Generalization of Higgs Effective Field Theory

Yoshiki Uchida (Nagoya U.)

Phys. Rev. D 100, 075020

In collaboration with
Ryo Nagai (INFN, Padua)
Masaharu Tanabashi (Nagoya U.)
Koji Tsumura (Kyusyu U.)

Goal

Make model-independent prediction about properties of H^0, $H^\pm$, $H^{\pm\pm}$, $\cdots$

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Make model-independent prediction about *properties* of H^0, $H^\pm$, $H^{\pm\pm}$, $\cdots$

- production cross section
- ϕ-V-V couplings ($\phi = H^0, H^\pm, H^{\pm\pm}\cdots$)

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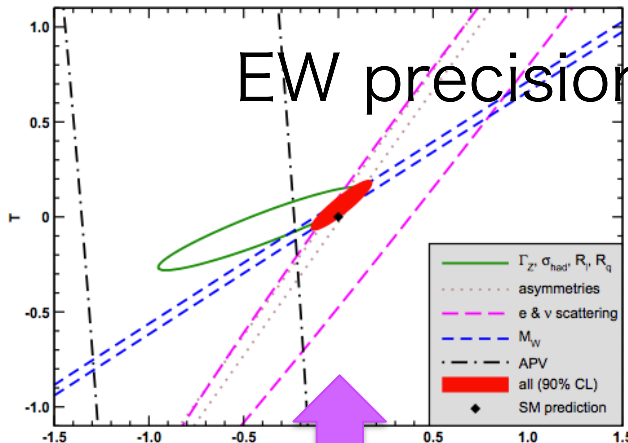
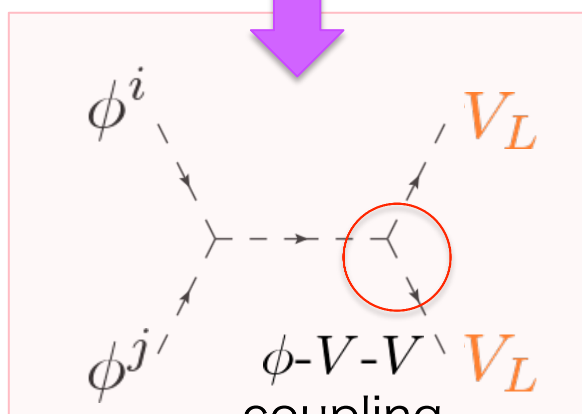
Make model-independent prediction about properties of H^0, $H^{\pm}$, $H^{\pm\pm}$, $\cdots$

Results

EW precision measurements



Tanabashi et al, (PDG),
Phys. Rev. D 98, 030001(2018)



ϕ-V-V coupling

$\phi = H^0, H^{\pm}, H^{\pm\pm} \cdots$

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EFT approach should be applied !!

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Make *model-independent* prediction about properties of $H^0, H^\pm, H^{\pm\pm}, \cdots$

EFT approach △ should be applied !!

Existing EFT : heavy particles are integrated-out

We need to extend existing EFT …

(Extension)

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What is Higgs Effective Field Theory ?

Higgs Effective Field Theory

(HEFT)

$$\mathcal{L}_{\rm HEFT} = \frac{v^2}{4} F(h) \mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + \cdots$$

$$F(h) = 1 + \kappa_1 \frac{h}{v} + \kappa_2 \left(\frac{h}{v}\right)^2 + \cdots \qquad U = \exp\left(\frac{i\pi^a\tau^a}{v}\right)$$

$$V(h) = m_h^2 h^2 + \lambda_3 h^3 + \cdots$$

- Symmetry : $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Matter contents : h , Z_μ , $W_\mu^\pm \cdots$

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cf.) SMEFT H , B_μ , W_μ^a $\cdots$ *classified under* $SU(2)_L \times U(1)_Y$

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$$m^2 \pi^+ \pi^- \qquad m'^2 \pi^0 \pi^0$$

In our vacuum, $SU(2)_L \times U(1)_Y$ is realized as the shift symmetry of $\pi^\pm, \pi^0$

and forbid their mass terms

Higgs Effective Field Theory

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In the theory with SSB ($\mathcal{G} \to \mathcal{H}$)

$\mathcal{G}$ is not really broken, but remains as non-linearly realized symmetry

Generalization of HEFT

From now on, we call generalized HEFT as " GHEFT "

~ GHEFT ~

・symmetry : $SU(3)_C \times \quad SU(2)_L \times U(1)_Y$

・matter contents : SM + $H^0, \ H^\pm, \ H^{\pm\pm}, \cdots$

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Non-trivial

Add $H^0, \ H^\pm, \ H^{\pm\pm}, \cdots$ respecting non-linearly realized $SU(2)_L \times U(1)_Y$

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→ CCWZ method

Coleman et. al., Phys. Rev. 177. 2239
Callan et. al., Phys. Rev. 177. 2247

Generalization of HEFT

We apply CCWZ to the extension of the HEFT !!

GHEFT

$$\mathcal{L}_{\text{GHEFT}} = \frac{1}{2} g_{ij}(\Phi) \partial_\mu \Phi^i \partial^\mu \Phi^j - V$$

$$\Phi^i = \{\pi^a, \phi^I\}$$

$$g_{1I} = G_{1I} + \frac{1}{2} G_{3I} \pi^2 - \frac{1}{6} G_{1I} \pi^2 \pi^2 + \frac{1}{6} G_{2I} \pi^1 \pi^2 + \mathcal{O}((\pi)^3),$$
$$g_{2I} = G_{2I} + \frac{1}{2} G_{3I} \pi^1 + \frac{1}{6} G_{1I} \pi^1 \pi^2 - \frac{1}{6} G_{2I} \pi^1 \pi^1 + \mathcal{O}((\pi)^3),$$
$$\vdots$$

R. Nagai, M. Tanabashi, K. Tsumura, Y.U.
Phys. Rev. D **100**, 075020

Phenomenology

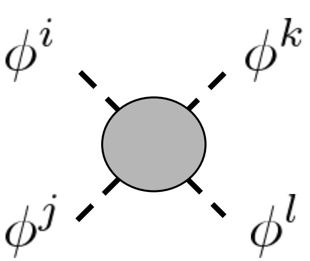
$$\mathcal{L}_{\mathrm{GHEFT}} = \frac{1}{2} g_{ij}(\Phi)\partial_\mu \Phi^i \partial^\mu \Phi^j - V \quad \Phi^i = \{\pi^a, \phi^I\}$$

GHEFT has common property to HEFT …

Nonrenormalizable

$$S_{\mathrm{div}} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

Perturbative non-unitary



$$\underset{E \gg m_\phi}{\sim} \frac{s}{3}(\bar{R}_{iklj} + \bar{R}_{ilkj}) + \frac{t}{3}(\bar{R}_{ijlk} + \bar{R}_{iljk}) + \frac{u}{3}(\bar{R}_{ijkl} + \bar{R}_{ikjl})$$

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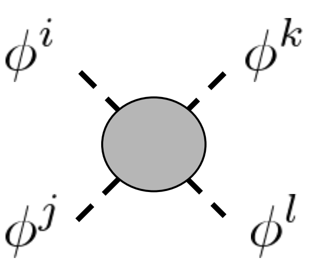

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Unitarity condition : $\bar{R}_{ijkl} = 0$

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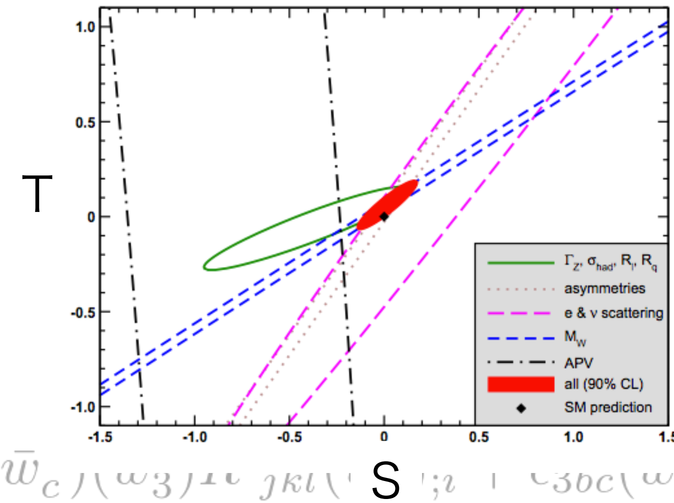
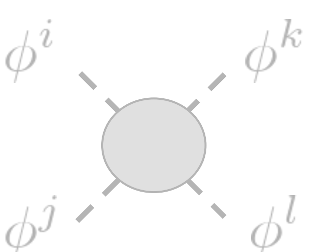


$$S_{\rm div} = -\frac{1}{12\pi}\Big(\epsilon_{3bc}(\bar w_c \ldots) \ldots \epsilon_{3bc}(\ldots)(\bar w_c^l)\bar R^i{}_{jkl}(\bar y^j)_{;i}\Big)\ln\frac{\Lambda^2}{\mu^2}$$



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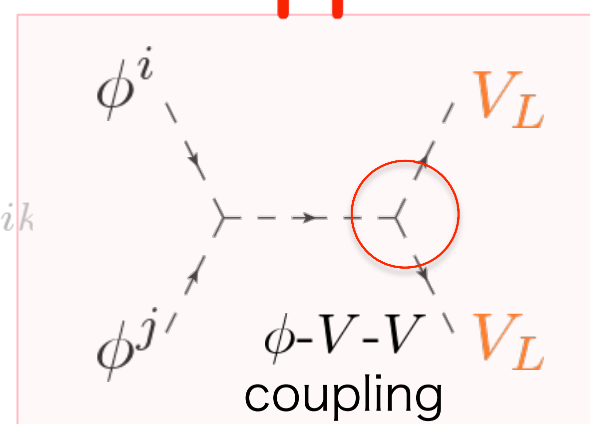
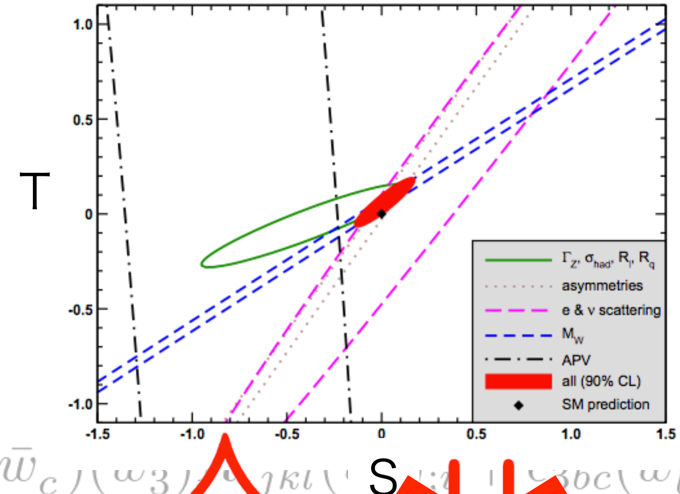


Results

$$\mathcal{L}_{\rm GHEFT} = \frac{1}{2} g_{ij}(\Phi)\partial_\mu \Phi^i \partial^\mu \Phi^j - V \qquad \Phi^i = \{\pi^a, \phi^I\}$$



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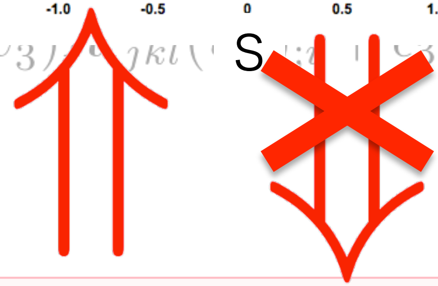

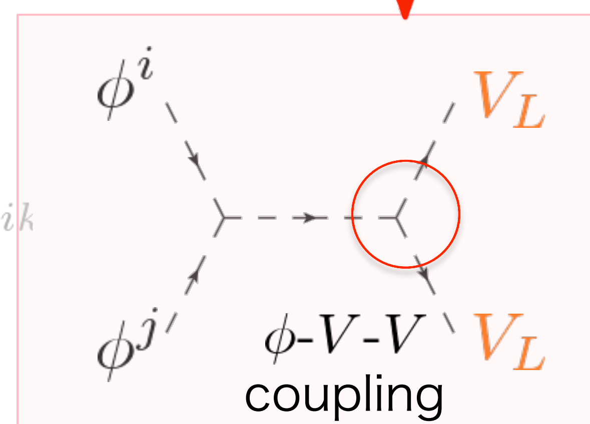

$$\sim_{E \gg m_\phi} \frac{s}{3}(\bar{R}_{ik} \ldots \bar{R}_{iljk}) + \frac{u}{3}(\bar{R}_{ijkl} + \bar{R}_{ikj})$$

Summary

Goal Make model-independent prediction about properties of H^0, $H^\pm$, $H^{\pm\pm}$, $\cdots$

- We formulate a generalization of HEFT (GHEFT) so that it includes additional scalar fields.
- GHEFT is invariant under nonlinear SU(2)xU(1) thanks to CCWZ method
- Take Home Message

Finiteness of S (*i.e.* $S_{\rm div} = 0$) $<$ perturbative unitarity (*i.e.* $\bar{R}_{ijkl} = 0$)

Back Up

Non-linearly realized sym.

Extension of HEFT

Mission

Add H^0, $H^\pm$, $H^{\pm\pm}$, $\cdots$ respecting non-linearly realized $SU(2)_L \times U(1)_Y$

→ CCWZ method

Coleman et. al., Phys. Rev. 177. 2239
Callan et. al., Phys. Rev. 177. 2247

Non-linearly realized $SU(2)_L \times U(1)_Y$

= $SU(2)_L \times U(1)_Y$ transformation depends on $\pi^\pm, \pi^0$

Ex.) $SU(2)_L$: $H^{+\prime} = H^+ + \frac{\theta_L}{2\sqrt{2}}(\pi^- - \pi^+)H^+ + \cdots$

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Even if we take $\theta_L(x) \to \theta_L$

It still depends on x_μ through $\pi^\pm(x), \pi^0(x)$

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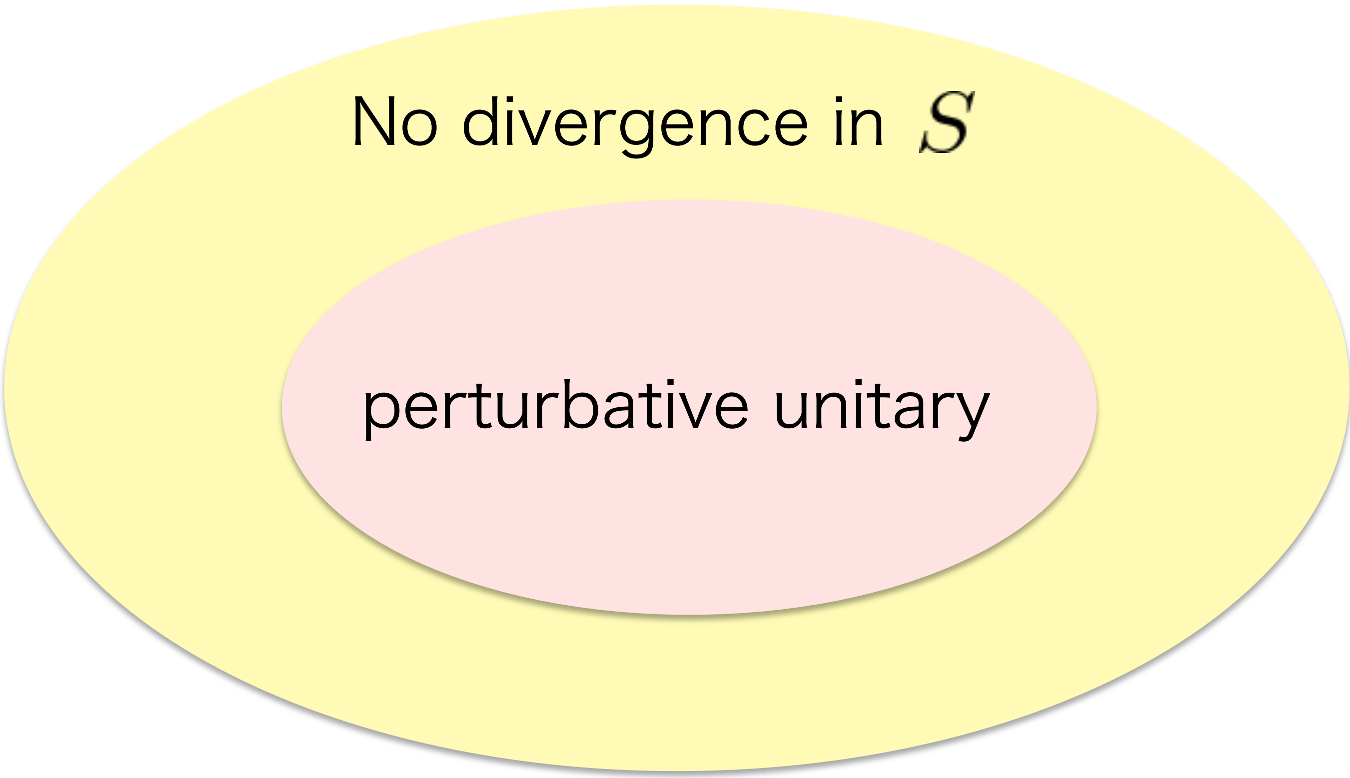
connection : W_μ, B_μ

connection : $\alpha_{\parallel\mu}$

Sufficient, not necessary

Phenomenological implication



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$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$\bar{w}_a^i$, $\bar{y}^i$: Killing vector for $SU(2)_L$, $U(1)_Y$

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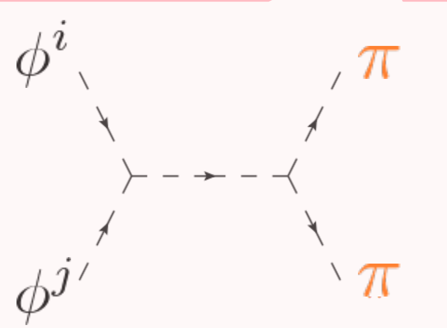

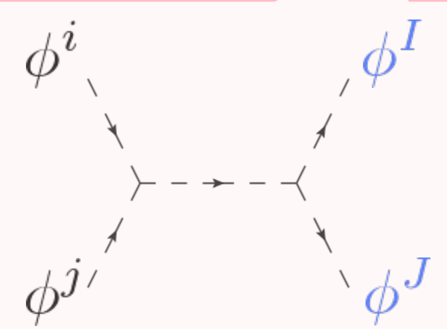

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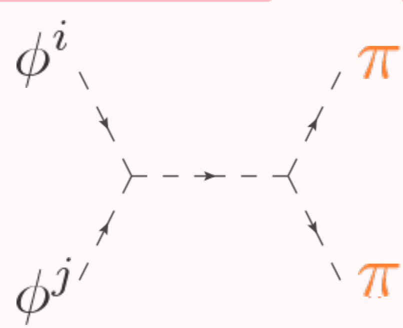

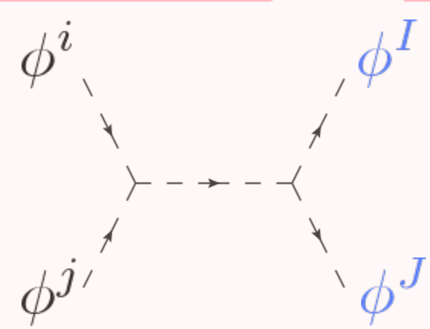

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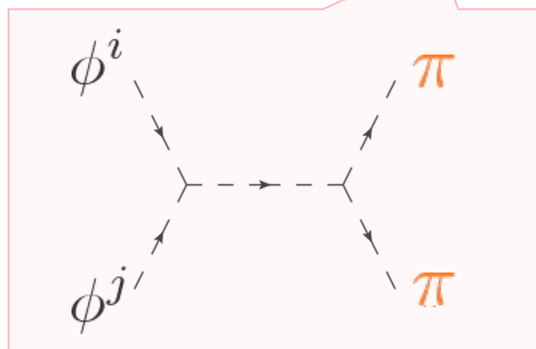
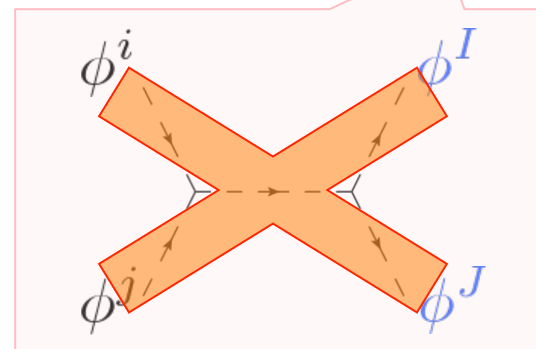
$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$\bar{w}^i_a$, $\bar{y}^i$: Killing vector for $SU(2)_L$, $U(1)_Y$

$$\sum_{k,l}\bar{w}^k\bar{w}^l\bar{R}^i{}_{jkl} = \bar{w}^\pi\bar{w}^\pi\bar{R}^i{}_{j\pi\pi} + \quad 0\times\ \bar{R}^i{}_{j\phi^I\phi^J}$$



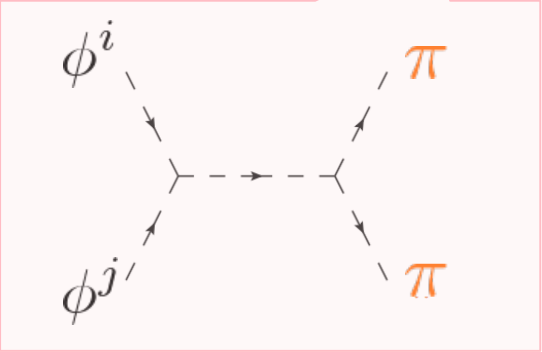


Phenomenological implication

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}_c^k)(\bar{w}_3^l)\bar{R}^i{}_{jkl}(\bar{w}_b^j)_{;i} + \epsilon_{3bc}(\bar{w}_b^k)(\bar{w}_c^l)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$\bar{w}_a^i$, $\bar{y}^i$: Killing vector for $SU(2)_L$, $U(1)_Y$

$$\sum_{k,l}\bar{w}^k\bar{w}^l\bar{R}^i{}_{jkl} = \bar{w}^\pi\bar{w}^\pi\bar{R}^i{}_{j\pi\pi}$$



Phenomenological implication

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$\bar{w}^i_a$, $\bar{y}^i$: Killing vector for $SU(2)_L$, $U(1)_Y$

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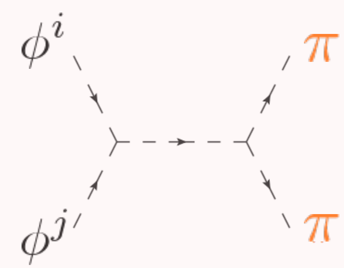


When we require $S_{\rm div} = 0$ $\cdots$

imposing $\bar{R}_{ijkl} = 0$ are too strict
(perturbative unitary)

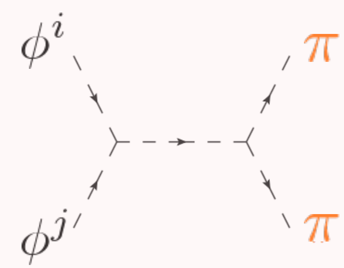
only $\bar{R}^i{}_{j\pi\pi} = 0$ is enough

Phenomenological implication

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$\bar{w}^i_a$, $\bar{y}^i$: Killing vector for $SU(2)_L$, $U(1)_Y$

$$\sum_{k,l}\bar{w}^k\bar{w}^l\bar{R}^i{}_{jkl} = \bar{w}^\pi\bar{w}^\pi\bar{R}^i{}_{j\pi\pi}$$



Finiteness of S
(*i.e.* $S_{\rm div} = 0$)

$<$

perturbative unitarity
(*i.e.* $\bar{R}_{ijkl} = 0$)

Loop expansion

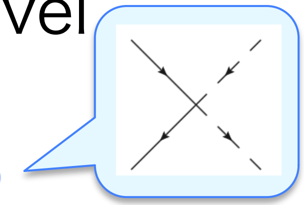
Loop expansion : tree level

tree level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO}$

$$\Delta\mathcal{L}_{\rm LO} = \frac{v^2}{4}F(h)\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h) + \cdots$$

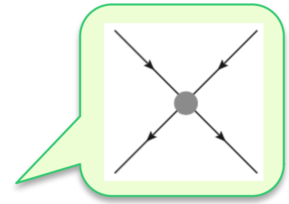
Loop expansion : 1-loop level

tree level $\mathcal{L} = \Delta\mathcal{L}_{\mathrm{LO}}$



$\bar{\psi}\psi \to \phi\phi$

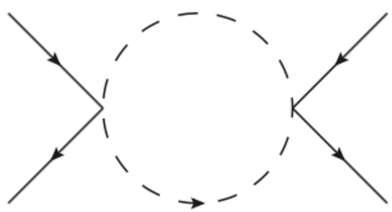
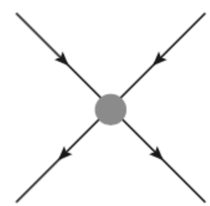
1-loop level $\mathcal{L} = \Delta\mathcal{L}_{\mathrm{LO}} + \Delta\mathcal{L}_{\mathrm{NLO}}$



(counterterm)

4-fermi

Renormalization condition

 $+$  $= \mathcal{O}_{\mathrm{exp}}$

1-loop diagram w/ $\Delta\mathcal{L}_{\mathrm{LO}}$ vertex

counterterm in $\Delta\mathcal{L}_{\mathrm{NLO}}$

Observables not used for R.C. can be predicted

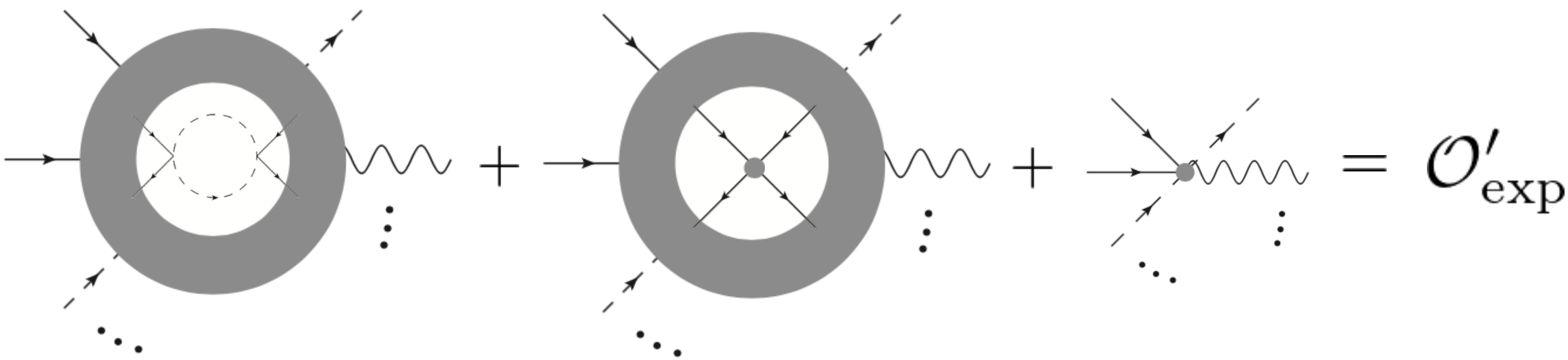
Loop expansion : 2-loop level

tree level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO}$

1-loop level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO} + \Delta\mathcal{L}_{\rm NLO}$

2-loop level $\mathcal{L} = \Delta\mathcal{L}_{\rm LO} + \Delta\mathcal{L}_{\rm NLO} + \Delta\mathcal{L}_{\rm NNLO}$

Renormalization condition



2-loop diagram w/ $\Delta\mathcal{L}_{\rm LO}$ vertex

1-loop diagram w/ $\Delta\mathcal{L}_{\rm NLO}$ vertex

counterterm in $\Delta\mathcal{L}_{\rm NNLO}$

Observables not used for R.C. can be predicted

Power counting

表 7.1: 相互作用の分類と、NLO に含まれる個数

LO の vertex	φ^{2i}	$\overline{\psi}_{L(R)}\psi_{R(L)}\varphi^k$	$X_\mu\varphi^l$	$X_\mu^2\varphi^s$	X_μ^4	X_μ^3	$\overline{\psi}_{L(R)}\psi_{L(R)}X_\mu$
因子	p^2/v^{2i-2}	y/v^{k-1}	gp/v^{l-2}	g^2/v^{s-2}	g^2	gp	g
$\mathcal{D}_L$ に含まれる個数	n_i	ν_k	m_l	r_s	x	u	$z_L(z_R)$

L-loop diagram : $\mathcal{D}_L$

$$\mathcal{D}_L \sim \frac{(yv)^\nu (gv)^{m+2r+2x+u+z}}{v^{F_L+F_R-2}} \frac{p^d}{\Lambda^{2L}} \bar{\psi}_L^{F_L^1} \psi_L^{F_L^2} \bar{\psi}_R^{F_R^1} \psi_R^{F_R^2} \left(\frac{X_{\mu\nu}}{v}\right)^V \left(\frac{\varphi}{v}\right)^B$$

$$d \equiv 2L + 2 - \frac{F_L + F_R}{2} - V - \nu - m - 2r - 2x - u - z$$

T, U parameter

1-loop corrections

Peskin-Takeuchi's S, T, U parameter

$$S_{\rm div} = -\frac{1}{12\pi}\left(\epsilon_{3bc}(\bar{w}^k_c)(\bar{w}^l_3)\bar{R}^i{}_{jkl}(\bar{w}^j_b)_{;i} + \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{y}^j)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$$U_{\rm div} = \frac{1}{12\pi}\left(\epsilon_{1bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{w}^j_1)_{;i} - \epsilon_{3bc}(\bar{w}^k_b)(\bar{w}^l_c)\bar{R}^i{}_{jkl}(\bar{w}^j_3)_{;i}\right)\ln\frac{\Lambda^2}{\mu^2}$$

$$\begin{aligned} T_{\rm div} \sim & \left[(\bar{w}^i_1)(\bar{w}^j_1) - (\bar{w}^i_3)(\bar{w}^j_3)\right] \\ & \times\Big\{\bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}^k_a\bar{w}^l_a + \bar{y}^k\bar{y}^l] \\ & \quad - 4g_W^2(\bar{w}^k_a)_{;i}(\bar{w}^l_a)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}\Big\}\ln\frac{\Lambda^2}{\mu^2} \end{aligned}$$

T parameter

$$T_{\rm div} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right]$$
$$\times \Big\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l]$$
$$- 4g_W^2 (\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2 (\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl} \Big\} \ln \frac{\Lambda^2}{\mu^2}$$

T parameter

$$
\begin{aligned}
T_{\text{div}} \sim\ & \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right] \\
& \times \left\{ \bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k \bar{w}_a^l + \bar{y}^k \bar{y}^l] \right. \\
& \qquad \left. - 4g_W^2 (\bar{w}_a^k)_{;i} (\bar{w}_a^l)_{;j} \bar{g}_{kl} - 4g_Y^2 (\bar{y}^k)_{;i} (\bar{y}^l)_{;j} \bar{g}_{kl} \right\} \ln \frac{\Lambda^2}{\mu^2}
\end{aligned}
$$

T parameter

$$T_{\rm div} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right] \times \Big\{\bar{R}_{ikjl}[\bar{g}^{km}\bar{g}^{ln}\bar{V}_{;mn} + \bar{w}_a^k\bar{w}_a^l + \bar{y}^k\bar{y}^l] - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}\Big\}\ln\frac{\Lambda^2}{\mu^2}$$

if $\bar{R}_{ikjl} = 0$

$$T_{\rm div} \sim \left[(\bar{w}_1^i)(\bar{w}_1^j) - (\bar{w}_3^i)(\bar{w}_3^j)\right] \times \Big\{ - 4g_W^2(\bar{w}_a^k)_{;i}(\bar{w}_a^l)_{;j}\bar{g}_{kl} - 4g_Y^2(\bar{y}^k)_{;i}(\bar{y}^l)_{;j}\bar{g}_{kl}\Big\}\ln\frac{\Lambda^2}{\mu^2}$$

= 0 (SM)

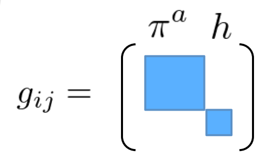
= 0 (2HDM)

$\neq$ 0 (Georgi Machacek Model)

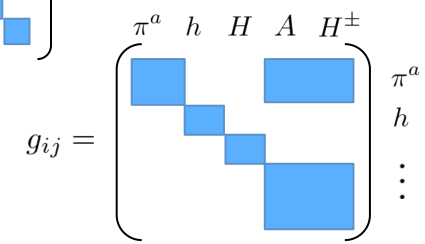
Geometry and symmetry

Killing eq. = Model-independent correlation

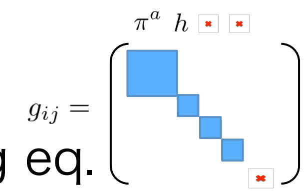
$$v_{i;j;k} = R^{l}{}_{kji} v_l$$



v_i and $R^{i}{}_{jkl}$ in SM should satisfy Killing eq.



v_i and $R^{i}{}_{jkl}$ in 2HDM should satisfy Killing eq.



v_i and $R^{i}{}_{jkl}$ in SM + Singlet Model should satisfy Killing eq.

Goal :

Model-independent constraints on new scalars

$$H^0, \; H^{\pm}, \; H^{\pm\pm}, \cdots$$

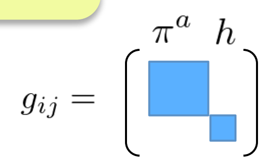
Goal :

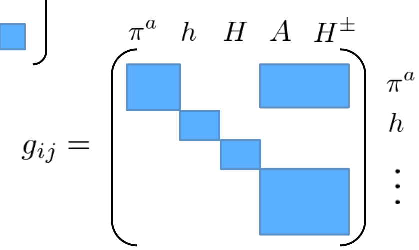
Model-independent prediction on new scalars

$$H^0,\ H^{\pm},\ H^{\pm\pm},\ \cdots$$

Killing eq. = Model-independent correlation

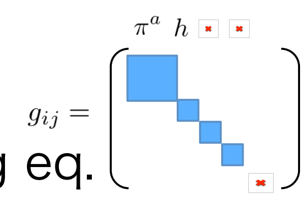
$$v_{i;j;k} = R^l{}_{kji} v_l$$





v_i and $R^i{}_{jkl}$ in SM should satisfy Killing eq.

v_i and $R^i{}_{jkl}$ in 2HDM should satisfy Killing eq.



v_i and $R^i{}_{jkl}$ in SM + Singlet Model should satisfy Killing eq.