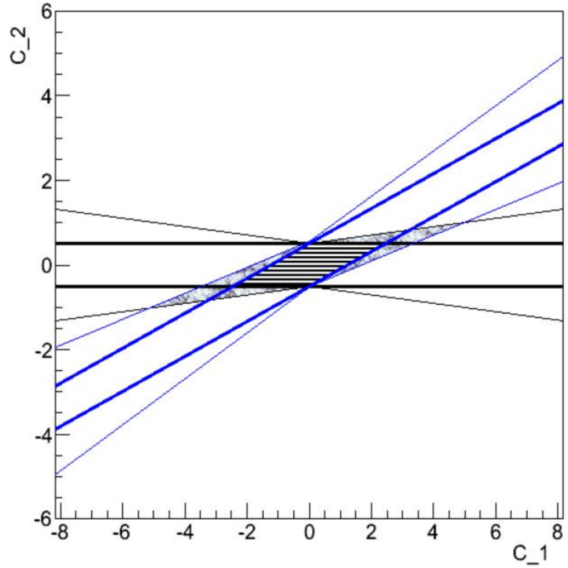
obs. 2 $\propto |c_2(1 \pm \epsilon) + \epsilon c_1|^2$

ϵ : theoretical uncertainty
(ϵ =0.1 in the figure)

Null results for both observables

$\rightarrow$ Parameters are constrained $c_1 \simeq 0$, $c_2 \simeq 0$

Relations among observables, parameter, and uncertainty



Realistic case

One of the observable depends on both c_1 and c_2

obs. 1 $\propto |c_2 \cos\theta - c_1 \sin\theta|^2$

$\theta = \pi/8 \pm \epsilon$ in the figure

obs. 2 $\propto |c_2(1 \pm \epsilon) + \epsilon c_1|^2$

Null results for both observables

$\rightarrow$ weaker constraint on c_1 due to mixing θ and uncertainty of c_2

Even though the constraint becomes weaker, since θ is not so small value, it is enclosed in a finite area

Relations among observables, parameter, and uncertainty

<u>Nightmare case</u>

One of the observable depends on both c_1 and c_2, and θ is small ($\theta = 2\epsilon \pm \epsilon$)

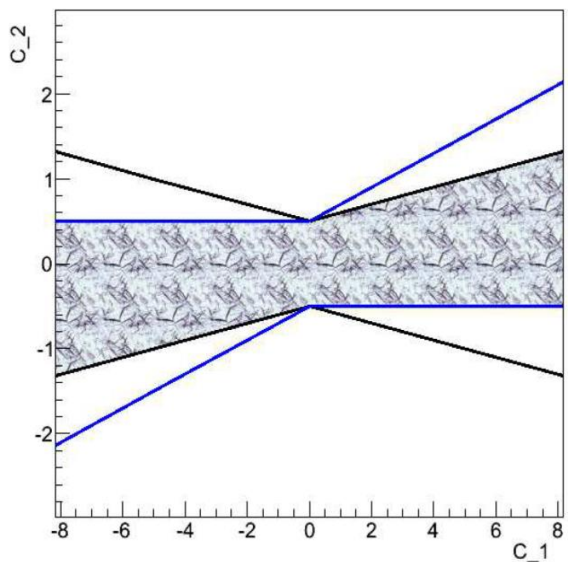
obs. 1 $\propto |c_2 \cos\theta - c_1 \sin\theta|^2$

obs. 2 $\propto |c_2(1 \pm \epsilon) + \epsilon c_1|^2$

Two observables are almost equated for $\theta \ll 1$

$$c_2 \cos\theta - c_1 \sin\theta \simeq c_2(1 \pm \epsilon) + \epsilon c_1$$

No constraint on c_1 ☹



How large θ is required to set bounds on both parameters?

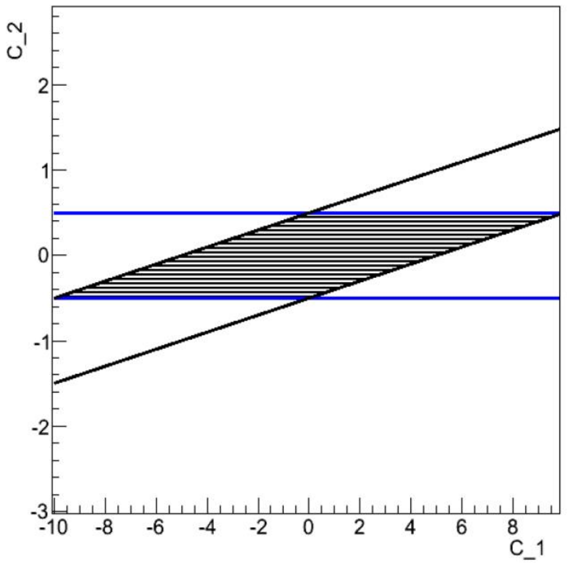
Relations among observables, parameter, and uncertainty

<u>Conclusion</u>

θ has to be larger than twice of uncertainty : $\theta > 2\epsilon$

Uncertainty from nucleus $\sim 10\%$ (heavy nucleus) $\rightarrow$ $\epsilon = 0.1$
$\sim 5\%$ (light nucleus) $\rightarrow$ $\epsilon = 0.05$

"different" target $\rightarrow$ angle between target vectors > 0.2 (0.1)

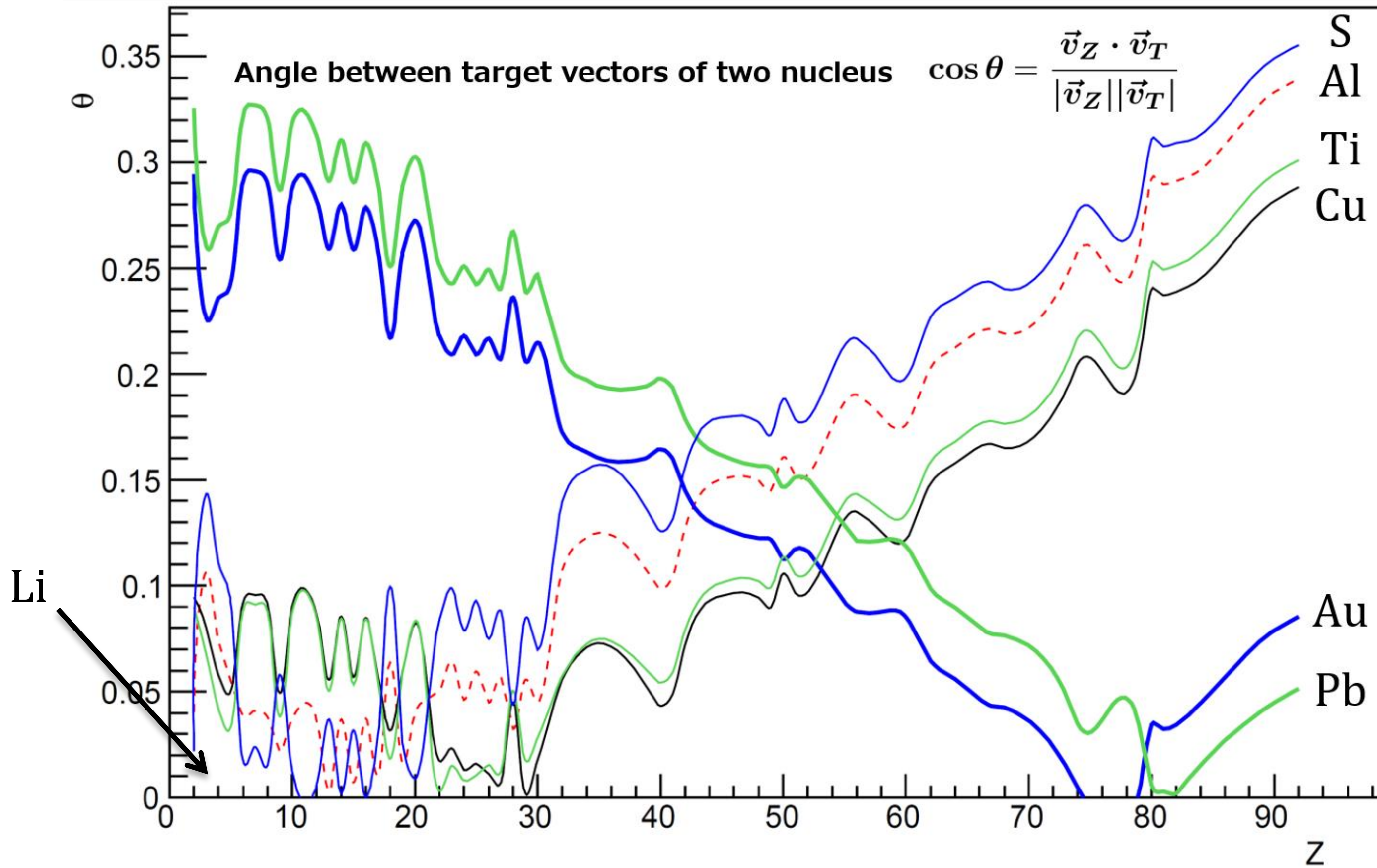


One of the observable depends on both c_1 and c_2, and $\theta = 2\epsilon$

obs. 1 $\propto |c_2 \cos\theta - c_1 \sin\theta|^2$

obs. 2 $\propto |c_2(1 \pm \epsilon) + \epsilon c_1|^2$

Which is independent set of targets?



- **Each set {Au,Pb} and {S,Ti,Cu} do not give independent constraints**
- **Next generation target Al is not independent of light nuclei**

There are bounds on these 5 targets

- **Li shows $\theta > 0.1$ for Al, and is a promising target**

(and other isotopes with higher n/p ratio)

Constraints on CLFV operators @ EW scale

$\widetilde{C}^{NN}_{O,X} = \sum_q G^{Nq}_O C^{qq}_{O,X}$ 82 independent operators in quark level

Coefficients @ experimental scale

$$
\begin{aligned}
\sqrt{\frac{BR^{exp}_{Al}}{33}} &\gtrsim \left|3C^{uu}_{V,L} + 3C^{dd}_{V,L} + 11C^{uu}_{S,R} + 11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R}\right| \\
&\gtrsim \Bigg|3C^{uu}_{V,L} + 3C^{dd}_{V,L} + \frac{\alpha}{\pi}\left[3C^{dd}_{A,L} - 6C^{uu}_{A,L}\right]\log + \frac{\alpha}{3\pi}[C^{ee}_{V,L} + C^{\mu\mu}_{V,L}]\log - \frac{\alpha}{3\pi}[C^{ee}_{A,L} + C^{\mu\mu}_{A,L}]\log \\
&\quad - \frac{2\alpha}{3\pi}\left[2(C^{uu}_{V,L} + C^{cc}_{V,L}) - (C^{dd}_{V,L} + C^{ss}_{V,L} + C^{bb}_{V,L}) - (C^{ee}_{V,L} + C^{\mu\mu}_{V,L} + C^{\tau\tau}_{V,L})\right]\log \\
&\quad + \lambda^{-a_S}\left(11C^{uu}_{S,R} + 11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R}\right) \\
&\quad + \lambda^{-a_S}\frac{\alpha}{\pi}\left[\frac{13}{6}(11C^{uu}_{S,R} + \frac{4m_N}{27m_c}C^{cc}_{S,R}) + \frac{5}{3}(11C^{dd}_{S,R} + 0.84C^{ss}_{S,R} + \frac{4m_N}{27m_b}C^{bb}_{S,R})\right]\log \\
&\quad - \lambda^{a_T} f_{TS}\frac{8\alpha}{\pi}\left[22C^{uu}_{T,R} + \frac{8m_N}{27m_c}C^{cc}_{T,R} - 11C^{dd}_{T,R} - 0.84C^{ss}_{T,R} - \frac{4m_N}{27m_b}C^{bb}_{T,R})\right]\log\Bigg|
\end{aligned}
$$

Coefficients @ EW scale

Combining with $\mu \to e\gamma$ and $\mu \to 3e$, there are 22 constraints. There remains “flat directions”.

Extensively search for other CLFV processes!

Summary

- ☑ COMET and Mu2e will improve the sensitivity to search for $\mu - e$ conversion which is a hopeful mode to discover CLFV
- ☑ Important to study how large an amount of information of CLFV can be obtained from experiments with taking into account theoretical uncertainties
- ☑ At least 4 independent target is required to identify types of CLFV operators
- ☑ **Independent target : {Au or Pb}, {S or Ti or Cu}, Al, Li**
- ☑ **Combining with $\mu \to e\gamma$ and $\mu \to 3e$, 22/82 fundamental CLFV operators at EW scale will be constrained.**
  Extensively search for other CLFV reactions.