Flavon Stabilization in Models with Discrete Flavor Symmetry

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PLB **788** (2019) 494 [arXiv:1810.05791]

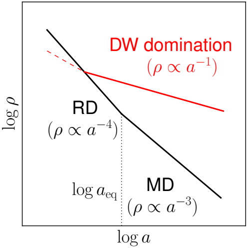
Summary of our work

We considered models with discrete flavor symmetry that explain lepton mixing angles

– Focus on SUSY A_4 model as an example

Discrete symmetry indicates existence of degenerate vacua

– Breaking of A_4 $\Leftrightarrow$ Domain wall (DW) formation that may overcloses the universe



We proposed a simple model without the domain wall problem

– A_4 symmetry is never restored after inflation

– Vacuum alignment (VA) is realized in a simple way (No need for "driving fields")

Lepton mixing matrix

In the basis m_ℓ is diagonalized,

$$U^T m_\nu U = \mathrm{diag}(m_1, m_2, m_3)$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix}$$

NuFIT 4.0 results

[1811.05487]

$$\sin^2\theta_{12} = 0.310^{+0.013}_{-0.012}$$

$$\sin^2\theta_{23} = 0.582^{+0.015}_{-0.019}$$

$$\sin^2\theta_{13} = 0.02240^{+0.00065}_{-0.00066}$$

$$\delta_{\rm CP}/^\circ = 217^{+40}_{-28}$$

Tri-bimaximal mixing (TB)

P. F. Harrison$^+$ (2002), Z. z. Xing (2002)

$$\sin^2\theta_{12} = 0.333$$

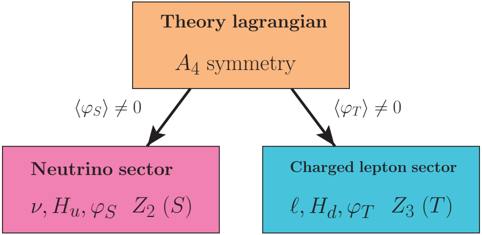
$$\sin^2\theta_{23} = 0.5$$

$$\sin^2\theta_{13} = 0$$

roughly approximate results

(SUSY) A_4 flavor model

A_4 triplet (**3**) flavons φ_S, φ_T breaks A_4 into Z_2, Z_3



Also accidental $\mu - \tau$ symmetry in the neutrino sector

$\Leftrightarrow$ TB mixing E. Ma+ (2001), K. S. Babu+ (2003), G. Altarelli+ (2005)

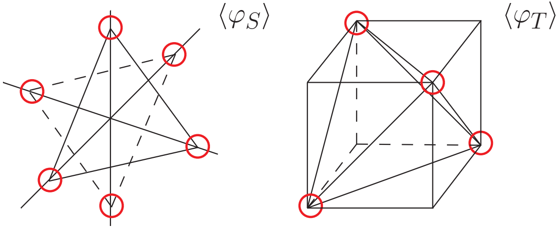
Vacuum alignment

$$\langle\varphi_S\rangle = (v_S, v_S, v_S) \quad ; \quad \langle\varphi_T\rangle = (v_T, 0, 0)$$

often realized by introducing “driving fields”

Degenerate vacua in A_4 model

There are degenerate vacua connected by A_4 transformations



This leads to the domain wall formation

Cosmology of flavor model : domain wall problem

There are several ways to avoid domain wall problem

- **Soft breaking of discrete symmetry**
  - Use of QCD anomaly — SC and K. Nakayama [1808.09601]
- **Embedding the symmetry into continuous one**

  S. F. King and Y. Zhou [1809.10292]
- **Discrete symmetry never restores (after inflation)**
  - Both $H_{\rm inf},\ T_R < m_\varphi$
  - Negative soft ~~SUSY~~ mass with non-renorm. potential

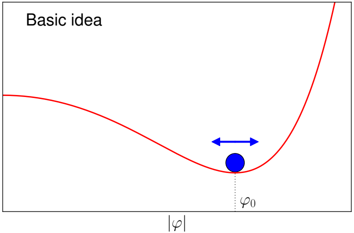
  $\Rightarrow$ This talk — SC, S. Kasuya, K. Nakayama [1810.05791]

Basic idea

Consider the flavon potential

$$V(\varphi) \sim -m^2|\varphi|^2 + \frac{|\varphi|^{2n-2}}{\Lambda^{2n-6}}$$

$$V'(\varphi_0) = 0$$



φ never crosses the origin $\Leftrightarrow$ Symmetry never restores

Questions

(a) Where negative mass term come from?

(b) Cosmological dynamics of φ?

(c) How to realize A_4 vacuum alignment?

(a)(b) : similar argument in Affleck-Dine baryogenesis M. Dine+ (1995)

in Thermal inflation D. H. Lyth+ (1995)

(a) Negative soft ~~SUSY~~ mass

Consider non-minimal contribution to Kähler potential

$$\delta K = \frac{c_1}{M_p^2} \chi^\dagger \chi \varphi^\dagger \varphi$$

δK generates mass term for scalar φ

$$\mathcal{L} \supset \frac{c_1}{M_p^2} \left\langle F_\chi^* F_\chi \right\rangle \varphi^\dagger \varphi$$

<u>(I) $H > m_{3/2}$</u>

χ is a field that dominates energy of the universe $\rho = 3H^2 M_p^2$,

$$\left\langle F_\chi^* F_\chi \right\rangle \sim \rho \;\; ; \;\; m_\varphi^2 \sim -3c_1 H^2$$

<u>(II) $H < m_{3/2}$</u>

χ is a field whose F-term breaks the SUSY

$$\langle F_\chi \rangle \sim m_{3/2} M_p \;\; ; \;\; m_\varphi^2 \sim -3c_1 m_{3/2}^2$$

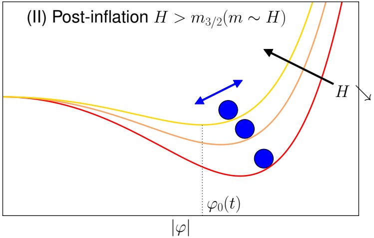
In any case, $c_1 > 0$ leads to negative contribution to m_φ^2

(b) Cosmological dynamics of flavons

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \qquad V(\varphi) \sim -m^2|\varphi|^2 + \frac{|\varphi|^{2n-2}}{\Lambda^{2n-6}}$$

Post-inflation era with $H > m_{3/2}$ $(m \sim H)$

If $n \geq 6$ (RD), $n \geq 4$ (MD), friction term decelerates φ



$\Rightarrow$ φ oscillates around the vacuum, never crosses the origin

(c) Model

	ℓ	e^c	μ^c	τ^c	H_u	H_d	φ_T	φ_S	ξ
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_{12}	ρ^5	ρ^7	ρ^7	ρ^7	1	1	1	ρ^2	ρ^2
$\mathrm{U}(1)_R$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

– $U(1)_R$ charge determines the superpotential exponent

$$W \sim \varphi^6/\Lambda^3 \quad \Leftrightarrow \quad n = 6$$

– Z_{12} charge forbids mixing between φ_T and $\{\varphi_S, \xi\}$

Vacuum alignment

Realized by linear combination of possible A_4 contractions

$$\varphi_T^6 : 11 \text{ terms}$$
$$\{\varphi_S, \xi\}^6 : 17 \text{ terms}$$

(c) Model : example

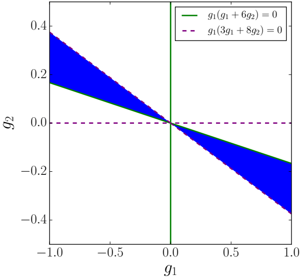
$$W_\mathrm{f} = \frac{1}{6\Lambda^3}\left[g_1(\varphi_T^2)^3 + g_2(\varphi_T^3)^2 + g_3(\varphi_S^2)^3 + g_4(\varphi_S^2)''^3 + g_5\xi^6\right]$$

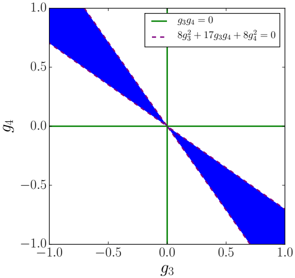
– Product rules for A_4 representations $\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{3}$

$$\mathbf{3}\otimes\mathbf{3} = \mathbf{1}\oplus\mathbf{1}'\oplus\mathbf{1}''\oplus\mathbf{3}\oplus\mathbf{3}$$

– Use $(\cdots)$, $(\cdots)''$ for products transformed as $\mathbf{1}$, $\mathbf{1}''$

Below are regions of parameter space consistent with VA





(c) Model : general arguments

Consider inclusion of all possible terms : g_i $(i = 1 \sim 28)$

A_4 invariant potential for $\varphi_T = (\varphi_{T1}, \varphi_{T2}, \varphi_{T3})$

$$V_{\rm f}^{(T)} \sim -m_T^2 \varphi_{T1}^2 + \frac{1}{\Lambda^6} \varphi_{T1}^{10} + \mathcal{O}\left(\{\varphi_{T2}, \varphi_{T3}\}^2\right)$$

$$\left.\frac{\partial V_{\rm f}^{(T)}}{\partial \varphi_{Ti}}\right|_{\varphi_{T2}=\varphi_{T3}=0} = 0 \quad (i = 2, 3)$$

Linear terms of $\{\varphi_{T2}, \varphi_{T3}\}$ are absent $\Leftrightarrow$ $(v_T, 0, 0)$ extremum

It is a minimum in the vicinity of the benchmark model

– Same follows for φ_S

For non-zero region of parameter space,
our model realizes required vacuum alignment

Conclusion

We proposed a SUSY A_4 model without DW problem

– A_4 symmetry is never restored after inflation

$$V(\varphi) \sim -m^2|\varphi|^2 + \frac{|\varphi|^{10}}{\Lambda^6}$$

– Vacuum alignment is realized in a simple way

$$W(\varphi) \sim \frac{1}{\Lambda^3} \sum_i g_i (\text{possible contractions of } \varphi^6)_i$$

$\Rightarrow$ Non-zero region of parameter space serves our purpose backup slides

Altarelli Model

	ℓ	e^c	μ^c	τ^c	$H_{u,d}$	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}'$	$\mathbf{1}''$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	ω	1	ω	ω
$\mathrm{U}(1)_R$	1	1	1	1	0	0	0	0	0	2	2	2

$$
\begin{aligned}
w_\ell = {} & \frac{y_e}{\Lambda} e^c H_d (\varphi_T \ell) + \frac{y_\mu}{\Lambda} \mu^c H_d (\varphi_T \ell)' + \frac{y_\tau}{\Lambda} \tau^c H_d (\varphi_T \ell)'' \\
& + \frac{x_a \xi + \tilde{x}_a \tilde{\xi}}{\Lambda^2} H_u H_u (\ell\ell) + \frac{x_b}{\Lambda^2} H_u H_u (\varphi_S \ell\ell) + \text{h.c.} + \cdots \\
w_d = {} & M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\
& + g_1 (\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi} (\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S) \\
& + g_4 \xi_0 \xi^2 + g_5 \xi_0 \xi \tilde{\xi} + g_6 \xi_0 \tilde{\xi}^2
\end{aligned}
$$

G. Altarelli and F. Feruglio [hep-ph/0512103]

TB from symmetries

(I) $\mu-\tau$ symmetry

$$A_{23}\equiv\begin{pmatrix}1&&\\&&1\\&1&\end{pmatrix}\quad;\quad m_\nu=A_{23}m_\nu A_{23}$$

(II) S parity ($S^2=\mathbf{1}$)

$$S\equiv\frac{1}{3}\begin{pmatrix}-1&2&2\\2&-1&2\\2&2&-1\end{pmatrix}\quad;\quad m_\nu=Sm_\nu S$$

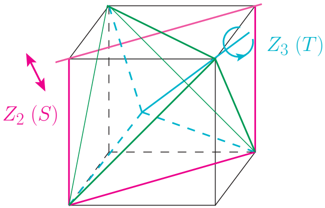
Imposing both of them, we get TB mixing matrix

$$m_\nu=\begin{pmatrix}x&y&y\\y&x+v&y-v\\y&y-v&x+v\end{pmatrix}\quad;\quad U_{\rm TB}^T m_\nu U_{\rm TB}=\text{(diagonal)}$$

(III) T symmetry ($T^3 = \mathbf{1}$)

$$T \equiv \mathrm{diag}\left(1, \omega, \omega^2\right) \quad ; \quad \omega \equiv e^{i2\pi/3} \quad (T^3 = \mathbf{1}),$$
$$m_\ell = (\text{diagonal}) \quad ; \quad m_\ell = T^\dagger m_\ell T$$

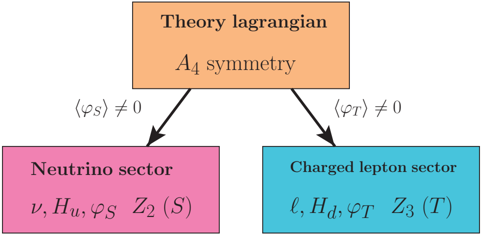
Group A_4 consists of products of S and T ($ST \neq TS$)



A_4	Z_3 (T)		
$Z_2(S) \times Z_2$	$\mathbf{1}$	T	T^2
	S	ST	ST^2
	TST^2	TS	TST
	T^2ST	T^2ST^2	T^2S

A_4 flavor model

Flavon triplets φ_S, φ_T breaks A_4 into Z_2 (S), Z_3 (T)



$\mu - \tau$ symmetry realized accidentally in the neutrino sector, and TB is realized E. Ma+ (2001), K. S. Babu+ (2003), G. Altarelli+ (2005)

Deviation from TB : $\theta_{13} > 0$, etc. can be achieved by

- Higher dimensional operators G. Altarelli+ (2006)
- additional $\mathbf{1}'$, $\mathbf{1}''$ flavons Y. Shimizu+ (2011), S. K. Kang+ (2018)

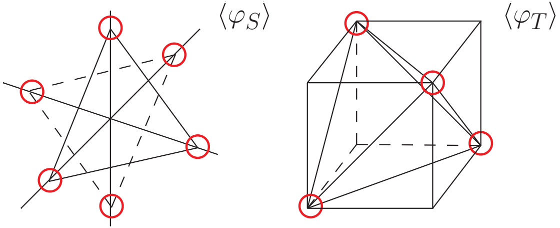
Vacuum alignment and degenerate vacua

Vacuum alignment required for the symmetry breaking pattern

$$\langle\varphi_S\rangle = (v_S, v_S, v_S) \quad ; \quad \langle\varphi_T\rangle = (v_T, 0, 0)$$

realized by introducing "driving fields" in popular models

Considering A_4 transformation, there are 12 degenerate vacua



This leads to the domain wall formation

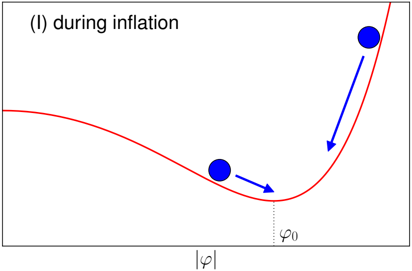
Scalar (= flavon) dynamics in our model

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \qquad V(\varphi) \sim -m^2|\varphi|^2 + \frac{|\varphi|^{2n-2}}{\Lambda^{2n-6}}$$

(I) during inflation $(m \sim H)$

friction: $H = H_I > 0 \cdots$ const

vacuum: $V'(\varphi) = 0 \Leftrightarrow |\varphi| \sim \varphi_0 \equiv \left(\frac{H_I^2 \Lambda^{2n-6}}{n-1}\right)^{\frac{1}{n-2}} \cdots$ const



$|\varphi|$ quickly settles into φ_0
$\cdots$ initial condition

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \qquad V(\varphi) \sim -m^2|\varphi|^2 + \frac{|\varphi|^{2n-2}}{\Lambda^{2n-6}}$$

(II) Post-inflation era $H > m_{3/2}$ $(m \sim H)$

friction: $H = p/t$, $p = 2/3$ (MD), $1/2$ (RD)

vacuum: $V'(\varphi) = 0 \Leftrightarrow |\varphi| \sim \varphi_0(t) \equiv \left(\frac{H^2\Lambda^{2n-6}}{n-1}\right)^{\frac{1}{n-2}}$

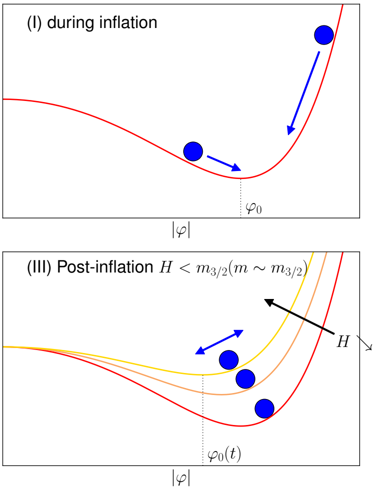
Change variables $z = \log t$, $|\varphi(t)| = \chi(t)\varphi_0(t)$

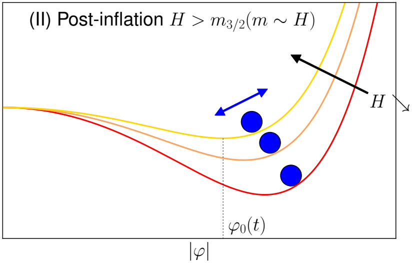
$$\ddot{\chi} + \left(3p - \frac{n}{n-2}\right)\dot{\chi} + \tilde{V}(\chi) = 0$$

If $\left(3p - \frac{n}{n-2}\right) \geq 0$, χ oscillates around $\chi \sim (1+\epsilon)^{\frac{1}{2n-4}} > 1$

(b) Illustration of the flavon dynamics

If $n \geq 4$ (MD) or $n \geq 6$ (RD)




(II) Post-inflation $H > m_{3/2} (m \sim H)$
$H \searrow$
$\varphi_0(t)$
$|\varphi|$