Matching scalar couplings between general renormalisable theories

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based on `arXiv:1810.09388`
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The need for Effective Field Theories (EFTs)

- Scale of New Physics M_{NP} is driven higher by experimental searches
  $\longrightarrow$ fixed-order calculations become plagued by **large logarithmic terms** $\propto \log M_{\text{MP}}/m_{\text{EW}}$
  $\longrightarrow$ accuracy of the calculation, or even perturbativity, can be spoilt when the logarithms grow!
- The perturbative expansion must be reorganised $\rightarrow$ **EFT calculation**

Effective Field Theory calculations

- **Integrate out heavy fields** at some scale $\Lambda \sim M_{\text{NP}}$ and **work in a low-energy EFT below** Λ
- Couplings in the EFT computed by **matching** effective actions between UV theory and EFT at scale $\Lambda \longrightarrow$ **threshold corrections**
- Use **RGEs** to run the couplings from the high input scale, to the low scale $(< M_{\text{NP}})$ at which the calculation is performed

$\Rightarrow$ Matching + RGE running $\rightarrow$ **large logs are resummed!**

Scalar couplings and Effective Field Theories

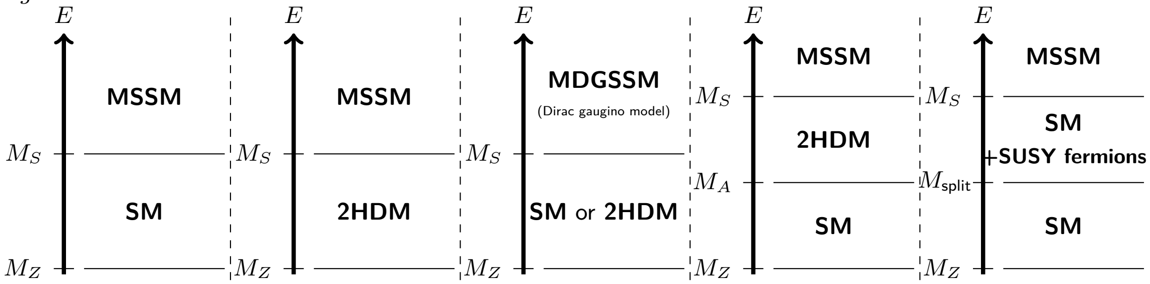
- In the context of Higgs mass calculations in SUSY models, heavy SUSY scenarios have been extensively investigated
  - → Important matching conditions: **scalar quartic couplings** needed to compute m_h in the EFT!
  - → UV theory has usually been the MSSM, and EFT is the SM

    see $e.g$ [Bernal, Djouadi, Slavich '07], [Draper, Lee, Wagner '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Pardo Vega, Villadoro '15], [Bagnaschi, Pardo Vega, Slavich '17], [Athron et al. '17], [Harlander, Klappert, Ochoa Franco, Voigt '18]

    but more and more scenarios are now being investigated!

    see $e.g$ [Benakli, Darmé, Goodsell, Slavich '13], [Bagnaschi, Giudice, Slavich, Strumia '14], [Lee, Wagner '15], [Benakli, Goodsell, Williamson '18], [Bahl, Hollik '18], etc.

$e.g.$



Matching of scalar couplings between generic theories

- Many possible scenarios → huge amount of work to compute all RGEs and matching conditions for each scenario!

  $\Rightarrow$ **Automation**

  i.e. compute RGEs and threshold corrections for general models, then apply the results to the scenario at hand.

- Two-loop RGEs are known for general QFTs, but for the thresholds, generic results have been obtained only at one-loop and mostly for the case of matching onto the SM or are difficult to implement in automated codes

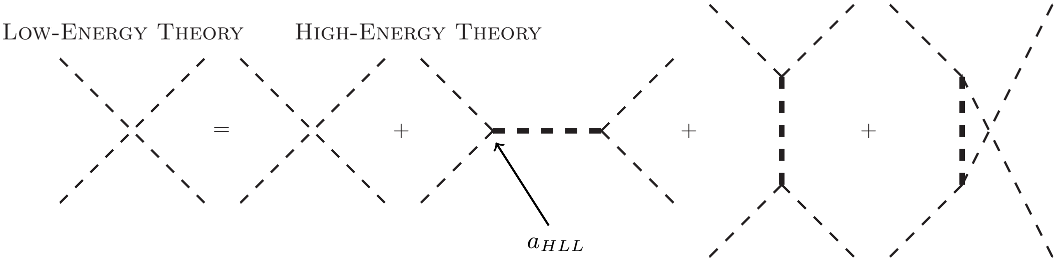
- **Our objective**: provide all necessary results to compute **threshold corrections** to scalar quartic (and Yukawa) couplings, when matching any high-energy model A onto any low-energy model B, and with the idea of going beyond one loop

→ however there are challenges to address already from **one-loop order**!

[JB, Goodsell, Slavich 1810.09388]

Matching of scalar couplings at tree-level

- Consider a general theory of scalars, fermions, and gauge bosons, with two mass scales: one light m_L and one heavy m_H
- Integrating out heavy fields ($i.e.$ of mass $\gtrsim m_H$), one finds at **tree-level**

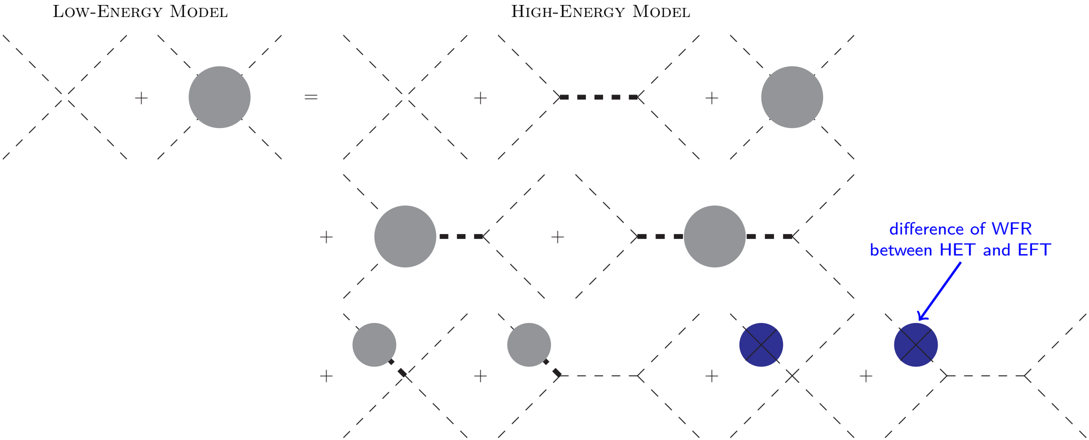


thin line: light state; *thick line*: heavy state

- Trilinear couplings between light states – a_{LLL} – receive **no threshold correction at tree-level**
- In any case, we will consider the limit $m_L \to 0$ in the following and then we must also take $a_{LLL} \to 0$

Matching of scalar couplings in a toy model at one loop

- Considering now the **one-loop** matching → many diagrams contribute!



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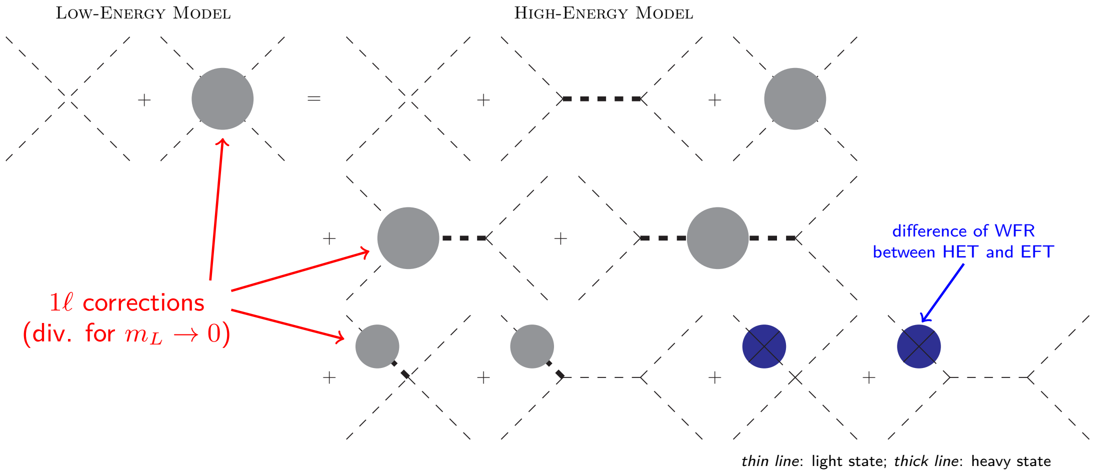
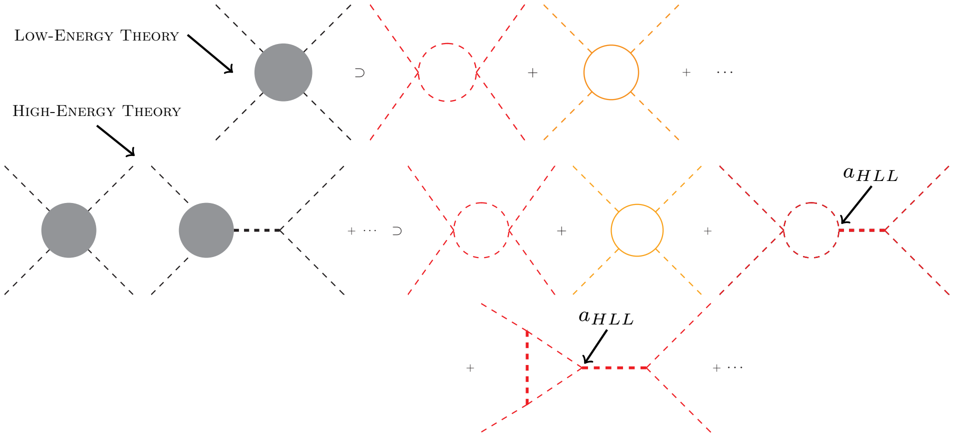


- Several diagrams are **IR divergent** in limit $m_L \to 0$, because of terms $\propto \log m_H/m_L$

Matching of scalar couplings at one loop

- IR parts in low and high energy theory must **exactly** cancel out, but because of a_{HLL}, divergent scalar diagrams are not in 1 to 1 correspondence $\rightarrow$ **automation impossible** as is!

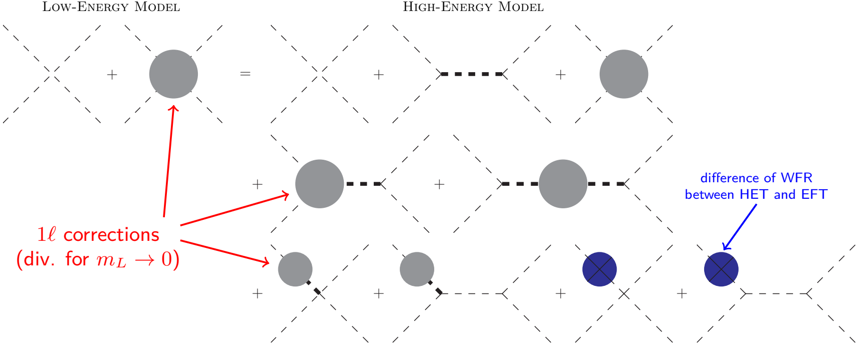


$\Rightarrow$ We have derived complete expressions for the matching of scalar couplings, at **one-loop** order, between two **generic** models*, and **eliminating the IR divergent logs**

* *however without heavy gauge bosons*

Matching quartic couplings between generic theories

The matching condition in the general case is



- ▷ Expressions can be regularised by using modified (Passarino-Veltmann) loop functions

$$B_0(0,0) \to 0, \quad C_0(0,0,X) \to -\frac{1}{X}B_0(0,X) = \frac{1}{X^2}A(X), \quad D_0(0,0,X,Y) \to -\frac{1}{X-Y}\left(\frac{1}{X^2}A(X) - \frac{1}{Y^2}A(Y)\right)$$

where $A(x) \equiv x(\log x/Q^2 - 1)$.

- ▷ In the absence of heavy gauge bosons, threshold corrections can be shown to be independent of the gauge couplings

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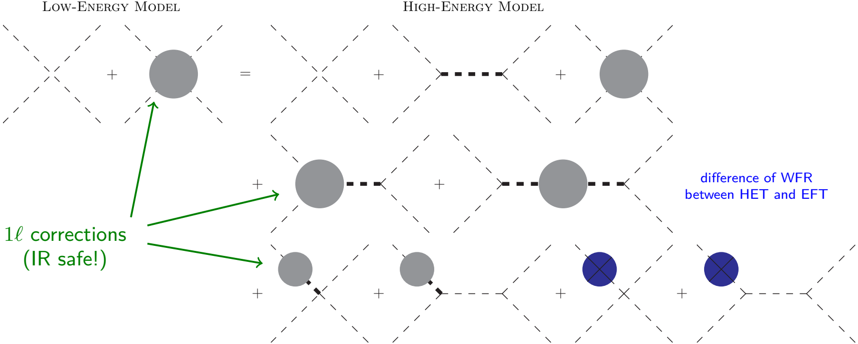

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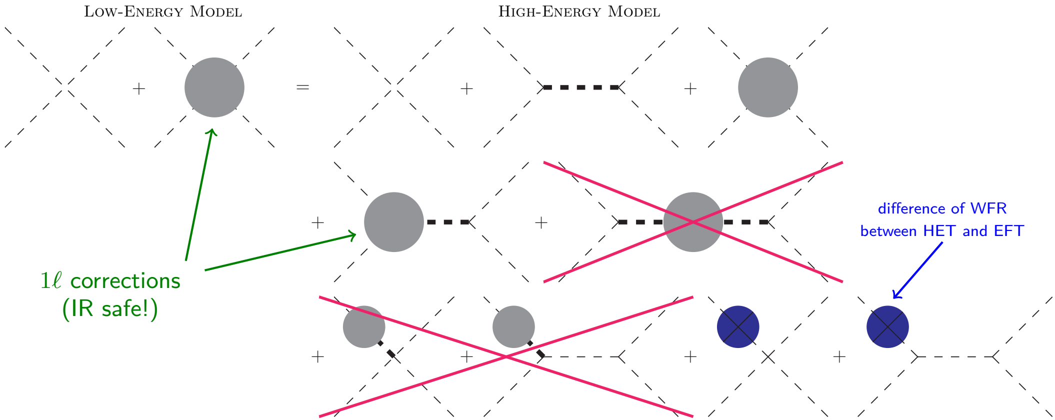

- ▷ Redefinition of (finite part of) mass counter-terms can allow eliminating δm^2_{KL} and δm^2_{iK} (*generalises a scheme devised in [Bagnaschi, Giudice, Slavich, Strumia '14] for models with 2 doublets*)

$\longrightarrow$ **mixing between heavy and light states eliminated from the matching condition!**

A simple approach to matching using two-point functions

Pole-mass matching (see $e.g.$ [Athron et al. '16])

- Extracting the threshold corrections to λ_{SM} from

$$\underbrace{2\lambda_{\text{SM}}v_{\text{SM}}^2+\Delta m_{\text{SM}}^2(p^2=m_h^2)}_{\text{Higgs pole mass in EFT (SM)}}=\underbrace{(m_{\text{HET}}^2)^{\text{tree}}+\Delta m_{\text{HET}}^2(p^2=m_h^2)}_{\text{Higgs pole mass in UV theory}}$$

$$\Rightarrow \lambda_{\text{SM}}=\frac{2}{v_{\text{HET}}^2}\left[m_{\text{HET}}^2\left(1+[\Pi_{hh}^{\text{HET}\,\prime}(0)-\Pi_{hh}^{\text{SM}\,\prime}(0)]\right)-\frac{m_{\text{HET}}^2}{m_Z^2}\left(\Pi_{ZZ}^{\text{HET}}(0)-\Pi_{ZZ}^{\text{SM}}(0)\right)+\left(\Delta m_{\text{HET}}^2(0)-\Delta m_{\text{SM}}^2(0)\right)\right]$$

$\Pi_{hh}(0)$, $\Pi_{ZZ}(0)$: Higgs and Z-boson self-energies at $p^2=0$, Δm^2: corrections to the Higgs mass

  - easier to extend beyond one-loop (as 2-point functions are easier to deal with)
  - only really tractable when EFT model does not have mixing in Higgs sector
  - as is, requires cancellation of large logs (as was our problem earlier)

- Formally equivalent to using the modified mass counterterms (*c.f. previous slide*)
- We obtain an efficient way to compute the threshold corrections to λ_{SM} as

$$\lambda_{\text{SM}}=\frac{2}{v_{\text{HET}}^2}\left[m_{\text{HET}}^2\left(1+2\underbrace{[\Pi_{hh}^{\text{HET}\,\prime}(0)-\Pi_{hh}^{\text{SM}\,\prime}(0)]}_{\text{w. light masses}\to 0}\right)+\underbrace{\hat{\Delta}m_{HET}^2(0)}_{\substack{\text{terms w. light masses only }\to 0\\ \text{logs of light masses }\to 0\\ \text{(gauge contributions }\to 0)}}\right]$$

Summary

- Use of **Effective Field Theories** becomes increasingly necessary as M_{NP} is driven higher by experimental searches

- When considering the calculation of a given observable in a wide range of scenarios or models
  $\longrightarrow$ **Automation** can provide fast and accurate predictions

- **Modified loop functions and renormalisation scheme choices** now allow simple matching of scalar quartic (and Yukawa) couplings between generic theories (similar results implemented in SARAH in [Gabelmann, Mühlleitner, Staub 1810.12326])

- **Efficient approach for pole mass matching**, that will be easier to extend beyond one-loop

- *Next*: going beyond one-loop $\longrightarrow$ use of modified scheme expected to become more important, consider pole-mass matching, ...

Thank you for your attention!

Backup

Previous results for the matching of scalar couplings between generic theories

- Two-loop RGEs known for general QFTs [Machacek, Vaughn '83,'84,'85], [Luo, Wang, Xiao '02], [Schienbein, Staub, Steudner, Svirina '18], [Sperling, Stöckinger, Voigt '13].

- General results (at one loop) exist for the matching of couplings in SMEFT studies with functional methods, but difficult to implement in automated codes
  see $e.g.$ [Henning, Lu, Murayama '14,'16], [Drozd, Ellis, Quevillon, You '15], [Ellis, Quevillon, You, Zhang '16,'17], [Fuentes-Martin, Portoles, Ruiz-Femenia '16], [Zhang '16], [Bumm, Voigt '18]

- Efforts ongoing on the matching of a generic model onto the SM at one loop, by the `FlexibleSUSY` collaboration [Athron et al. '17] and in `SARAH` [Staub, Porod '17], via *pole mass matching* $i.e.$ extracting the threshold corrections to λ_{SM} from

$$2\lambda_{\text{SM}} v_{\text{SM}}^2 + \Delta m_{\text{SM}}^2(m_h^2) = (m_{\text{HET}}^2)^{\text{tree}} + \Delta m_{\text{HET}}^2(m_h^2)$$