Non-perturbative effects of loop-induced mixing: Majorana fields

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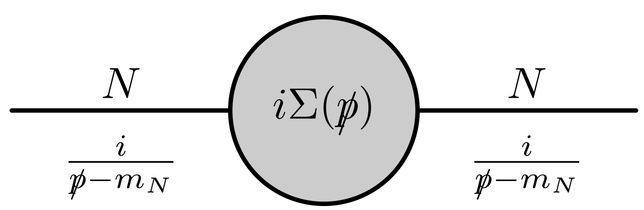
Single flavor

RH Majorana field — Yukawa coupling — Scalar SU(2) doublet — Lepton SU(2) doublet

- Lagrangian

$$\mathcal{L} = \frac{1}{2}\overline{N} i\partial\!\!\!/ N - \frac{1}{2} m_N \overline{N} N - f \overline{L}\widetilde{\phi} \mathrm{R} N - f^* \overline{N} \widetilde{\phi}^\dagger \mathrm{L} L$$
$$+ \frac{1}{2}\delta_N \overline{N} i\partial\!\!\!/ N - \frac{1}{2} m_N(\delta_N + \delta_M + \delta_N\delta_M)\overline{N} N - \delta_V f \overline{L}\widetilde{\phi} \mathrm{R} N - \delta_V^* f^* \overline{N}\widetilde{\phi}^\dagger \mathrm{L} L$$

- Self-energy of the RH neutrino: quantum correction to the propagator



$$i\Sigma(p\!\!\!/) = i p\!\!\!/ \Sigma_R(p^2)$$

$$\Sigma_R(p^2) := \frac{|f|^2}{16\pi^2}\left[-\log\left(\frac{p^2}{\mu^2}\right) + i\pi\Theta(p^2)\right]$$

Up to the LO

- Resummed propagator: propagator with quantum corrections

$$i\Delta(p\!\!\!/) = \frac{i}{p\!\!\!/ - m_N} + \frac{i}{p\!\!\!/ - m_N}[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/ - m_N} + \frac{i}{p\!\!\!/ - m_N}[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/ - m_N}[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/ - m_N} + \cdots$$
$$= \sum_{n=0}^{\infty}\frac{i}{p\!\!\!/ - m_N}\left\{[i\Sigma(p\!\!\!/)]\frac{i}{p\!\!\!/ - m_N}\right\}^n$$
$$= i\{p\!\!\!/ - m_N + \Sigma(p\!\!\!/)\}^{-1} = i\{[1+\Sigma_R(p^2)]p\!\!\!/ - m_N\}^{-1}. \quad ($$

Resummation by geometric series

$$\Delta(p^2) = Z_N^{-1} Z_M^{-1} \frac{P(p^2)}{m_N}\frac{p\!\!\!/ + P(p^2)}{p^2 - P^2(p^2)}$$

Single flavor

- Expanding the propagator around the pole

$$i\Delta_{RR}(p^2) = i\Delta_{LL}(p^2) = R^{\widehat{N}}\,\frac{ip_{\widehat{N}}}{p^2 - p_{\widehat{N}}^2} + \cdots\,,$$
$$i\not p\Delta_{LR}(p^2) = i\not p\Delta_{RL}(p^2) = R^{\widehat{N}}\,\frac{i\not p}{p^2 - p_{\widehat{N}}^2} + \cdots\,,$$
$$\begin{aligned} i\Delta(\not p) &= i\mathsf{R}\Delta_{RR}(p^2) + i\mathsf{R}\not p\Delta_{RL}(p^2) + i\mathsf{L}\not p\Delta_{LR}(p^2) + i\mathsf{L}\Delta_{LL}(p^2) \\ &= R^{\widehat{N}}\,\frac{i(\not p + p_{\widehat{N}})}{p^2 - p_{\widehat{N}}^2} + \cdots\,, \end{aligned}$$

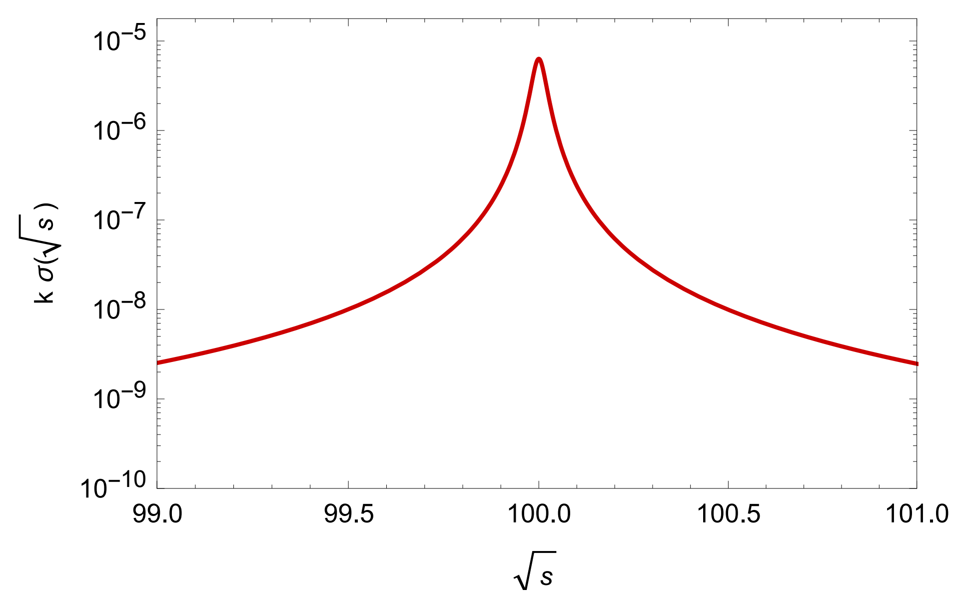
Physical pole

$$p_{\widehat{N}}^2 = m_{\widehat{N}}^2 - i m_{\widehat{N}}\Gamma_{\widehat{N}}$$

- Physical mass and decay width

$$\sigma_{\mathrm{CM}}(L\phi \to L\phi) \propto |\mathcal{M}(L\phi \to L\phi)|^2$$

$$\begin{aligned} i\mathcal{M}(L\phi \to L\phi) &= \overline{u_L}(\mathbf{p}'_L)(-i\widetilde{f}\mathsf{R})[i\Delta(\not p)](-i\widetilde{f}^*\mathsf{L})u_L(\mathbf{p}_L) \\ &= -i|\widetilde{f}|^2\overline{u_L}(\mathbf{p}'_L)\not p\Delta_{RL}(p^2)\mathsf{L}u_L(\mathbf{p}_L) \\ &= -i|\widetilde{f}|^2\overline{u_L}(\mathbf{p}'_L)\frac{R^{\widehat{N}}\not p}{p^2 - p_{\widehat{N}}^2}\mathsf{L}u_L(\mathbf{p}_L) + \cdots \end{aligned}$$



Breit-Wigner resonance

Multiple flavors

- Kinetic part in the Lagrangian

$$\mathcal{L}_{\text{kin}} = \frac{1}{2}\sum_{\alpha}\overline{N'_{R\alpha}}\not{\partial}N'_{R\alpha} + \frac{1}{2}\sum_{\alpha}\overline{N'^{c}_{R\alpha}}\not{\partial}N'^{c}_{R\alpha} - \frac{1}{2}\sum_{\alpha}(M_N)_{\beta\alpha}\overline{N'^{c}_{R\beta}}N'_{R\alpha} - \frac{1}{2}\sum_{\alpha}(M_N)_{\beta\alpha}\overline{N'_{R\beta}}N'^{c}_{R\alpha}$$

- Diagonalization of the mass matrix

$$N_\alpha = U_{\alpha\beta}N'_\beta, \qquad M_N^{\text{diag}} = U^{\mathsf{T}} M_N U$$

Unitary matrix

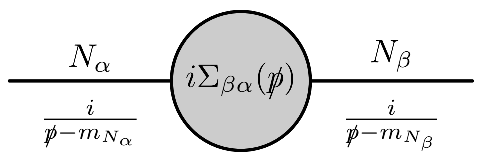
$$\begin{aligned}\mathcal{L}_{\text{kin}} &= \frac{1}{2}\sum_{\alpha}\overline{N_{R\alpha}}i\not{\partial}N_{R\alpha} + \frac{1}{2}\sum_{\alpha}\overline{N^{c}_{R\alpha}}i\not{\partial}N^{c}_{R\alpha} - \frac{1}{2}\sum_{\alpha}m_{N_\alpha}\overline{N^{c}_{R\alpha}}N_{R\alpha} - \frac{1}{2}\sum_{\alpha}m_{N_\alpha}\overline{N_{R\alpha}}N^{c}_{R\alpha} \\ &= \frac{1}{2}\sum_{\alpha}\overline{N_\alpha}i\not{\partial}N_\alpha - \frac{1}{2}\sum_{\alpha}m_{N_\alpha}\overline{N_\alpha}N_\alpha \qquad (4\end{aligned}$$

- Lagrangian

$$\begin{aligned}\mathcal{L} = &\frac{1}{2}\sum_{\alpha}\overline{N_{R\alpha}}i\not{\partial}N_{R\alpha} + \frac{1}{2}\sum_{\alpha}\overline{N^{c}_{R\alpha}}i\not{\partial}N^{c}_{R\alpha} - \frac{1}{2}\sum_{\alpha}m_{N_\alpha}\overline{N^{c}_{R\alpha}}N_{R\alpha} - \frac{1}{2}\sum_{\alpha}m_{N_\alpha}\overline{N_{R\alpha}}N^{c}_{R\alpha} \\ &- \sum_{i,\alpha}(fC_V^z)_{i\alpha}\overline{L_i}\widetilde{\phi}N_{R\alpha} - \sum_{i,\alpha}(fC_V^z)^*_{i\alpha}\overline{N_{R\alpha}}\widetilde{\phi}^\dagger L_i \\ &+ \frac{1}{2}\sum_{\alpha,\beta}(\delta_N)_{\beta\alpha}\overline{N_{R\beta}}i\not{\partial}N_{R\alpha} + \frac{1}{2}\sum_{\alpha,\beta}(\delta_N)^*_{\beta\alpha}\overline{N^{c}_{R\beta}}i\not{\partial}N^{c}_{R\alpha} \\ &- \sum_{i,\alpha}[(\delta_V^v f + f\delta_V^w + \delta_V^v f\delta_V^w)C_V^z]_{i\alpha}\overline{L_i}\widetilde{\phi}N_{R\alpha} - \sum_{i,\alpha}[(\delta_V^v f + f\delta_V^w + \delta_V^v f\delta_V^w)C_V^z]^*_{i\alpha}\overline{N_{R\alpha}}\widetilde{\phi}^\dagger L_i \\ = &\frac{1}{2}\sum_{\alpha}\overline{N_\alpha}i\not{\partial}N_\alpha - \frac{1}{2}\sum_{\alpha}m_{N_\alpha}\overline{N_\alpha}N_\alpha - \sum_{i,\alpha}(fC_V^z)_{i\alpha}\overline{L_i}\widetilde{\phi}\mathsf{R}N_\alpha - \sum_{i,\alpha}(fC_V^z)^*_{i\alpha}\overline{N_\alpha}\widetilde{\phi}^\dagger\mathsf{L}L_i \\ &+ \frac{1}{2}\sum_{\alpha}(\delta_N)_{\beta\alpha}\overline{N_\beta}i\not{\partial}\mathsf{R}N_\alpha + \frac{1}{2}\sum_{\alpha}(\delta_N)^*_{\beta\alpha}\overline{N_\beta}i\not{\partial}\mathsf{L}N_\alpha - \sum_{i,\alpha}[(\delta_V^v f + f\delta_V^w + \delta_V^v f\delta_V^w)C_V^z]_{i\alpha}\overline{L_i}\widetilde{\phi}\mathsf{R}N_\alpha \\ &- \sum_{i,\alpha}[(\delta_V^v f + f\delta_V^w + \delta_V^v f\delta_V^w)C_V^z]^*_{i\alpha}\overline{N_\alpha}\widetilde{\phi}^\dagger\mathsf{L}L_i, \qquad (4.200)\end{aligned}$$

Multiple flavors

- Self-energy matrix



$$\Sigma(\not{p}) = \not{p}\mathsf{R}\Sigma_R(p^2) + \not{p}\mathsf{L}\Sigma_R^{\mathsf{T}}(p^2)$$

$$(\Sigma_R)_{\beta\alpha}(p^2) := \sum_i \frac{f_{i\beta}^* f_{i\alpha}}{16\pi^2}\left[-\log\left(\frac{p^2}{\mu_{\beta\alpha}^2}\right) + i\pi\Theta(p^2)\right]$$

Non-diagonal in general

- Resummed propagator matrix

$$\Delta(\not{p}) = \mathsf{R}\Delta_{RR}(p^2) + \mathsf{R}\not{p}\Delta_{RL}(p^2) + \mathsf{L}\not{p}\Delta_{LR}(p^2) + \mathsf{L}\Delta_{LL}(p^2).$$

$$\Delta_{RR}(p^2) = \left\{[1+\Sigma_R^{\mathsf{T}}(p^2)]M_N^{-1}[1+\Sigma_R(p^2)]p^2 - M_N\right\}^{-1},$$
$$\Delta_{LR}(p^2) = M_N^{-1}[1+\Sigma_R(p^2)]\Delta_{RR}(p^2),$$
$$\Delta_{LL}(p^2) = \left\{[1+\Sigma_R(p^2)]M_N^{-1}[1+\Sigma_R^{\mathsf{T}}(p^2)]p^2 - M_N\right\}^{-1},$$
$$\Delta_{RL}(p^2) = M_N^{-1}[1+\Sigma_R^{\mathsf{T}}(p^2)]\Delta_{LL}(p^2).$$

Non-diagonal in general

- Diagonalization of the resummed propagator

$$i\mathcal{M}(L_i\phi \to L_j\phi) = \sum_{\beta,\alpha} \overline{u_{L_j}}(\mathbf{p}_{L_j})(-i\widetilde{f}_{i\beta}\mathsf{R})[i\Delta_{\beta\alpha}(\not{p})](-i\widetilde{f}_{i\alpha}^*\mathsf{L})u_{L_i}(\mathbf{p}_{L_i})$$
$$= \sum_{\alpha} \overline{u_{L_j}}(\mathbf{p}_{L_j})(-i\widehat{f}_{i\alpha}\mathsf{R})[i\widehat{\Delta}_{\alpha\alpha}(\not{p})](-i\widehat{f}_{i\alpha}^c\mathsf{L})u_{L_i}(\mathbf{p}_{L_i})$$

Resummed Yukawa couplings

Diagonalized resummed propagator

Find mixing matrices that give the resummed propagator of unstable fermion

Multiple flavors

- Diagonalized propagator matrix

$$\Delta_{RR}(p^2) = C_R(p^2)\widehat{\Delta}_{RR}(p^2)C_R^{\mathsf{T}}(p^2) = C_R(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{P_1(p^2)}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{P_2(p^2)}{p^2-P_2^2(p^2)} \end{pmatrix} C_R^{\mathsf{T}}(p^2),$$

$$\Delta_{LL}(p^2) = C_L(p^2)\widehat{\Delta}_{LL}(p^2)C_L^{\mathsf{T}}(p^2) = C_L(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{P_1(p^2)}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{P_2(p^2)}{p^2-P_2^2(p^2)} \end{pmatrix} C_L^{\mathsf{T}}(p^2),$$

$$\Delta_{LR}(p^2) = C_L(p^2)\widehat{\Delta}_{LR}(p^2)C_R^{\mathsf{T}}(p^2) = C_L(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{\not{p}}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{\not{p}}{p^2-P_2^2(p^2)} \end{pmatrix} C_R^{\mathsf{T}}(p^2),$$

$$\Delta_{RL}(p^2) = C_R(p^2)\widehat{\Delta}_{RL}(p^2)C_L^{\mathsf{T}}(p^2) = C_R(p^2)\begin{pmatrix} \frac{P_1(p^2)}{m_{N_1}}\frac{\not{p}}{p^2-P_1^2(p^2)} & 0 \\ 0 & \frac{P_2(p^2)}{m_{N_2}}\frac{\not{p}}{p^2-P_2^2(p^2)} \end{pmatrix} C_L^{\mathsf{T}}(p^2),$$

Mixing matrices

Mixing matrices are non-unitary.

$$C_R(p^2) \neq C_L^*(p^2)$$

$$i(\widehat{\Delta}_{RR})_{\beta\alpha}(p^2) = i(\widehat{\Delta}_{LL})_{\beta\alpha}(p^2) = \delta_{\beta\alpha}R^{\widehat{N}_\alpha}\frac{ip_{\widehat{N}_\alpha}}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots,$$

$$i\not{p}(\widehat{\Delta}_{LR})_{\beta\alpha}(p^2) = i\not{p}(\widehat{\Delta}_{RL})_{\beta\alpha}(p^2) = \delta_{\beta\alpha}R^{\widehat{N}_\alpha}\frac{i\not{p}}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots,$$

$$i\widehat{\Delta}_{\beta\alpha}(\not{p}) = \left[i\mathsf{R}\widehat{\Delta}_{RR}(p^2) + i\mathsf{R}\not{p}\widehat{\Delta}_{RL}(p^2) + i\mathsf{L}\not{p}\widehat{\Delta}_{LR}(p^2) + i\mathsf{L}\widehat{\Delta}_{LL}(p^2)\right]_{\beta\alpha}$$
$$= \delta_{\beta\alpha}R^{\widehat{N}_\alpha}\frac{i(\not{p}+p_{\widehat{N}_\alpha})}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots,$$

Did not Introduce any nontrivial assumptions for diagonalization!

Geometric series: loop-effects + Linear algebra

$$p^2_{\widehat{N}_\alpha} = m^2_{\widehat{N}_\alpha} - im_{\widehat{N}_\alpha}\Gamma_{\widehat{N}_\alpha}$$ Physical pole

Multiple flavors

- Effective Yukawa couplings

$$C_R^{\widehat{N}_\alpha} := C_R(p^2_{\widehat{N}_\alpha}), \qquad C_L^{\widehat{N}_\alpha} := C_L(p^2_{\widehat{N}_\alpha})$$

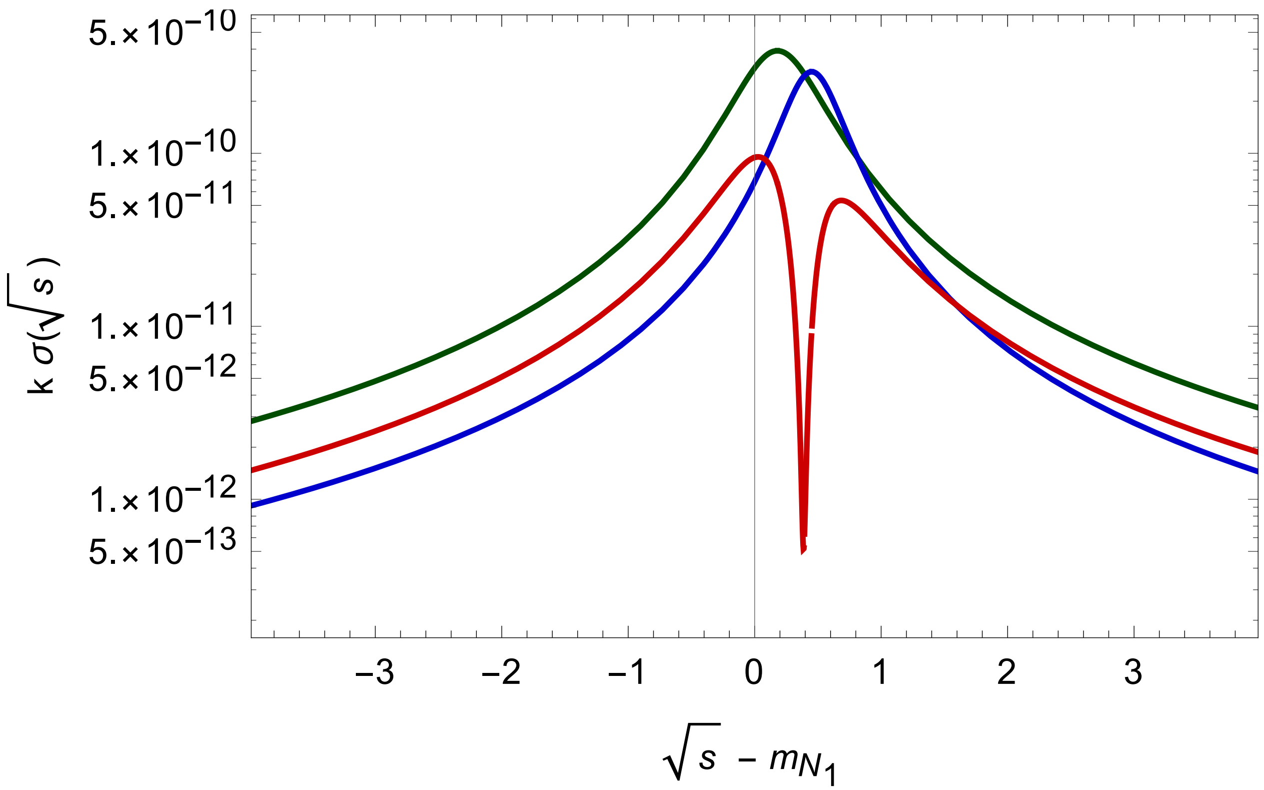
$$\widehat{f}_{i\alpha} := (D_R^{\widehat{N}_\alpha} f C_R^{\widehat{N}_\alpha})_{i\alpha}, \qquad \widehat{f}^c_{i\alpha} := (D_L^{\widehat{N}_\alpha} f^* C_L^{\widehat{N}_\alpha})_{i\alpha}.$$

Vertex-loop correction

- Physical masses and decay widths

$$p^2_{\widehat{N}_\alpha} = m^2_{\widehat{N}_\alpha} - i m_{\widehat{N}_\alpha} \Gamma_{\widehat{N}_\alpha}$$

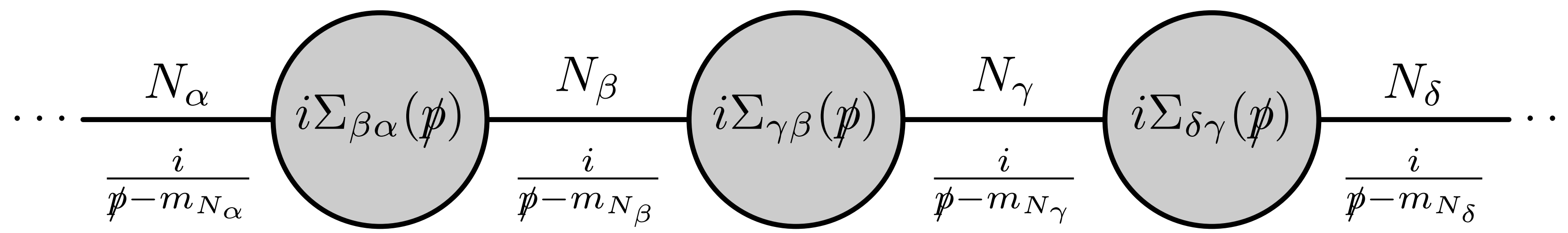
Mass and decay width of an effective particle



Two flavors of RH Majorana neutrinos with an intermediate mass difference

Non-perturbative effects

- **Non-perturbative effects** of loop-induced mixing



$\Sigma(\not{p}) \sim \mathcal{O}(f^2/16\pi^2)$

$\not{p} - m_{N_\alpha} \sim \not{p} - m_{N_\beta} \sim \mathcal{O}(f^2/16\pi^2)$

Mass differences are small (or intermediate)

Easy to add up by **resummation**, but easy to be **neglected**!

Diagonalization exact at least up to the NLO made the proper analysis of them possible!

$C_{R,L}(p^2) \neq 1 + \mathcal{O}(f^2/16\pi^2)$

Physical particles as asymptotic states **cannot** satisfy Dirac equation even up the LO!

- Physical particles should be interpreted as **quasiparticles.**

Quasiparticle

- Definition

Emergent phenomena that occur when a microscopically complicated system such as a solid behaves **as if** it contained different weakly interacting particles in free space - *Wikipedia*

- Example

A. An **electron traveling through a semiconductor** behaves like an electron with a different mass (**effective mass**) traveling unperturbed through free space - *Wikipedia*

B. Particles in the **early universe** are thought to have been in a thermal environment and have acquired **thermal masses**

Large mass difference

- Large mass difference

$$m_{N_2} - m_{N_1} \gg m_{N_1}\mathcal{O}(\Sigma), \qquad m_{N_2} - m_{N_1} \gg m_{N_2}\mathcal{O}(\Sigma).$$

- Perturbatively calculable

$$\left|\frac{4b_{12}^2}{(a_2 - a_1)^2}\right| \ll 1$$

Small expansion parameter

- Results

$$C_R(p^2) = M_N^{-\frac{1}{2}} O_R M_N^{\frac{1}{2}} = \begin{pmatrix} 1 & -\frac{m_{N_2}[m_{N_2}(\Sigma_R)_{12}+m_{N_1}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} \\ \frac{m_{N_1}[m_{N_2}(\Sigma_R)_{12}+m_{N_1}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} & 1 \end{pmatrix},$$

$$C_L(p^2) = M_N^{-\frac{1}{2}} O_L M_N^{\frac{1}{2}} = \begin{pmatrix} 1 & -\frac{m_{N_2}[m_{N_1}(\Sigma_R)_{12}+m_{N_2}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} \\ \frac{m_{N_1}[m_{N_1}(\Sigma_R)_{12}+m_{N_2}(\Sigma_R)_{21}]}{m_{N_2}^2-m_{N_1}^2} & 1 \end{pmatrix}.$$

$$i\Delta_{\alpha\alpha}(\not{p})\big|_{p^2\approx p^2_{\widehat{N}_\alpha}} = R^{\widehat{N}_\alpha}\frac{i(\not{p}+p_{\widehat{N}_\alpha})}{p^2-p^2_{\widehat{N}_\alpha}} + \cdots,$$

$$i\Delta_{11}(\not{p}) = \begin{cases} R^{\widehat{N}_1}\dfrac{i(\not{p}+p_{\widehat{N}_1})}{p^2-p^2_{\widehat{N}_1}}, & p^2 \approx p^2_{\widehat{N}_1}, \\ 0, & p^2 \approx p^2_{\widehat{N}_2}, \end{cases}$$

$$\widehat{f}_{i\alpha} = (fC_R^{\widehat{N}_\alpha})_{i\alpha} = f_{i\alpha} + f_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$

$$\widehat{f}^c_{i\alpha} = (f^*C_L^{\widehat{N}_\alpha})_{i\alpha} = f^*_{i\alpha} + f^*_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$

Identical to the decoupled case

Identical to those obtained by one-loop calculation

Large mass difference

- In the literature, some **nontrivial assumptions** were always introduced to **diagonalize** the propagator, and those assumptions are in fact valid only when the **mass difference is large**.

- As a result, they could see only the case of a large mass difference, and could **never** see the **non-perturbative effects**.

- **No** need to interpret the effective particles as **quasiparticles**.

Small mass difference

- Small mass difference $$m_{N_2}-m_{N_1}\ll m_{N_1}\mathcal{O}(\Sigma),\qquad m_{N_2}-m_{N_1}\ll m_{N_2}\mathcal{O}(\Sigma)$$

- Non-perturbative in general

- Consider an extreme case $$N_1\neq N_2,\qquad m_{\widehat{N}}\coloneqq m_{\widehat{N}_1}=m_{\widehat{N}_2},\qquad m_N\coloneqq m_{N_1}=m_{N_2},\qquad f_i\coloneqq f_{i1}=f_{i2},$$

- Results

$$C_R(p^2)=M_N^{-\frac{1}{2}}O_RM_{\widehat{N}}^{\frac{1}{2}}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\-1&1\end{pmatrix},\qquad C_L(p^2)=M_N^{-\frac{1}{2}}O_LM_{\widehat{N}}^{\frac{1}{2}}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\-1&1\end{pmatrix}.$$

$$\begin{aligned}i\Delta(\not p)&=\frac{P_1(p^2)}{m_{\widehat{N}}}\frac{i[\not p+P_1(p^2)]}{p^2-P_1^2(p^2)}\frac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}+\frac{P_2(p^2)}{m_{\widehat{N}}}\frac{i[\not p+P_2(p^2)]}{p^2-P_2^2(p^2)}\frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}\\&=\begin{cases}R^{\widehat{N}_1}\dfrac{i(\not p+p_{\widehat{N}_1})}{p^2-p^2_{\widehat{N}_1}}\dfrac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}+\cdots, & p^2\approx p^2_{\widehat{N}_1},\\ R^{\widehat{N}_2}\dfrac{i(\not p+p_{\widehat{N}_2})}{p^2-p^2_{\widehat{N}_2}}\dfrac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix}+\cdots, & p^2\approx p^2_{\widehat{N}_2}.\end{cases}\end{aligned}$$

$$i\Delta_{11}(\not p)=\begin{cases}\dfrac{1}{2}\dfrac{i(\not p+m_{\widehat{N}})}{p^2-m^2_{\widehat{N}}}, & p^2\approx m^2_{\widehat{N}},\\ \dfrac{1}{2}R^{\widehat{N}_2}\dfrac{i(\not p+p_{\widehat{N}_2})}{p^2-p^2_{\widehat{N}_2}}, & p^2\approx p^2_{\widehat{N}_2},\end{cases}$$

Equal contributions from two propagators, which clearly shows that the current analysis in the literature is wrong.

$$\begin{aligned}p_{\widehat{N}_1}&=P_1(m^2_{\widehat{N}})=m_N=m_{\widehat{N}}-i\frac{\Gamma_{\widehat{N}_1}}{2},\\ p_{\widehat{N}_2}&=P_2(m^2_{\widehat{N}})=m_N[1-2\Sigma^a_R(m^2_{\widehat{N}})]=m_{\widehat{N}}-i\frac{\Gamma_{\widehat{N}_2}}{2},\end{aligned}$$

$$\begin{aligned}\widehat{f}_{i1}&=(fC_R^{\widehat{N}_1})_{i1}=\frac{1}{\sqrt{2}}(1-1)f_i=0, & \widehat{f}_{i2}&=(fC_R^{\widehat{N}_2})_{i2}=\frac{1}{\sqrt{2}}(1+1)f_i=\sqrt{2}f_i,\\ \widehat{f}^c_{i1}&=(f^*C_L^{\widehat{N}_1})_{i1}=\frac{1}{\sqrt{2}}(1-1)f_i^*=0, & \widehat{f}^c_{i2}&=(f^*C_L^{\widehat{N}_2})_{i2}=\frac{1}{\sqrt{2}}(1+1)f_i^*=\sqrt{2}f_i^*.\end{aligned}$$

Intermediate mass difference

- Intermediate mass difference $m_{N_2} - m_{N_1} \sim m_{N_1}\mathcal{O}(\Sigma), \qquad m_{N_2} - m_{N_1} \sim m_{N_2}\mathcal{O}(\Sigma).$

- Only numerically calculable

$m_{N_1} = 1000.$

$m_{N_2} = 1000.$

$$\Sigma_R(m_{N_1}^2) = \begin{pmatrix} 0. + 3.24469\times10^{-10}\,i & 6.1477\times10^{-11} - 2.88915\times10^{-10}\,i \\ -6.1477\times10^{-11} - 2.88915\times10^{-10}\,i & 1.11725\times10^{-19} + 2.78145\times10^{-10}\,i \end{pmatrix}$$

$$\Sigma_R(m_{N_2}^2) = \begin{pmatrix} -1.30333\times10^{-19} + 3.24469\times10^{-10}\,i & 6.1477\times10^{-11} - 2.88915\times10^{-10}\,i \\ -6.1477\times10^{-11} - 2.88915\times10^{-10}\,i & 0. + 2.78145\times10^{-10}\,i \end{pmatrix}$$

$p_{N_1} = P_{11} = 1000. - 3.5515\times10^{-7}\,i$

$p_{N_2} = P_{22} = 1000. - 2.47464\times10^{-7}\,i$

$$f = \begin{pmatrix} 0.0000642625 + 0.0000466894\,i & -0.0000510455 - 0.0000370867\,i \\ 0.0000951057 + 0.0000309017\,i & -0.0000809017 - 0.0000587785\,i \end{pmatrix}$$

$$\hat{f} = \begin{pmatrix} 0.0000687768 + 0.0000965131\,i & -0.0000902131 + 0.0000193361\,i \\ 0.0000949879 + 0.00010018\,i & -0.000111624 + 0.0000163194\,i \end{pmatrix}$$

$$\hat{f}^c = \begin{pmatrix} 0.000113043 - 0.0000355864\,i & -9.48769\times10^{-6} + 0.0000917729\,i \\ 0.000176382 - 1.85878\times10^{-6}\,i & -0.0000492687 + 0.000149514\,i \end{pmatrix}$$

$$C_L(m_{N_1}^2) = \begin{pmatrix} 1.24519 - 0.130381\,i & 0.213356 + 0.760928\,i \\ -0.213356 - 0.760928\,i & 1.24519 - 0.130381\,i \end{pmatrix}$$

$$C_R(m_{N_1}^2) = \begin{pmatrix} 1.24519 - 0.130381\,i & 0.213356 + 0.760928\,i \\ -0.213356 - 0.760928\,i & 1.24519 - 0.130381\,i \end{pmatrix}$$

$$C_R^\dagger(m_{N_1}^2)\,C_R(m_{N_1}^2) = \begin{pmatrix} 2.19202 + 0.\,i & 1.05028\times10^{-10} + 1.95063\,i \\ 1.05028\times10^{-10} - 1.95063\,i & 2.19202 + 0.\,i \end{pmatrix}$$

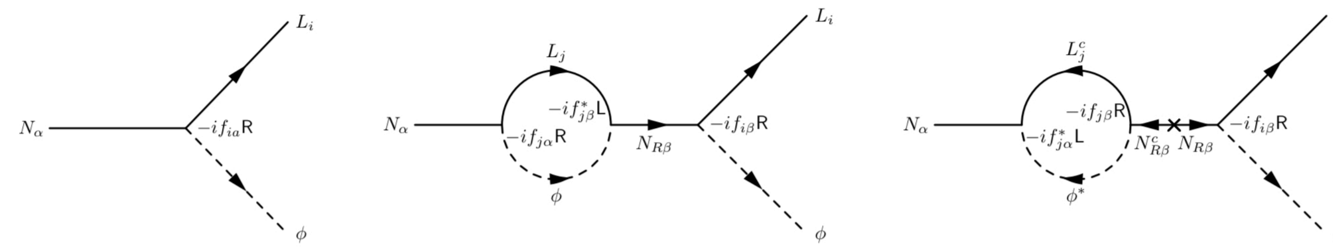
$2^{-1/2}\,\|C_R^\dagger(m_{N_1}^2)\,C_R(m_{N_1}^2)\|_F = 2.93426$

$$\|A\|_F := \sqrt{\mathrm{tr}[AA^\dagger]}$$

If A is unitary, the norm is 1.

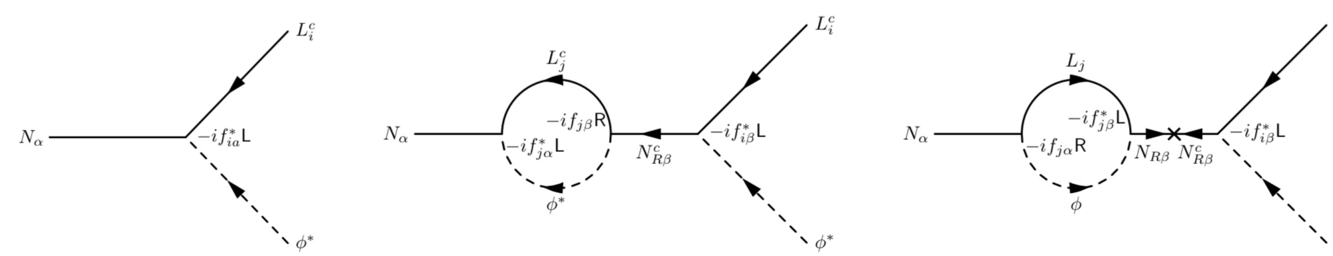
CP violation in the decay of heavy neutrinos

- CP asymmetry



(a) $-if_{i\alpha}\mathsf{R}$ (b) $-i(\delta f^{D}_{wR})_{i\alpha}\mathsf{R}$ (c) $-i(\delta f^{M}_{wR})_{i\alpha}\mathsf{R}$

Figure 5. Feynman diagrams that contribute to $N_\alpha \to L_i\phi$ through the wavefunction renormalization.



(a) $-if^{*}_{i\alpha}\mathsf{L}$ (b) $-i(\delta f^{D}_{wL})_{i\alpha}\mathsf{L}$ (c) $-i(\delta f^{M}_{wL})_{i\alpha}\mathsf{L}$

Figure 6. Feynman diagrams that contribute to $N_\alpha \to L^c_i\phi^*$ through the wavefunction renormalization.

One-loop calculation

$$\delta^i_{N_\alpha} := \frac{\Gamma(N_\alpha \to L_i\phi) - \Gamma(N_\alpha \to L^c_i\phi^*)}{\sum_{i=1}^{3}\left[\Gamma(N_\alpha \to L_i\phi) + \Gamma(N_\alpha \to L^c_i\phi^*)\right]}$$
$$= \frac{1}{8\pi(f^\dagger f)_{\alpha\alpha}} \sum_{\beta\neq\alpha} \left\{ \mathrm{Im}\left[(f^\dagger f)_{\beta\alpha} f_{i\beta} f^*_{i\alpha}\right] m_{N_\alpha} + \mathrm{Im}\left[(f^\dagger f)_{\alpha\beta} f_{i\beta} f^*_{i\alpha}\right] m_{N_\beta} \right\} \frac{m_{N_\alpha}}{m^2_{N_\alpha} - m^2_{N_\beta}},$$

$$\varepsilon_{N_\alpha} = \delta_{N_\alpha} := \sum_i \delta^i_{N_\alpha} = \frac{1}{8\pi(f^\dagger f)_{\alpha\alpha}} \sum_{\beta\neq\alpha} \mathrm{Im}\left[\left\{(f^\dagger f)_{\alpha\beta}\right\}^2\right] \frac{m_{N_\alpha} m_{N_\beta}}{m^2_{N_\alpha} - m^2_{N_\beta}}$$

Covi, Roulet, Vissani, 1996

Consistent with

$$\widehat{f}_{i\alpha} = (fC^{\widehat{N}_\alpha}_R)_{i\alpha} = f_{i\alpha} + f_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$
$$\widehat{f}^c_{i\alpha} = (f^*C^{\widehat{N}_\alpha}_L)_{i\alpha} = f^*_{i\alpha} + f^*_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$

CP violation in the decay of heavy neutrinos

- Discrepancy among the expressions of CP asymmetry

$$\delta_{N_i} \approx \varepsilon_{N_i} = \frac{\mathrm{Im}(h^{\nu\dagger}h^{\nu})^2_{ij}}{(h^{\nu\dagger}h^{\nu})_{ii}(h^{\nu\dagger}h^{\nu})_{jj}} \frac{(m^2_{N_i} - m^2_{N_j}) m_{N_i} \Gamma^{(0)}_{N_j}}{(m^2_{N_i} - m^2_{N_j})^2 + m^2_{N_i} \Gamma^{(0)2}_{N_j}},$$

Pilaftsis, Underwood, 2004

$$\varepsilon_1(\hat{M}_1^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{11}} \frac{\hat{M}_1 \hat{M}_2 (\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_2\Gamma_2 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2},$$

$$\varepsilon_2(\hat{M}_2^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{22}} \frac{\hat{M}_1 \hat{M}_2 (\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_1\Gamma_1 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2}.$$

Anisimov, Broncano, Plümacher, 2006

- Common error

$$\frac{\Sigma_2 + \Sigma_3\Sigma_4}{A + \Sigma_1} = \frac{\Sigma_2}{A + \Sigma_1}$$

Inconsistent expansion

$$\frac{\Sigma_2 + \Sigma_3\Sigma_4}{A + \Sigma_1} = \frac{\Sigma_2\left(1 + \frac{\Sigma_3\Sigma_4}{\Sigma_2}\right)}{A\left(1 + \frac{\Sigma_1}{A}\right)} = \frac{\Sigma_2}{A}\left(1 + \frac{\Sigma_3\Sigma_4}{\Sigma_2} - \frac{\Sigma_1}{A}\right)$$

Consistent expansion

CP violation in the decay of heavy neutrinos

- Pilaftsis, Underwood, 2004

$$\Delta_{11}\big|_{p^2\approx p^2_{N_1}} = \frac{Z_1}{\not p - p_{N_1}}, \qquad \Delta_{22}\big|_{p^2\approx p^2_{N_2}} = \frac{Z_2}{\not p - p_{N_2}},$$

Implicitly assumed a large mass difference

$$\Delta_{11} = \begin{cases} \frac{Z_1(\not p + p_{N_1})}{p^2 - p^2_{N_1}} + \cdots, & p^2 \approx p^2_{N_1}, \\ (D_{11})^{-1} D_{12} \frac{Z_2(\not p + p_{N_2})}{p^2 - p^2_{N_2}} D_{21}(D_{11})^{-1} + \cdots, & p^2 \approx p^2_{N_2} \end{cases}$$

$$D_{\beta\alpha}(\not p) = \delta_{\beta\alpha}(\not p - m_{N_\alpha}) - \Sigma_{\beta\alpha}(\not p)$$

$$[D_{22}(D_{12})^{-1} - D_{21}(D_{11})^{-1}]_{p^2=p^2_{N_\alpha}} = 0$$

$$(\bar h^\nu_+)_{l1} = h^\nu_{l1} + iB_{l1} - \frac{ih^\nu_{l2} m_{N_1}(m_{N_1}A_{12} + m_{N_2}A_{21})}{m^2_{N_1} - m^2_{N_2} + 2iA_{22}m^2_{N_1}},$$

$$(\bar h^\nu_+)_{l2} = h^\nu_{l2} + iB_{l2} - \frac{ih^\nu_{l1} m_{N_2}(m_{N_2}A_{21} + m_{N_1}A_{12})}{m^2_{N_2} - m^2_{N_1} + 2iA_{11}m^2_{N_2}},$$

$$(\bar h^\nu_-)_{l1} = h^{\nu *}_{l1} + iB^*_{l1} - \frac{ih^{\nu *}_{l2} m_{N_1}(m_{N_1}A^*_{12} + m_{N_2}A^*_{21})}{m^2_{N_1} - m^2_{N_2} + 2iA_{22}m^2_{N_1}},$$

$$(\bar h^\nu_-)_{l2} = h^{\nu *}_{l2} + iB^*_{l2} - \frac{ih^{\nu *}_{l1} m_{N_2}(m_{N_2}A^*_{21} + m_{N_1}A^*_{12})}{m^2_{N_2} - m^2_{N_1} + 2iA_{11}m^2_{N_2}}.$$

Correct only up to NLO

Up to NLO, consistent with

$$\widehat{f}_{i\alpha} = (f C_R^{\widehat N_\alpha})_{i\alpha} = f_{i\alpha} + f_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$

$$\widehat{f}^c_{i\alpha} = (f^* C_L^{\widehat N_\alpha})_{i\alpha} = f^*_{i\alpha} + f^*_{i\beta}\frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m^2_{N_\alpha}) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m^2_{N_\alpha})]}{m^2_{N_\beta} - m^2_{N_\alpha}},$$

CP violation in the decay of heavy neutrinos

- Anisimov, Broncano, Plümacher, 2006

future use, we introduce here an expansion parameter α related to the largest of the couplings K_{ij},

$$\alpha = \mathrm{Max}\left[\frac{K_{ij}}{16\pi^2}\right]. \tag{19}$$

In the interesting case that the masses of the right-handed neutrinos are quasi-degenerate, i.e., $\hat{M}_2 - \hat{M}_1 \ll \hat{M}_1$, one can define an additional small expansion parameter

$$\Delta \equiv \frac{\hat{M}_2 - \hat{M}_1}{\hat{M}_1}. \tag{20}$$

Our results, to be presented in the following, will only be valid as long as $\Delta \gg \alpha$, since otherwise perturbation theory breaks down.

$$\varepsilon_1(\hat{M}_1^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{11}} \frac{\hat{M}_1 \hat{M}_2(\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_2 \Gamma_2 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2},$$

$$\varepsilon_2(\hat{M}_2^2) = \frac{\mathrm{Im}(K_{12}^2)}{8\pi K_{22}} \frac{\hat{M}_1 \hat{M}_2(\hat{M}_2^2 - \hat{M}_1^2)}{(\hat{M}_2^2 - \hat{M}_1^2 - \frac{1}{\pi}\hat{M}_1 \Gamma_1 \ln(\hat{M}_2^2/\hat{M}_1^2))^2 + (\hat{M}_2\Gamma_2 - \hat{M}_1\Gamma_1)^2}.$$

$$\widehat{f}_{i\alpha} = (f C_R^{\widehat{N}_\alpha})_{i\alpha} = f_{i\alpha} + f_{i\beta} \frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m_{N_\alpha}^2) + m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m_{N_\alpha}^2)]}{m_{N_\beta}^2 - m_{N_\alpha}^2},$$

$$\widehat{f}_{i\alpha}^c = (f^* C_L^{\widehat{N}_\alpha})_{i\alpha} = f_{i\alpha}^* + f_{i\beta}^* \frac{m_{N_\alpha}[m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m_{N_\alpha}^2) + m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m_{N_\alpha}^2)]}{m_{N_\beta}^2 - m_{N_\alpha}^2},$$

→

$$\widehat{f}_{i\alpha} = f_{i\alpha} + f_{i\beta} \frac{m_{N_\alpha}[m_{N_\alpha}(\Sigma_R)_{\beta\alpha}(m_{N_\alpha}^2) + m_{N_\beta}(\Sigma_R)_{\alpha\beta}(m_{N_\alpha}^2)]}{p_{N_\alpha}^2 - p_{N_\beta}^2},$$

$$\widehat{f}_{i\alpha}^c = f_{i\alpha}^* + f_{i\beta}^* \frac{m_{N_\alpha}[m_{N_\alpha}(\Sigma_R)_{\alpha\beta}(m_{N_\alpha}^2) + m_{N_\beta}(\Sigma_R)_{\beta\alpha}(m_{N_\alpha}^2)]}{p_{N_\alpha}^2 - p_{N_\beta}^2},$$

Physical implications

- The **intermediate** mass difference allows **maximal CP violation**. Applicable to the resonant leptogenesis. The current analysis should be corrected.
- The **quasiparticles** of heavy neutrinos are **not Majorana**.
- The correlation between the CP violation and the generation of quasiparticles. (Incomplete)
- **Dirac** fields
  - Implication for the CKM matrix: mixing matrices are non-unitary.
- **Scalar** fields
  - **Meson mixing**: might resolve the discrepancy between the current analysis and experimental results for heavier mesons.

Summary

- **Loop-induced mixing** generates **quasiparticles** which are effective particles generated by RH Majorana neutrinos interacting with each other through Yukawa interaction.

- In the cases of **intermediate** mass difference, those quasiparticles **cannot** be treated perturbatively or analytically.

- The **non-perturbative effects** of loop-induced mixing can be properly treated by **carefully diagonalizing** the propagator matrix.

- Correct calculation of the **CP violation** effects is made possible.