

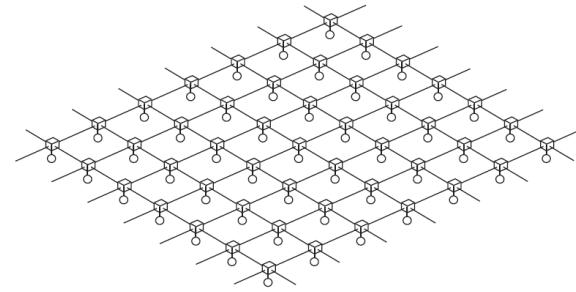


CBSM²

テンソルネットワーク法による 量子格子系の解法とその周辺

2019.12.17 川島直輝 (物性研)

Tensor Network (TN)



(1) 古典統計力学モデルはもともとテンソルネットワークである.

--- 角転送行列 (Baxter, Nishino, Okunishi, ...)

(2) テンソルネットワークは実空間繰り込み群に向いた表現

--- 「スケール不変テンソル」や様々な手法 (TRG, TNR, loop-TNR, MERA, HOTRG, ...) (Gu, Levin, Wen, Vidal, Evenbly, Xiang, ...)

(3) テンソルネットワークは量子状態のトポロジーを議論するのに向いている

--- 射影表現, 対称性に保護されたトポロジカル状態 (SPT)

(Chen, Gu, Wen, Verstraete, Cirac, Schuch, Perez-Garcia, Oshikawa, ...)

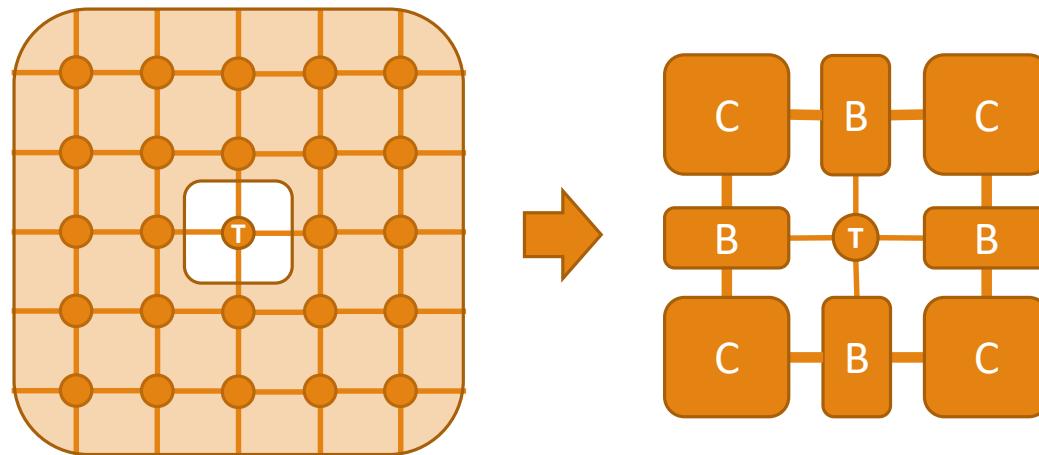
(4) テンソルネットワークは境界領域の議論に多く使われている

--- テンソルネットワークのデータ科学・機械学習への応用, 機械学習, 自動微分, 格子QCD, AdS/CFT-対応, etc.

古典統計力学からの転用

Example: Corner transfer matrix (CTM)

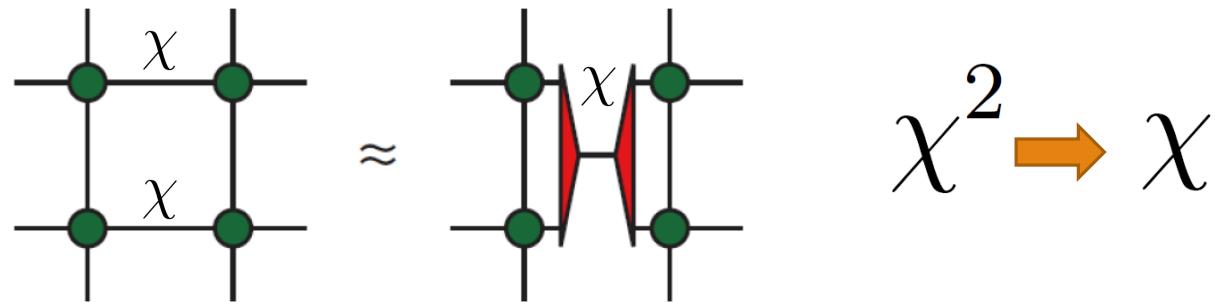
Baxter: J. Math. Phys 9, 650 (1968); J. Stat. Phys. 19, 461 (1978)
Nishino, Okunishi: JPSJ **65**, 891 (1996)
R. Orus *et al*: Phys. Rev. B **80**, 094403 (2009)



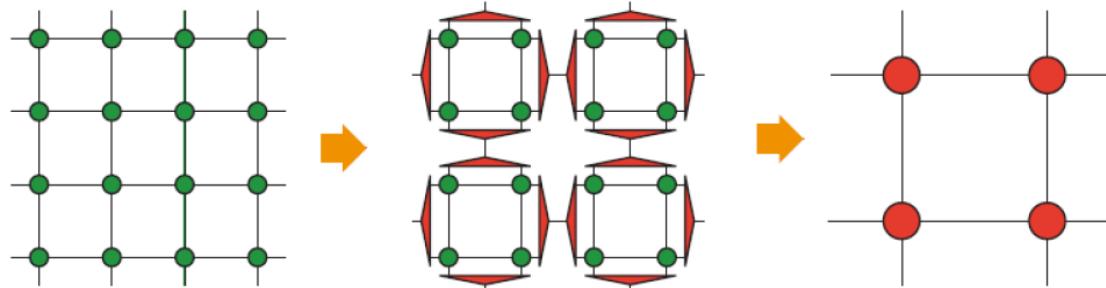
Effect of the infinite environment is approximated by B and C,
which are obtained by iteration/self-consistency.

TN表現に基づく実空間繰込み

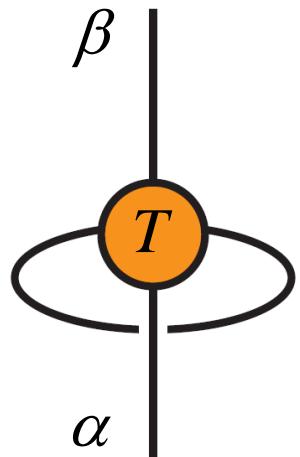
"HOTRG"



実空間繰り込み群に使うと自由エネルギーが10桁以上の精度で求まる。



スケール不变テンソル



Gu-Wen: PRB80, 155131 (2009)

$$\lambda_\mu = e^{-2\pi\left(\Delta_\mu - \frac{c}{12}\right)}$$

eigenvalues of the
partially contracted
scale invariant tensor

$$\zeta_s \equiv e^{-f_s} = \sum_\mu e^{-2\pi\left(\Delta_\mu - \frac{c}{12}\right)\text{Im}\tau + i\sigma_\mu \text{Re}\tau}$$

$$\Delta_\mu \equiv h_\mu^R + h_\mu^L, \quad \sigma_\mu \equiv h_\mu^R - h_\mu^L$$

$\tau \equiv (\text{complex aspect ratio parameter})$

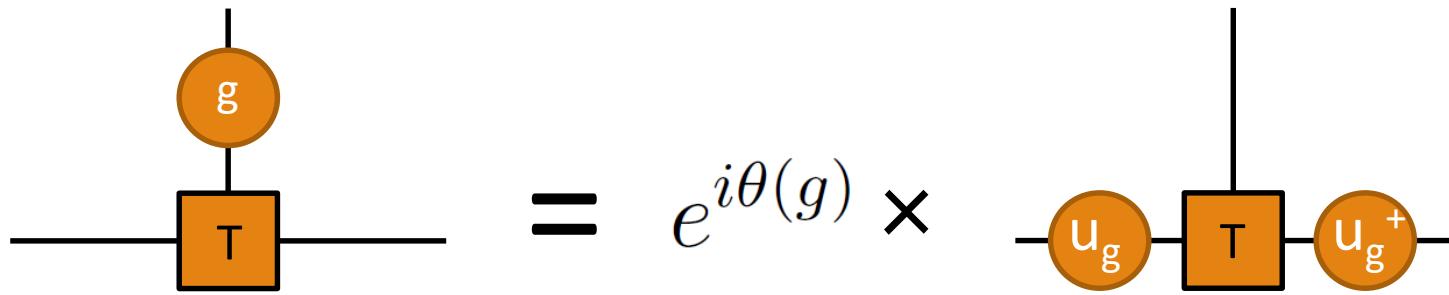
スケール不变テンソルは
臨界現象の完全な情報を
含んでいる。

Cardy: Nucl. Phys. B 270 (1986)

Topological/gauge structure in TN

Example: Characterization of SPT phase

Chen,Gu,Liu,Wen (2012)



$$u_g u_h = \boxed{\omega(g, h)} u_{gh}$$

For the symmetry group G, an SPT phase is characterized by the 2nd cohomology group $H_2(G, U(1))$ of the projective representation of G.

TN for data science

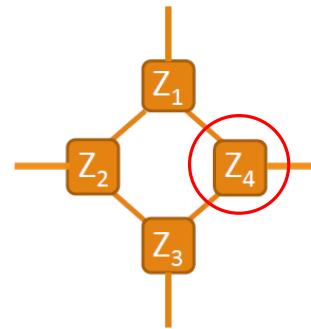
Example: Image classification by TN

Zhao, Cichocki et al, arXiv:1606.05535

COIL100 2D image classification task
128 x 128 x 3 x 7200 bits



Decomposed the whole data into a tensor ring, and applied KNN classifier ($K=1$) to the image-identifying core (Z_4).



Ring decomposition shows better performance than open chain (TT).

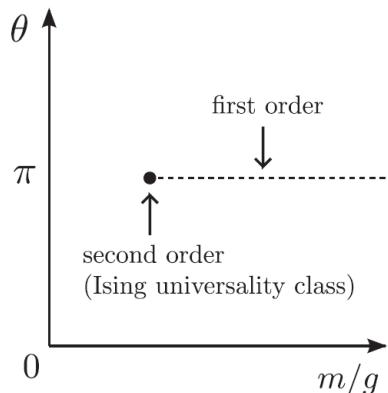
We may use TN for identification of subtle characters.

TNRG for 2D Schwinger model

Shimizu and Kuramashi:
PRD90, 014508 and 074503 (2014)

$$Z = \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{-S[A] + \frac{\theta}{4\pi}} \int d^2x \epsilon_{\mu\nu} F_{\mu\nu}$$

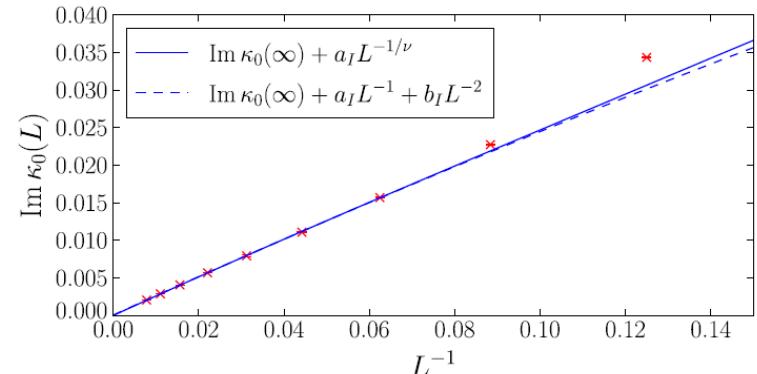
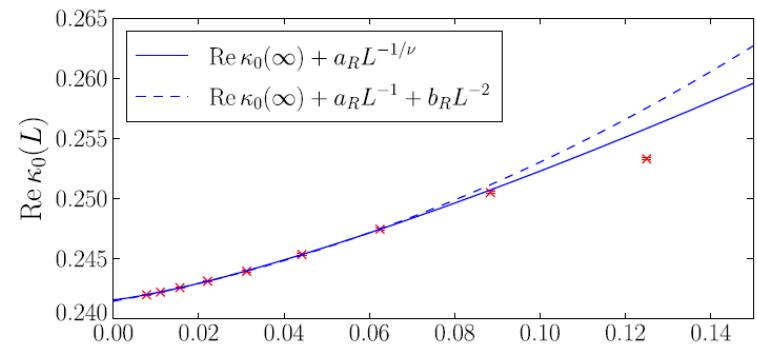
$$S = \int d^2x \left\{ \bar{\psi} (\gamma_\mu \partial_\mu + i\gamma_\mu A_\mu + m) \psi + \frac{1}{4g^2} F_{\mu\nu} F_{\mu\nu} \right\}$$



Lee-Yang zero of the
partition function

Ising universality
class is confirmed.

$$1/\kappa = 2(m + 2)$$



TNRG for 4D Ising model

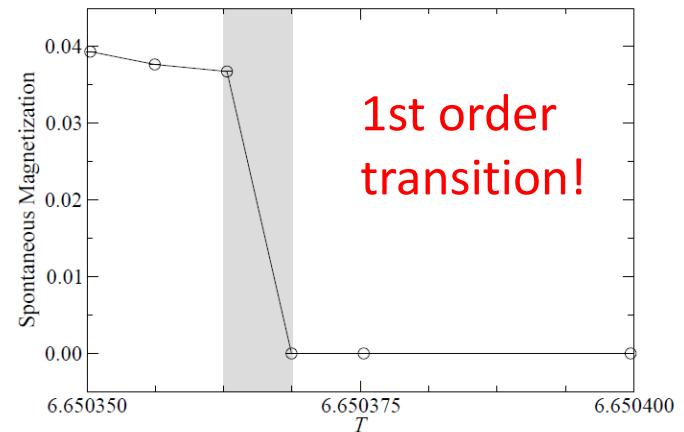
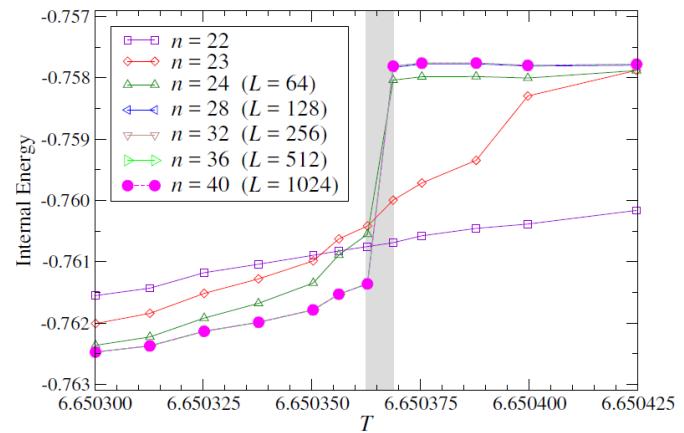
Akiyama, Kuramashi, Yamashita, Yoshimura:
PRD 100, 054510 (2019)

HOTRG calculation

computational cost $\propto D^{4d-1}$

expensive!

CF: Monte Carlo result: $T_c=6.68026(2)$
[Lundow-Markström, PRE80, 031104 (2009)]



Conquest for Lighter algorithm

"Triad" TRG

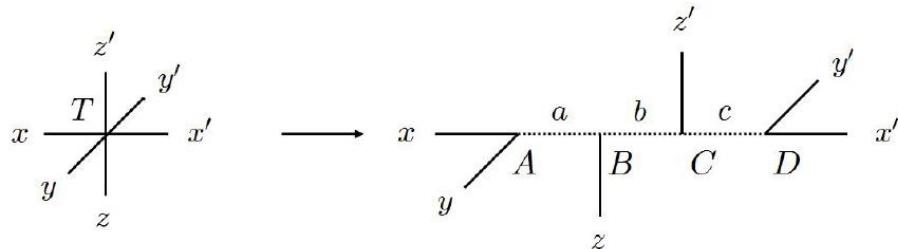
HOTRG: $O(D^{4d-1})$

[Xie, et al, PRB86, 045139 (2012)]

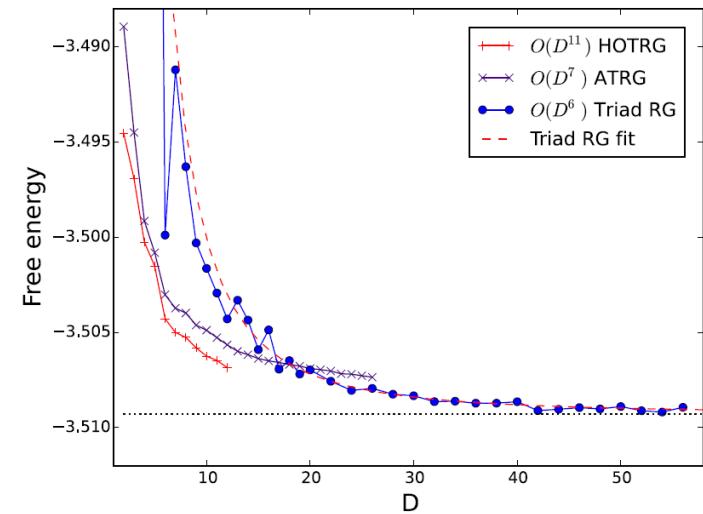
ATRG: $O(D^{2d+1})$

[Adachi, Okubo, Todo, arXiv:1906.02007]

Triad TRG: $O(D^{d+3})$ [Kadoh, Nakayama, arXiv:1912.02414]



The accuracy reduction is not so bad.



Collaborators

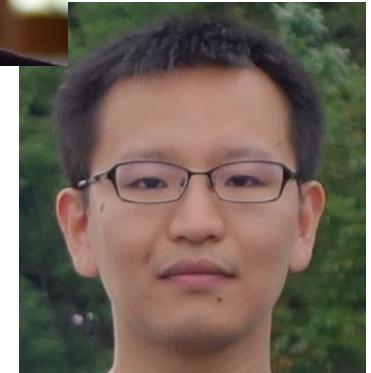
Hyun-Yong LEE (ISSP)

Tsuyoshi OKUBO (U. Tokyo)

Ryui KANEKO (ISSP)

Yohei YAMAJI (U. Tokyo)

Yong-Baek Kim (Toronto)



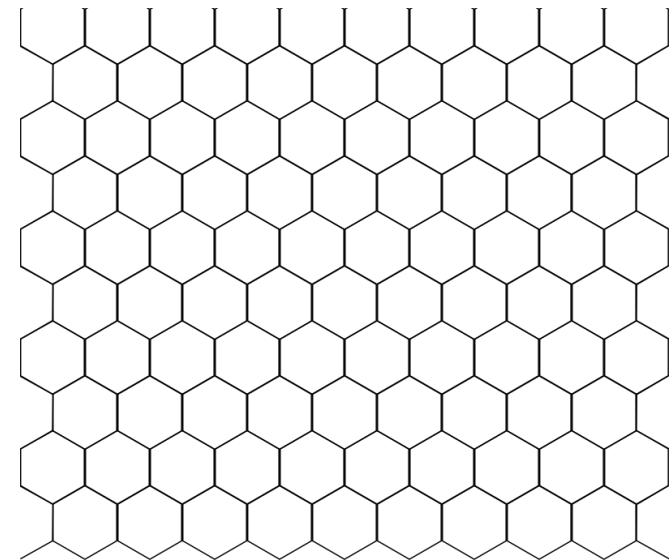
Kitaev Model

Kitaev, Ann. Phys. 321 (2006) 2

$$H = \sum_{(ij)} \sum_{\mu=x,y,z} J_{ij}^{\mu} \sigma_i^{\mu} \sigma_j^{\mu}$$

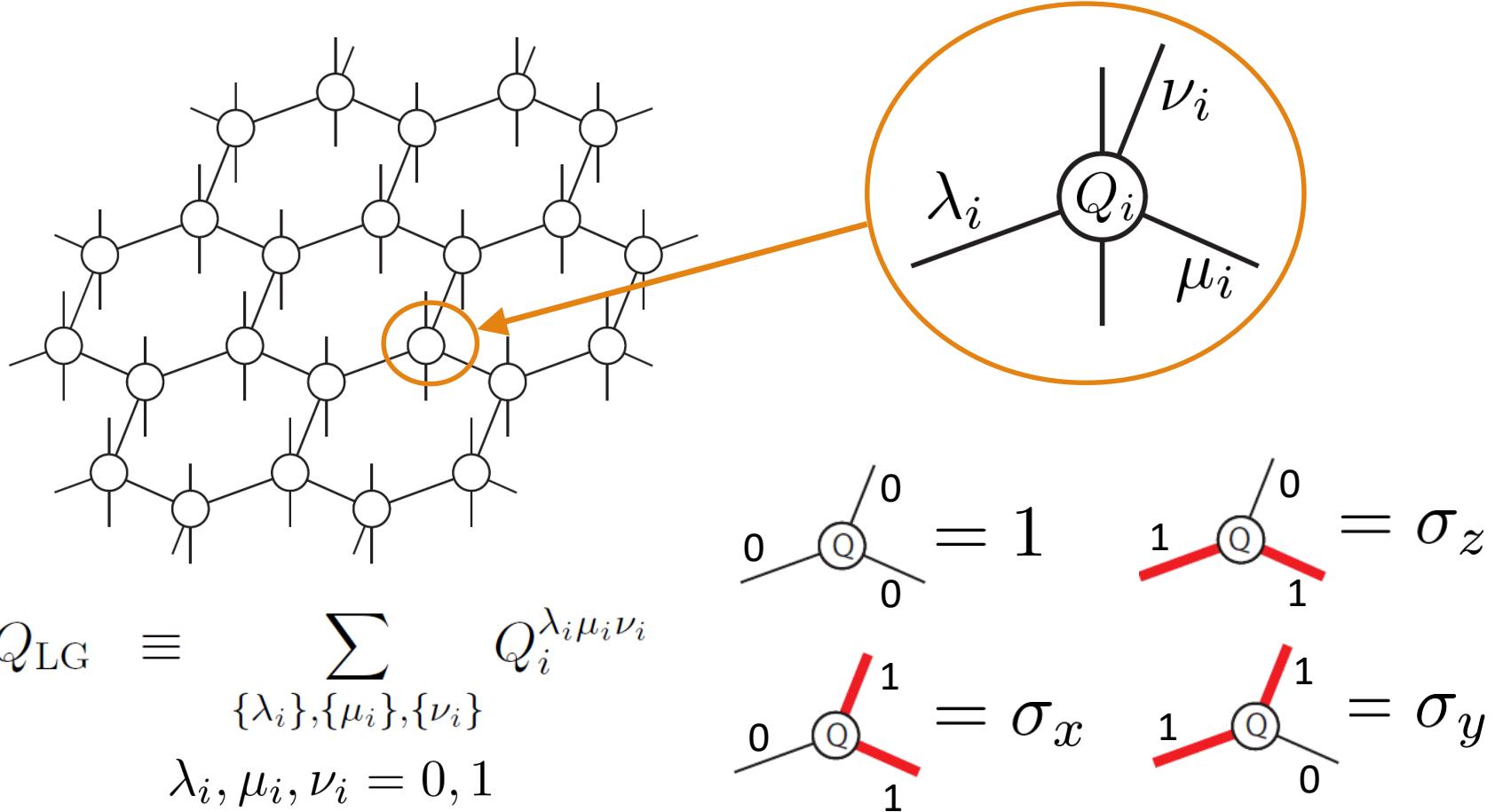
$$J_{ij}^{\mu} = \begin{cases} J & ((ij) \parallel \mu\text{-axis}) \\ 0 & (\text{otherwise}) \end{cases}$$

- Local conservation of flux operators:
- $W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$
- Exact decomposition into flux degrees of freedom and Majorana fermions
- Robust quantum operations by "braiding"



Projection to flux-free sector

--- Loop Gas Projector (LGP)



Loop Gas State

$$|\text{LGS}\rangle = \sum_{G: \text{loop config.}} Q(G) |(111)\rangle$$

$$= \left| \begin{array}{c} \text{hexagonal lattice} \\ \text{no loops} \end{array} \right\rangle + \left| \begin{array}{c} \text{hexagonal lattice} \\ \text{1 loop} \end{array} \right\rangle + \left| \begin{array}{c} \text{hexagonal lattice} \\ \text{2 loops} \end{array} \right\rangle + \dots$$

$$\langle \text{LGS} | \text{LGS} \rangle = Z_{\text{LG}} \left(\zeta = \frac{1}{3} \right)$$

Gapped

Critical

LGS = Critical Loop Gas

$$Z_{\text{LG}}(n, \zeta) \equiv \sum_{G: \text{loop config.}} n^{N_{\text{loop}}(G)} \zeta^{|G|}$$

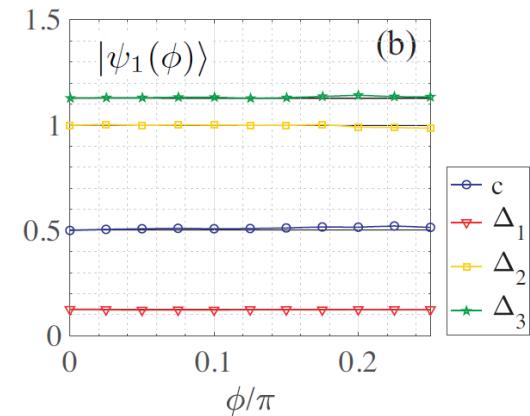
$$\zeta_c(n=1) = \frac{1}{\sqrt{3}} \quad \text{B. Nienhuis, PRL 49, 1062 (1982).}$$

Series of Ansatzes

$$|\psi_0\rangle \equiv P_{\text{LG}}|(111)\rangle = |\text{LGS}\rangle$$

$$|\psi_1\rangle \equiv P_{\text{LG}}R_{\text{DG}}(\phi_1)|(111)\rangle$$

$$|\psi_2\rangle \equiv P_{\text{LG}}R_{\text{DG}}(\phi_1)R_{\text{DG}}(\phi_2)|(111)\rangle$$



	$\Psi_0 = \text{LGS}$	Ψ_1	Ψ_2	KHM gr. st.
# of prmtrs.	0	1	2	
E/J	-0.16349	-0.19643	-0.19681	-0.19682
$\Delta E/E$	0.17	0.02	0.00007	-

Best accuracy by numerical calculation is achieved
with only two tunable parameters

Kitaev Model on Star Lattice

$$\hat{\mathcal{H}} = -\frac{J}{4} \sum_{\langle ij \rangle \in \gamma} \hat{\sigma}_i^\gamma \hat{\sigma}_j^\gamma - \frac{J'}{4} \sum_{\langle ij \rangle \in \gamma'} \hat{\sigma}_i^{\gamma'} \hat{\sigma}_j^{\gamma'}$$

The ground state is CSL

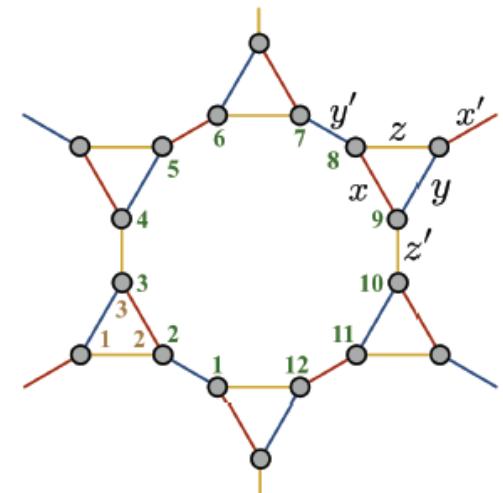
H. Yao and S. A. Kivelson, PRL99, 247203 (2007)

S. B. Chung, et al, PRB 81, 060403 (2010)

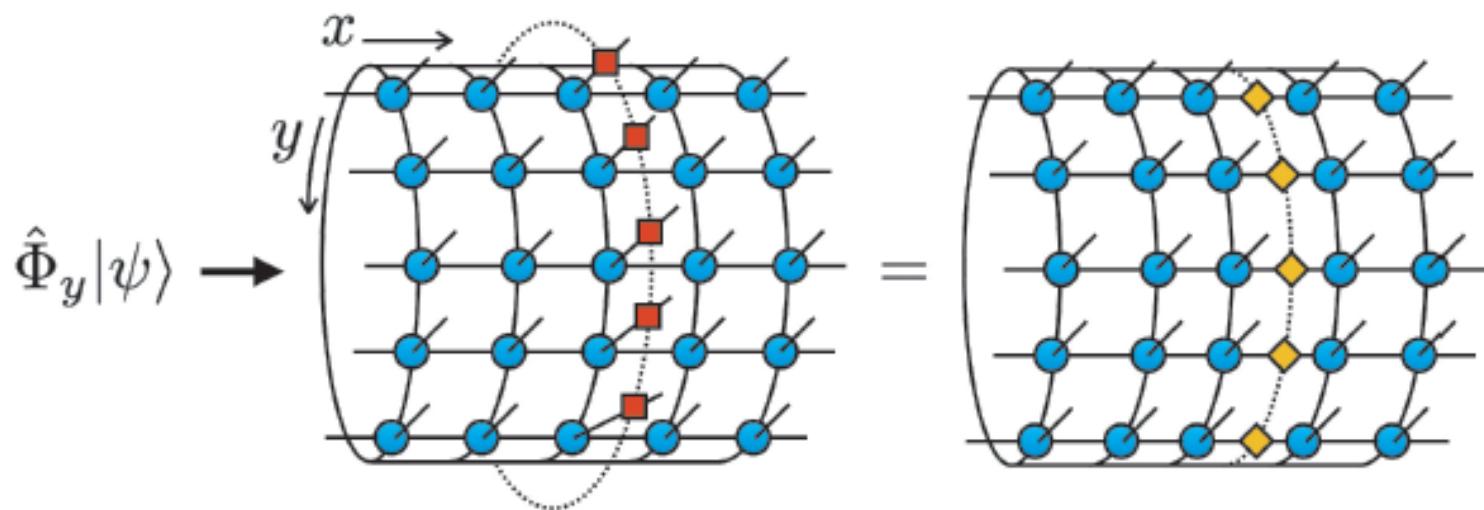
On the torus,

Abelian CSL ... 4-fold degenerate

non-Abelian CSL ... 3-fold degenerate

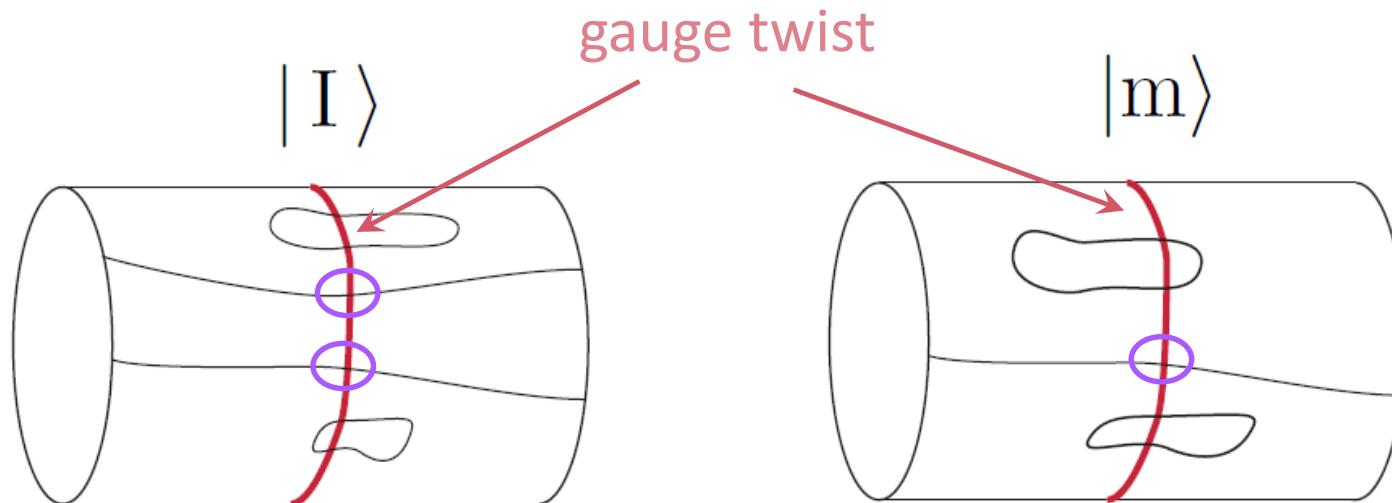


Effect of Global-Flux Operator



Global flux op. is equivalent to the global gauge twist.

Minimally Entangled States (MES)



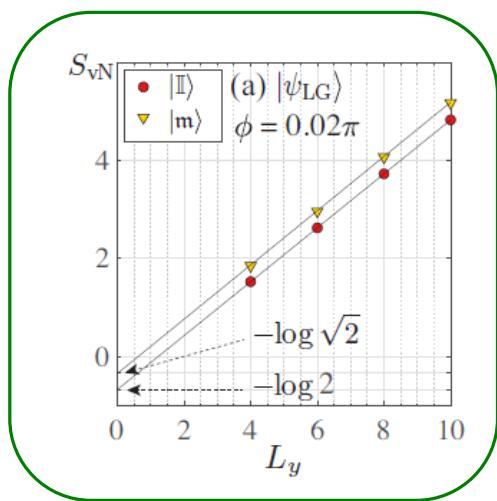
$$\hat{\Phi}_y = +1$$

$$\hat{\Phi}_y = -1$$

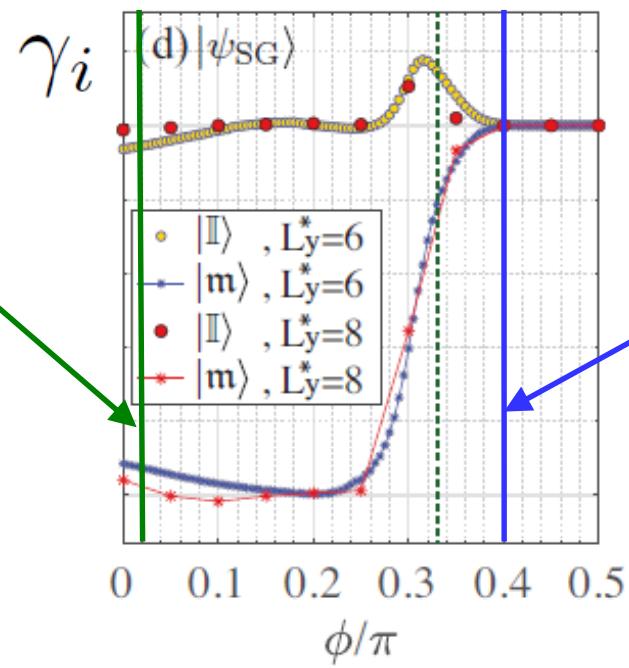
Topological Entanglement Entropy

$$S = \alpha L_y - \gamma_i$$

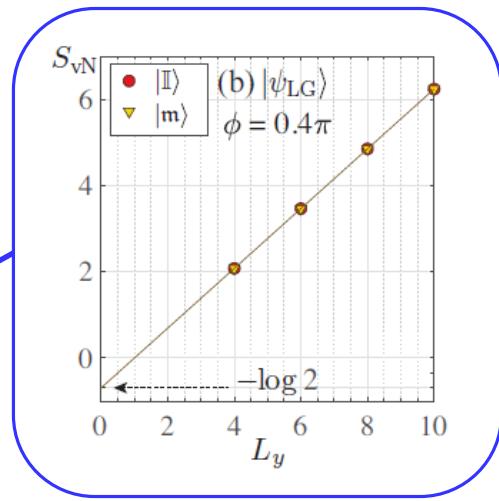
non-Abelian



TEE depends on
the top. excitation
(Ising anyon)



Abelian



TEE does not depends
on the top. excitation
(the same as toric code)

Summary

キタエフスピニ液体状態を扱う新しい枠組み

(1) $S=1/2$ Kitaev honeycomb model (KHM)

- 臨界的な場合もギャップのある場合も表現できた
- KHM と LGS の関係
(cf: ハイゼンベルク鎖と AKLT 状態の関係)

(2) $S=1/2$ star-lattice Kitaev model

- ギャップのある Abelian/non-Abelian カイラルスピニ液体状態が表現できた (CF: the no-go theorem by Dubail-Read (2015))