Modular symmetry in Type IIB flux vacua

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Based on： Tatsuo Kobayashi (Hokkaido Univ.)

arXiv:2001.07972

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Motivation

- Quark sector (Weak mixing):

PDG ('18)

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}$$

- Lepton sector (Large mixing):

NuFIT 3.2 (2018)

$$|U|_{3\sigma} = \begin{pmatrix} 0.799 \to 0.844 & 0.516 \to 0.582 & 0.141 \to 0.156 \\ 0.242 \to 0.494 & 0.467 \to 0.678 & 0.639 \to 0.774 \\ 0.284 \to 0.521 & 0.490 \to 0.695 & 0.615 \to 0.754 \end{pmatrix}$$

What is the principle to control flavors of quarks/leptons ?
It gives us a hint of beyond SM

Non-abelian discrete flavor symmetry

- Non-Abelian discrete flavor symmetries (such as S_3 , S_4, A_4, A_5)

  ー Useful to reproduce the data, in particular, the lepton sector

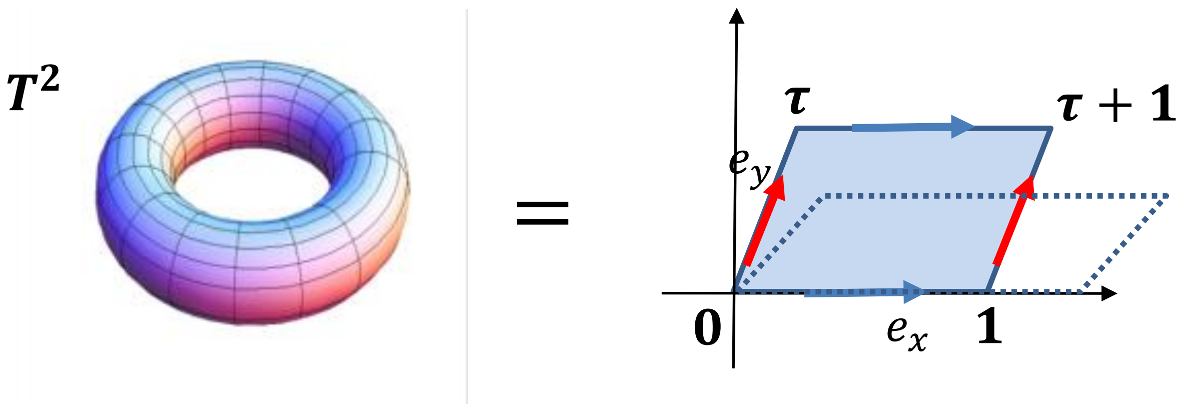
- Discrete flavor models require the existence of gauge singlet scalars (so-called flavons) whose VEVs determine the flavor structures.

$$y_{ij}(\Phi)Q_iHU_j$$

- Alternative approach to derive such flavor symmetries ?

  ー <u>Modular symmetry</u>

$SL(2,Z)$ modular symmetry



$\tau \equiv e_y/e_x$: modulus parameter of $T^2 = R^2/\Lambda$

- Lattice vectors are related under the modular trf.:

$$\begin{pmatrix} e'_y \\ e'_x \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_y \\ e_x \end{pmatrix}$$

$a, b, c, d \in \boldsymbol{Z}, \qquad ad - bc = 1$

$SL(2,Z)$ modular trf.

$$\tau \equiv \frac{e_y}{e_x} \rightarrow \tau' \equiv \frac{e'_y}{e'_x} = \frac{a\tau + b}{c\tau + d} = R(\tau)$$

Two generators :

$$S : \tau \rightarrow -1/\tau$$

$$T : \tau \rightarrow \tau + 1$$

$$\Gamma \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1\}$$

Non-abelian discrete groups in $SL(2,Z)$ modular group

$$\Gamma \simeq \{S, T \mid S^2 = 1, (ST)^3 = 1\}$$

- $SL(2,Z)$ modular group includes finite non-abelian discrete groups

  — After imposing $T^N = 1$,

  S_3, A_4, S_4, A_5 for $N = 2,3,4,5$ Feruglio, 1706.08749

- Differences between flavon models and modular flavor models

  — Yukawa couplings have non-trivial representations under modular flavor symmetries

  — Modulus VEV breaks the modular flavor symmetry

Congruence subgroups

$$SL(2,\boldsymbol{Z}) \simeq \{\begin{bmatrix} a & b \\ c & d \end{bmatrix}, ad - bc = 1\}$$

$a, b, c, d \in \boldsymbol{Z}$

$$\Gamma(N) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SL(2,\mathbb{Z}) \middle| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \equiv \begin{bmatrix} 1(\mathrm{mod}\, N) & 0(\mathrm{mod}\, N) \\ 0(\mathrm{mod}\, N) & 1(\mathrm{mod}\, N) \end{bmatrix} \right\}$$

S_3, A_4, S_4, A_5 for $N = 2,3,4,5$

Feruglio, 1706.08749

$$\Gamma^0(N) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SL(2,\mathbb{Z}) \middle| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \equiv \begin{bmatrix} * & 0(\mathrm{mod}\, N) \\ * & * \end{bmatrix} \right\}$$

$$\Gamma_0(N) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SL(2,\mathbb{Z}) \middle| \begin{bmatrix} a & b \\ c & d \end{bmatrix} \equiv \begin{bmatrix} * & * \\ 0(\mathrm{mod}\, N) & * \end{bmatrix} \right\}$$

After compactifying on toroidal background (orientifold), we would obtain the modular invariant effective action.

Question :

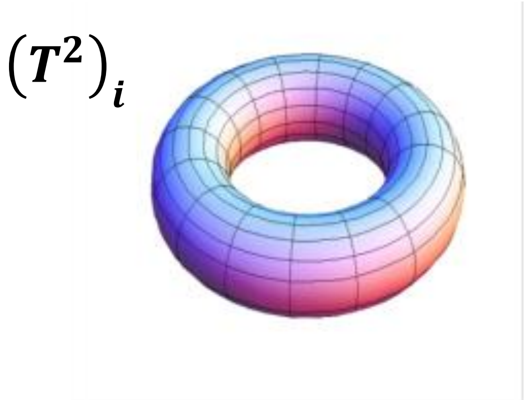
<u>Discrete modular group in the string effective action ?</u>

○ Modular Symmetry in Type IIB flux vacua

arXiv:2001.07972 with T. Kobayashi

Keyword : Flux compactification

Setup : Type IIB string on toroidal orientifold $\Pi_{i=1}^{3}(T^2)_i/Z_2$



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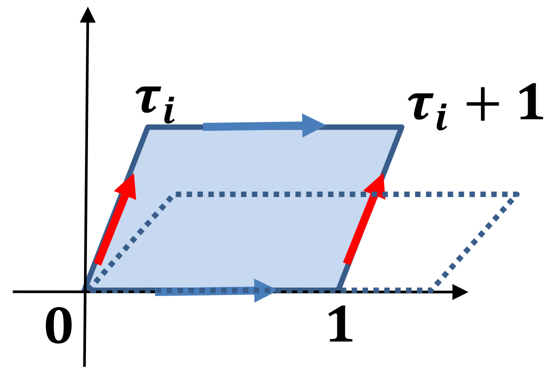

τ_i : Complex structure of $(T^2)_i$

- Let us consider the following flux-induced superpotential

A. Hebecker, P. Henkenjohann, L. T. Witkowski, 1708.06761

$$W = (M\tau_1 - N\tau_2)(\tau - \tau_3)$$

M, N : Flux quanta
τ : Axio-dilaton

⇨ Minimum: $\tau_3 = \tau$ and $M\tau_1 = N\tau_2.$

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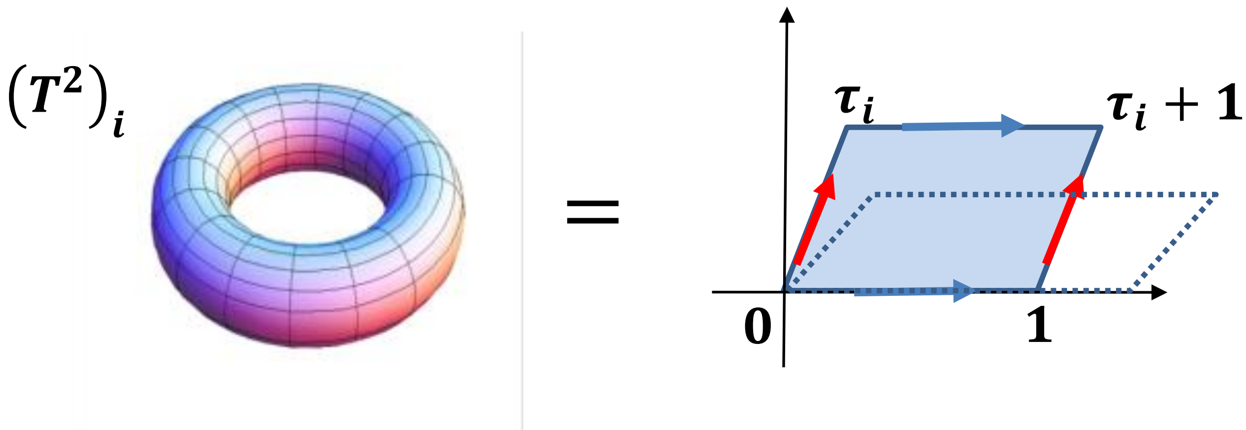

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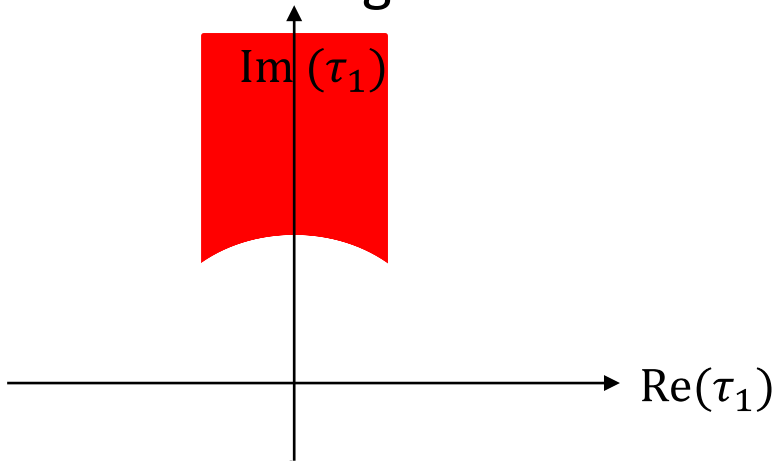
⇒ Minimum: $\tau_3 = \tau$ and $M\tau_1 = N\tau_2.$

When $M = 1$, the moduli space is enlarged to $0 \leq \mathrm{Re}(\tau_1) \leq N$ due to $0 \leq \mathrm{Re}(\tau_2) \leq 1$

- Indeed, the modular symmetry of torus is partially broken by fluxes

  — Fundamental region of the first torus:



$$\tau_1 = R_1(\tau_1') = \frac{a\tau_1' + b}{c\tau_1' + d}$$

for some $R_1 \in \mathrm{SL}(2, \mathbb{Z})$.

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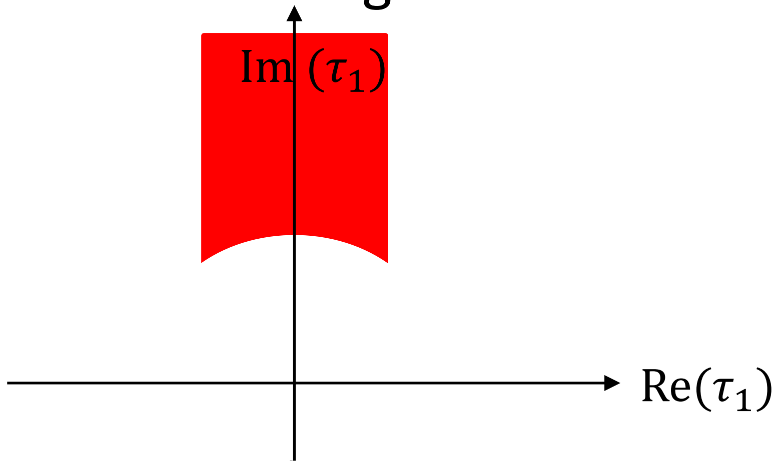
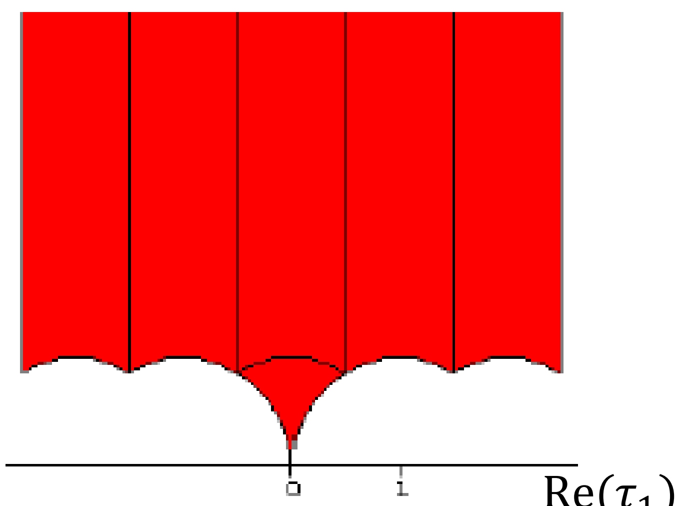


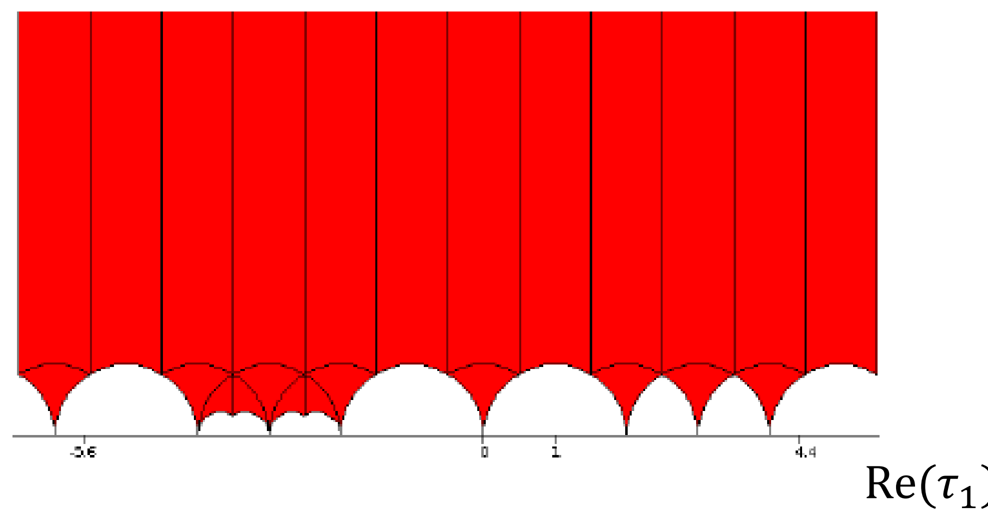
$$\tau_1 = R_1(\tau_1') = \frac{a\tau_1' + b}{c\tau_1' + d}$$

for some $R_1 \in \mathrm{SL}(2, \mathbb{Z})$.

- When we introduce the flux N $(SL(2,Z) \to \Gamma^0(N))$ $\quad R_1 = \begin{pmatrix} 1 & N \\ 0 & 1 \end{pmatrix}$



$N = 5$



$N = 12$

From 1708.06761

Consisntency conditions for fluxes

- Cancellation condition of D3-brane charge:

$$\int C_4 \qquad \qquad \int_{CY} C_4 \wedge G_3 \wedge \overline{G_3}$$

$$32 - 2n_{\mathrm{D3}} = \frac{1}{(2\pi)^4(\alpha')^2} \int H_3 \wedge F_3 = c^0 b_0 + c^{ii} b_{ii} - d_{ii} a^{ii} - d_0 a^0$$

$$MN = 32 - 2n_{D3} \leq 32$$

- Fluxes are even integers:

(3-cycle volume on T^6 is divided by the corresponding cycle on T^6/Z_2)

$$M, N \in 2\boldsymbol{Z}$$

- Possible congruence subgroups on T^6/Z_2

$$MN = 32 - 2n_{D3} \leq 32$$

$$M, N \in 2\boldsymbol{Z}$$

$\{\Gamma^{(1)}, \Gamma^{(2)}\}$
$\{\Gamma_0(2), \Gamma^0(2)\}$, $\{\Gamma_0(3), \Gamma^0(3)\}$, $\{\Gamma_0(4), \Gamma^0(4)\}$

Table 1. Possible congruence subgroups of $\{SL(2,\mathbb{Z})_1, SL(2,\mathbb{Z})_2\}$

- What happened when we consider D-brane configurations accommodating the SM?

$T^6/(Z_2 \times Z_2)$ Toroidal Orientifolds with D3/D7-branes

- Possible congruence subgroups of $\{SL(2,\mathbb{Z})_1, SL(2,\mathbb{Z})_2\}$

$\{\Gamma^{(1)}, \Gamma^{(2)}\}$
$\{\Gamma_0(2), \Gamma^0(2)\}$, $\{\Gamma_0(3), \Gamma^0(3)\}$, $\{\Gamma_0(4), \Gamma^0(4)\}$, $\{\Gamma_0(5), \Gamma^0(5)\}$, $\{\Gamma_0(6), \Gamma^0(6)\}$, $\{\Gamma(6), \Gamma(6)\}$

- Quantization condition of fluxes

$$M, N \in 4\boldsymbol{Z}$$

- Cancellation conditions of D-brane charges severely constrain the possible congruence subgroups

Conclusion

- Discrete modular group can be realized in the EFT of Type IIB string on toroidal orientifolds with fluxes

  —severely constrained by **the quantization of fluxes** and **cancellation conditions of D-brane charges**

Discussion

- Higher-dimensional orbifold (T^4 or T^6 etc.)
  (Discrete groups of $SL(4,\boldsymbol{Z})$ and $SL(6,\boldsymbol{Z})$?)

- Modular symmetry beyond toroidal orbifold ?

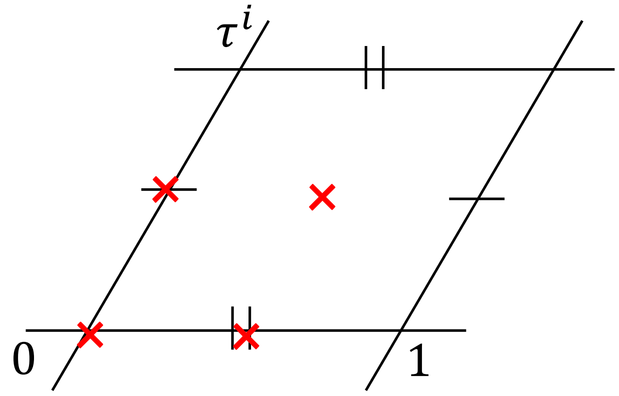
T^6/Z_2 Toroidal Orientifold

- Coordinates on $T^6 \simeq T_1^2 \times T_2^2 \times T_3^2$

  x^i, y^i $(i = 1,2,3)$ with $x^i \equiv x^i + 1$, $y^i \equiv y^i + 1$

T_i^2 :

(i=1,2,3)



- τ^i parametrizes the shape of torus (complex structure moduli)

- $Z_2 : (x^i, y^i) \rightarrow -(x^i, y^i)$

  64 O3-planes at 64 fixed points of Z_2 action

T^6/Z_2 Toroidal Orientifold

- Background three-form fluxes :

$$\frac{1}{(2\pi)^2\alpha'}F_3 = \underline{a^0}\alpha_0 + \underline{a^{ij}}\alpha_{ij} + \underline{b_{ij}}\beta^{ij} + \underline{b_0}\beta^0,$$

$$\frac{1}{(2\pi)^2\alpha'}H_3 = \underline{c^0}\alpha_0 + \underline{c^{ij}}\alpha_{ij} + \underline{d_{ij}}\beta^{ij} + \underline{d_0}\beta^0,$$

Flux quanta (Input data)

α_*, β^*: Basis of $H^3(T^6, Z)$

e.g., $\alpha_0 = dx^1 \wedge dx^2 \wedge dx^3$

- Flux-induced scalar potential, depending on τ^i and axio-dilaton S

$$V_{\text{flux}} = \frac{1}{4\kappa_{10}^2(\text{Im } S)}\int_{T^6} G_3 \wedge * G_3 \qquad G_3 = F_3 - SH_3$$

$T^6/(Z_2 \times Z_2)$ Toroidal Orientifold

- Coordinates on $T^6 \simeq T_1^2 \times T_2^2 \times T_3^2$ $\quad z^i = x^i + \tau^i y^i$

$$\theta \,:\, (z_1, z_2, z_3) \to (-z_1, -z_2, z_3), \qquad \theta' \,:\, (z_1, z_2, z_3) \to (z_1, -z_2, -z_3)$$

$$\mathcal{R} \,:\, (z_1, z_2, z_3) \to (-z_1, -z_2, -z_3)$$

- 64 O3-planes at 64 fixed points of $\mathcal{R}$
- 4 $O7_1$-planes at the fixed locus of $\mathcal{R}\theta'$
- 4 $O7_2$-planes at the fixed locus of $\mathcal{R}\theta\theta'$
- 4 $O7_3$-planes at the fixed locus of $\mathcal{R}\theta$

$T^6/(Z_2\times Z_2)$ Toroidal Orientifold

- Magnetized D(3+2n)-branes

$$\frac{m_a^i}{2\pi}\int_{T_i^2} F_a^i = n_a^i.$$

(D3-, D5-, D7-, D9-branes can consider 0, 1, 2, 3 nonvanishing fluxes)

- # of chiral zero-modes at the intersection of D-branes

$$I_{ab} = \Pi_{i=1}^3 (n_a^i m_b^i - n_b^i m_a^i),$$

$T^6/(Z_2 \times Z_2)$ Toroidal Orientifold

- Cancellation conditions of D-brane charges

$$
\begin{aligned}
D3 &: \sum_a N_a n_a^1 n_a^2 n_a^3 + \frac{1}{2} N_{\text{flux}} = 16, \\
D7_1 &: \sum_a N_a n_a^1 m_a^2 m_a^3 = -16, \\
D7_2 &: \sum_a N_a n_a^2 m_a^1 m_a^3 = -16, \\
D7_3 &: \sum_a N_a n_a^3 m_a^1 m_a^2 = -16.
\end{aligned}
$$

- Fluxes are quantized in multiples of 4 (without discrete torsion) (due to the $Z_2 \times Z_2$ orbifold)

$$M, N \in 4Z$$