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# Pseudo-Nambu-Goldstone dark matter from *gauged* $U(1)_{B-L}$ symmetry


**Yoshihiko Abe**

(Kyoto U)



Based on arXiv:2001.03954 [hep-ph]

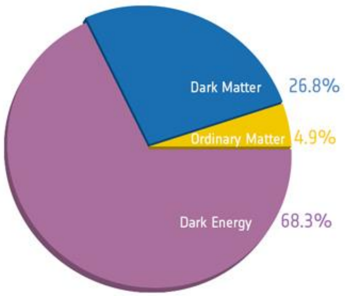


With

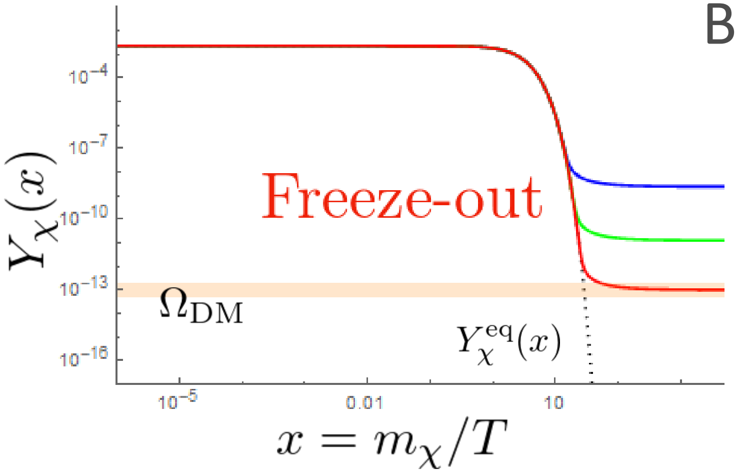
**Takashi Toma** (McGill U) and **Koji Tsumura** (Kyushu U)

# WIMP dark matter

- Dark matter
  - The existence of dark matter is inferred from various observations.
  - The nature of dark matter is still unknown.
  - Identification of dark matter ⇒ BSM




- WIMP dark matter
  - Dark matter relic abundance is realized as the thermal relic



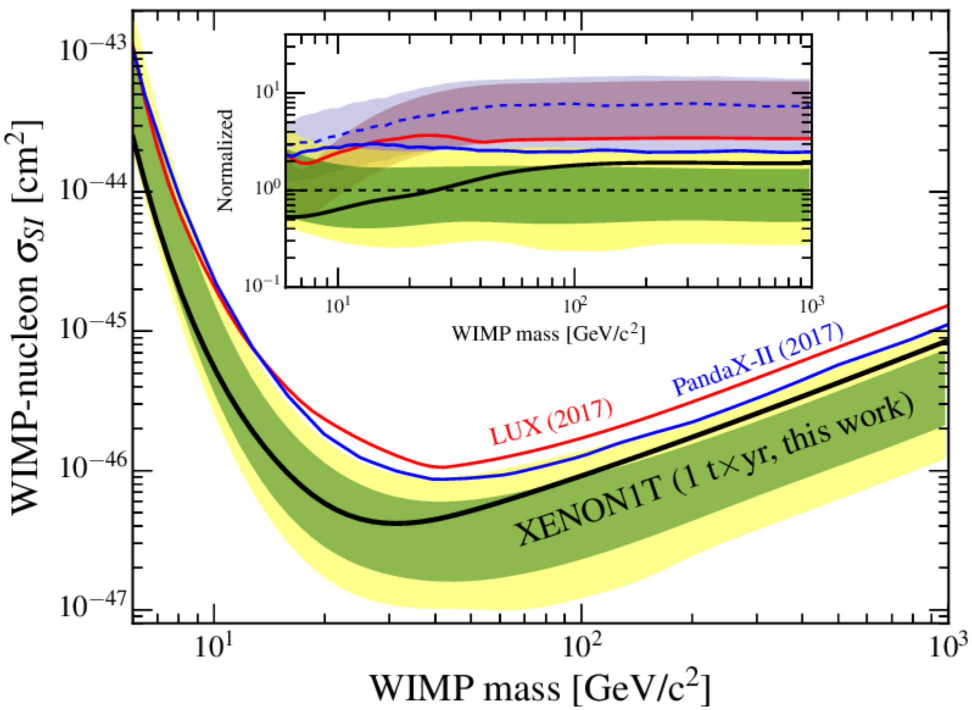

Boltzmann equation for WIMP

$$\frac{dY_\chi(x)}{dx} = \frac{\langle \sigma v \rangle\, s}{xH}\left(Y_\chi(x)^2 - Y_\chi^{\rm eq}(x)^2\right)$$

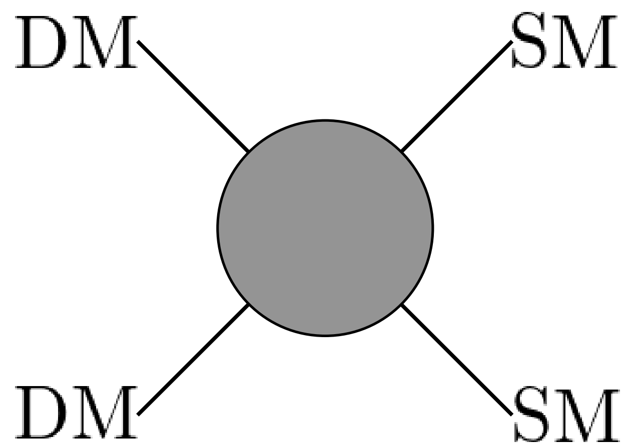
$$\boxed{\Gamma_\chi \ \text{vs}\ H}$$

# Direct detection experiments

- Direct detection experiments
  LUX, PandaX-II, XENON
  ⇒ **Severe constraints** on the WIMP-nucleon cross section




**XENON collaboration (2018)**




Nuclear recoil by dark matter

# Pseudo-Naumbu-Goldstone boson dark matter

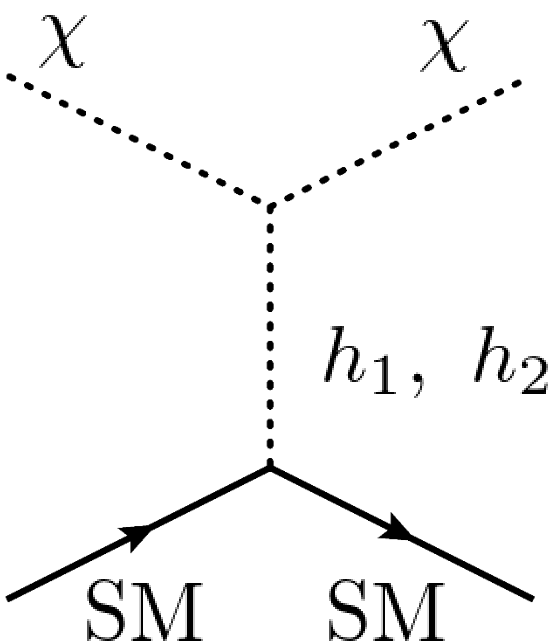
- PNGB dark matter model

[Gross-Lebedev-Toma (2017),…]

SM + SM singlet scalar $S$

$$V(H,S) = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2$$
$$\underline{-\frac{m^2}{4}(S^2 + S^{*2})}$$

Soft breaking term ⇒ creating dark matter mass




$$\kappa_{h_1\chi\chi} = -\frac{m_{h_1}^2 \sin\theta}{v_s}, \quad \kappa_{h_2\chi\chi} = +\frac{m_{h_2}^2 \cos\theta}{v_s}$$

Scattering amplitude

$$i\mathcal{M} \propto \frac{\sin\theta\cos\theta}{v_s}\left(-\frac{m_{h_1}^2}{q^2 - m_{h_1}^2} + \frac{m_{h_2}^2}{q^2 - m_{h_2}^2}\right)$$
$$\sim -\frac{\sin\theta\cos\theta}{v_s}\frac{q^2(m_{h_1}^2 - m_{h_2}^2)}{m_{h_1}^2 m_{h_2}^2} \to 0$$

# Our motivation

- What is the origin for the soft breaking term?

$$V_{\mathrm{soft}}(H,S) = -\frac{m^2}{4}(S^2 + S^{*2})$$

Other term? Renormalizability? Symmetry?

- What is the UV physics of the pNGB dark matter model?

## Our assumptions

- Renormalizable field theoretic description
- The *symmetry* of the UV physics maybe *gauge symmetry*

No global symmetry

# Gauged $U(1)_{B-L}$ model

- Gauged $U(1)_{B-L}$ model

| | $Q_L$ | $u_R^c$ | $d_R^c$ | $L$ | $e_R^c$ | $H$ | $\nu_R^c$ | $S$ | $\Phi$ |
|---|---|---|---|---|---|---|---|---|---|
| $SU(3)_c$ | $\mathbf{3}$ | $\overline{\mathbf{3}}$ | $\overline{\mathbf{3}}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $SU(2)_L$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $U(1)_Y$ | $+1/6$ | $-2/3$ | $+1/3$ | $-1/2$ | $+1$ | $+1/2$ | $0$ | $0$ | $0$ |
| $U(1)_{B-L}$ | $+1/3$ | $-1/3$ | $-1/3$ | $-1$ | $+1$ | $0$ | $+1$ | $+1$ | $+2$ |

**Our gauged $U(1)_{B-L}$ model**

Ordinary $U(1)_{B-L}$ model

SM

Giving Majorana masses

+ RHν $\nu_R$ + New gauge boson $X_\mu$ + Singlet scalar $\Phi$

+ Singlet scalar $S$ ← **New !!**

# Gauged $U(1)_{B-L}$ model

- Gauged $U(1)_{B-L}$ model

| | $Q_L$ | $u_R^c$ | $d_R^c$ | $L$ | $e_R^c$ | $H$ | $\nu_R^c$ | $S$ | $\Phi$ |
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| $U(1)_{B-L}$ | $+1/3$ | $-1/3$ | $-1/3$ | $-1$ | $+1$ | $0$ | $+1$ | $+1$ | $+2$ |

RHνs couple to $\Phi$

- Lagrangian
  - Kinetic term

$$D_\mu = \partial_\mu + i g_{B-L} X_\mu, \;\; X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$$

$$\mathcal{L}_K = |D_\mu S|^2 + |D_\mu \Phi|^2 + \overline{\nu_R} i \not{D} \nu_R - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{\sin\epsilon}{2} X_{\mu\nu} B^{\mu\nu}$$

  - Yukawa interactions

$$\mathcal{L}_Y = -(y_\nu)_{ij} \tilde{H}^\dagger \overline{\nu_{Ri}} L_j \underbrace{- \frac{(y_\Phi)_{ij}}{2} \Phi \overline{\nu_{Ri}^c} \nu_{Rj}}_{\text{Giving Majorana masses}} + \text{h.c.}$$

# Gauged $U(1)_{B-L}$ model

- Scalar potential

$$V(H,S,\Phi) = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 - \frac{\mu_\Phi^2}{2}|\Phi|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \frac{\lambda_\Phi}{2}|\Phi|^4 + \lambda_{HS}|H|^2|S|^2 + \lambda_{H\Phi}|H|^2|\Phi|^2 + \lambda_{S\Phi}|S|^2|\Phi|^2 - \left(\frac{\mu_c}{\sqrt{2}}\Phi^* S^2 + \text{c.c.}\right)$$

- Parametrization

$$H = \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix}, \quad S = \frac{v_s + s + i\eta_s}{\sqrt{2}}, \quad \Phi = \frac{v_\phi + \phi + i\eta_\phi}{\sqrt{2}}$$

- Type-I see-saw ⇒ the scale of $v_\phi$ is determined

$$\boxed{v_\phi \sim 4.3 \times 10^{14}\ \text{GeV}\left(\frac{y_\nu^2}{y_\Phi}\right)} \qquad \gg v,\ v_s$$

Masses of the heaviest scalar and new gauge boson $\sim v_\phi$

# Gauged $U(1)_{B-L}$ model

- Scalar potential

$$V(H,S,\Phi) = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 - \frac{\mu_\Phi^2}{2}|\Phi|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \frac{\lambda_\Phi}{2}|\Phi|^4$$
$$+ \lambda_{HS}|H|^2|S|^2 + \lambda_{H\Phi}|H|^2|\Phi|^2 + \lambda_{S\Phi}|S|^2|\Phi|^2 \boxed{- \left(\frac{\mu_c}{\sqrt{2}}\Phi^* S^2 + \text{c.c.}\right)}$$

## Symmetry breaking

- Symmetry of the scalar potential

$$U(1)_S \times U(1)_\Phi \xrightarrow[\mu_c \Phi^* S^2]{} U(1)_{B-L}$$

If $\mu_c \to 0$, there are tow NGBs

- Symmetry breaking

$$\cancel{U(1)_{B-L}}$$

- NGB → eaten by the gauge boson
- pNGB → $\text{mass}^2 \propto \mu_c$

# Gauged $U(1)_{B-L}$ model

- Mass eigenstates $\tan 2\theta \approx \frac{2vv_s(\lambda_{HS}\lambda_\Phi - \lambda_{H\Phi}\lambda_{S\Phi})}{v^2(\lambda_{H\Phi}^2 - \lambda_H\lambda_\Phi) - v_s^2(\lambda_{S\Phi}^2 - \lambda_S\lambda_\Phi)}$
  - CP-even scalars

SM-like Higgs boson

$$\begin{pmatrix} h \\ s \\ \phi \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & \frac{\lambda_{H\Phi}v}{\lambda_\Phi v_\phi} \\ 0 & 1 & \frac{\lambda_{S\Phi}v_s}{\lambda_\Phi v_\phi} \\ -\frac{\lambda_{H\Phi}v}{\lambda_\Phi v_\phi} & -\frac{\lambda_{S\Phi}v_s}{\lambda_\Phi v_\phi} & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}$$

$$m_{h_1}^2 \approx \lambda_H v^2 - \frac{\lambda_{H\Phi}^2\lambda_S - 2\lambda_{HS}\lambda_{H\Phi}\lambda_{S\Phi} + \lambda_\Phi\lambda_{HS}^2}{\lambda_{S\Phi} - \lambda_{S\Phi}^2}v^2, \quad \leftarrow \text{125 GeV}$$

$$m_{h_2}^2 \approx \frac{\lambda_S\lambda_\Phi - \lambda_{S\Phi}^2}{\lambda_\Phi}v_s^2 + \frac{(\lambda_\Phi\lambda_{HS} - \lambda_{H\Phi}\lambda_{S\Phi})^2}{\lambda_\Phi(\lambda_S\lambda_\Phi - \lambda_{S\Phi}^2)}v^2, \quad m_{h_3}^2 \approx \lambda_\Phi v_\phi^2$$

  - CP-odd scalars

**pNGB (dark matter)**

$$\begin{pmatrix} \eta_s \\ \eta_\phi \end{pmatrix} = \frac{1}{(v_s^2 + 4v_\phi^2)^{1/2}} \begin{pmatrix} 2v_\phi & v_s \\ -v_s & 2v_\phi \end{pmatrix} \begin{pmatrix} \chi \\ \boxed{\tilde{\chi}} \end{pmatrix}$$

Eaten by $X_\mu$

$$\boxed{m_\chi^2 = \frac{\mu_c(v_s^2 + 4v_\phi^2)}{4v_\phi}}$$

# pNGB dark matter from gauged $U(1)_{B-L}$ model

- Naïve understanding

$$V(H,S,\langle\Phi\rangle) = -\frac{\mu_H^2}{2}|H|^2 + \frac{\lambda_{H\Phi}v_\phi^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_{S\Phi}v_\phi^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4$$
$$+ \lambda_{HS}|H|^2|S|^2 - \left(\frac{\mu_c v_\phi}{2}S^2 + \frac{\mu_c v_\phi}{2}S^{*2}\right)$$

Energy scale

$v_\phi \sim 10^{13}$ GeV: $U(1)_{B-L}$ is broken by $v_\phi$

$\tilde{\chi}$ is eaten by $X_\mu \Rightarrow X_\mu$ becomes massive $m_X \sim v_\phi$

$\approx$ Large VEV hierarchy ~ heavy particles decouple $X_\mu, \phi$

$v_s \sim$ TeV

SM + singlet scalar $S$ with $S^2$ term

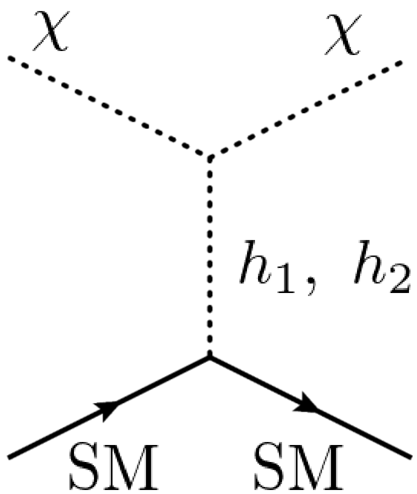
$\Rightarrow$ pNGB dark matter (Check!!)

$v \sim 246$ GeV

(+ heavy particles effects through the mixings)

# pNGB dark matter from gauged $U(1)_{B-L}$ model

- Amplitude for DM + SM → DM +SM



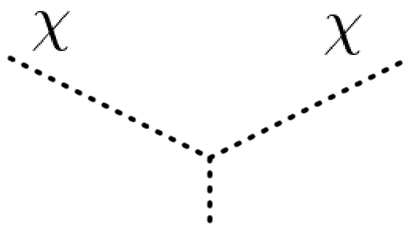

$$\kappa_{\chi\chi h_1} \approx -\frac{m_{h_1}^2 \sin\theta}{v_s}, \quad \kappa_{\chi\chi h_2} \approx +\frac{m_{h_2}^2 \cos\theta}{v_s},$$

$$\kappa_{\chi\chi h_3} \approx +\frac{m_{h_3}^2}{v_s}\frac{\lambda_{S\Phi} v_s}{\lambda_\Phi v_\phi}$$

$$i\mathcal{M} \propto \frac{\sin\theta\cos\theta}{v_s}\left(-\frac{m_{h_1}^2}{q^2 - m_{h_1}^2} + \frac{m_{h_2}^2}{q^2 - m_{h_2}^2}\right) + \mathcal{O}(1/v_\phi)$$

The scattering amplitudes are suppressed in the same way as pNGB model in order $\mathcal{O}(1/v_\phi)$.

- Amplitude for DM + SM → DM +SM

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**Pseudo-Nambu-Goldstone dark matter from gauged $U(1)_{B-L}$ symmetry**

The scattering amplitudes are suppressed in the same way as pNGB model in order $\mathcal{O}(1/v_\phi)$.

# Long-lived dark matter

- Our dark matter $\chi$ is not stabilized due to the new interactions and scalar mixing.

- Constraints of our model from a conservative limit of the dark matter life time [Baring-Ghosh-Queiroz-Sinha (2015)]

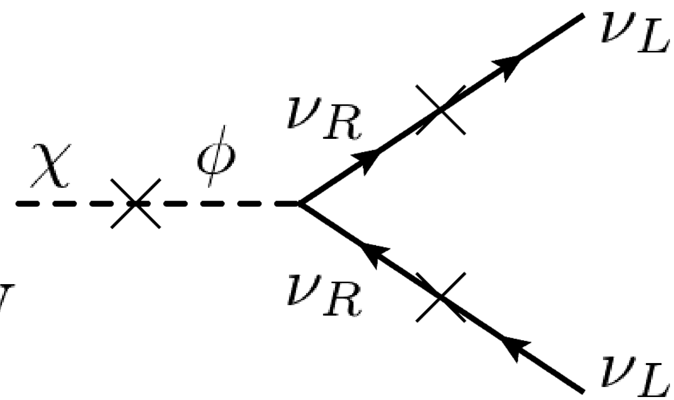
$$\boxed{\tau_{\rm DM} \gtrsim 10^{27}\ {\rm s} \quad \Leftrightarrow \quad \Gamma_{\rm DM} \lesssim 6.6 \times 10^{-52}\ {\rm GeV}}$$

- We have to check the decay cannels of this pNGB dark matter

# Long-lived dark matter

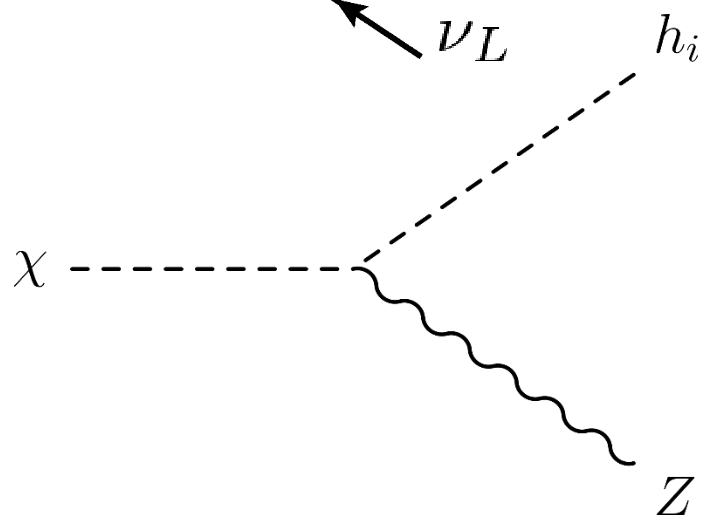
## Two body decay

- $\chi \to \nu\nu$




$$\Gamma_{\chi\to\nu\nu} \lesssim 10^{-67}\ \mathrm{GeV}$$

- $\chi \to h_i\, Z$



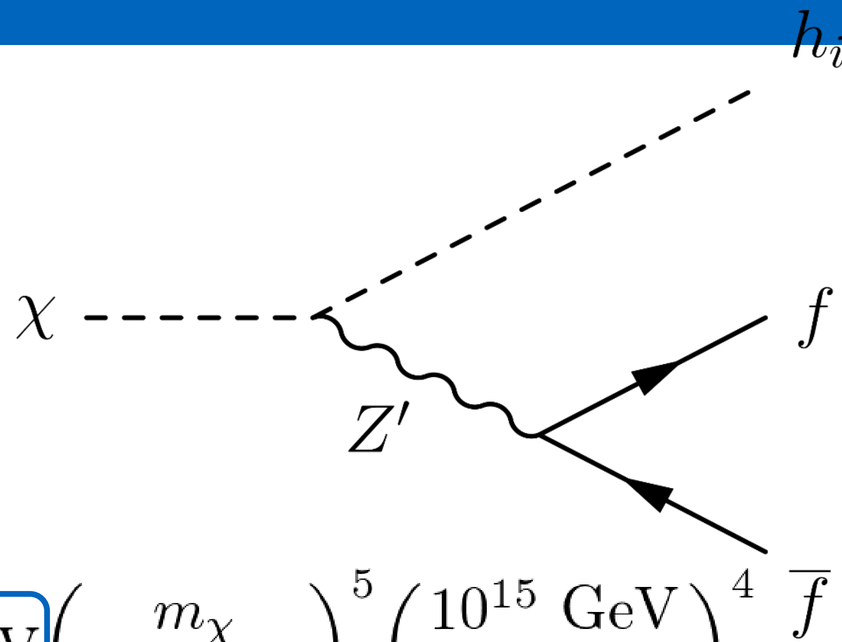

$$\Gamma_{\text{2-body}} \approx \frac{g_{B-L}^2}{16\pi m_{Z'}^4} m_Z^2 m_\chi^3 \sin^2\theta_W \sin^2\epsilon$$

$$= \boxed{5.8\times 10^{-52}\ \mathrm{GeV}} \left(\frac{m_\chi}{0.5\ \mathrm{TeV}}\right)^3 \left(\frac{10^{15}\ \mathrm{GeV}}{m_{Z'}}\right)^2 \left(\frac{10^{15}\ \mathrm{GeV}}{v_\phi}\right)^2 \left(\frac{\sin\epsilon}{1/\sqrt{2}}\right)^2$$

# Long-lived dark matter

## Three body decay

- $\chi \to h_i f \bar{f}$




$$\Gamma_{\text{3-body}}\Big|_{\sin\epsilon\to 0} \approx \frac{13}{16}\frac{g_{B-L}^4}{1536\pi^3}\frac{m_\chi^5}{m_{Z'}^4}$$

$$\approx \boxed{5.3\times 10^{-52}\ \text{GeV}} \left(\frac{m_\chi}{0.5\ \text{TeV}}\right)^5 \left(\frac{10^{15}\ \text{GeV}}{v_\phi}\right)^4$$

$$\Gamma_{\text{3-body}}\Big|_{\sin\epsilon\to 1/\sqrt{2}} \approx \frac{g_{B-L}^2}{768\pi^3}\frac{m_\chi^5}{m_{Z'}^4}\left(10g_1^2 - 8\sqrt{2}g_1 g_{B-L} + 26 g_{B-L}^2\right)$$

$$\approx \boxed{4.1\times 10^{-52}\ \text{GeV}} \left(\frac{m_\chi}{0.5\ \text{TeV}}\right)^5 \left(\frac{10^{15}\ \text{GeV}}{m_{Z'}}\right)^2 \left(\frac{10^{15}\ \text{GeV}}{v_\phi}\right)^2$$

$$\times \left[1 - \frac{2\sqrt{2}}{5}\frac{m_{Z'}}{g_1 v_\phi} + \frac{13}{20}\frac{m_{Z'}^2}{g_1^2 v_\phi^2}\right]$$

# Numerical Results

# Numerical Analysis

- Parameter sets

$$m_{h_2} = 300 \text{ or } 1000 \text{ GeV}, \quad m_{h_3} = 10^{13} \text{ GeV},$$

$$\sin\theta = 0.1, \quad \lambda_{H\Phi} = \lambda_{S\Phi} = 10^{-6}$$

$$m_{Z'} = 10^{14} \text{ or } 10^{15} \text{ GeV},$$

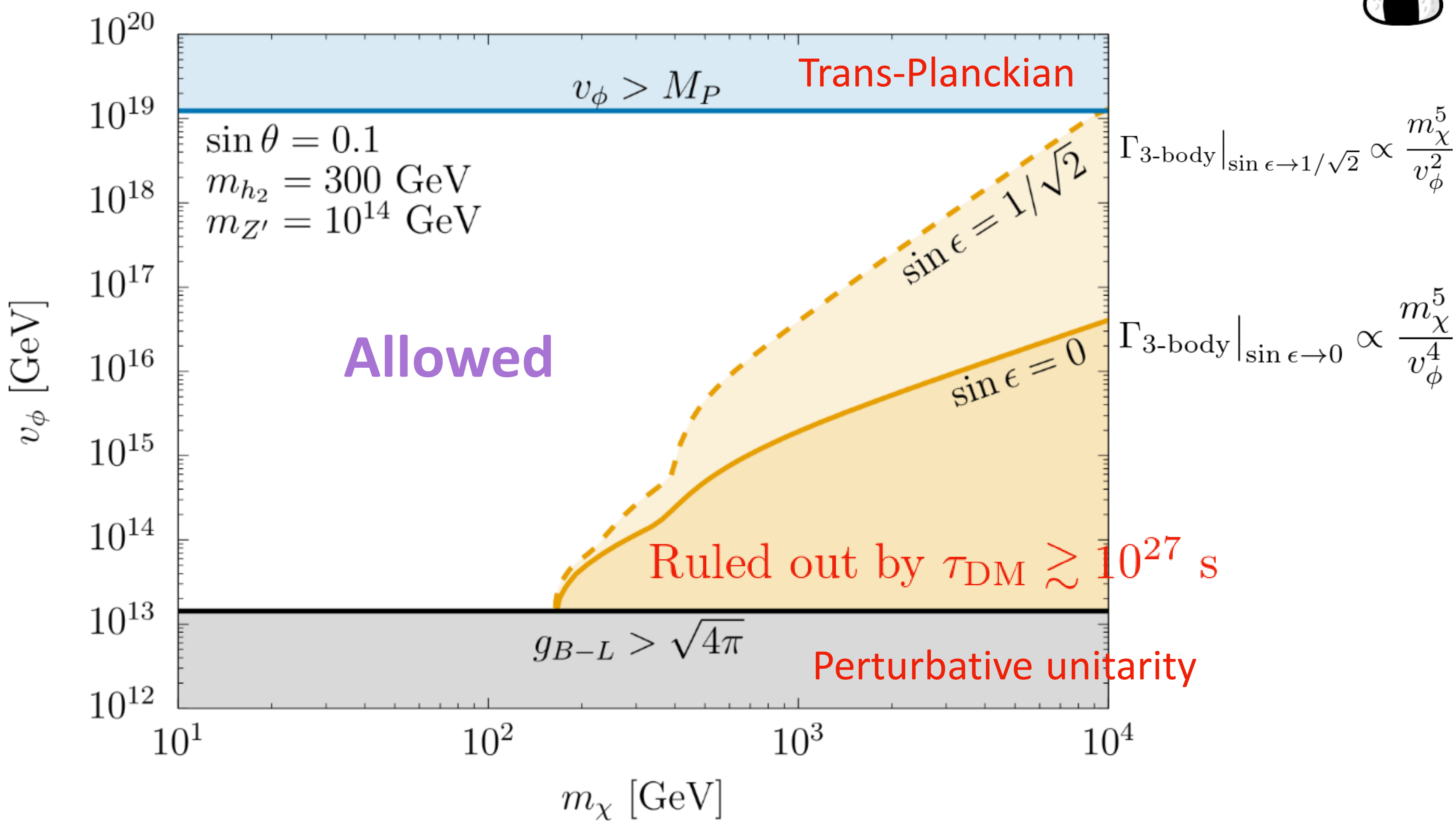
$$\sin\epsilon = 0 \text{ or } \frac{1}{\sqrt{2}}$$

Gauge coupling and quartic coupling are fixed by

$$g_{B-L}^2 \approx \frac{m_{Z'}^2}{4v_\phi^2}, \quad \lambda_\Phi \approx \frac{m_{h_3}^2}{v_\phi^2}$$

# Allowed region in the $(m_\chi, v_\phi)$ plane

$$m_{h_2} = 300\ \text{GeV}, \quad m_{Z'} = 10^{14}\ \text{GeV}$$



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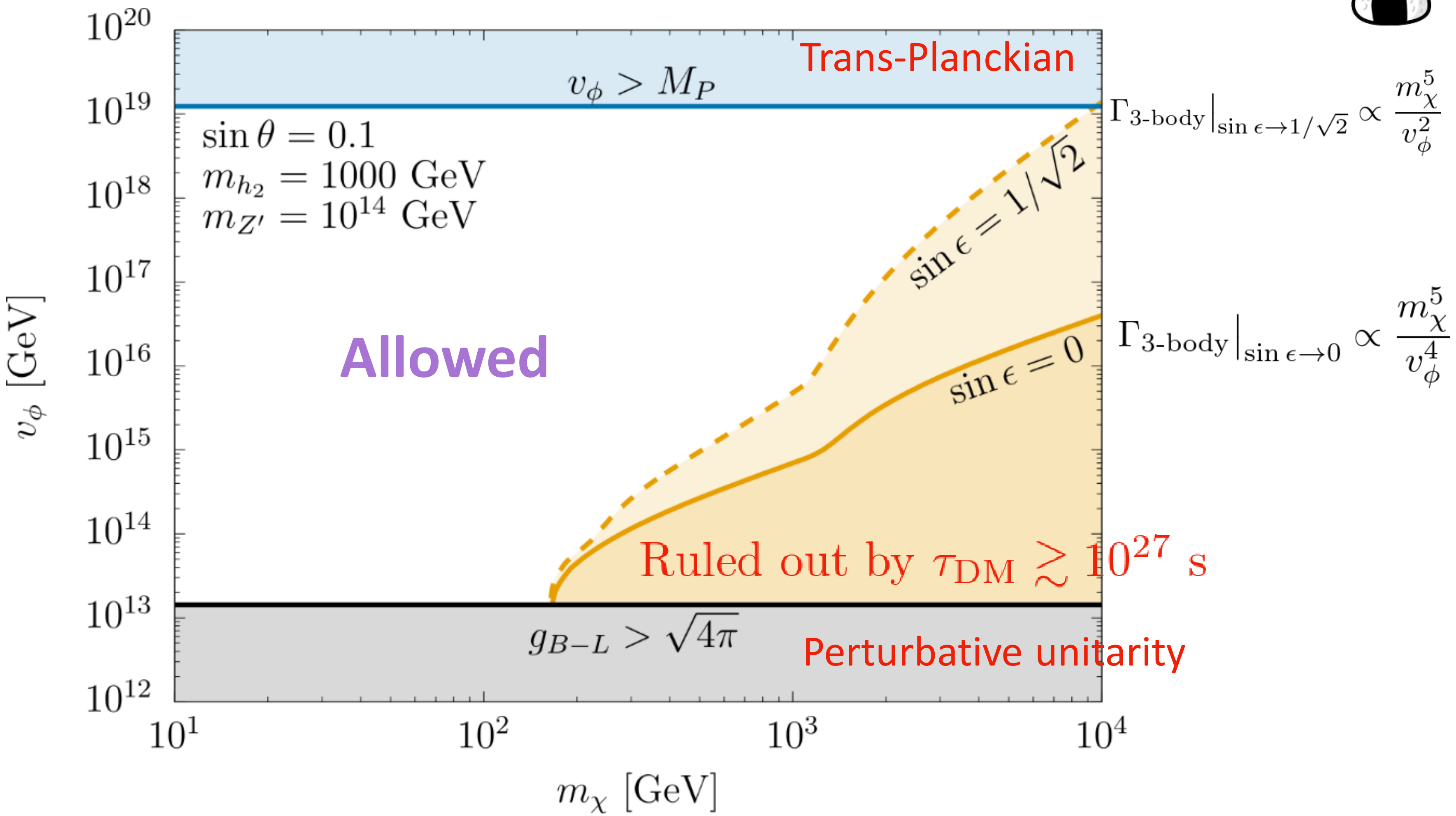
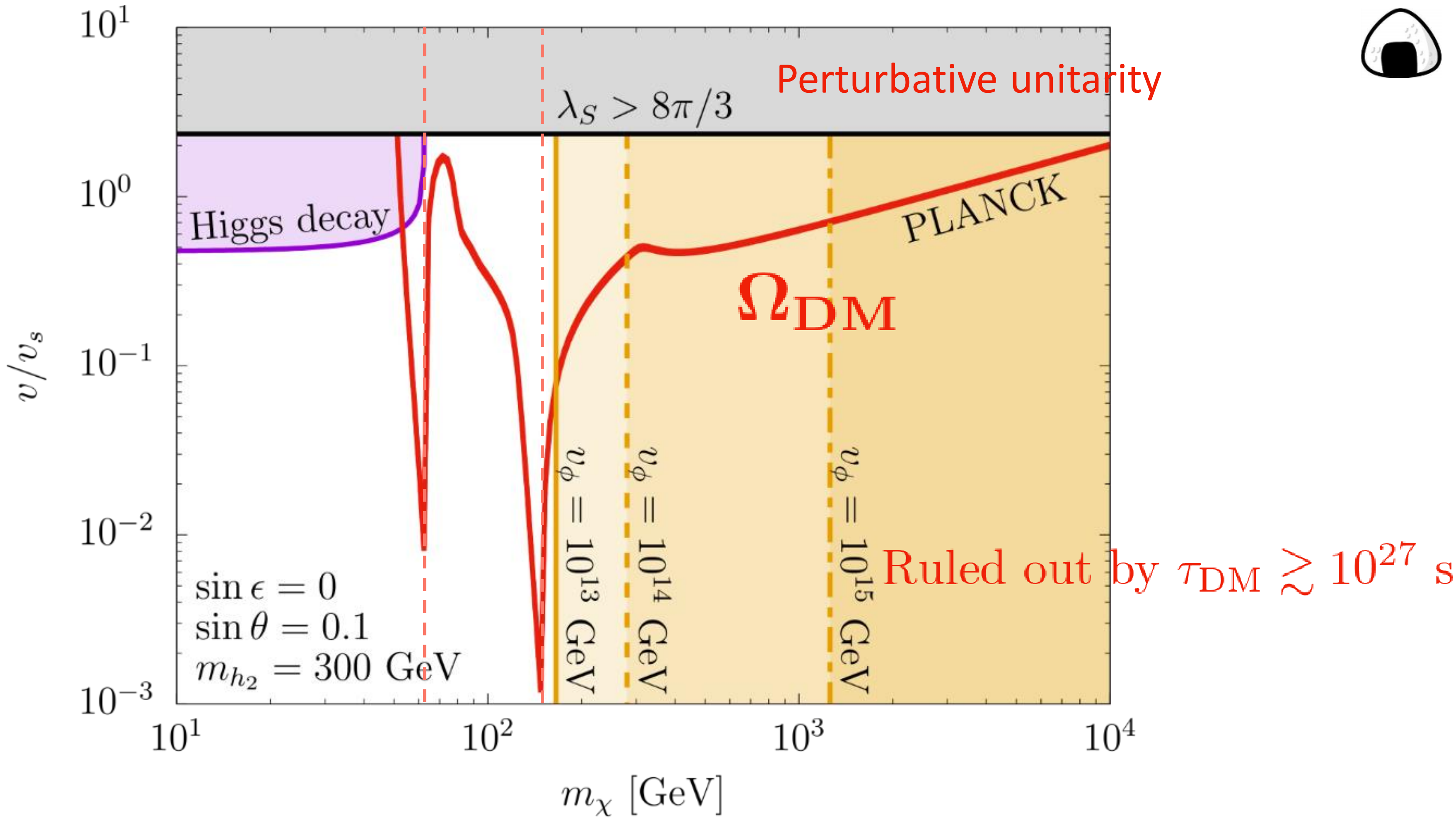


# Allowed region in the $(m_\chi, v/v_s)$ plane

$$m_{h_2} = 300 \text{ GeV}, \quad \sin\epsilon = 0$$



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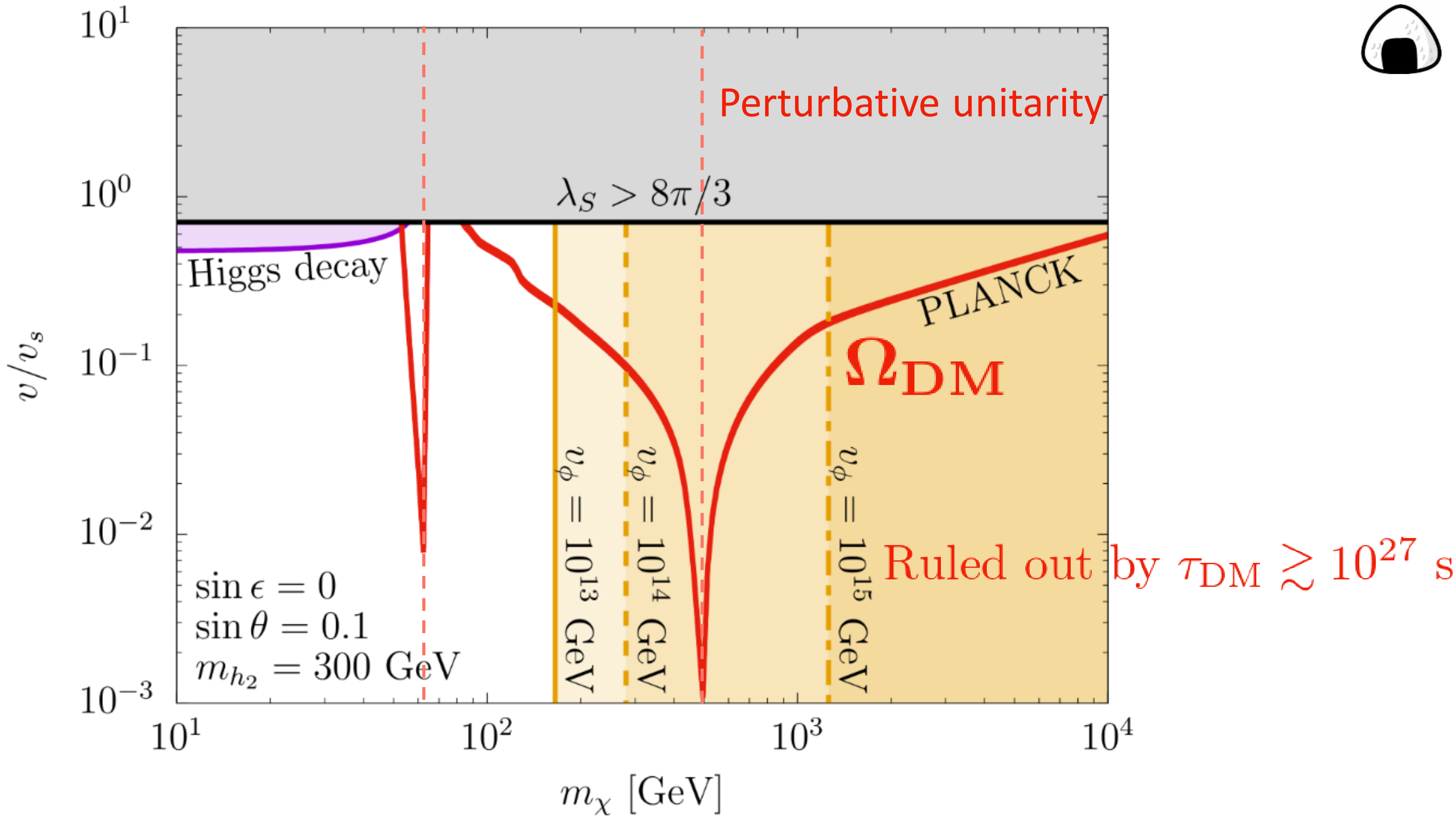
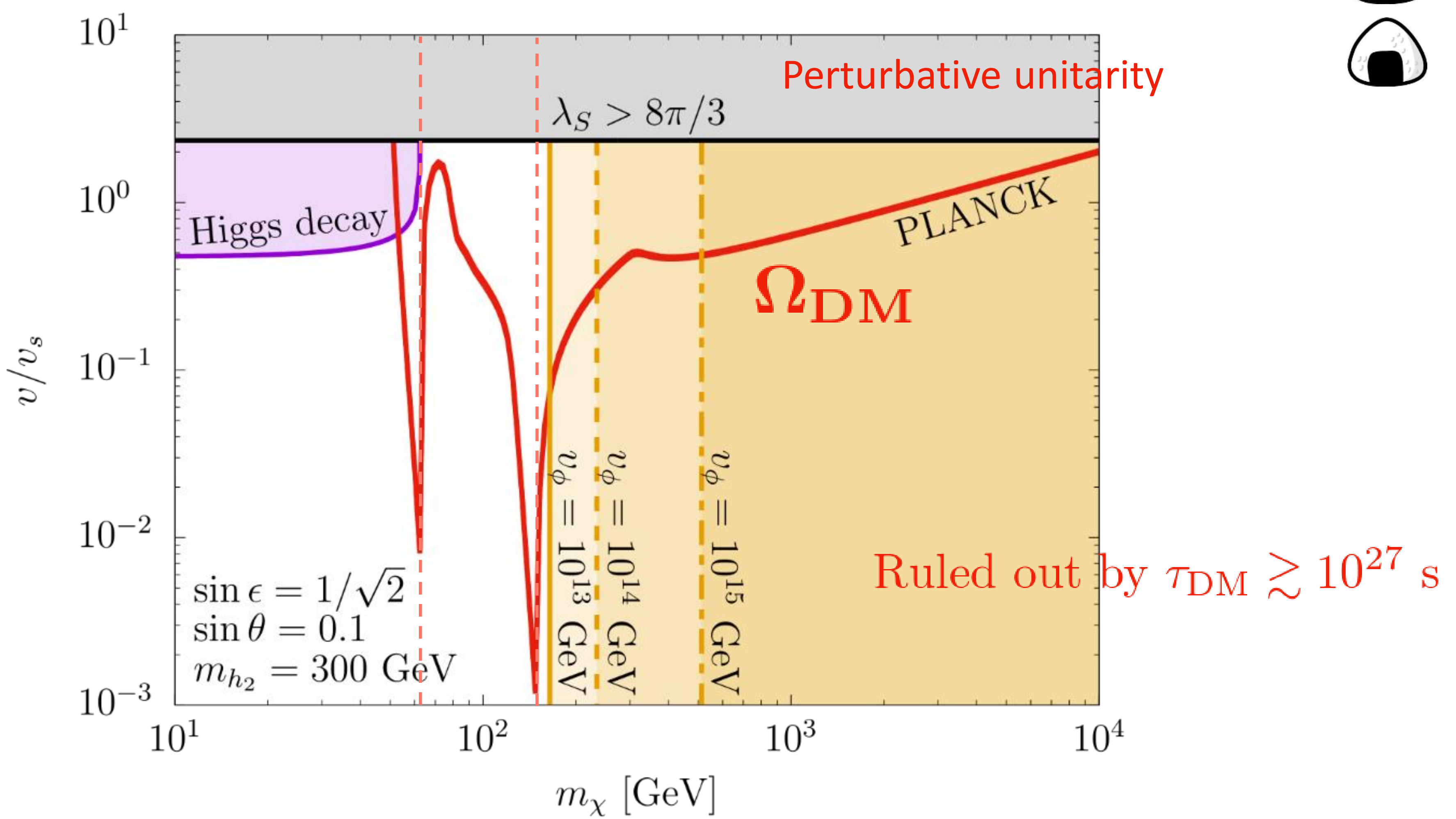


# Allowed region in the $(m_\chi, v/v_s)$ plane

$$m_{h_2} = 300\ \text{GeV}, \quad \sin\epsilon = 1/\sqrt{2}$$



# Summary

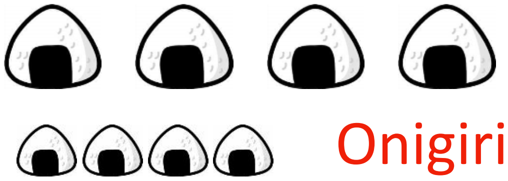
- We studied the pNGB dark matter scenario derived from the *gauged* $U(1)_{B-L}$ model.

- This is the decaying dark matter then we showed the life time is long enough to be dark matter.

- We have found the parameter space consistent with the relevant constraints.

- This model can be explored by the planned gamma-ray observations.

e.g. CTA, LHASSO



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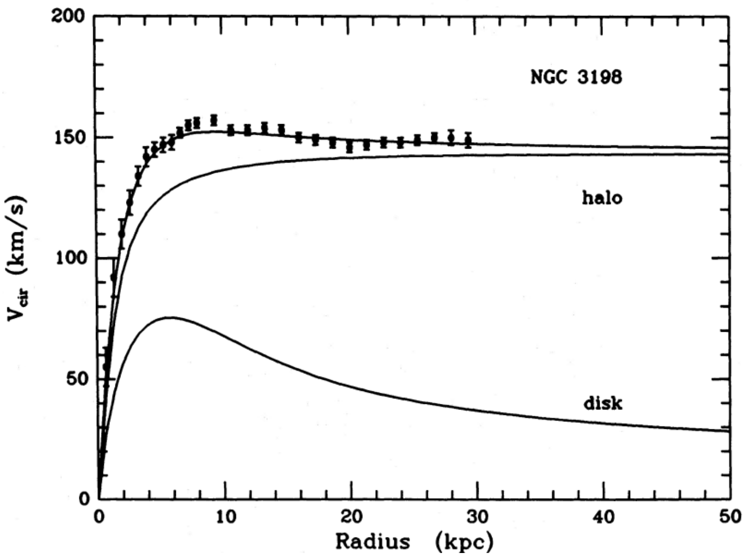
# Back Up

# Dark matter

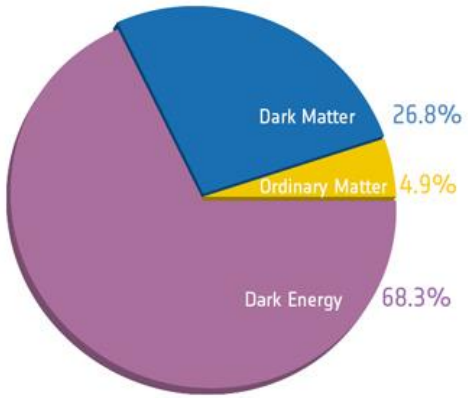
There is a lot of evidence of dark matter.

- Rotation curve of spiral galaxies
- CMB observations
- Gravitational lensing
- Large scale structure of the universe
- Bullet cluster



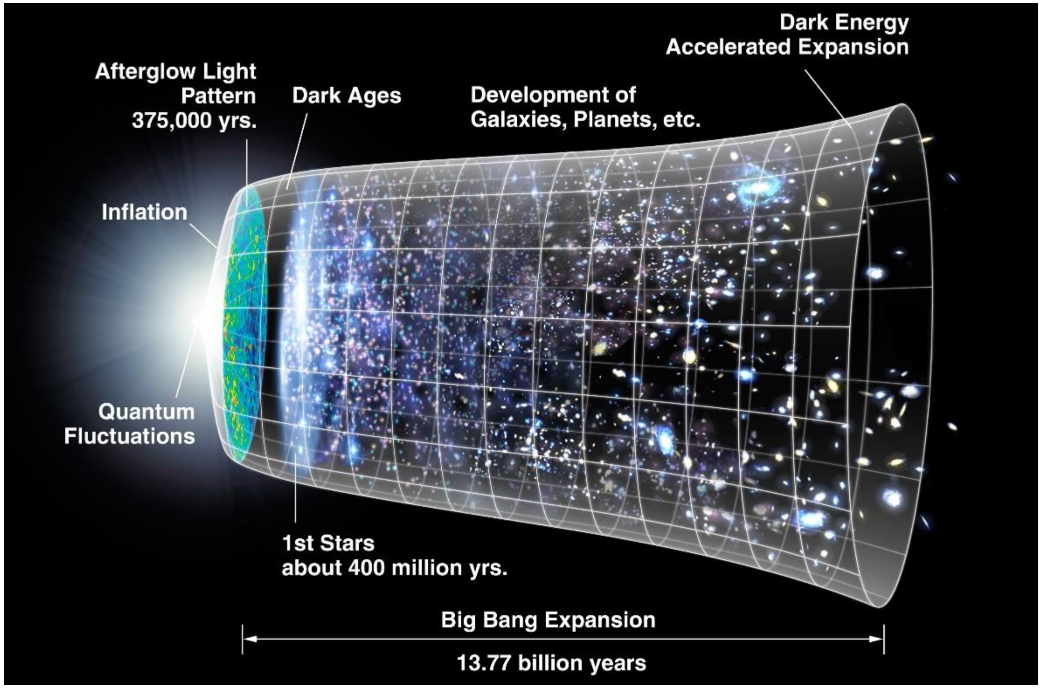

Dark matter existence is crucial.

Alabada et al. ApJ (1985)




# Dark matter

- Nature of dark matter
  - Stable (at least longer than the age of the universe)
  - Electrically neutral (may have very small charge)
  - Occupy 27% of energy density of the universe
  - Gravitationally interacting
  - Non-relativistic (cold)



# Boltzmann equation

- Boltzmann equation for dark matter

$$Y_\chi(x) = n_\chi/s, \quad x = m_\chi/T$$

$$\boxed{\frac{dY_\chi(x)}{dx} = -\frac{\langle\sigma v\rangle}{x^2}\frac{s(m_\chi)}{H(m_\chi)}\left(Y_\chi(x)^2 - Y_\chi^{\rm eq}(x)^2\right)}$$

$$s(T) = \frac{2\pi^2}{45}g_*^S T^3, \quad H(T) = \sqrt{\frac{4\pi^3}{45}g_*}\frac{T^2}{M_P}$$

$$Y_\chi^{\rm eq} = n_\chi^{\rm eq}/s, \quad n_\chi^{\rm eq} = \frac{m_\chi^2 T^2}{2\pi^2}K_2(m_\chi/T)$$

$$\langle\sigma v\rangle = \frac{1}{n_\chi^{\rm eq2}}\frac{1}{2^5\pi^4}\left(\frac{m_\chi}{x}\right)\int_{4m_\chi^2}^{\infty} ds\,(s - 4m_\chi^2)\sqrt{s}K_1(x\sqrt{s}/m_\chi)\sigma(s)$$

$\sigma$: Total dark matter annihilation cross section

# Another choice of $Q_{B-L}(S)$

- If another choice is taken, the suppression of scattering amplitude among dark matter and SM particles is non-trivial.

e.g.

$$V(H,S,\Phi) \supset -\frac{2\mu_q}{3}\left(\Phi^* S^3 + S^{*3}\Phi\right)$$

→ dark matter – dark matter – CP-even scalars vertices

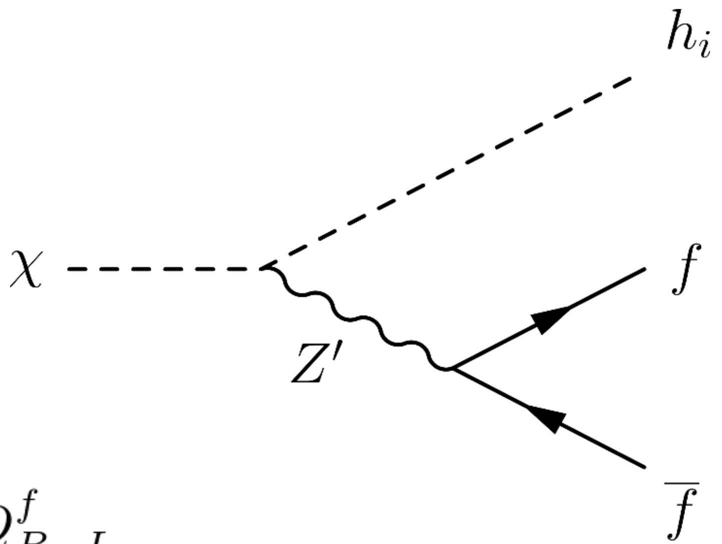
$$\kappa_{\chi\chi h_1} \sim -\frac{m_{h_1}^2 + m_\chi^2}{v_s}\sin\theta, \quad \kappa_{\chi\chi h_2} \sim +\frac{m_{h_2}^2 + m_\chi^2}{v_s}\cos\theta$$

$$i\mathcal{M} \sim \frac{\sin\theta\cos\theta\, m_\chi^2}{v_s}\left[\frac{1}{q^2 - m_{h_1}^2} - \frac{1}{q^2 - m_{h_2}^2}\right]$$

# Long-lived dark matter

## Three body decay

- $\chi \to h_i f \bar{f}$




$$\mathcal{L}_{Z' f\bar{f}} = -Z'_\mu \bar{f} \gamma^\mu \left[ g_V^f + g_A^f \gamma_5 \right] f$$

$$g_A^f \approx 0$$

$$g_V^f \approx -g_1 (Q_{\rm em}^f - T_3^f) \tan\epsilon + \frac{g_{B-L}}{\cos\epsilon} Q_{B-L}^f$$

$$\Gamma_{\chi \to h_i f\bar{f}} = \frac{g_{B-L} U_{si}^2 m_\chi^5}{768\pi^3 m_{Z'}^4} \frac{\cos^2\zeta}{\cos^2\epsilon} \left( {g_V^f}^2 + {g_A^f}^2 \right) \left[ 1 - 8\xi_i + 8\xi_i^3 - \xi_i^4 - 12\xi^2 \log\xi_i \right]$$

$$\xi_i \equiv m_{h_i}^2 / m_\chi^2$$

$$\boxed{\Gamma_{3\text{-body}} \equiv \sum_i \sum_f \Gamma_{\chi \to h_i f\bar{f}}}$$

# Reference values

- Dark matter relic [Aghanim *et al.* [Planck Collaboration] (2018)]

$$\boxed{\Omega_{\rm DM} h^2 = 0.120 \pm 0.001}$$

- Dark matter lifetime [Baring-Ghosh-Queiroz-Sinha (2015)]

$$\boxed{\tau_{\rm DM} \gtrsim 10^{27}\ {\rm s} \quad \Leftrightarrow \quad \Gamma_{\rm DM} \lesssim 6.6 \times 10^{-52}\ {\rm GeV}}$$

- Constraints on the second Higgs

[Flakowski-Gross-Lebedev (2015)]

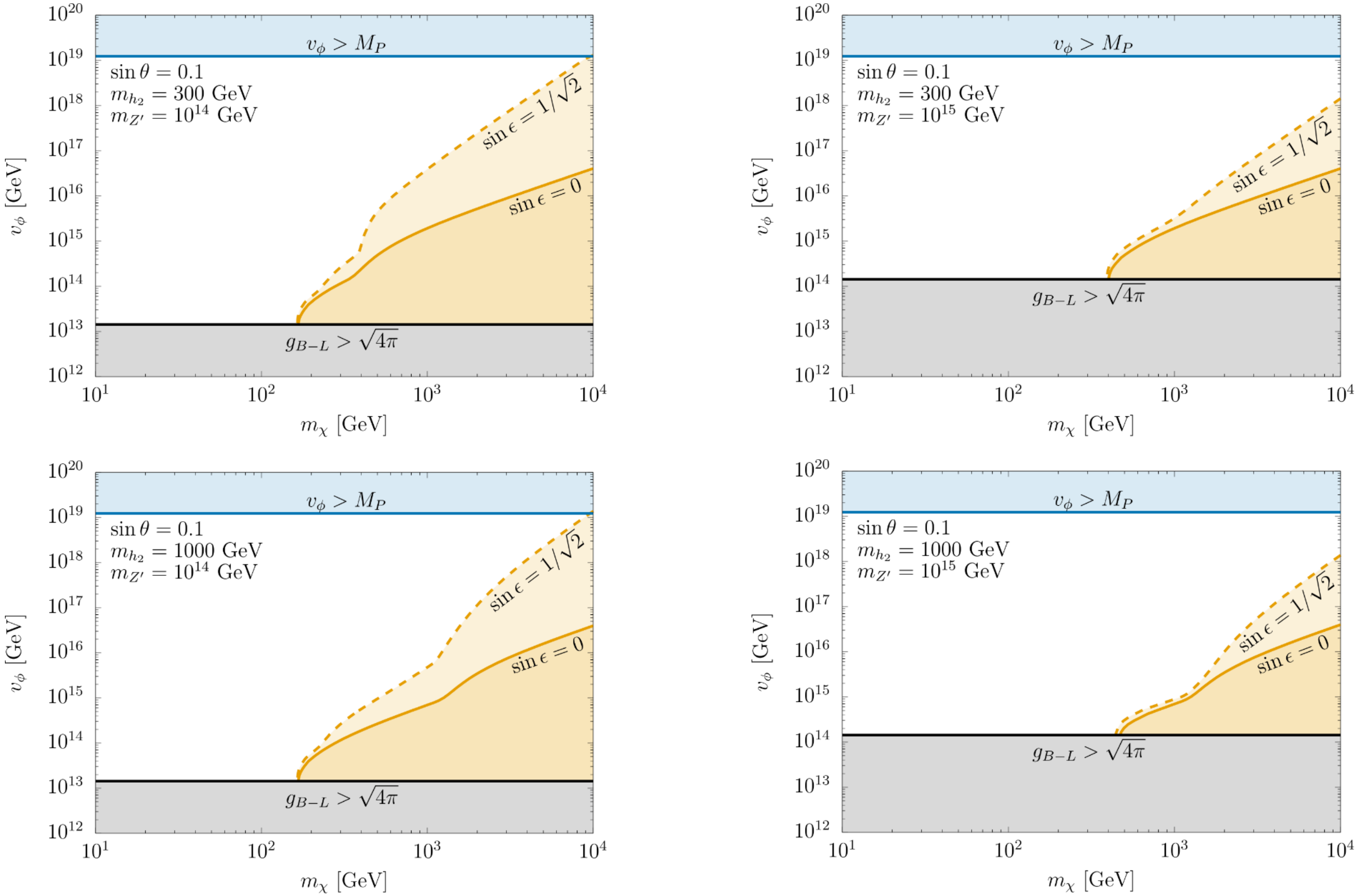
$$\sin\theta \lesssim 0.3 \quad {\rm for} \quad m_{h_2} \gtrsim 100\ {\rm GeV}$$

[Chen-Dawson-Lewis (2015)]

$$\lambda_S \leq 8\pi/3$$

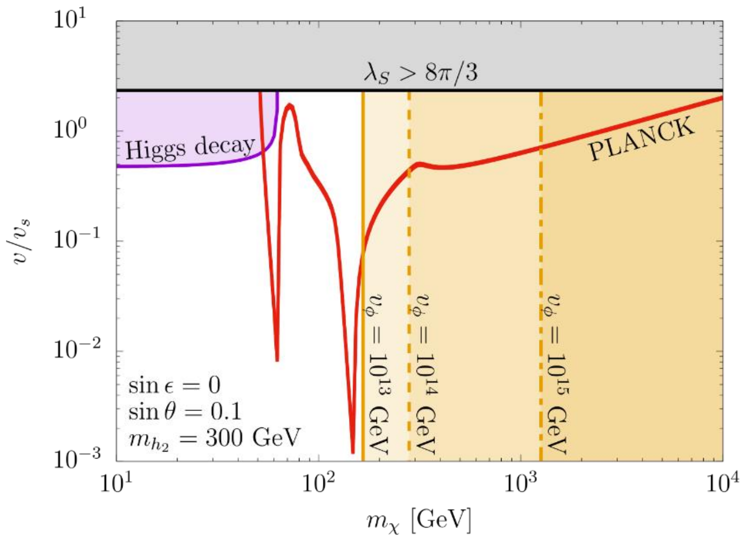
# Numerical Analysis

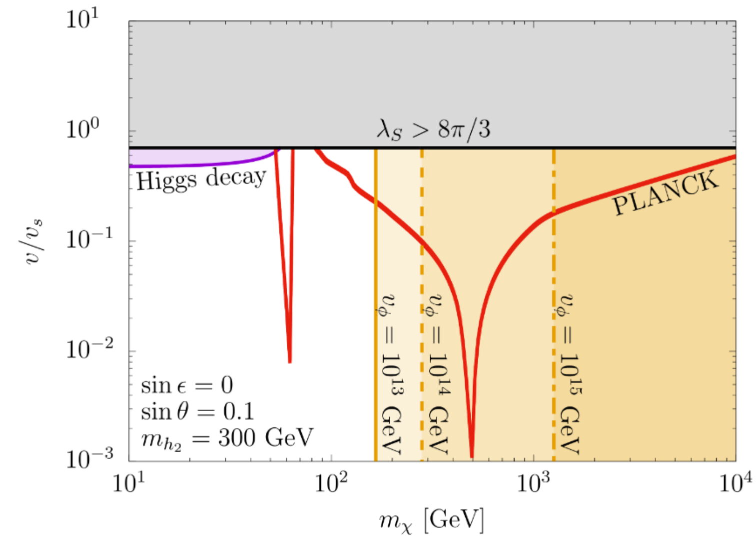
- Allowed regions in the $(m_\chi, v_\phi)$ plane

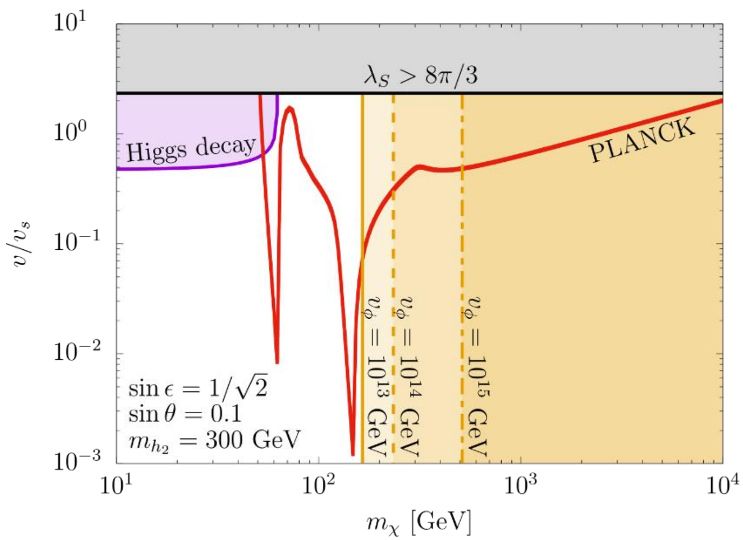


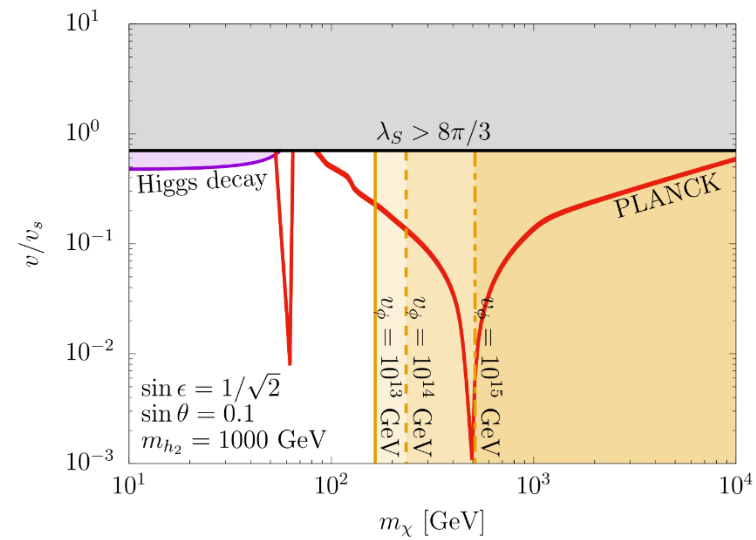
# Numerical Analysis

- Allowed regions in the $(m_\chi, v/v_s)$ plane



# Gauge kinetic mixing

## $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge group

- Kinetic term

$$\mathcal{L}_{GK} = -\frac{1}{2}\mathrm{tr}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\sin\epsilon}{2}B_{\mu\nu}X^{\mu\nu}$$

- Mass term

$$\mathcal{L}_M = \frac{1}{2}\begin{pmatrix} B_\mu & W^3_\mu X_\mu \end{pmatrix}\begin{pmatrix} \sin^2\theta_W m_{\tilde{Z}}^2 & -\sin\theta_W\cos\theta_W m_{\tilde{Z}}^2 & 0 \\ -\sin\theta_W\cos\theta_W m_{\tilde{Z}}^2 & \cos^2\theta_W m_{\tilde{Z}}^2 & 0 \\ 0 & 0 & m_X^2 \end{pmatrix}\begin{pmatrix} B^\mu \\ W^{3\mu} \\ X^\mu \end{pmatrix}$$

$$\sin\theta_W \equiv \frac{g_1}{\sqrt{g_1^2+g_2^2}}, \quad \cos\theta_W \equiv \frac{g_2}{\sqrt{g_1^2+g_2^2}},$$

$$m_{\tilde{Z}}^2 \equiv \frac{g_1^2+g_2^2}{4}v^2, \quad m_X^2 \equiv g_{B-L}^2(v_s^2+4v_\phi^2)$$

# Gauge kinetic mixing

## $SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ gauge group

- Mixing

$$\tilde{V}_{GK} = \begin{pmatrix} 1 & 0 & -\tan\epsilon \\ 0 & 1 & 0 \\ 0 & 0 & 1/\cos\epsilon \end{pmatrix}, \qquad \tan 2\zeta = \frac{-m_{\tilde{Z}}^2 \sin_W \sin 2\epsilon}{m_X^2 - m_{\tilde{Z}}^2(\cos^2\epsilon - \sin^2\theta_W \sin^2\epsilon)}$$

$$U_G = \begin{pmatrix} \cos\theta_W & -\sin\theta_W & 0 \\ \sin\theta_W & \cos\theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\zeta & -\sin\zeta \\ 0 & \sin\zeta & \cos\zeta \end{pmatrix}$$

$$\begin{pmatrix} B_\mu \\ W_\mu^3 \\ X_\mu \end{pmatrix} = \tilde{V}_{GK} U_G \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

- Mass eigenvalues

$$m_Z^2 = \frac{1}{2}\left[\overline{M}^2 - \sqrt{\overline{M}^4 - \frac{4m_{\tilde{Z}}^2 m_X^2}{\cos^2\epsilon}}\right], \quad m_{Z'}^2 = \frac{1}{2}\left[\overline{M}^2 - \sqrt{\overline{M}^4 + \frac{4m_{\tilde{Z}}^2 m_X^2}{\cos^2\epsilon}}\right]$$

$$\overline{M}^2 \equiv m_{\tilde{Z}}^2(1 + \sin^2\theta_W \tan^2\epsilon) + m_X^2/\cos^2\epsilon$$

# Interactions

- Scalar-dark matter-massive gauge boson

$$\mathcal{L}_{Zh_i\chi} = \sum_i g_{B-L}\frac{\sin\zeta}{\cos\epsilon}\frac{U_{si}}{\sqrt{1+v_s^2/4v_\phi^2}}Z_\mu(h_i\partial^\mu\chi - \chi\partial^\mu h_i),$$

$$\mathcal{L}_{Z'h_i\chi} = \sum_i g_{B-L}\frac{\cos\zeta}{\cos\epsilon}\frac{U_{si}}{\sqrt{1+v_s^2/4v_\phi^2}}Z'_\mu(h_i\partial^\mu\chi - \chi\partial^\mu h_i)$$

- Massive gauge boson-fermion

$$\mathcal{L}_{Z'\overline{f}f} = -Z'_\mu\overline{f}\gamma^\mu\left[g_V^f + g_A^f\gamma_5\right]f$$

$$\begin{aligned} g_V^f =& -\frac{g_2}{2}T_3^f\sin\zeta\cos\theta_W + g_1(Q_{\rm em}^f - F_3^f)(\sin\zeta\sin\theta_W - \cos\zeta\tan\epsilon) \\ &+ g_{B-L}Q_{B-L}^f\frac{\cos\zeta}{\cos\epsilon}, \\ g_A^f =& \frac{g_2}{2}T_3^f\sin\zeta\cos\theta_W \end{aligned}$$