Two-loop corrections to the Higgs trilinear coupling in models with extended scalar sectors

Johannes Braathen

based on
Phys. Lett. B796 (2019) 38–46
and 1911.11507 (to appear in EPJC)
with Shinya Kanemura

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Probing the shape of the Higgs potential

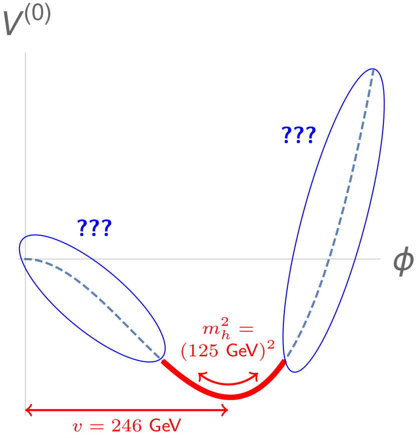
- Since the Higgs discovery, the existence of the Higgs potential is confirmed, but at the moment we only know:

  → the location of the EW minimum: $v \simeq 246$ GeV
  → the curvature of the potential around the EW minimum: $m_h \simeq 125$ GeV

  However what we still don't know is the **shape** of the Higgs potential, which **depends on λ_{hhh}**

- λ_{hhh} determines the nature of the EWPT!

  $\Rightarrow \mathcal{O}(20-30\%)$ deviation of λ_{hhh} from its SM prediction needed to have a strongly first-order EWPT
  → necessary for EWBG
  [Grojean, Servant, Wells '04], [Kanemura, Okada, Senaha '04]



Alignment with or without decoupling

- Aligned scenarios already seem to be favoured $\rightarrow$ Higgs couplings are SM-like at **tree-level**
- Non-aligned scenarios ($e.g.$ in 2HDMs) could be almost entirely excluded in the close future using synergy of HL-LHC and ILC!

  $\rightarrow$ Alignment **through decoupling**? or alignment **without** decoupling?
- If alignment without decoupling, Higgs couplings like λ_{hhh} can still exhibit **large deviations** from SM predictions because of **BSM loop effects**
- Current best limit (at 95% CL): $-3.7 < \lambda_{hhh}/\lambda_{hhh}^{\text{SM}} < 11.5$ [ATLAS-CONF-2019-049]
- Improvement at future colliders: **HL-LHC**: $\lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ within $\sim 50 - 100\%$; **lepton colliders** (ILC, CLIC) within some tens of $\%$; even down to $5 - 7\%$ at **100-TeV hadron collider** (*details in backup*)

c.f. talk of Dr. J. Park on Tuesday, and see also [de Blas et al., 1905.03764], [Cepeda et al., 1902.00134], [Di Vita et al. 1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130, 1908.00753], etc.

Non-decoupling effects in λ_{hhh}

The Two-Higgs-Doublet Model (2HDM)

[$c.f.$ also previous talk by M. Aiko]

- CP-conserving 2HDM, with softly-broken $\mathbb{Z}_2$ symmetry $(\Phi_1 \to \Phi_1, \Phi_2 \to -\Phi_2)$ to avoid tree-level FCNCs
- 2 $SU(2)_L$ doublets $\Phi_{1,2} = \begin{pmatrix} \Phi_{1,2}^+ \\ \Phi_{1,2}^0 \end{pmatrix}$ of hypercharge $1/2$

$$V_{\text{2HDM}}^{(0)} = m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - m_3^2(\Phi_2^\dagger\Phi_1 + \Phi_1^\dagger\Phi_2) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_2^\dagger\Phi_1|^2 + \frac{\lambda_5}{2}\Big((\Phi_2^\dagger\Phi_1)^2 + \text{h.c.}\Big)$$

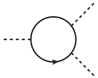
- 7 free parameters in scalar sector: m_3^2, λ_i $(i = 1 \cdots 5)$, $\tan\beta \equiv \langle\Phi_2^0\rangle/\langle\Phi_1^0\rangle$
  (m_1^2, m_2^2 eliminated with tadpole equations, and $\langle\Phi_1^0\rangle + \langle\Phi_2^0\rangle = v^2 = (246\text{ GeV})^2$)
- Doublets expanded in terms of mass eigenstates:
  h, H: CP-even Higgses, A: CP-odd Higgs, $H^\pm$: charged Higgs
- λ_i $(i = 1 \cdots 5)$ traded for mass eigenvalues m_h, m_H, m_A, $m_{H^\pm}$ and CP-even mixing angle α
- m_3^2 replaced by a soft-breaking mass scale $M^2 = 2m_3^2/s_{2\beta}$

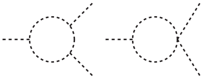
Non-decoupling effects in λ_{hhh} at one loop

First studies of the one-loop corrections to λ_{hhh} in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan '02] and [Kanemura, Okada, Senaha, Yuan '04]

- Leading one-loop corrections to λ_{hhh} (for $s_{\beta-\alpha}=1$)

$$\delta^{(1)}\lambda_{hhh} = \underbrace{-\frac{48m_t^4}{v^3}}_{\text{SM-like}} + \sum_{\Phi=H,A,H^\pm} \underbrace{\frac{4n_\Phi m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3}_{\text{BSM}}$$





(recall $\lambda_{hhh}^{(0)} = 3m_h^2/v$)

- Masses of additional scalars $\Phi = H, A, H^\pm$ in 2HDM can be written as $m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ ($\tilde{\lambda}_\Phi$: some combination of λ_i)
- Power-like dependence of BSM terms $\propto m_\Phi^4$, and

$$\left(1-\frac{M^2}{m_\Phi^2}\right)^3 \to \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2 \end{cases}$$

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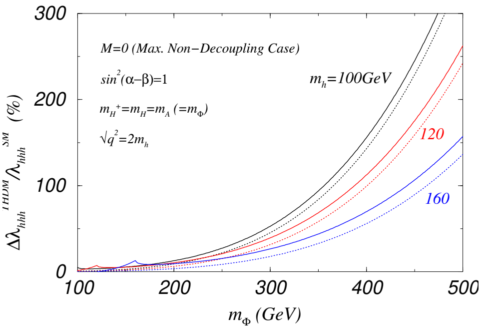


figure from [Kanemura, Okada, Senaha, Yuan '04]

- **Huge deviations possible, without violating unitarity!** $\rightarrow$ **non-decoupling effects**

Non-decoupling effects in λ_{hhh} at one loop

First studies of the one-loop corrections to λ_{hhh} in the 2HDM in [Kanemura, Kiyoura, Okada, Senaha, Yuan '02] and [Kanemura, Okada, Senaha, Yuan '04]

- Leading one-loop corrections to λ_{hhh} (for $s_{\beta-\alpha}=1$)

> Such non-decoupling effects are found at one loop for various Higgs couplings and for a wide range of BSM models (2HDM, IDM, HSM, etc.)
>
> (see $e.g.$ results in `H-COUP` [Kanemura, Kikuchi, Sakurai, Yagyu '17], [Kanemura, Kikuchi, Mawatari, Sakurai, Yagyu '19])
>
> $\Rightarrow$ What happens at two loops? New huge corrections?
>
> $\Rightarrow$ We derive the **dominant** two-loop corrections to λ_{hhh} in several BSM models [J.B., Kanemura '19]

($\tilde{\lambda}_\Phi$: some combination of λ_i)

- Power-like dependence of BSM terms $\propto m_\Phi^4$, and

$$\left(1-\frac{M^2}{m_\Phi^2}\right)^3 \to \begin{cases} 0, & \text{for } M^2 \gg \tilde{\lambda}_\Phi v^2 \\ 1, & \text{for } M^2 \ll \tilde{\lambda}_\Phi v^2 \end{cases}$$

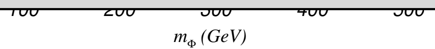


figure from [Kanemura, Okada, Senaha, Yuan '04]

- **Huge deviations possible, without violating unitarity!** $\rightarrow$ **non-decoupling effects**

Our two-loop calculation

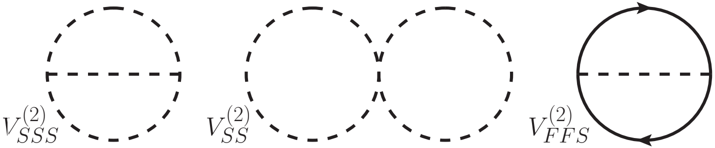
Setup of our effective-potential calculation

Step 1: calculate $\underbrace{V_{\text{eff}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 2:** $\underbrace{\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\Big|_{\text{min.}}}_{\overline{\text{MS}}}$ $\rightarrow$ **Step 3:** convert from $\overline{\text{MS}}$ to OS scheme

- $\overline{\text{MS}}$-renormalised two-loop effective potential is

$$V_{\text{eff}} = V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)} \qquad \left(\kappa \equiv \frac{1}{16\pi^2}\right)$$

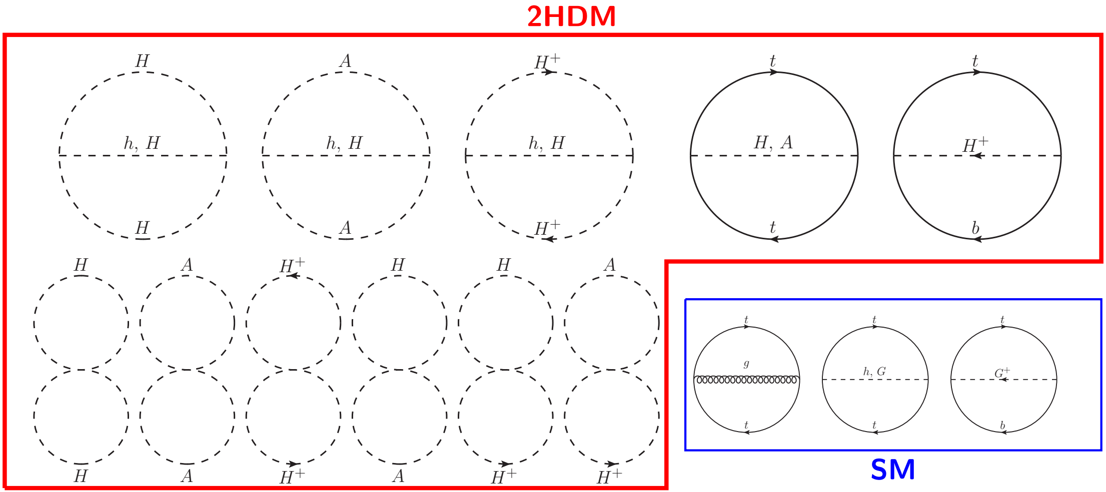
- $V^{(2)}$: 1PI vacuum bubble diags., and we want to study the leading two-loop BSM corrections from **additional scalars** and **top quark**, so we only need



- Also, we **neglect subleading contributions** from h, $G, G^{\pm}$, and light fermions $\Rightarrow$ no need to specify type of 2HDM + greatly simplifies the $\overline{\text{MS}} \rightarrow$ OS scheme conversion (*details in backup*)
- **Scenarios without mixing**: aligned 2HDM ($s_{\beta-\alpha} = 1$) $\Rightarrow$ evade exp. constrains!
(loop-induced deviations from alignment also neglected)

λ_{hhh} at two loops in the 2HDM

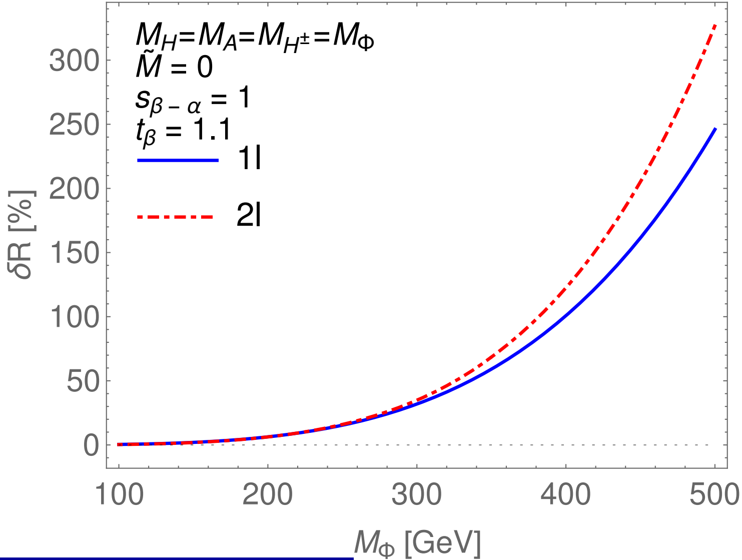
2HDM $\rightarrow$ 15 new BSM diagrams appearing in $V^{(2)}$ w.r.t. the SM case



In the following: some results for the BSM deviation

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{BSM}} - \lambda_{hhh}^{\text{SM}}}{\lambda_{hhh}^{\text{SM}}}$$

Non-decoupling effects



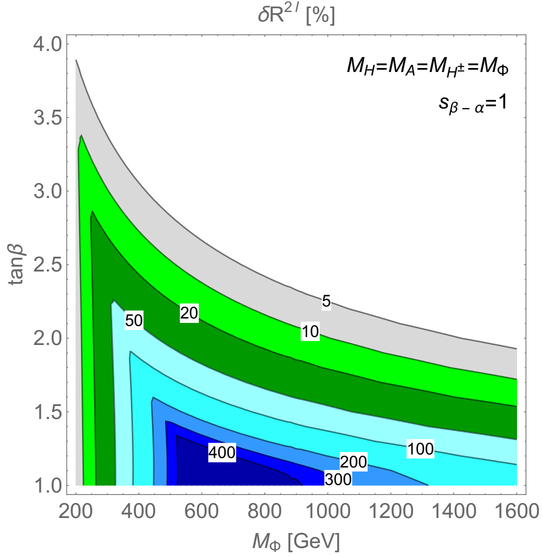
- $\tilde{M} = 0 \to$ maximal non-decoupling effects

  [$\tilde{M}$: "OS" version of M, defined to ensure proper decoupling for $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$ (c.f. backup)]

- $\delta^{(1)}\hat{\lambda}_{hhh} \to \propto M_\Phi^4$
- $\delta^{(2)}\hat{\lambda}_{hhh} \to \propto M_\Phi^6$

- For $\tilde{M} = 0$, $\tan\beta = 1.1$, tree-level unitarity is lost around $M_\Phi \approx 600$ GeV [Kanemura, Kubota, Takasugi '93]

Maximal BSM allowed deviations



$$\delta R \equiv \frac{\lambda^{\text{2HDM}}_{hhh}}{\lambda^{\text{SM}}_{hhh}} - 1$$

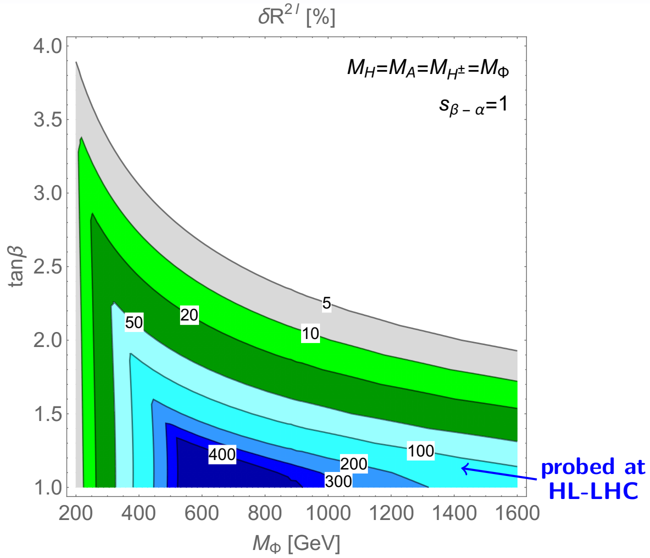
- Here: Maximal deviation δR $(1\ell{+}2\ell)$ while fulfilling perturbative unitarity, in $(\tan\beta, M_\Phi)$ plane

$$M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$$

- One cannot take $M_\Phi \to \infty$ with $\tilde{M} = 0$ without breaking unitarity
- At some point $\tilde{M}$ must be non-zero $\to$ reduction factor

$$\left(1 - \frac{\tilde{M}^2}{M_\Phi^2}\right)^n < 1$$

Maximal BSM allowed deviations



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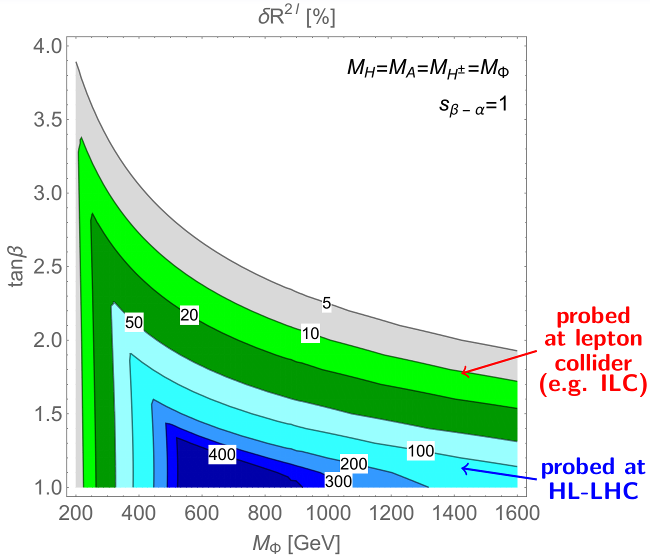

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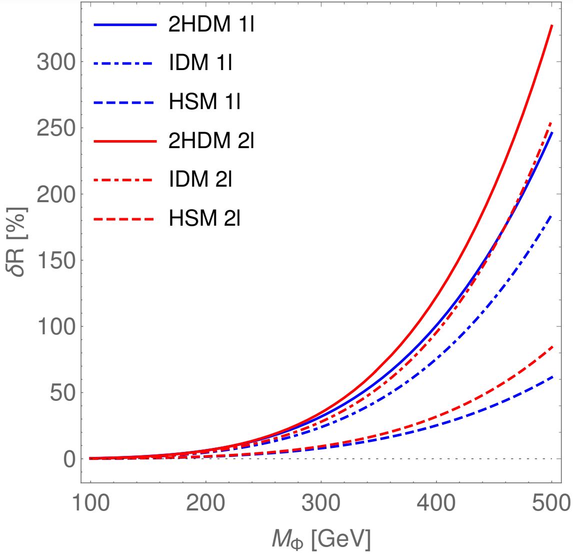
Two-loop calculation for more models



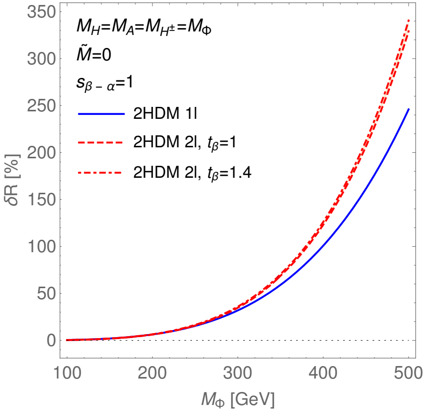
- ▶ We considered several more models [J.B., Kanemura 1911.11507]
- ▷ 2HDM $\rightarrow$ previously presented
- ▷ Inert Doublet Model (IDM), in DM-inspired model (H light; A, $H^\pm$ heavy)
- ▷ Real-singlet extension of the SM (HSM)

- ▶ Size of BSM deviation $\propto$ # heavy d.o.f.
- ▷ 2HDM $\rightarrow$ 4 (H, A, $H^\pm$)
- ▷ IDM $\rightarrow$ 3 (A, $H^\pm$)
- ▷ HSM $\rightarrow$ 1 (S)

Two-loop calculation for more models

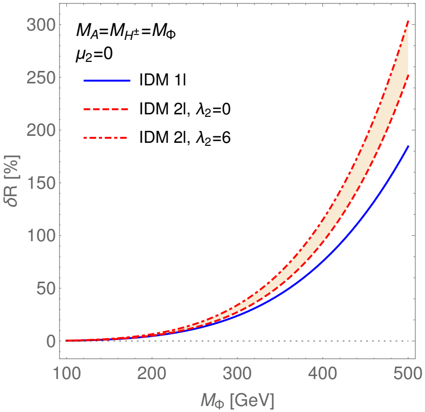
Each model contains a new parameter appearing from two loops:

Aligned 2HDM $\rightarrow \tan\beta$



$\tan\beta$ constrained by perturbative unitarity
$\rightarrow$ only small effects

IDM $\rightarrow \lambda_2$ (inert doublet quartic coupling)



λ_2 is less contrained $\rightarrow$ enhancement is possible
(but 2ℓ effects remains <u>well smaller</u> than 1ℓ ones)

Summary

- **First two-loop calculation of λ_{hhh} in 2HDM**, in a scenario with alignment $+$ also **IDM** and **HSM**

- Two-loop corrections to λ_{hhh} remain smaller than one-loop contributions, at least as long as perturbative unitarity is maintained $\rightarrow$ **typical size $10-20\%$ of one-loop contributions**

$\Rightarrow$ non-decoupling effects found at one loop are **not drastically changed**

$\Rightarrow$ in the future perspective of a precise measurement of λ_{hhh}, computing corrections beyond one loop will be **necessary**

- Precise calculation of Higgs couplings (λ_{hhh}, etc.) can allow distinguishing aligned scenarios **with or without decoupling**

see also 1903.05417 and 1911.11507 *for details*

Thank you for your attention!

Backup

Investigating the Higgs trilinear coupling λ_{hhh}

Current experimental limits

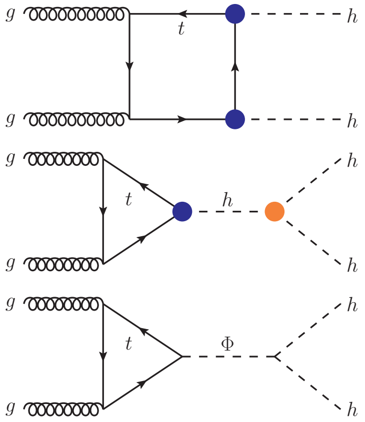
- Current limits on $\kappa_\lambda \equiv \lambda_{hhh}/\lambda_{hhh}^{\text{SM}}$ are (at 95% CL)

  $-3.2 < \kappa_\lambda < 11.9$ (ATLAS) and $-11 < \kappa_\lambda < 17$ (CMS)

  see [ATL-PHYS-PUB-2019-009] (ATLAS), [CMS-HIG-17-008] (CMS)

Future prospects

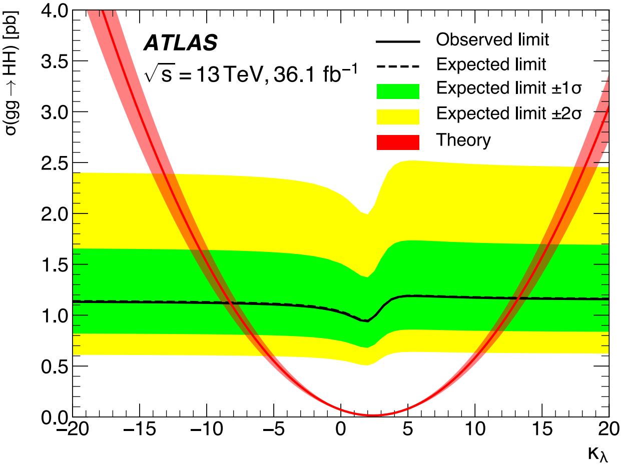
- HL-LHC with 3 ab^{-1} could reach $0.1 < \kappa_\lambda < 2.3$, and a 27-TeV HE-LHC with 15 ab^{-1} $0.58 < \kappa_\lambda < 1.45$
- ILC-250 cannot measure λ_{hhh}, but 500-GeV and 1-TeV extensions could obtain measurements with precisions of 27% and 10% respectively
- CLIC 1.4 TeV + 3 TeV $\rightarrow$ 20% accuracy
- 100-TeV hadron collider with 30 ab^{-1} $\rightarrow$ 5-7% accuracy



Double Higgs production

see $e.g.$ [de Blas et al., 1905.03764], [Cepeda et al., 1902.00134], [Di Vita et al. 1711.03978], [Homiller and Meade, 1811.02572], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Abramowicz et al., 1608.07538], [Charles et al., 1812.06018], [Gonçalves et al. 1802.04319], [Chang et al. 1804.07130, 1908.00753]

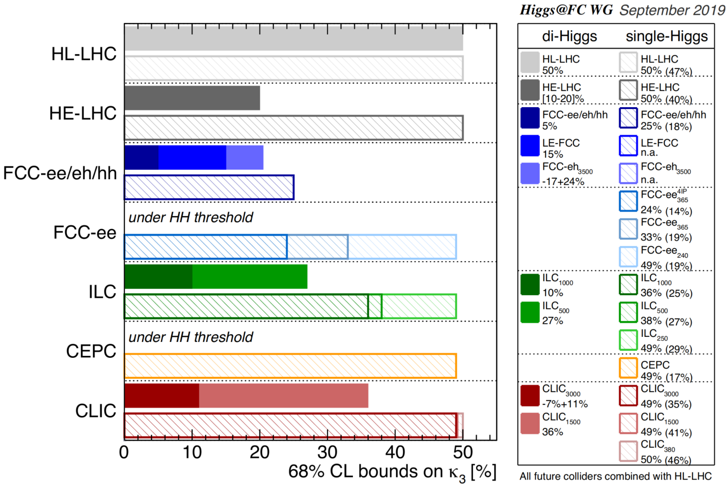
An example of experimental limits on λ_{hhh}



Example of current limits on κ_λ from the ATLAS search of $hh \to b\bar{b}\gamma\gamma$
(*taken from* [ATLAS collaboration 1807.04873])

Future measurements prospects for the Higgs trilinear coupling λ_{hhh}

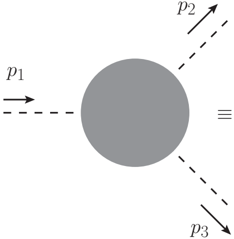
$$\left(\kappa_3 = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM}}}\right)$$



[Higgs@FC report, 1905.03764]

Radiative corrections to the Higgs trilinear coupling

- Higgs three-point function, $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$, requires a diagrammatic calculation, with non-zero external momentum
- Instead it is much more convenient to work with an effective Higgs trilinear coupling λ_{hhh}



$\equiv \Gamma_{hhh}(p_1^2, p_2^2, p_3^2)$

$$\mathcal{L} \supset -\frac{1}{6}\lambda_{hhh}h^3 \quad \to \quad \underbrace{\lambda_{hhh} = \frac{\partial^3 V_{\text{eff}}}{\partial h^3}\bigg|_{\text{min.}}}_{\overline{\text{MS}}\text{ result}}$$

$V_{\text{eff}} = V^{(0)} + \Delta V_{\text{eff}}$: effective potential (calculated in $\overline{\text{MS}}$ scheme)

- In effective-potential calculations, one should usual fix conditions for the lower derivatives of V_{eff}

$$\underbrace{\frac{\partial V_{\text{eff}}}{\partial h}\bigg|_{\text{min.}} = 0,}_{\text{tadpole condition}} \qquad \underbrace{[M_h^2]_{V_{\text{eff}}} = \frac{\partial^2 V_{\text{eff}}}{\partial h^2}\bigg|_{\text{min.}} - \frac{1}{v}\frac{\partial V_{\text{eff}}}{\partial h}\bigg|_{\text{min.}}}_{\text{curvature mass of the Higgs}}$$

- Using these, we obtain

$$\lambda_{hhh} = \frac{3[M_h^2]_{V_{\text{eff}}}}{v} + \mathcal{D}_3 \Delta V_{\text{eff}}\Big|_{\text{min.}}, \qquad \text{with } \mathcal{D}_3 \equiv \frac{\partial^3}{\partial h^3} - \frac{3}{v}\left[-\frac{1}{v}\frac{\partial}{\partial h} + \frac{\partial^2}{\partial h^2}\right]$$

Radiative corrections to the Higgs trilinear coupling (detailed)

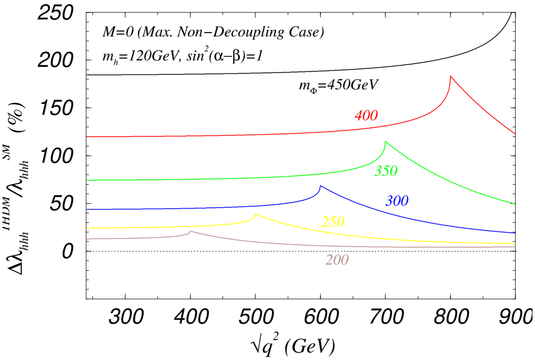
- Γ_{hhh} and λ_{hhh} can be related as

$$-\Gamma_{hhh}(0,0,0) = \underbrace{\hat{\lambda}_{hhh}}_{\text{OS result}} = \left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2} \underbrace{\lambda_{hhh}}_{\overline{\text{MS}}\text{ result}} = \left(1 + \frac{3}{2}\frac{d}{dp^2}\Pi_{hh}(p^2)\Big|_{p^2=M_h^2}\right)\lambda_{hhh}$$

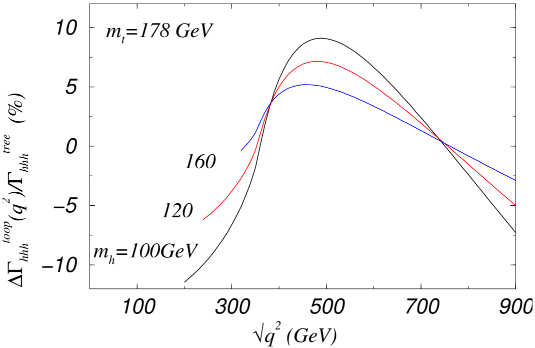
$\delta Z_h^{\text{OS},\overline{\text{MS}}} = Z_h^{\text{OS},\overline{\text{MS}}} - 1$: wave-function renormalisation counterterms in OS/$\overline{\text{MS}}$ scheme,
$\Pi_{hh}(p^2)$: finite part of Higgs self-energy at ext. momentum p^2

- Taking $\Gamma_{hhh}(p_1^2, p_2^2, p_3^2) \simeq \Gamma_{hhh}(0,0,0)$ is a good approximation
  - $\rightarrow$ shown for λ_{hhh} at one loop in [Kanemura, Okada, Senaha, Yuan '04] (difference is only a few %)
  - $\rightarrow$ no study including external momentum exists at two loops, but in the case of two-loop Higgs mass calculations, momentum effects are known to be subleading

Momentum dependence (at one loop)



(*scalar part*)



(*top quark loop*)

figures from [Kanemura, Okada, Senaha, Yuan '04]

Setup of our effective-potential calculation (detailed)

- OS result is obtained as

$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\mathsf{OS}}}{Z_h^{\overline{\mathrm{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\mathrm{MS}}\text{ parameters} \\ \text{translated to OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\mathrm{MS}}}) + \kappa f^{(1)}(x^{\overline{\mathrm{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\mathrm{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

and

$$x^{\overline{\mathrm{MS}}} = X^{\mathsf{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\mathsf{OS}}) + \kappa\left[f^{(1)}(X^{\mathsf{OS}}) + \frac{\partial f^{(0)}}{\partial x}(X^{\mathsf{OS}})\delta^{(1)}x\right]$$
$$+ \kappa^2\left[f^{(2)}(X^{\mathsf{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\mathsf{OS}})\delta^{(1)}x + \frac{\partial f^{(0)}}{\partial x}(X^{\mathsf{OS}})\delta^{(2)}x + \frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\mathsf{OS}})(\delta^{(1)}x)^2\right]$$

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$$\hat{\lambda}_{hhh} = \underbrace{\left(\frac{Z_h^{\text{OS}}}{Z_h^{\overline{\text{MS}}}}\right)^{3/2}}_{\text{inclusion of WFR}} \times \underbrace{\lambda_{hhh}}_{\substack{\overline{\text{MS}}\text{ parameters} \\ \text{replaced by OS ones}}}$$

- Let's suppose (for simplicity) that λ_{hhh} only depends on one parameter x, as

$$\lambda_{hhh} = f^{(0)}(x^{\overline{\text{MS}}}) + \kappa f^{(1)}(x^{\overline{\text{MS}}}) + \kappa^2 f^{(2)}(x^{\overline{\text{MS}}}) \qquad \left(\kappa = \frac{1}{16\pi^2}\right)$$

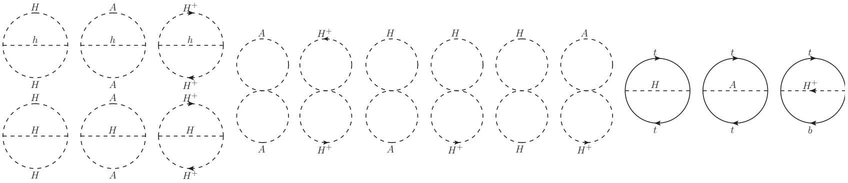
and

$$x^{\overline{\text{MS}}} = X^{\text{OS}} + \kappa\delta^{(1)}x + \kappa^2\delta^{(2)}x$$

then in terms of OS parameters

$$\lambda_{hhh} = f^{(0)}(X^{\text{OS}}) + \kappa\left[f^{(1)}(X^{\text{OS}}) + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x}\right]$$
$$+ \kappa^2\left[f^{(2)}(X^{\text{OS}}) + \frac{\partial f^{(1)}}{\partial x}(X^{\text{OS}})\delta^{(1)}x + \cancel{\frac{\partial f^{(0)}}{\partial x}(X^{\text{OS}})\delta^{(2)}x} + \cancel{\frac{\partial^2 f^{(0)}}{\partial x^2}(X^{\text{OS}})(\delta^{(1)}x)^2}\right]$$

because we neglect m_h in the loop corrections and $\lambda^{(0)}_{hhh} = 3m_h^2/v$ (in absence of mixing)



- In the $\overline{\mathrm{MS}}$ scheme

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2-m_\Phi^2+(M^2+2m_\Phi^2)\,\overline{\log}\,m_\Phi^2\right]$$
$$+\frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[1+2\,\overline{\log}\,m_\Phi^2\right]$$
$$+\frac{96m_\Phi^4 m_t^2\cot^2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\left[-1+2\,\overline{\log}\,m_\Phi^2\right]+\mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

(Recall: aligned scenario, degenerate masses, dominant corrections only)

Decoupling behaviour of the $\overline{\mathrm{MS}}$ expressions

- Seeing whether corrections from additional BSM states decouple if said state is taken to be very massive is a good way to check the consistency of the calculation

$$\delta^{(1)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^3}\left(1-\frac{M^2}{m_\Phi^2}\right)^3$$

$$\delta^{(2)}\lambda_{hhh} = \frac{16m_\Phi^4}{v^5}\left(4+9\cot^2 2\beta\right)\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[-2M^2-m_\Phi^2+(M^2+2m_\Phi^2)\,\overline{\log}\, m_\Phi^2\right]$$
$$+\frac{192m_\Phi^6\cot^2 2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^4\left[1+2\,\overline{\log}\, m_\Phi^2\right]$$
$$+\frac{96m_\Phi^4 m_t^2\cot^2\beta}{v^5}\left(1-\frac{M^2}{m_\Phi^2}\right)^3\left[-1+2\,\overline{\log}\, m_\Phi^2\right]+\mathcal{O}\left(\frac{m_\Phi^2 m_t^4}{v^5}\right)$$

where $m_\Phi^2 = M^2+\tilde{\lambda}_\Phi v^2$

- To have $m_\Phi \to \infty$, then we must take $M\to\infty$, otherwise the quartic couplings grow out of control
- Fortunately all of these terms go like

$$(m_\Phi^2)^{n-1}\left(1-\frac{M^2}{m_\Phi^2}\right)^n \underset{m_\Phi^2=M^2+\tilde{\lambda}_\Phi v^2}{=} \frac{(\tilde{\lambda}_\Phi v^2)^n}{M^2+\tilde{\lambda}_\Phi v^2} \xrightarrow[\tilde{\lambda}_\Phi v^2 \text{ fixed}]{M\to\infty} 0$$

Decoupling behaviour and $\overline{\text{MS}}$ to OS scheme conversion

- To obtain $\hat{\lambda}_{hhh} = -\Gamma_{hhh}(0,0,0)$, we must express our results in terms of physical parameters

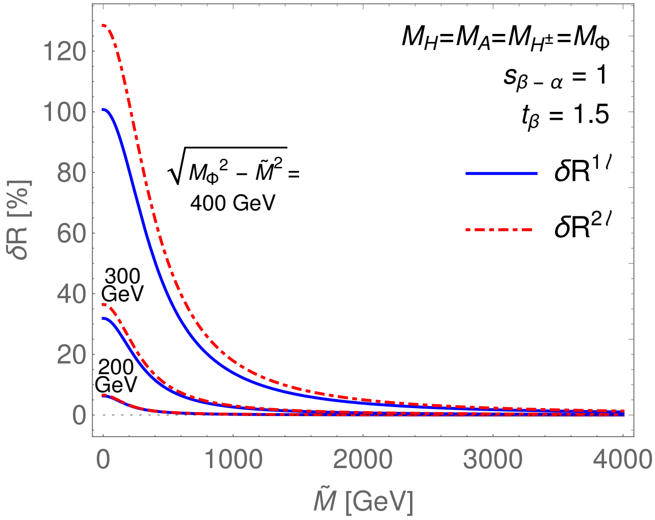
$$\overline{\text{MS}}\text{ scheme:}\Big\{\underbrace{m_H, m_A, m_{H^\pm}}_{m_\Phi}, m_t, v\Big\} \longrightarrow \text{OS scheme:}\Big\{\underbrace{M_H, M_A, M_{H^\pm}}_{M_\Phi}, M_t, v_{\text{phys}} = (\sqrt{2}G_F)^{-1/2}\Big\}$$

- A priori, M is still renormalised in $\overline{\text{MS}}$ scheme, because it is difficult to relate to physical observable

  ... but then, two-loop expressions do not decouple for $M_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$ and $M \to \infty$!
- This is because we should relate M_Φ, renormalised in OS scheme, and M, renormalised in $\overline{\text{MS}}$ scheme, with a **one-loop relation** $\to$ then the two-loop corrections decouple properly
- We give a new "OS" prescription for the finite part of the counterterm for M be requiring that the decoupling of $\delta^{(2)}\hat{\lambda}_{hhh}$ (in OS scheme) is apparent using a relation $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$

$$\delta^{(2)}\hat{\lambda}_{hhh} = \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4\left\{4+3\cot^2 2\beta\left[3-\frac{\pi}{\sqrt{3}}\left(\frac{\tilde{M}^2}{M_\Phi^2}+2\right)\right]\right\} + \frac{576M_\Phi^6\cot^2 2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^4$$
$$+\frac{288M_\Phi^4M_t^2\cot^2\beta}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3 + \frac{168M_\Phi^4M_t^2}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^3 - \frac{48M_\Phi^6}{v_{\text{phys}}^5}\left(1-\frac{\tilde{M}^2}{M_\Phi^2}\right)^5 + \mathcal{O}\left(\frac{M_\Phi^2M_t^4}{v_{\text{phys}}^5}\right)$$

Decoupling behaviour



- δR size of BSM contributions to λ_{hhh}:

$$\delta R \equiv \frac{\lambda_{hhh}^{\text{2HDM}}}{\lambda_{hhh}^{\text{SM}}} - 1$$

- $\tilde{M}$: "OS" version of M, defined so as to ensure proper decoupling for $M_\Phi^2 = \tilde{M}^2 + \tilde{\lambda}_\Phi v^2$ and $\tilde{M} \to \infty$

- Radiative corrections from additional scalars + top quark indeed decouple properly for $\tilde{M} \to \infty$

Existing works at two loops

Model [ref.]	Included Corrections	Eff. pot. approx.	Typical size	Motivation
MSSM [Brucherseifer, Gavin, Spira '14]	$\mathcal{O}(\alpha_s\alpha_t)$	Yes	$\mathcal{O}(\sim 10\%)$	Reach similiar accuracy as m_h
NMSSM [Mühlleitner, Nhung, Ziesche '15]	$\mathcal{O}(\alpha_s\alpha_t)$	Yes	$\mathcal{O}(\sim 5-10\%)$	Reach similiar accuracy as m_h
IDM [Senaha '18]	$\mathcal{O}(\lambda_\Phi^3)$ (partial)	Yes	$\mathcal{O}(\sim 2\%)$	Effect on strength of EWPT