Gravitational waves induced by scalar perturbations at second order

~ as probes of the primordial power spectrum on small scales ~

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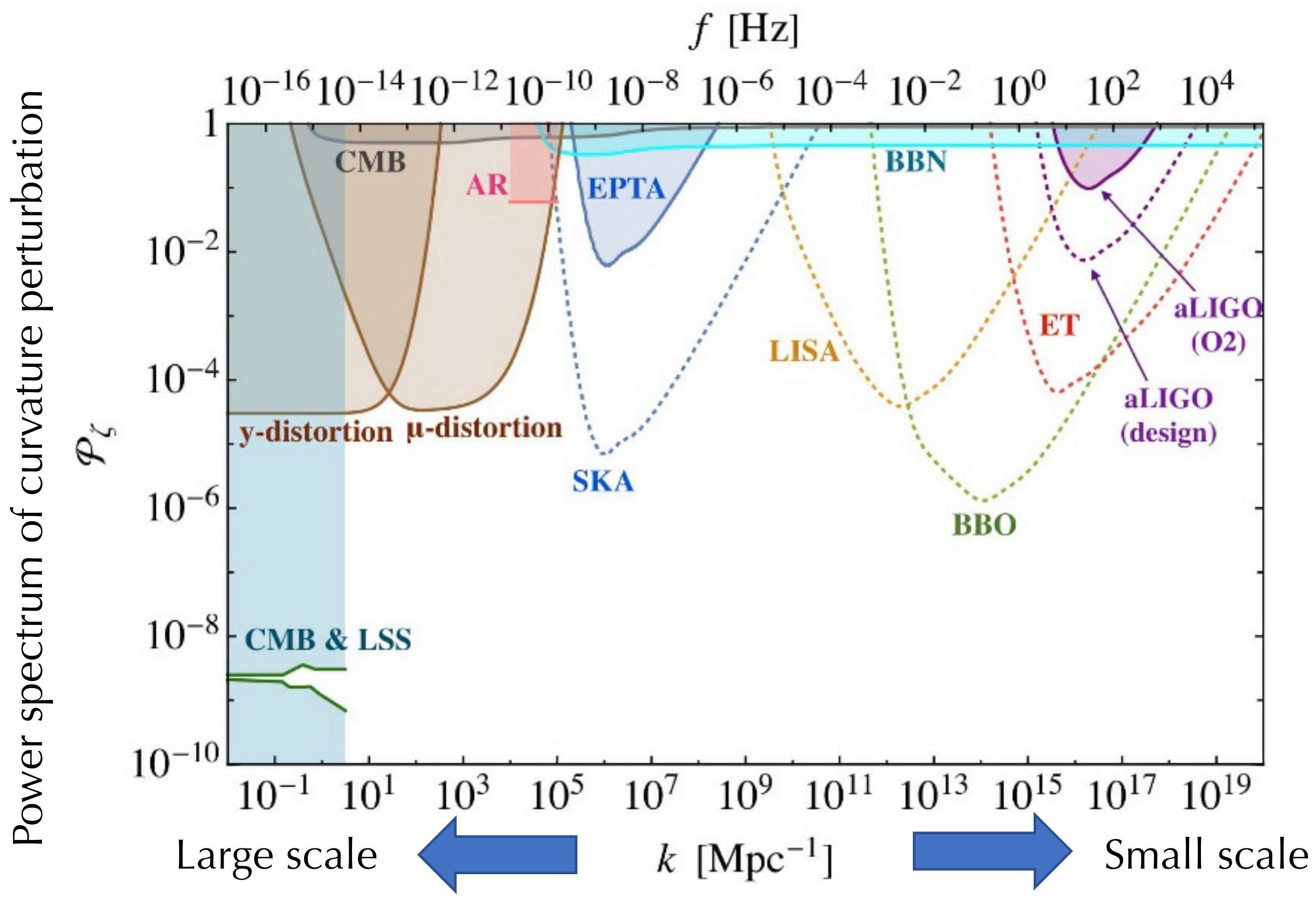
arXiv: 1812.00674

What we do



We derive constraints on small scale curvature perturbations using GWs induced by scalar perturbations.

Outline

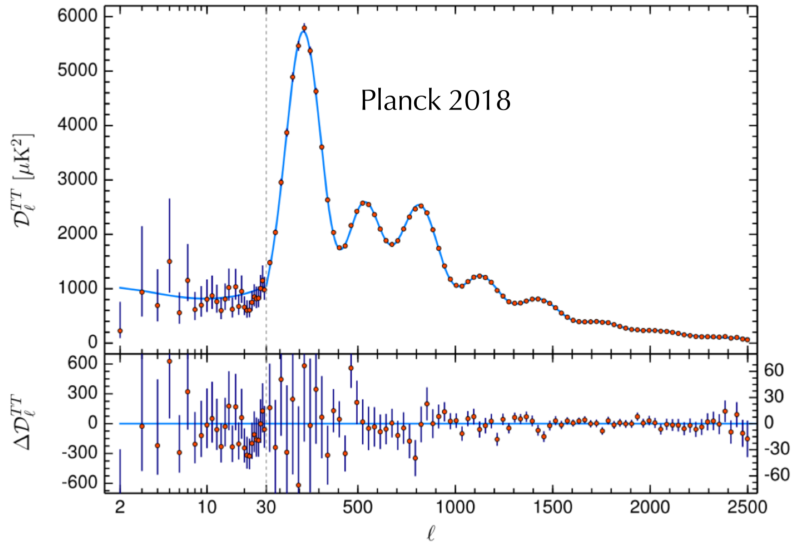
- <u>Introduction</u>
- Gravitational waves induced by scalar perturbations
- Constraints on the power spectrum on small scale
- Summary

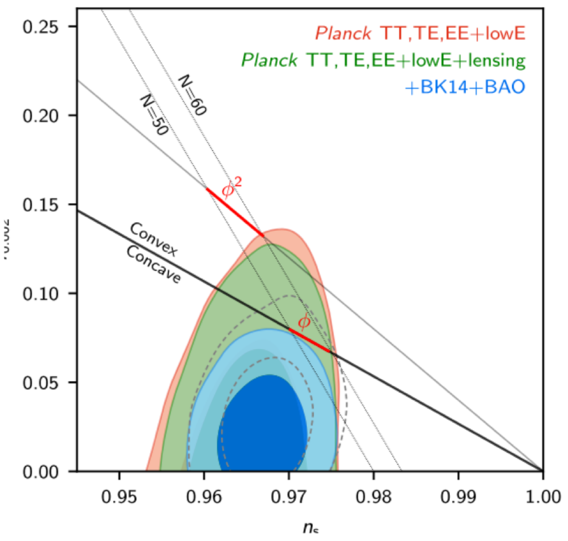
Perturbations and Cosmology

The inflation theory predicts power spectra of perturbations, including scalar (curvature), tensor (GWs) perturbations.

Therefore

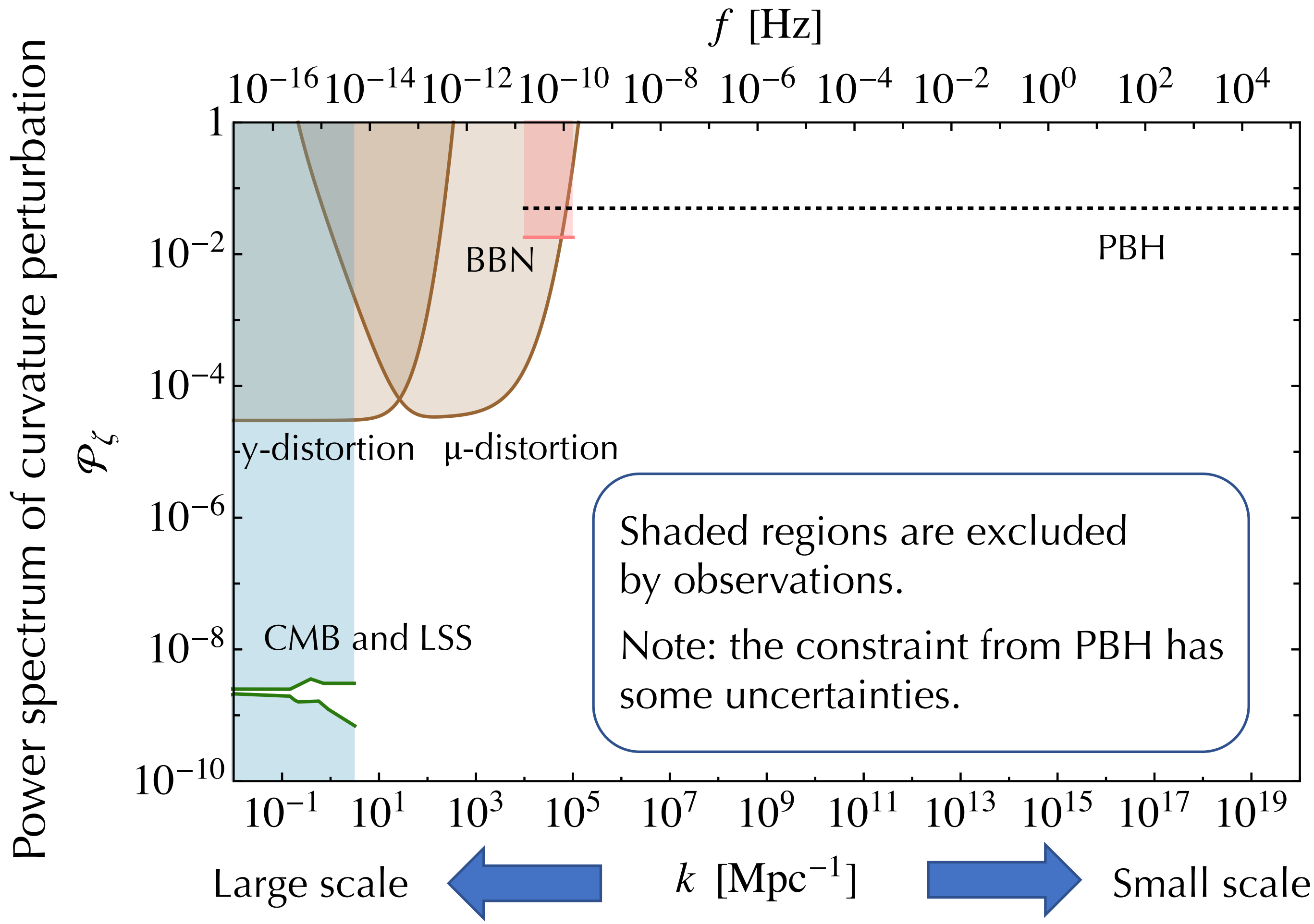
To understand the nature of the inflation theory, it is important to investigate the perturbations.





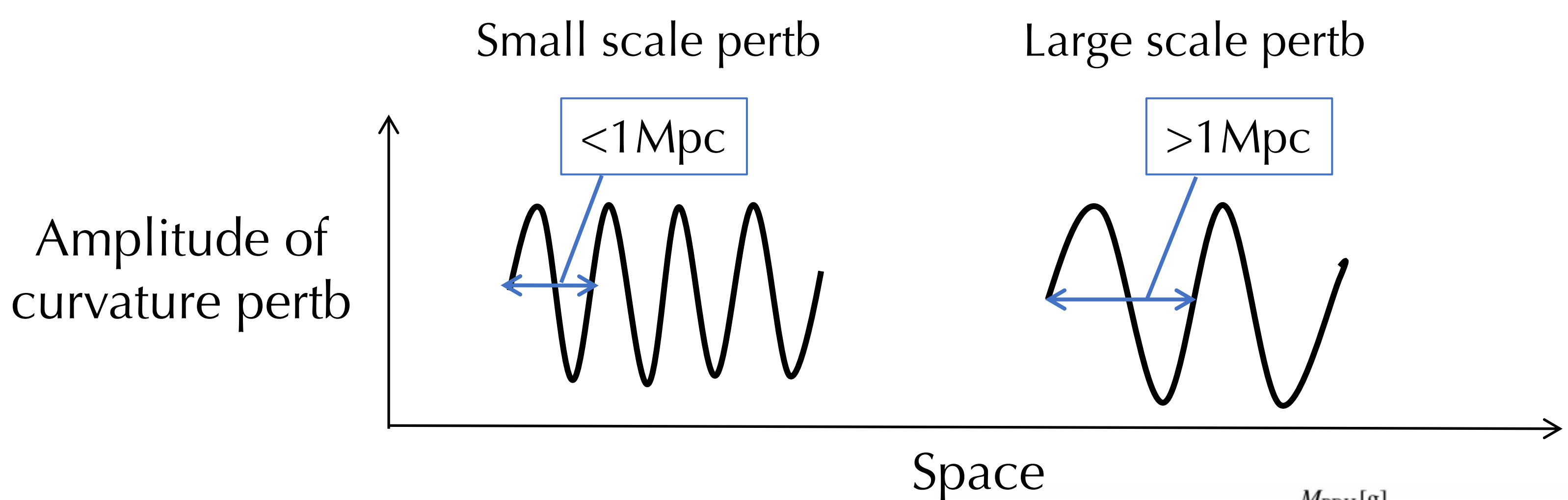
On large scale (>1Mpc), curvature perturbations are observed very well.

Current constraints on the small scale



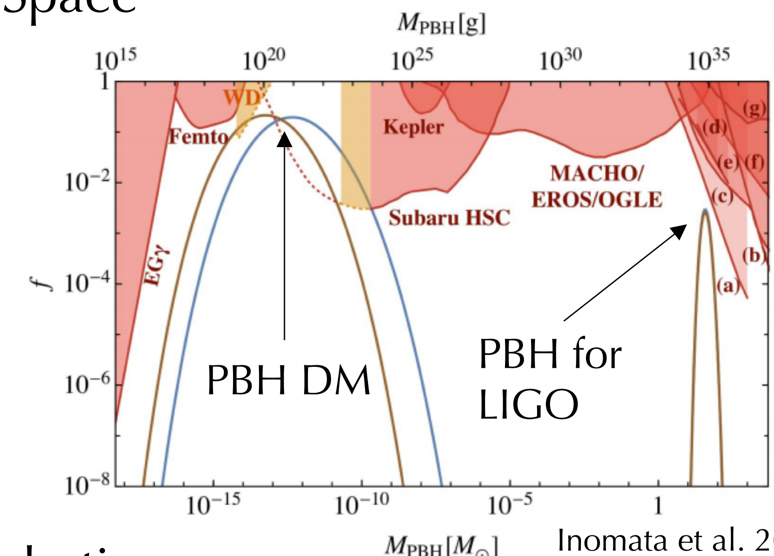
It is difficult to observe small scale perturbations due to Silk damping or non-linear growth of large scale structure(LSS).

Small scale perturbation and PBH



Some inflation models predict large perturbations on small scale and sizable amount of PBHs.

PBH is one of the candidate of DM and the heavy BHs detected by LIGO.



Inomata et al. 2017

Therefore

It is important to discuss small scale perturbations.

What we do in this work

GWs can be induced by scalar (curvature) perturbations at second order. (Terada-san's talk)

If there is a large perturbation on small scale, the induced GWs can be constrained by observations.

We can constrain the small scale perturbations using the induced GWs!

Related at second order

Scalar perturbations $\longleftrightarrow$ GWs

Constraints on small scale perturbation from induced GWs $\longleftarrow$ Constraints on GWs from observations

We do this!

Related work: Assadullahi & Wands, 2009

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Brief review of the induced GWs

GWs can be induced by scalar (curvature) perturbations at second order. (Terada-san's talk)

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{a(\eta)H(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)},$$

Energy density of the induced GWs

$$\overline{\mathcal{P}_h(\eta,k)} \simeq 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \times \overline{I^2(v,u,k\eta)}\mathcal{P}_\zeta(kv)\mathcal{P}_\zeta(ku)$$

Power spectrum of the induced GWs

$$\overline{I^2(v,u,x\to\infty)} = \frac{1}{2}\left(\frac{3(u^2+v^2-3)}{4u^3v^3x}\right)^2 \left(\left(-4uv+(u^2+v^2-3)\log\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2 + \pi^2(u^2+v^2-3)^2\Theta(v+u-\sqrt{3})\right)$$

$$\Omega_{\rm GW}(\eta_0,k) = 0.83\left(\frac{g_c}{10.75}\right)^{-1/3}\Omega_{r,0}\,\underline{\Omega_{\rm GW}(\eta_c,k)} \qquad (1/k \ll \eta_c \ll \eta_{\rm eq})$$

What we calculate with above formula

Constraints on the induced GWs

When multiple detectors or pulsars are available, it is beneficial to use a cross-correlation between them to look for the correlated signal due to stochastic GWs.

Signal to noise ratio is defined as (Allen & Romano, 1997)

Induced GWs

$$\rho = \sqrt{2T}\left[\int_{f_{\rm min}}^{f_{\rm max}} df \left(\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm GW,eff}(f)}\right)^2\right]^{1/2}$$

Observation time

Effective sensitivity

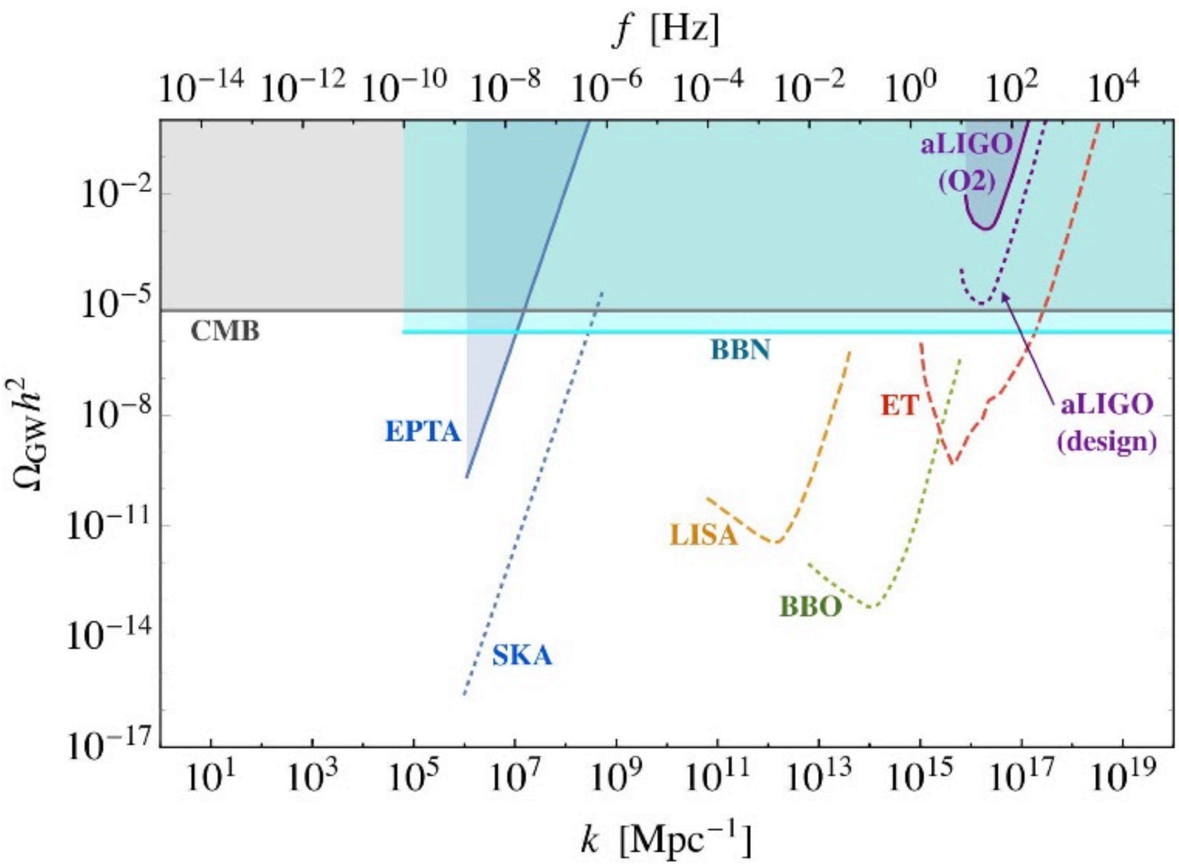
The effective sensitivity is given by

Overlap reduction function, coming from cross correlation

$$\Omega_{\rm GW,eff}(f) H_0^2 = \frac{2\pi^2}{3} f^3 \left[\sum_{I=1}^{M}\sum_{J>I}^{M} \frac{\Gamma_{IJ}^2(f)}{P_{nI}(f)P_{nJ}(f)}\right]^{-1/2}$$

Noise power spectrum

Summary of GW constraints



Shaded regions are excluded by current observations.

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How to derive the constraints

To be specific, we assume the peak-like profile for the power spectrum

$$\mathcal{P}_\zeta(k) = A \exp\left(-\frac{(\log k - \log k_\mathrm{p})^2}{2\sigma^2}\right).$$

1. Fix σ.
2. Fix T (observation time) for each project.

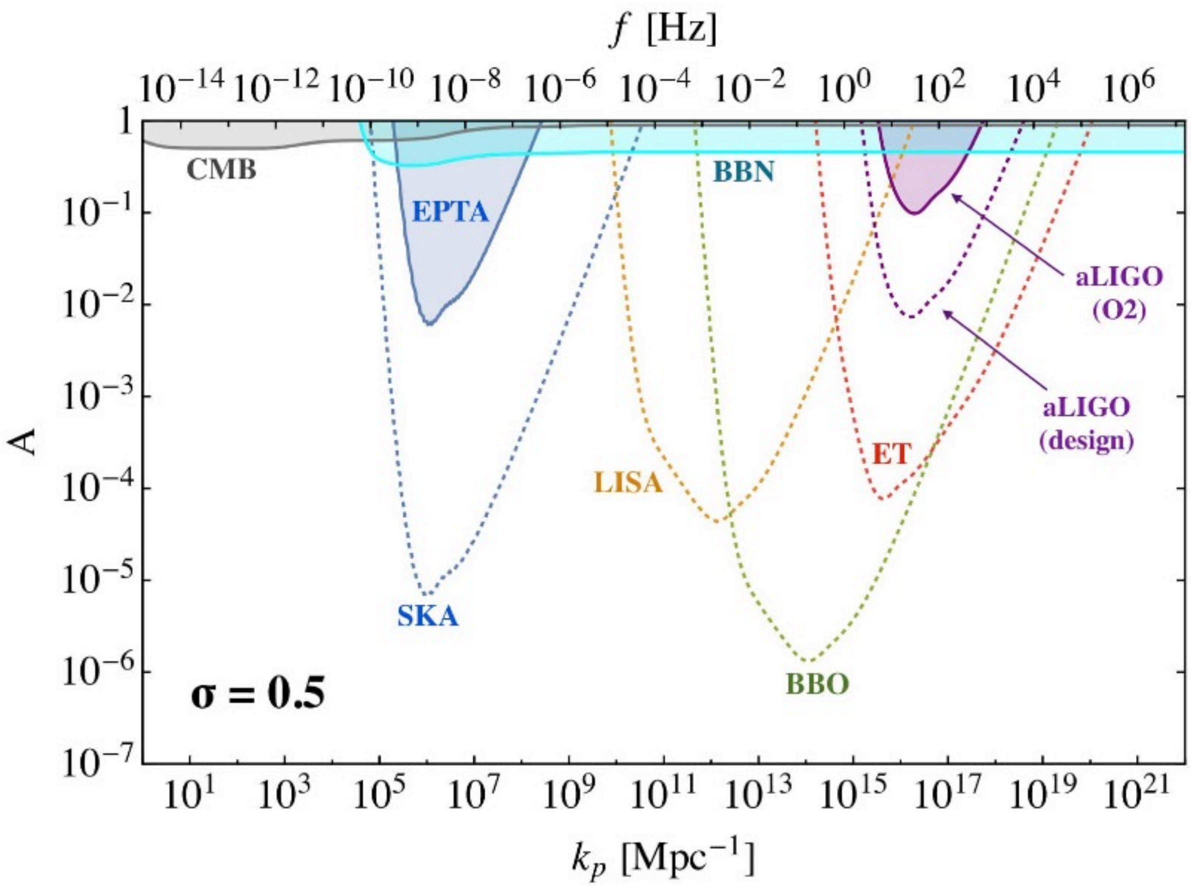
   EPTA:18 years, SKA: 20 years, aLIGO (O2): 4 months, Others: 1 year.

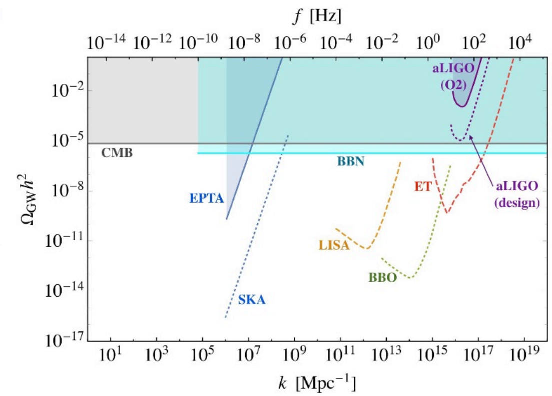
3. Fix k_p and calculate A that yields signal-to-noise ratio unity (ρ=1) for each project.

$$\rho = \sqrt{2T}\left[\int_{f_\mathrm{min}}^{f_\mathrm{max}} df \left(\frac{\Omega_\mathrm{GW}(f)}{\Omega_\mathrm{GW,eff}(f)}\right)^2\right]^{1/2}$$

4. Change k_p and calculate A repeatedly and get the constraint curves on A. $(A \propto \rho^{1/2} T^{-1/4})$

Constraints on small scale perturbations





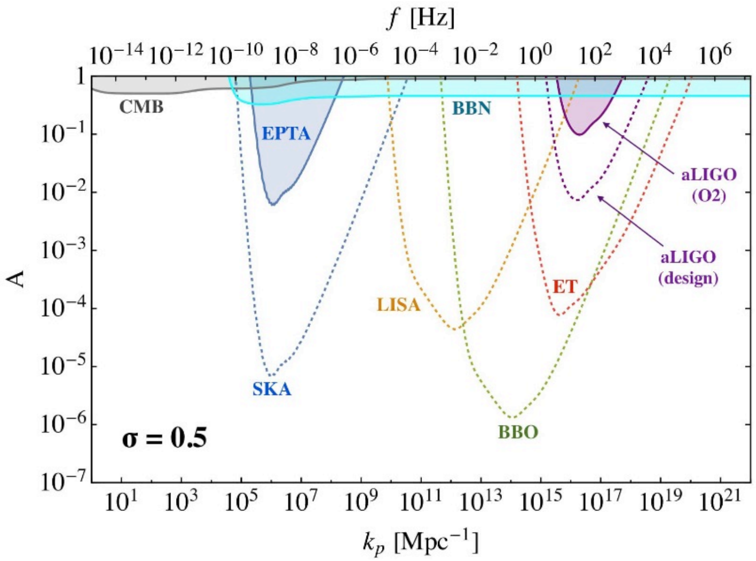
$$\mathcal{P}_\zeta(k) = A \exp\left(-\frac{(\log k - \log k_{\rm p})^2}{2\sigma^2}\right).$$

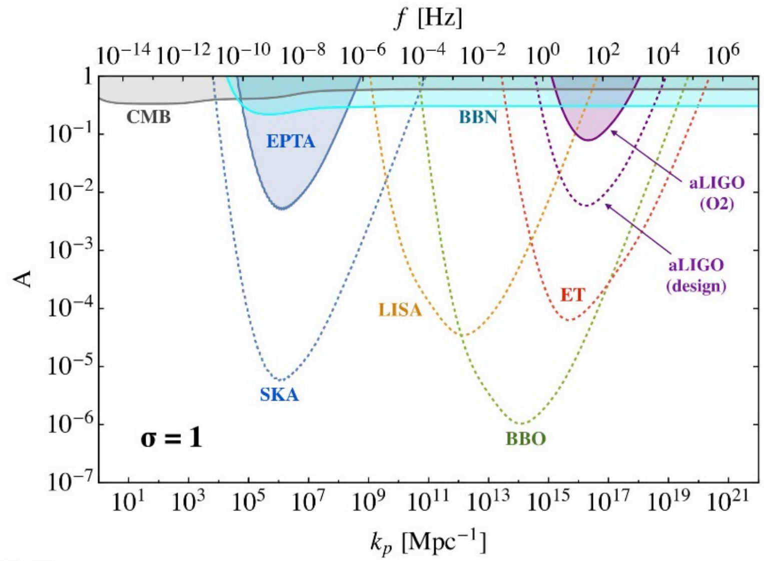
$$\rho = \sqrt{2T}\left[\int_{f_{\rm min}}^{f_{\rm max}} df \left(\frac{\Omega_{\rm GW}(f)}{\Omega_{\rm GW,eff}(f)}\right)^2\right]^{1/2}$$

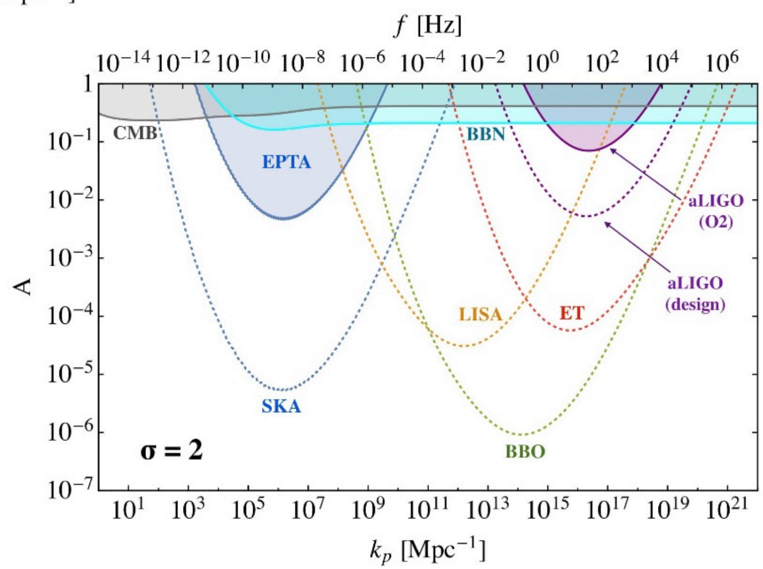
The constraints on the smaller-scale are stronger compared to the effective sensitivities to GWs.

This is because of the frequency integration in the definition of signal-to-noise ratio.

Profile dependence







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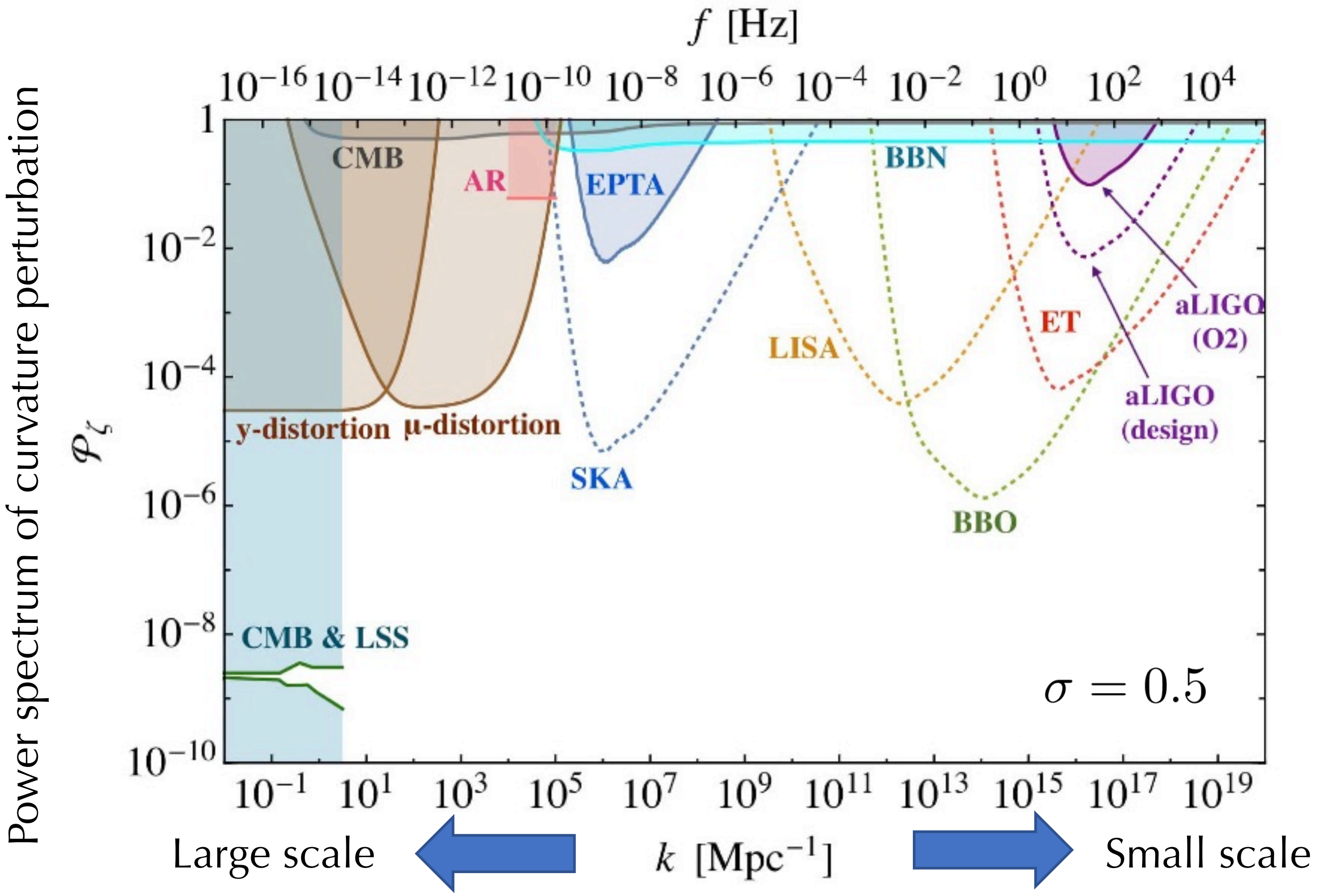
What we have done



We have derived the constraints on small scale perturbations using GWs induced by scalar perturbations at 2nd order.

Backup

Foregrounds

In future, stochastic GWs of astrophysical origin or cosmological origins that are different from the induced GWs, such as those from phase transition, may be detected.
In that case, the constraints would become weak.

Foreground example:

Around PTA target frequency
GWs from supermassive-black-hole binaries

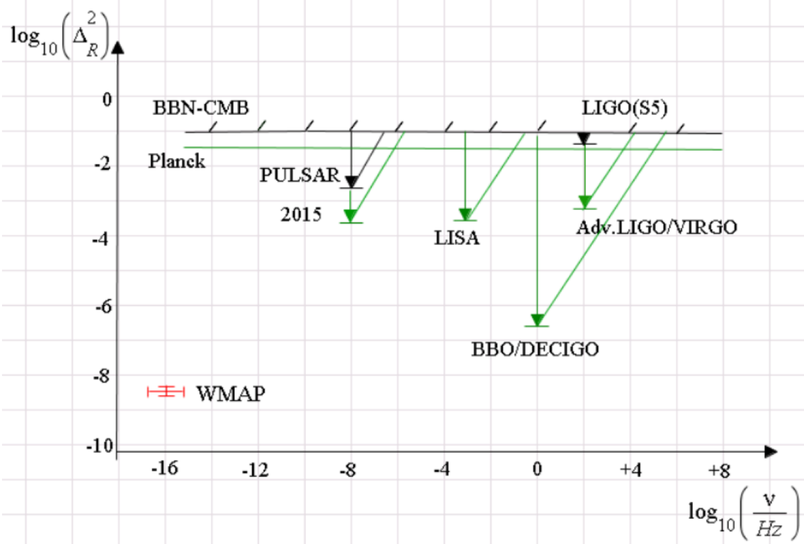
Around LISA target frequency
GWs from galactic white-dwarf binaries

Around BBO target frequency
GWs from neutron star binaries

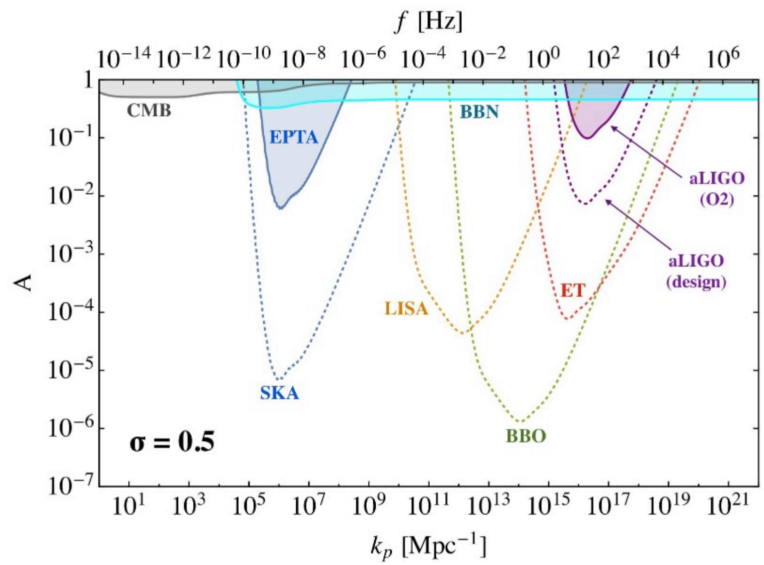
We may use non-Gaussianity of the induced GWs to discriminate them from other sources. (Bartolo et al., 2018)

Difference between our work and previous work

Assadullahi & Wands, 2009



Our work



In previous work, they use the old formula for the induced GWs.

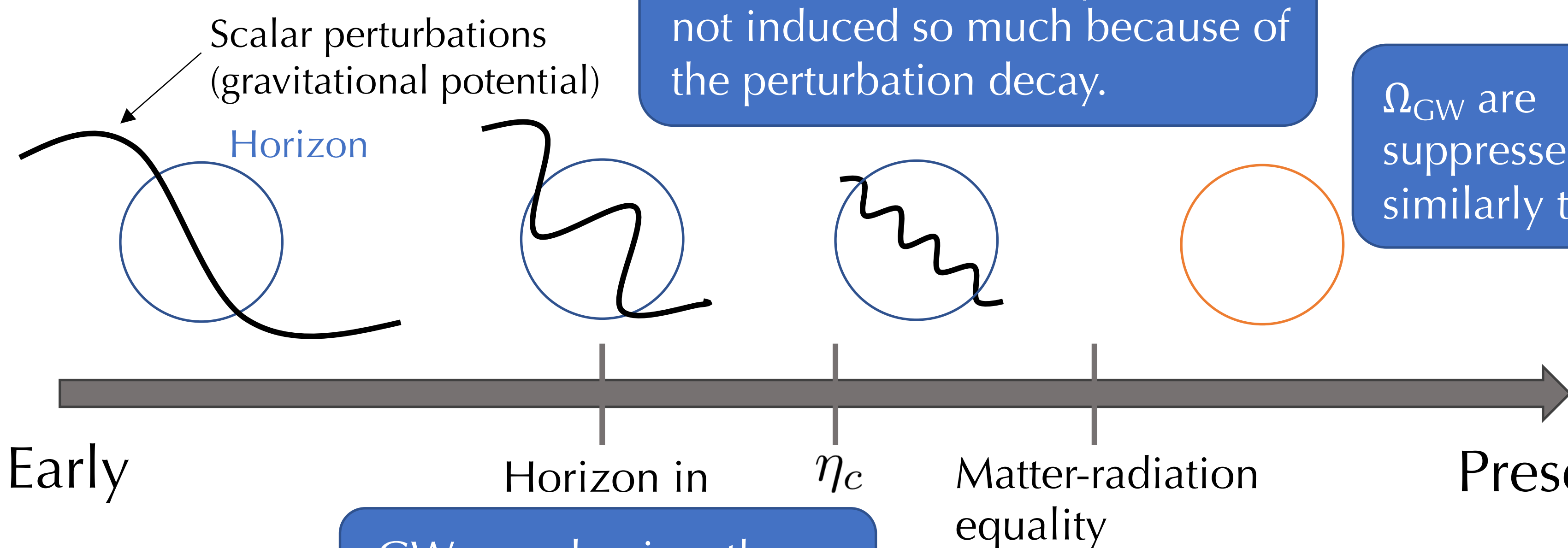
For scale invariant spectrum: ($P_\zeta = A$)

$$\Omega_{\rm GW}(\eta_c, k) \simeq \begin{cases} 33.3\, A^2 & \text{(for previous work)} \\ 0.8222\, A^2 & \text{(for Kohri\&Terada, 2018)} \end{cases}$$

In addition, they do not take into account frequency dependence of the sensitivity curves.

Evolution of the induced GWs

Comoving scale



After the horizon entry, GWs are not induced so much because of the perturbation decay.

$\Omega_{\rm GW}$ are suppressed similarly to Ω_r.

GWs are dominantly induced at horizon entry.

$$(1/k \ll \eta_c \ll \eta_{\rm eq})$$

$$\Omega_{\rm GW}(\eta_0, k) = 0.83 \left(\frac{g_c}{10.75} \right)^{-1/3} \Omega_{r,0} \, \underline{\Omega_{\rm GW}(\eta_c, k)}$$

What we calculate with the formula