December 6 2018

KEK-PH 2018 winter @KEK

Phys.Lett.B785(2018) 536-542 [arXiv:1805.10793]

Tomography by neutrino pair beam

Niigata Univ., Hisashi Okui
Niigata Univ.[A], Takehiko Asaka[A]
Osaka Univ.[B], Minoru Tanaka[B]
Okayama Univ.[C], Motohiko Yoshimura[C]

NIIGATA UNIVERSITY

Thanks to the remarkable efforts of various experiments neutrino oscillations have been measured accurately.

So we consider seriously

the application of neutrino physics

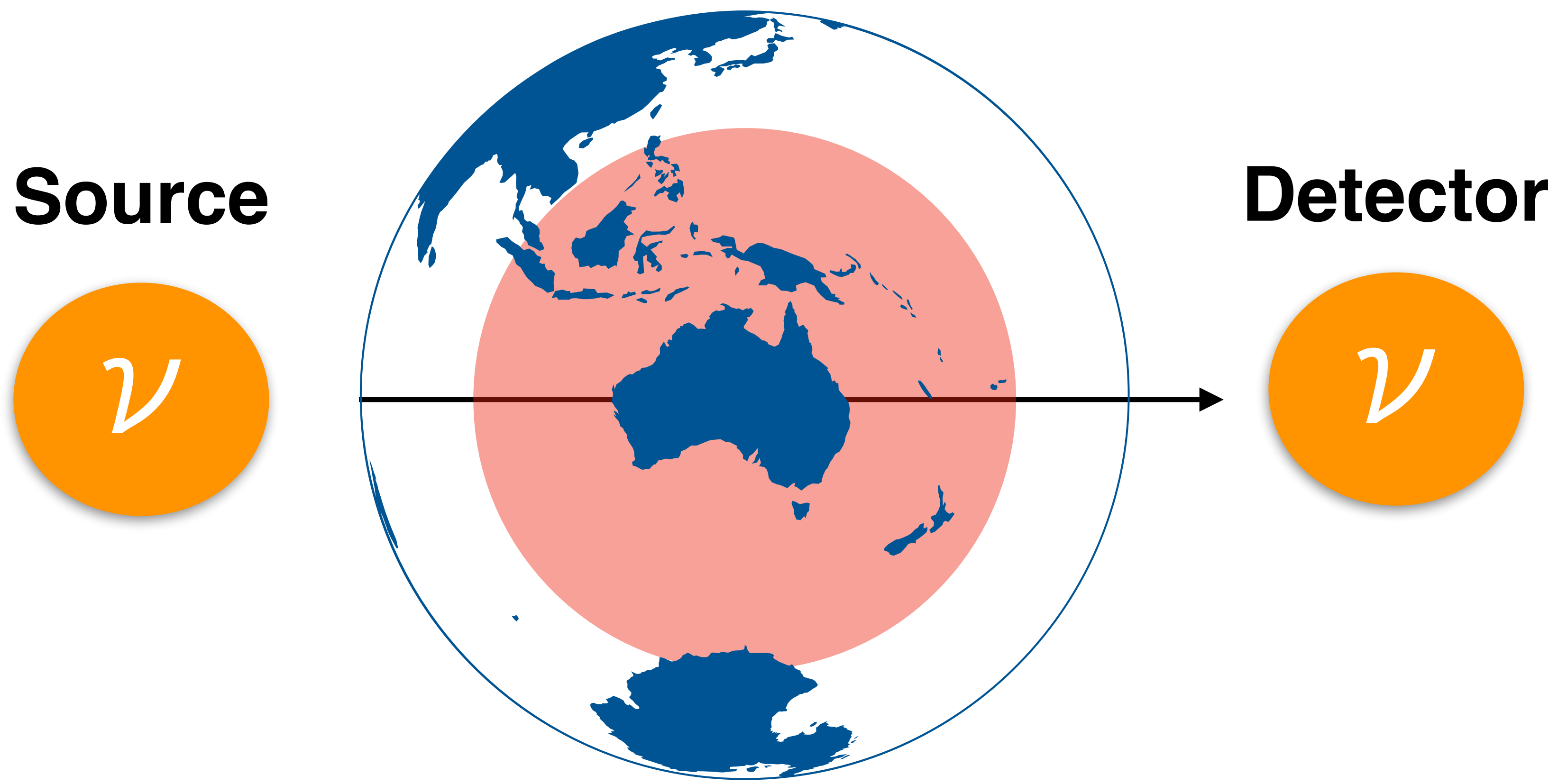
to various fields of basic science.

One of the applications of neutrino physics is

Neutrino Tomography

The idea of neutrino tomography

Imaging of the Earth's interior structure using neutrino.



Neutrino can easily transmit the Earth due to the weakness of its interaction.

Neutrino Tomography

There are three different methods.

- **Neutrino Absorption Tomography**
  - L. V. Volkova and G. T. Zatsepin, Bull. Acad. Sci. USSR, Phys. Ser. 38 (1974) 151.
    And more …
- **Neutrino Diffraction Tomography**
  - A.D. Fortes, I. G.Wood, and L. Oberauer, Astron. Geophys. 47(2006) 5.31–5.33.
  - R. Lauter, Astron. Nachr. 338 (2017) no.1, 111.
- **Neutrino Oscillation Tomography**
  - T. Ohlsson andW.Winter, Europhys. Lett. 60 (2002) 34
  - E. K. Akhmedov, M. A. Tortola and J.W. F. Valle, JHEP 0506, 053 (2005)
  - W.Winter, Nucl. Phys. B 908 (2016) 250
  - A.N. Ioannisian and A. Y. Smirnov, Phys. Rev. D 96 (2017) no.8, 083009
    And more …

There is no realistic tomography method.

$\because$

- **There is no powerful source.**
- **There is no established reconstruction method.**

We discuss the neutrino oscillation tomography precisely from now on.

Neutrino Oscillation in Matter

Neutrino oscillation in matter

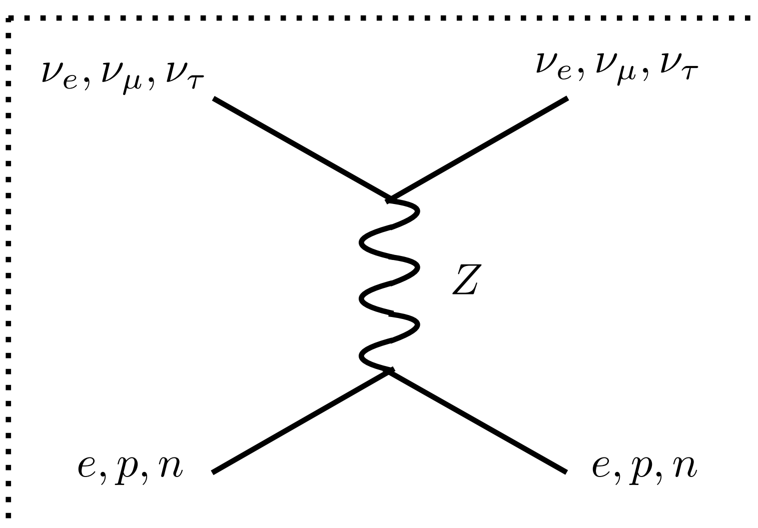
In matter, effective potential is added to the vacuum Hamiltonian.

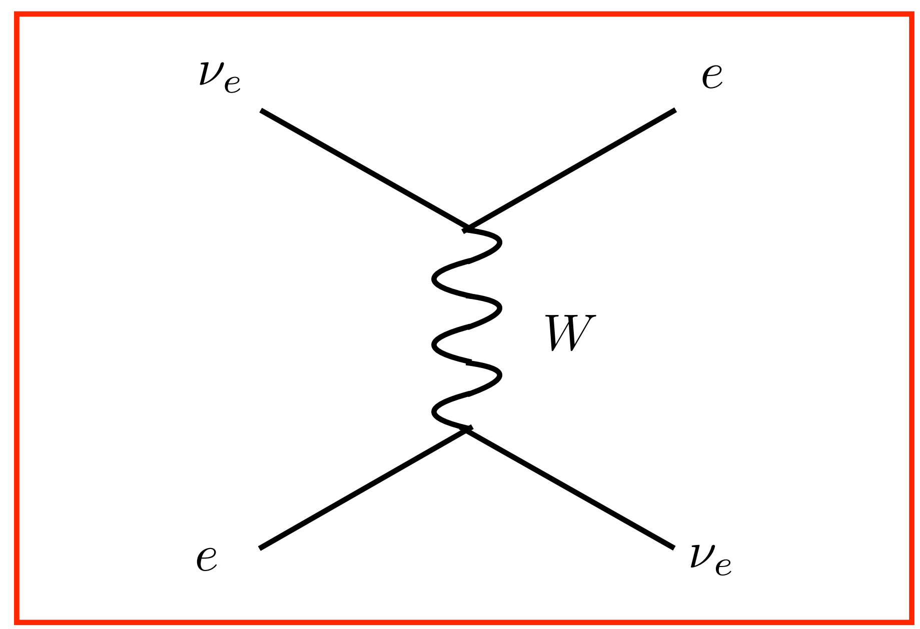
For simplicity, we consider 2 flavor neutrino oscillation.

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \nu_\alpha(x) \rangle$$

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e\to\nu_e} \\ A_{\nu_e\to\nu_\mu} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} A_{\nu_e\to\nu_e} \\ A_{\nu_e\to\nu_\mu} \end{pmatrix}$$

Vacuum contribution **Additional effective potential**





The main contribution to the potential is the CC interaction and effective potential depends on the electron number density.

Neutrino Oscillation in Matter

For simplicity, we consider the 2 flavor neutrino oscillation.

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \nu_\alpha(x) \rangle$$

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix}$$

Effective potential is written as

$$V_{CC}(x) = \sqrt{2} G_F n_e(x)$$

The electron number density is translated into the matter density.

$$n_e(x) \simeq \frac{\rho(x)}{2m_p}$$

$$\begin{aligned} \rho &= m_p n_p + m_n n_n + m_e n_e \\ &\simeq m_N (n_p + n_n) \\ &\simeq m_N 2 n_e \\ \therefore n_e &\simeq \frac{\rho}{2m_N} \end{aligned}$$

$$m_p \simeq m_n \gg m_e$$

$$n_e = n_p = n_n$$

Neutrino Oscillation in Matter

For simplicity, we consider the 2 flavor neutrino oscillation.

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta | \nu_\alpha(x) \rangle$$

$$i\frac{d}{dx}\begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix} = \left[U \begin{pmatrix} 0 & 0 \\ 0 & \frac{\Delta m^2}{2E} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC}(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} A_{\nu_e \to \nu_e} \\ A_{\nu_e \to \nu_\mu} \end{pmatrix}$$

Effective potential is written as

$$V_{CC}(x) = \sqrt{2} G_F n_e(x)$$

The electron number density is translated into the matter density.

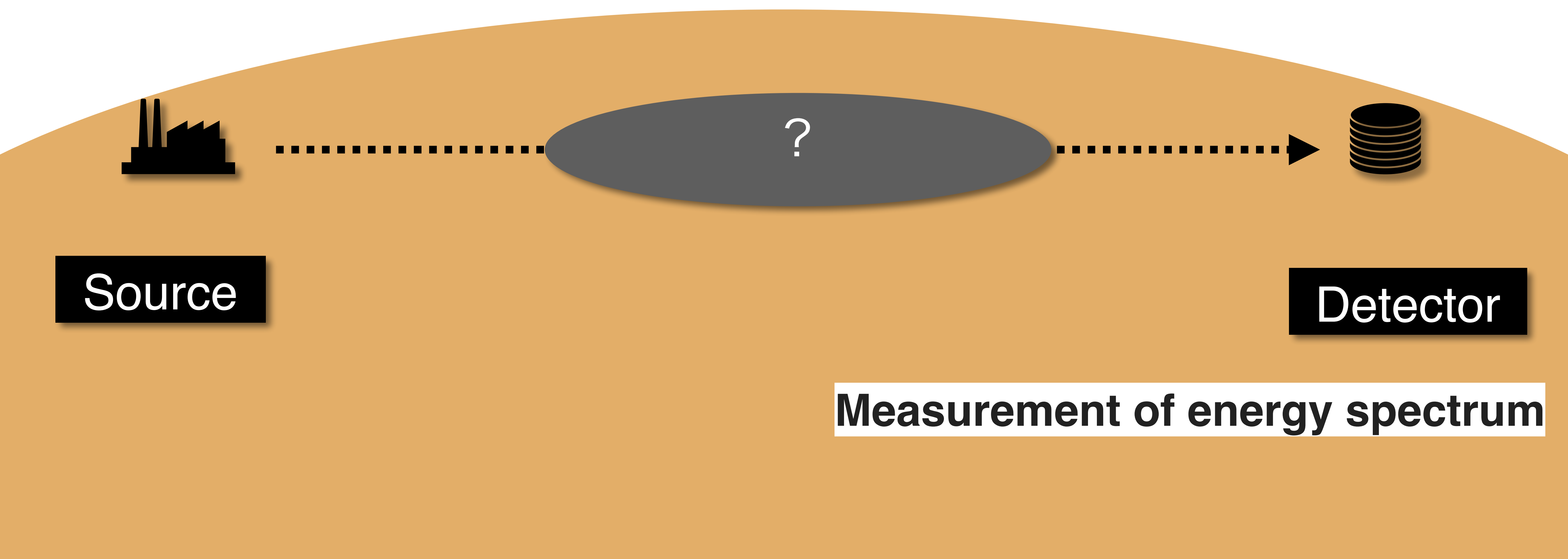
$$n_e(x) \simeq \frac{\rho(x)}{2m_p}$$

Probability is calculated as follow

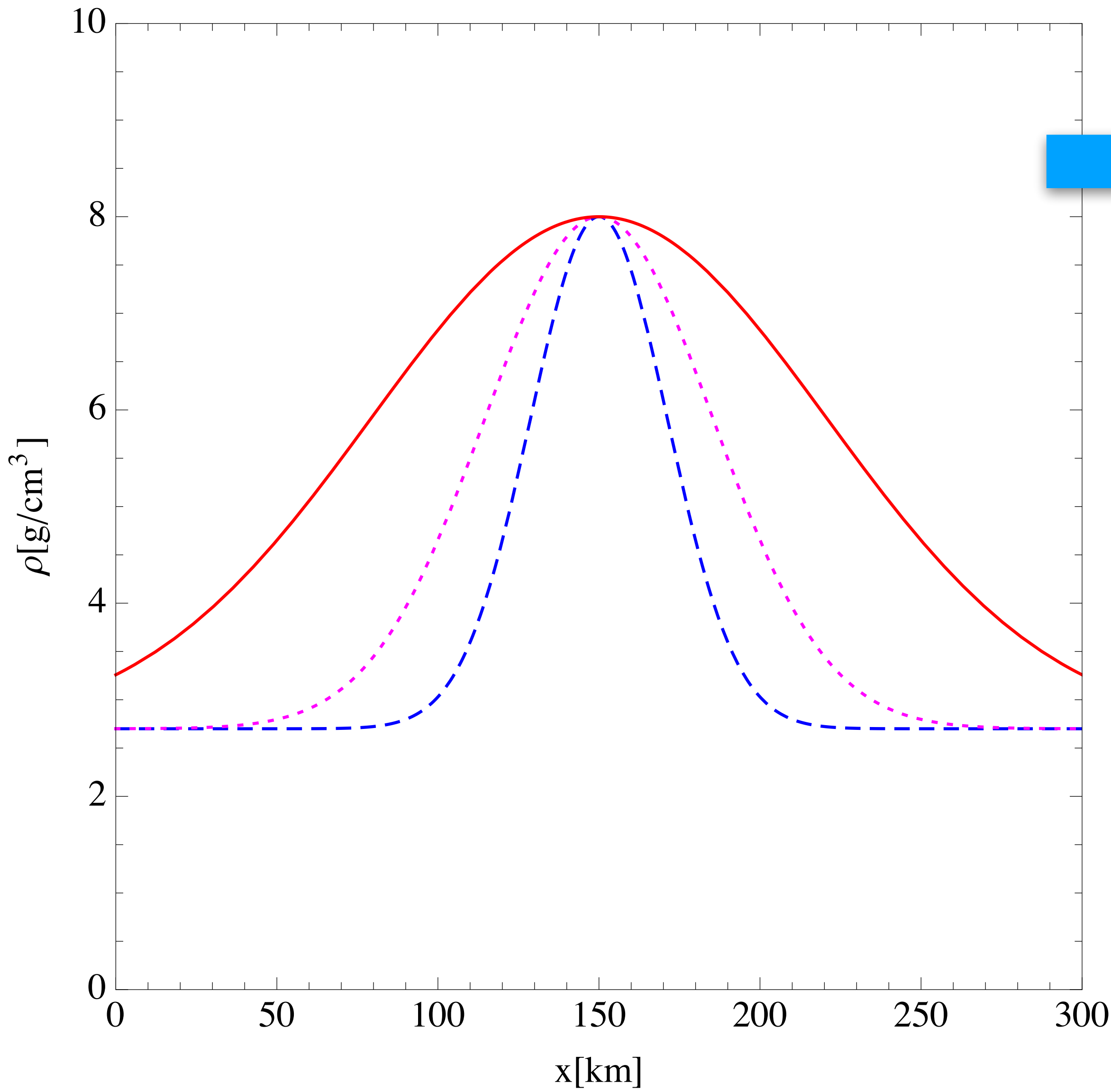
$$P_{\nu_\alpha \to \nu_\beta}(E_\nu, x) = |A_{\nu_\alpha \to \nu_\beta}(E_\nu, x)|^2$$

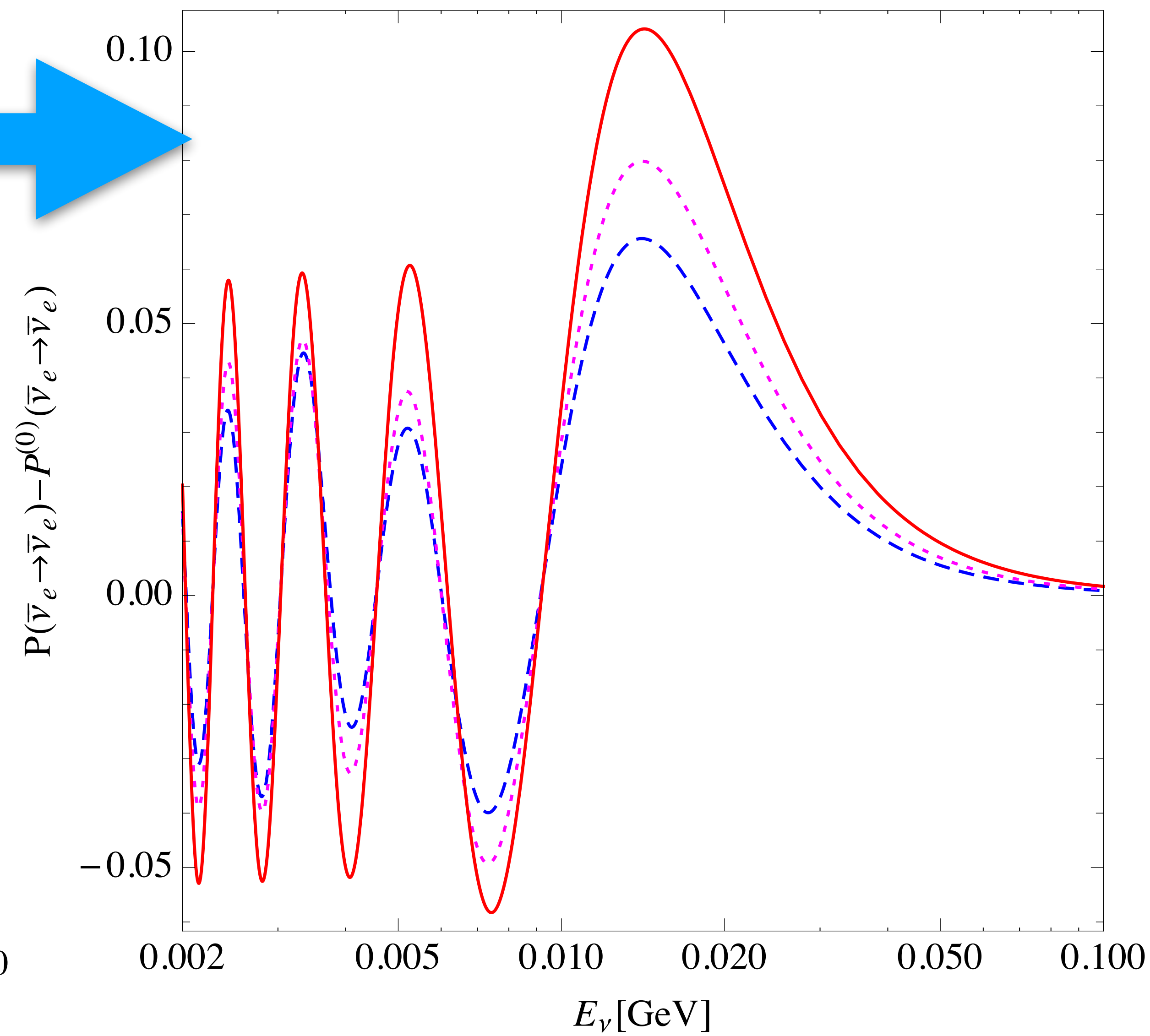
Neutrino path length

We consider such as gedanken experiment.



Influence of the density profile





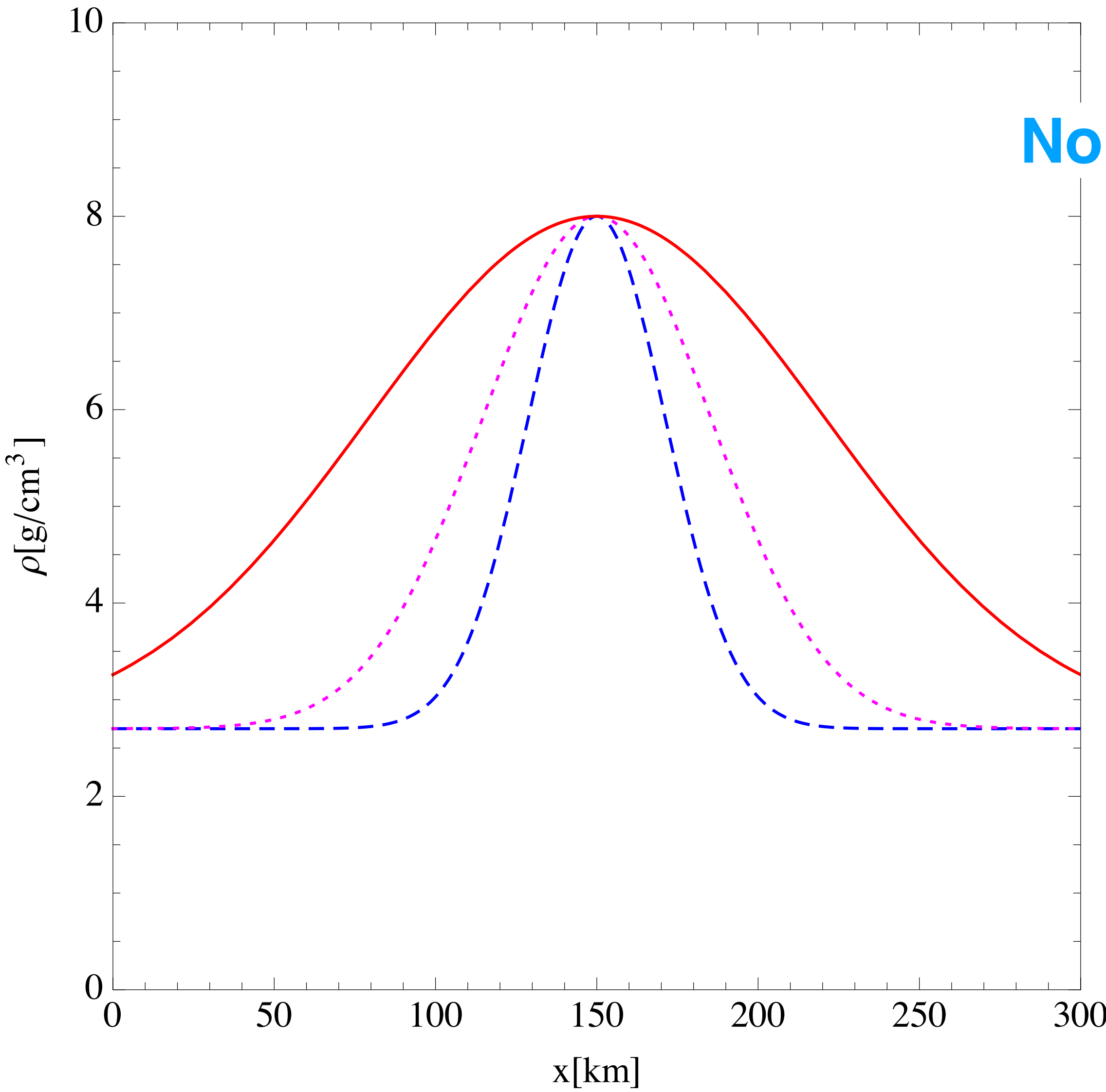
Density Profile

Oscillation Probability (subtracted the vacuum contribution)

Energy spectrum of the oscillation probability changes according to the density profile.

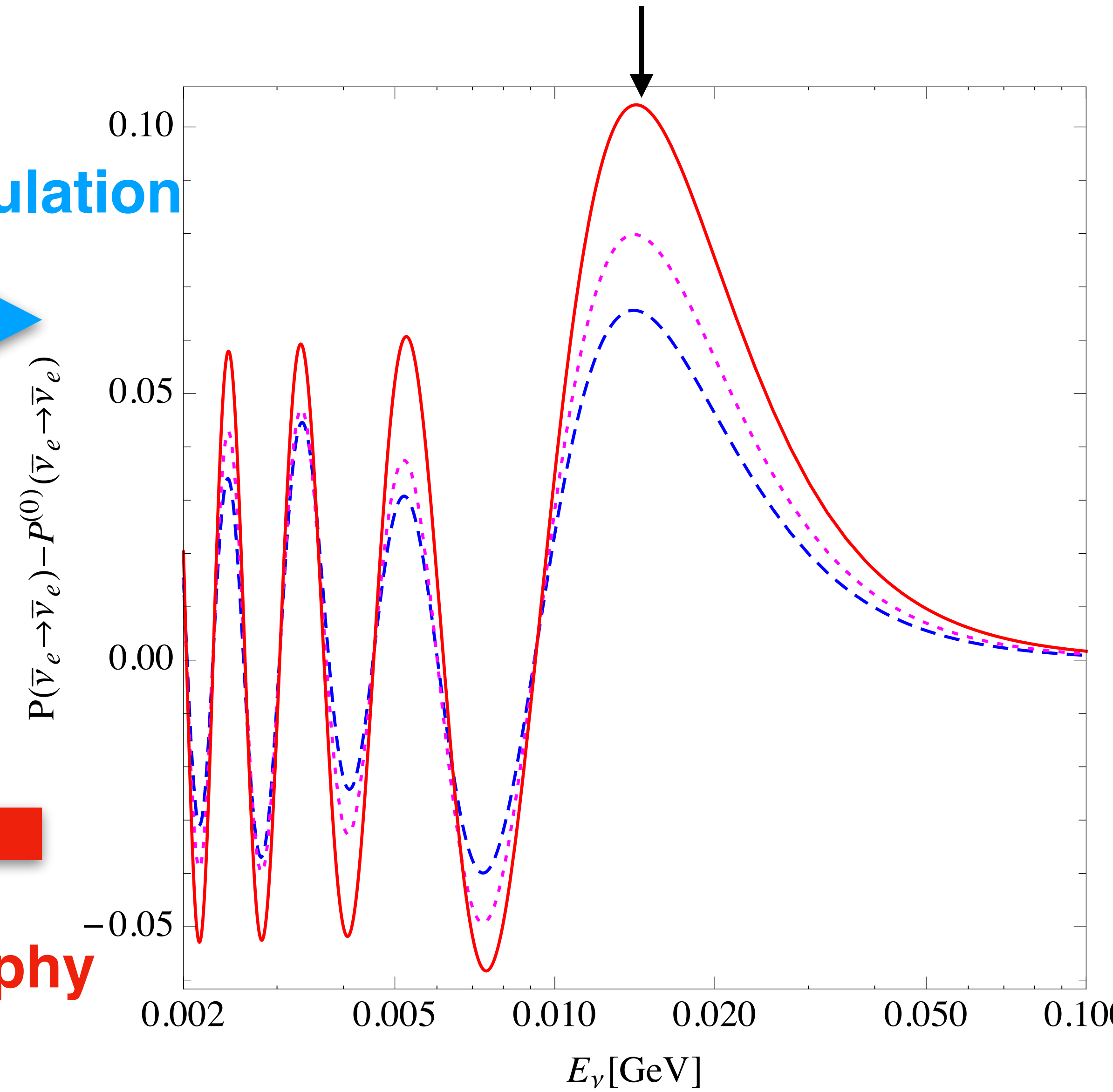
Neutrino Oscillation Tomography

Information of the density profile



Normal calculation

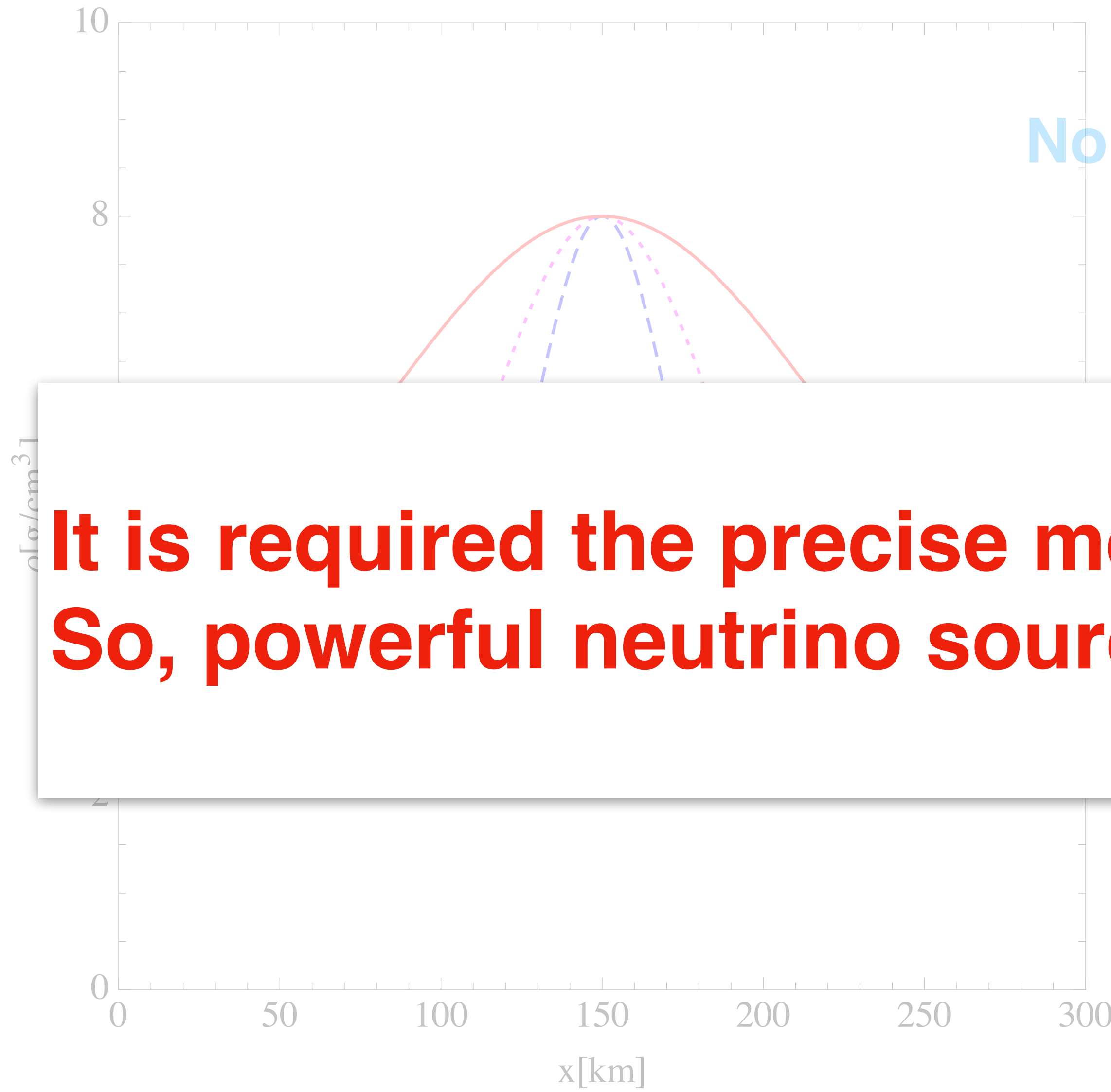
Tomography



But this effect is a few percent !

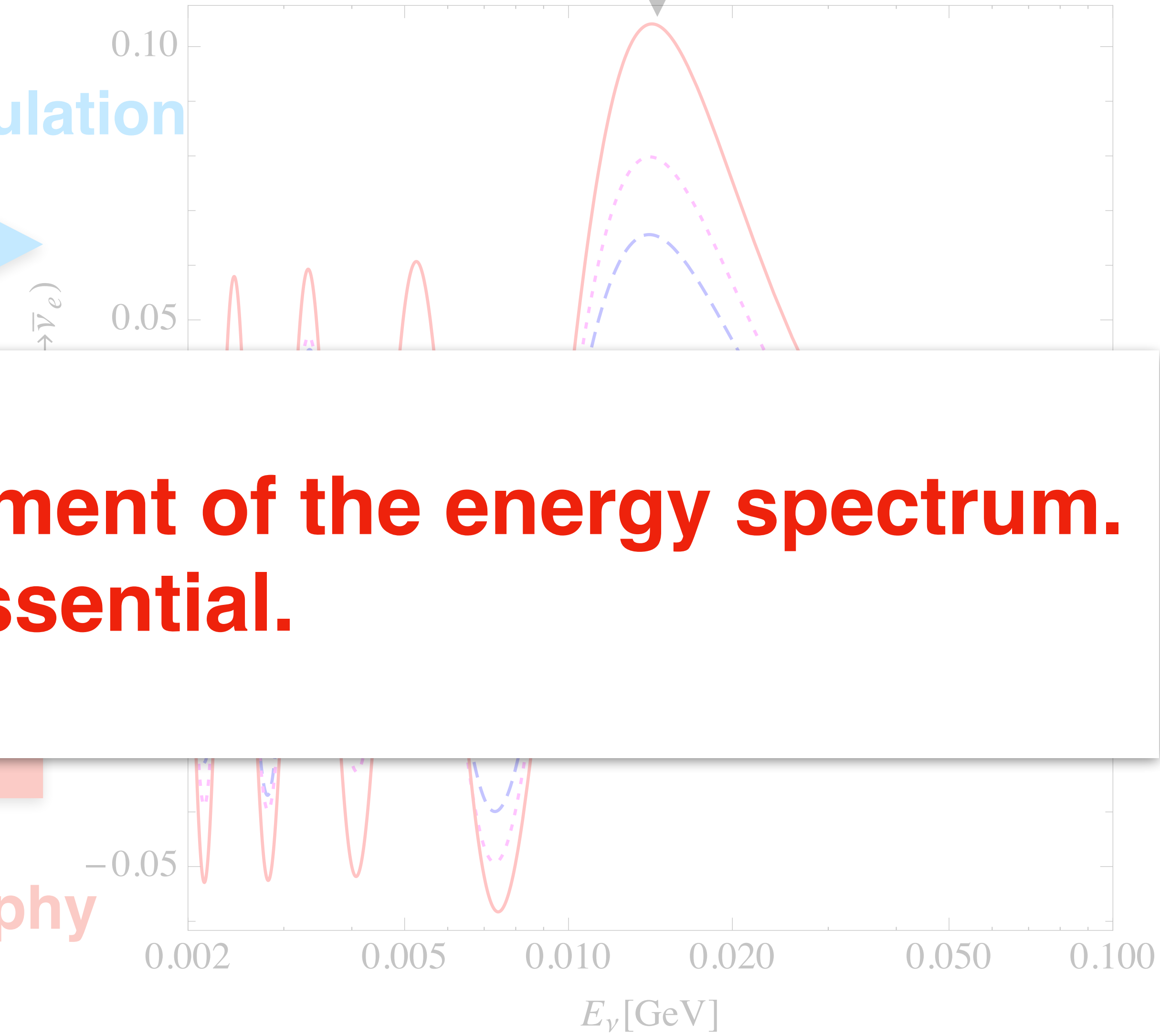
Neutrino Oscillation Tomography

Information of the density profile



Normal calculation

Tomography



It is required the precise measurement of the energy spectrum.
So, powerful neutrino source is essential.

But this effect is few percent !

Open Questions

- How do we realize accurate energy spectrum measurement ?
- How do we reconstruct the Earth's density distribution ?

Tomography by neutrino pair beam

Phys.Lett.B785 (2018) 536-542 [arXiv:1805.10793]

T Asaka, HO, M Tanaka, M Yoshimura

Open Questions

- How do we realize accurate energy spectrum measurement ?

  → Powerful source (Neutrino pair beam)

- How do we reconstruct the Earth's density distribution ?

  → Reconstruction method with 2nd order perturbation

Neutrino Source

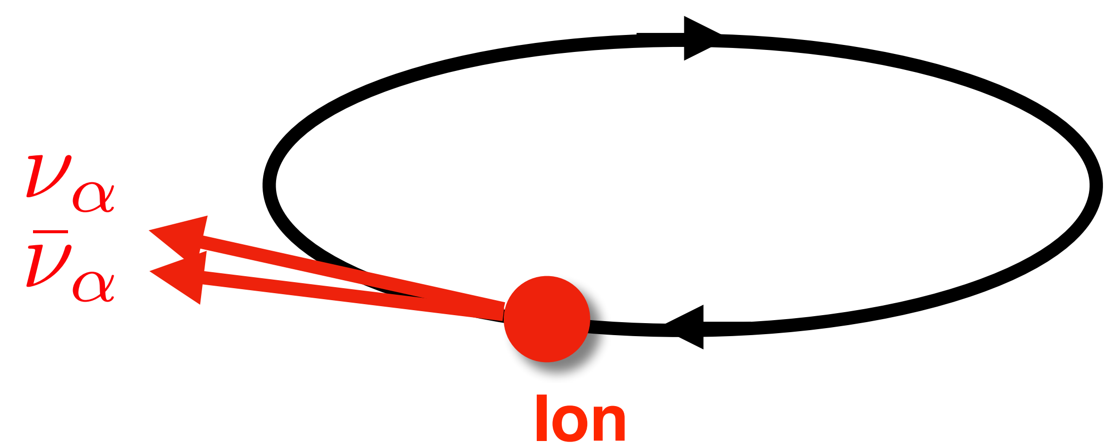
Neutrino Pair Beam

The pair beam, which has been proposed recently, can produced a large amount of neutrino pairs from the circulating partially stripped ions.

[Yoshimura, Sasao, Phys. Rev. D 92, 073015 (2015)]

Characteristics of the Neutrino Pair Beam

- **It generates the all flavor neutrino pairs**
  $$(\nu_e, \overline{\nu_e}), (\nu_\mu, \overline{\nu_\mu}), (\nu_\tau, \overline{\nu_\tau})$$
- **Very high intensity flux of neutrino beam**
- **High beam directivity**
- **Mainly electron neutrino pair being generated**



Neutrino tomography requires the precise measurement of the energy spectrum for the precise reconstruction of the density profile.

This high event rate (high flux) is essential.

Comparison of neutrino flux

Source	Energy	Flux
Atmospheric : ν_μ $(\cos\theta_Z = 0)$	3.2 GeV	$3.6 \times 10^2 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Solar	10MeV	$10^4 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
T2K at SK : ν_μ	1GeV	$2 \times 10^4 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Beta beam at 100 km : $\bar{\nu}_e$	581 MeV (average)	$2.1 \times 10^5 [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Neutrino Pair Beam at 100 km	100 MeV	$\sim 10^{10}\ [\mathrm{m}^{-2}\mathrm{s}^{-1}]$
Neutrino Pair Beam at 300 km	100 MeV	$\sim 10^{9}\ [\mathrm{m}^{-2}\mathrm{s}^{-1}]$

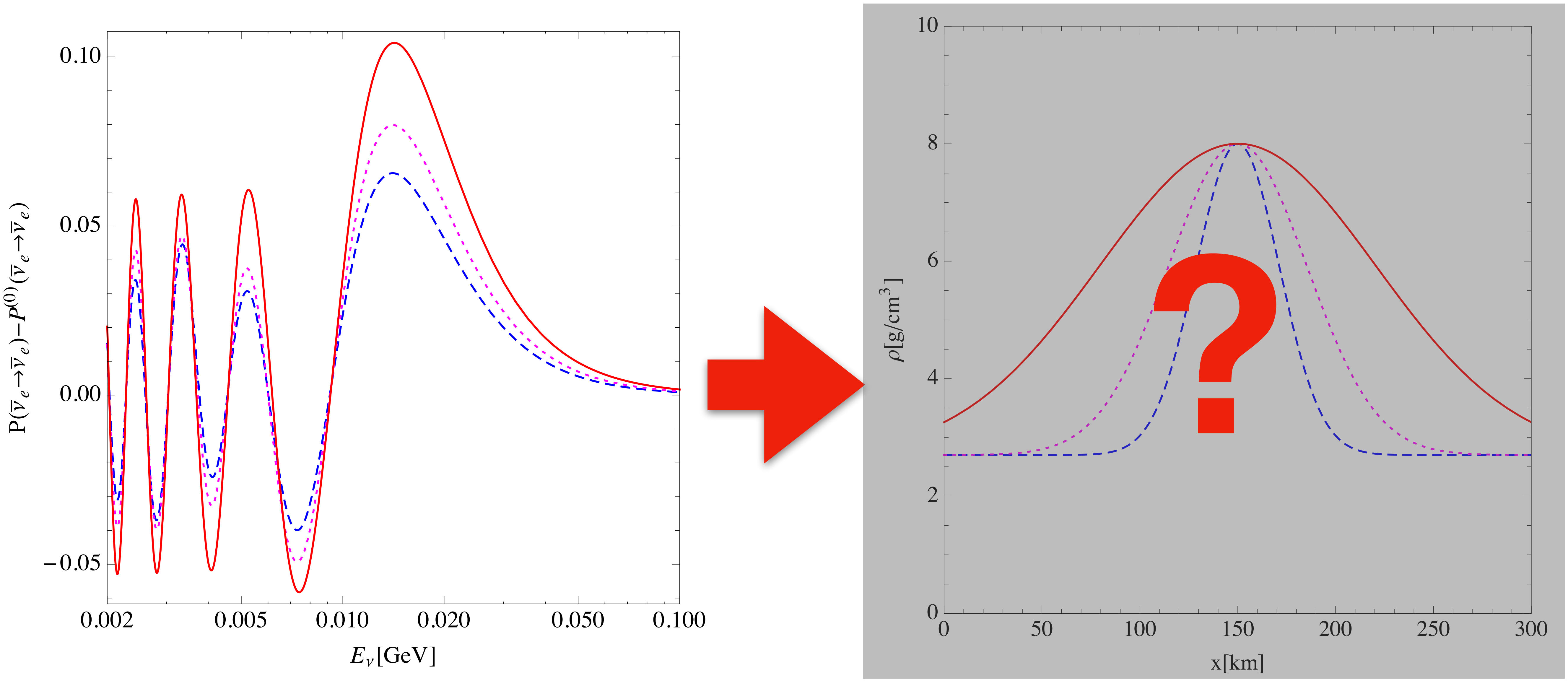
Atmospheric : M. Honda et.al., PhysRevD.92.023004
Solar : J. N. Bahcall et.al., New J. Phys. 6 (2004) 63
T2K : K. Abe et al. Phys. Rev. D 87 (2013) no.1, 012001
Beta beam : P. Zucchelli, Phys. Lett. B 532 (2002) 166
Neutrino Pair Beam : M.Yoshimura, N.Sasao, Phys.Rev.D92(2015) no.7, 073015

Production amount of neutrino (estimation)

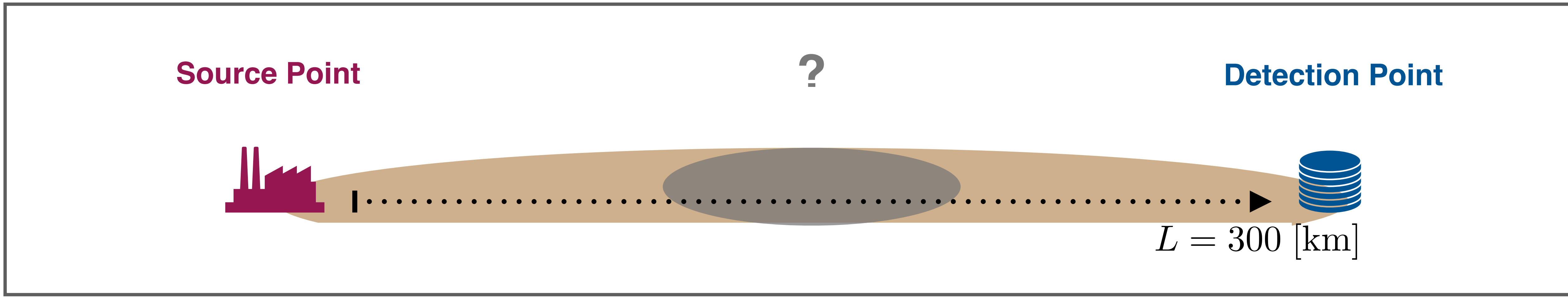
nuMAX (Neutrino Factory) : $\sim 10^{20}$ / yr
NPB : $\sim 10^{22}$ / yr

Reconstruction Method

How reconstruct the density profile from the energy spectrum of the neutrino oscillation?



Toy model



- We consider the symmetric density profile.

e.g.

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{(x - \frac{L}{2})^2}{D_l^2} \right]$$

L : length of the baseline
D_l : width of the lump

- We consider the low energy $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillation.

$$E_\nu : 2 \sim 100 \text{ [MeV]}$$

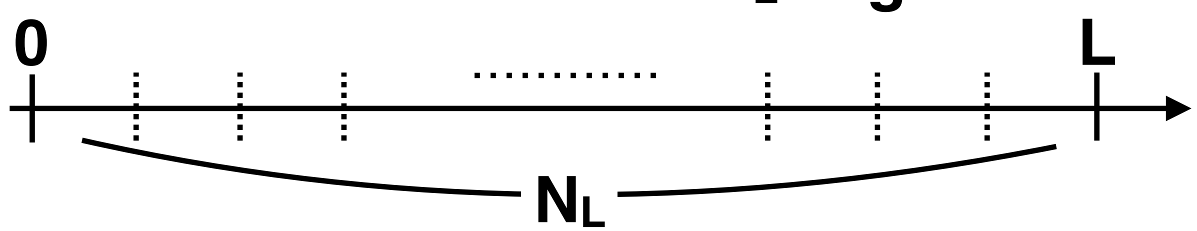
- We assume the huge liquid Argon as the neutrino detector.

Fiducial volume 10^5 m^3

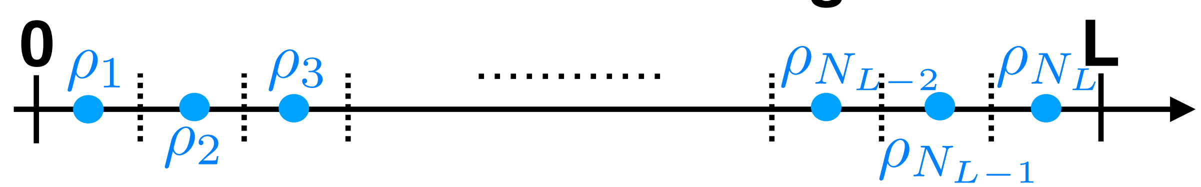
We don't discuss about systematic error.

Density Profile Reconstruction Method

1. We discretize the neutrino baseline into the N_L segments.

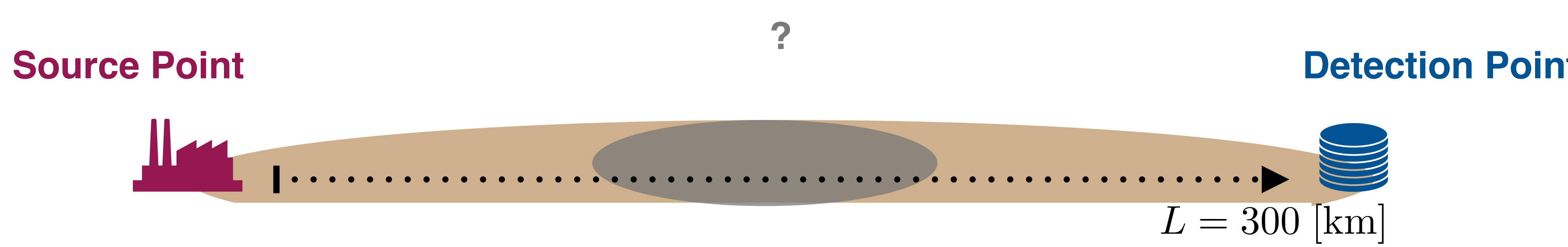


2. We consider the matter densities for these segments as free parameters ρ_j .



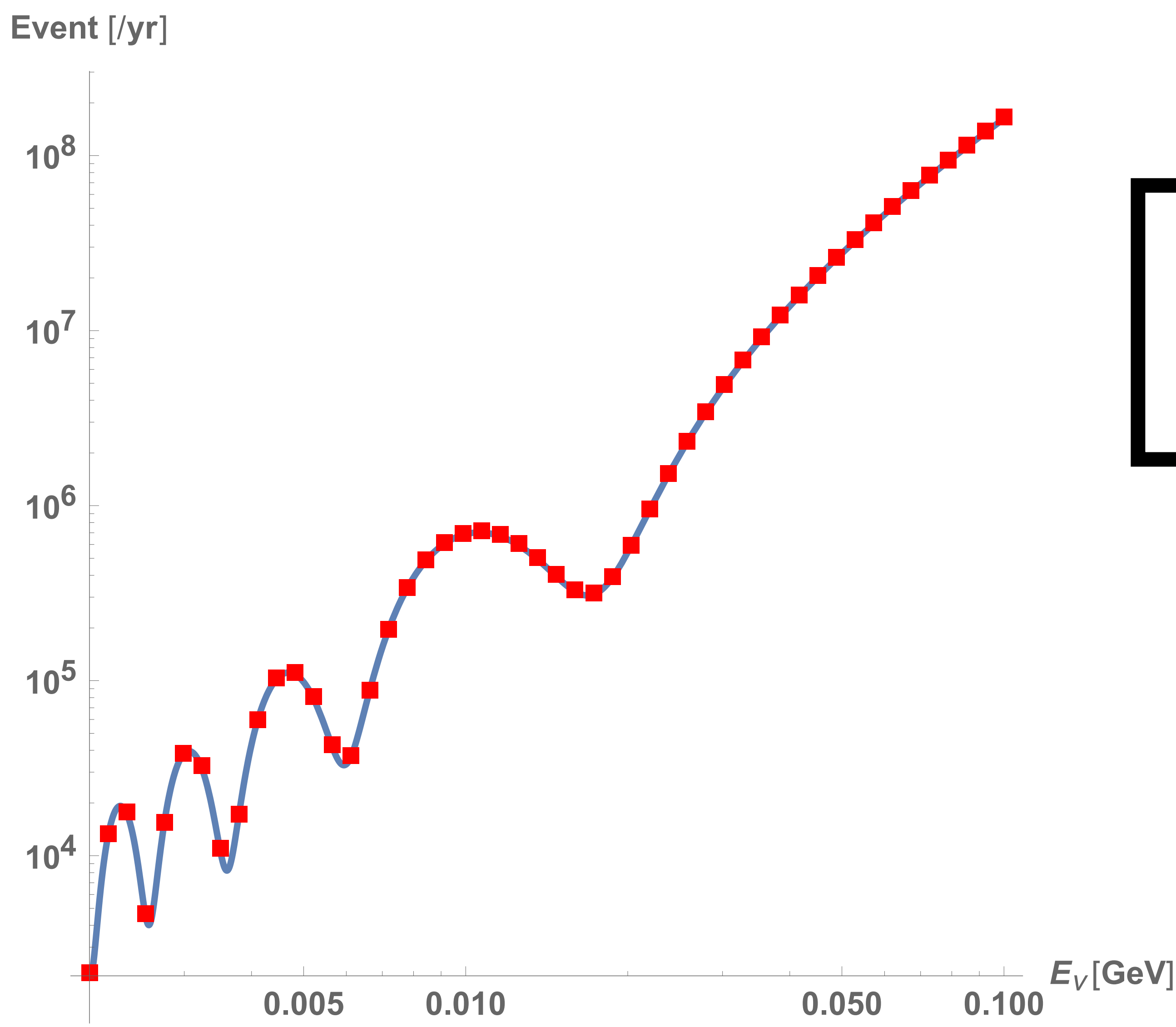
$j = 1, \cdots, N_L$

We assume that the each density is constant within each segment.



Density Profile Reconstruction Method

3. We also divide the energy range into the N_E parts, and define the χ^2 function



$$\chi^2 = \sum_{i=1,N_E} \frac{\left[N^{\rm obs}(E_i) - N^{\rm th}(E_i)\right]^2}{\sigma^2(E_i)}$$

$$\sigma(E_i) = \sqrt{N^{\rm obs}(E_i)}.$$

$$N(E_\nu) \simeq 4.73 \times 10^7 \times P(\bar{\nu}_e \to \bar{\nu}_e; E_\nu) \times \left(\frac{E_\nu}{100\ \text{MeV}}\right)^{\frac{5}{2}} \left(\frac{\Delta E_\nu}{1\ \text{MeV}}\right) \left(\frac{300\ \text{km}}{L}\right)^2 \times \left(\frac{V_d}{10^5\ \text{m}^3}\right)\left(\frac{T}{1\ \text{year}}\right).$$

4. We determine those density by minimizing the χ^2 function by comparing the observational data $N^{\rm obs}(E_i)$ for a given original profile ρ(x) with the theoretical prediction $N^{\rm th}(E_i)$ from unknown parameters ρ_j .

Perturbation Formula

$$N^{\rm th}(E_i) = \text{flux} \times P_{\bar{\nu}_e \to \bar{\nu}_e}(E, L) \times \text{detection rate}$$

Neutrino oscillation probability is calculated from the evolution equation.

$$i\frac{d}{dx}\vec{A}(x) = [H_0^F + V^F]\vec{A}(x)$$

Then we assume the relation $H_0^F > V^F$

And calculate the oscillation probability by perturbation.

$$P_{\alpha\beta} = |A_{\beta\alpha}^{(0)} + A_{\beta\alpha}^{(1)} + A_{\beta\alpha}^{(2)} + \cdots|^2$$

$$= \underbrace{|A_{\beta\alpha}^{(0)}|^2}_{\text{0th}} + \underbrace{A_{\beta\alpha}^{(0)*}A_{\beta\alpha}^{(1)} + A_{\beta\alpha}^{(0)}A_{\beta\alpha}^{(1)*}}_{\text{1st}} + \underbrace{|A_{\beta\alpha}^{(1)}|^2 + A_{\beta\alpha}^{(0)*}A_{\beta\alpha}^{(2)} + A_{\beta\alpha}^{(0)}A_{\beta\alpha}^{(2)*}}_{\text{2nd}} + \cdots$$

Ex) perturbation formula at 1st order is written as

$$P^{(1)}(E_i) \propto \sum \rho(x_j)\left[\sin\left\{\frac{\Delta m^2}{2E_i}L\right\} - \sin\left\{\frac{\Delta m^2}{2E_i}L\right\}x_j - \sin\left\{\frac{\Delta m^2}{2E_i}(L - x_j)\right\}\right]$$

Reconstruction of density profile

4. We determine these densities by minimizing the χ² function by comparing the experimental data $N^{\mathrm{obs}}(E_i)$ for a given original profile ρ(x) with the theoretical prediction $N^{\mathrm{th}}(E_i)$ from unknown parameters ρ_j.

$$\chi^2 = \sum_{i=1,N_E} \frac{\left[N^{\mathrm{obs}}(E_i) - N^{\mathrm{th}}(E_i)\right]^2}{\sigma^2(E_i)}$$

$$N^{\mathrm{th}}(E_i) = \mathrm{flux} \times \boxed{P_{\bar{\nu}_e \to \bar{\nu}_e}(E, L)} \times \mathrm{detection\ rate}$$

Neutrino Oscillation Probability of the perturbation formulae

We find the 2nd order perturbation is important for the successful reconstruction.

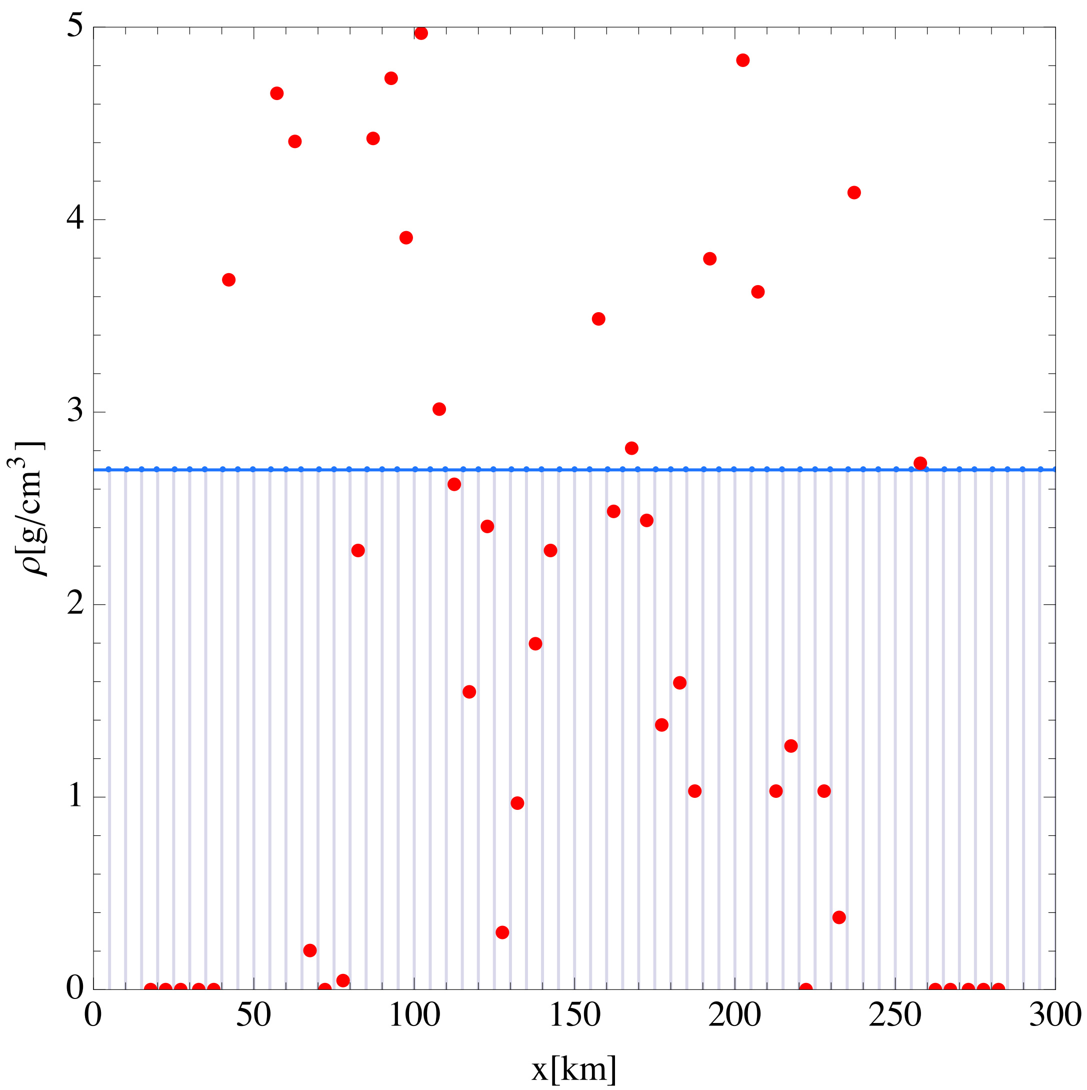
Results of Reconstruction

Result of the flat density

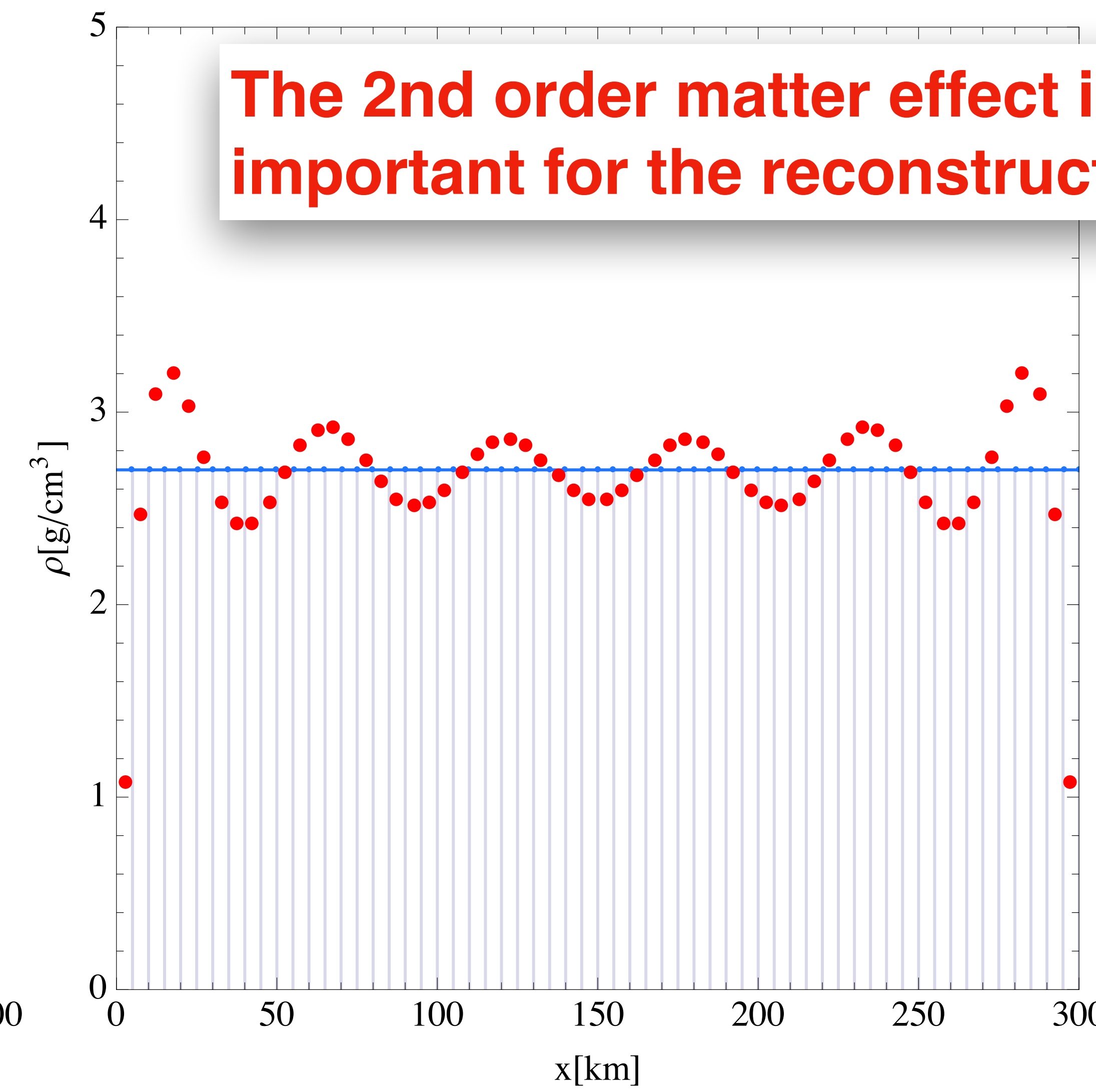
Reconstruction of 60 segment's densities with 100 energy bins

$$\bar{\rho} = 2.7\ [\mathrm{g/cm^3}]$$

Result with using the 1st order formula



Result with using the 2nd order formula



- : Original density profile

• : Reconstructed density profile

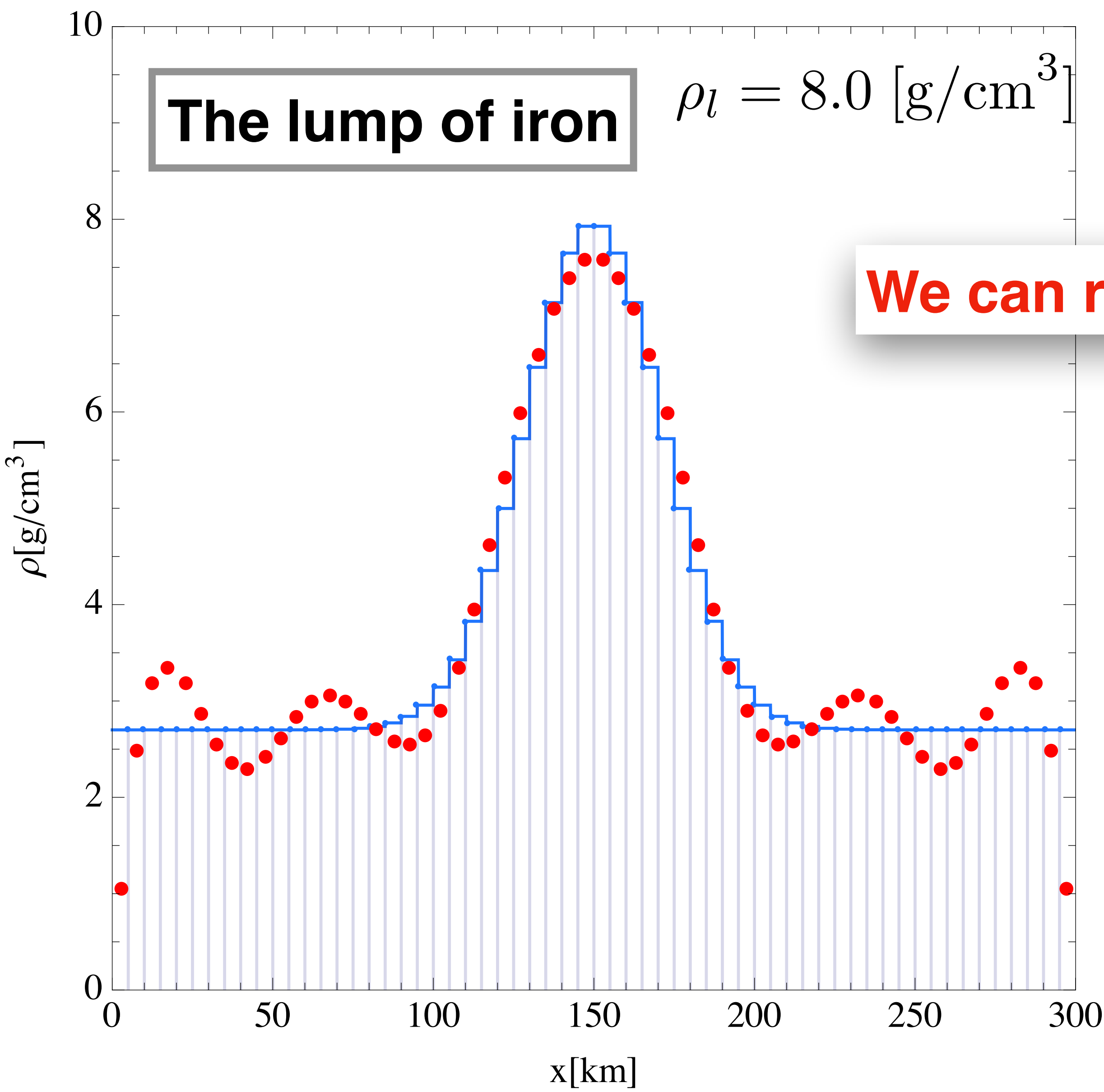
Result with using the 2nd order formula

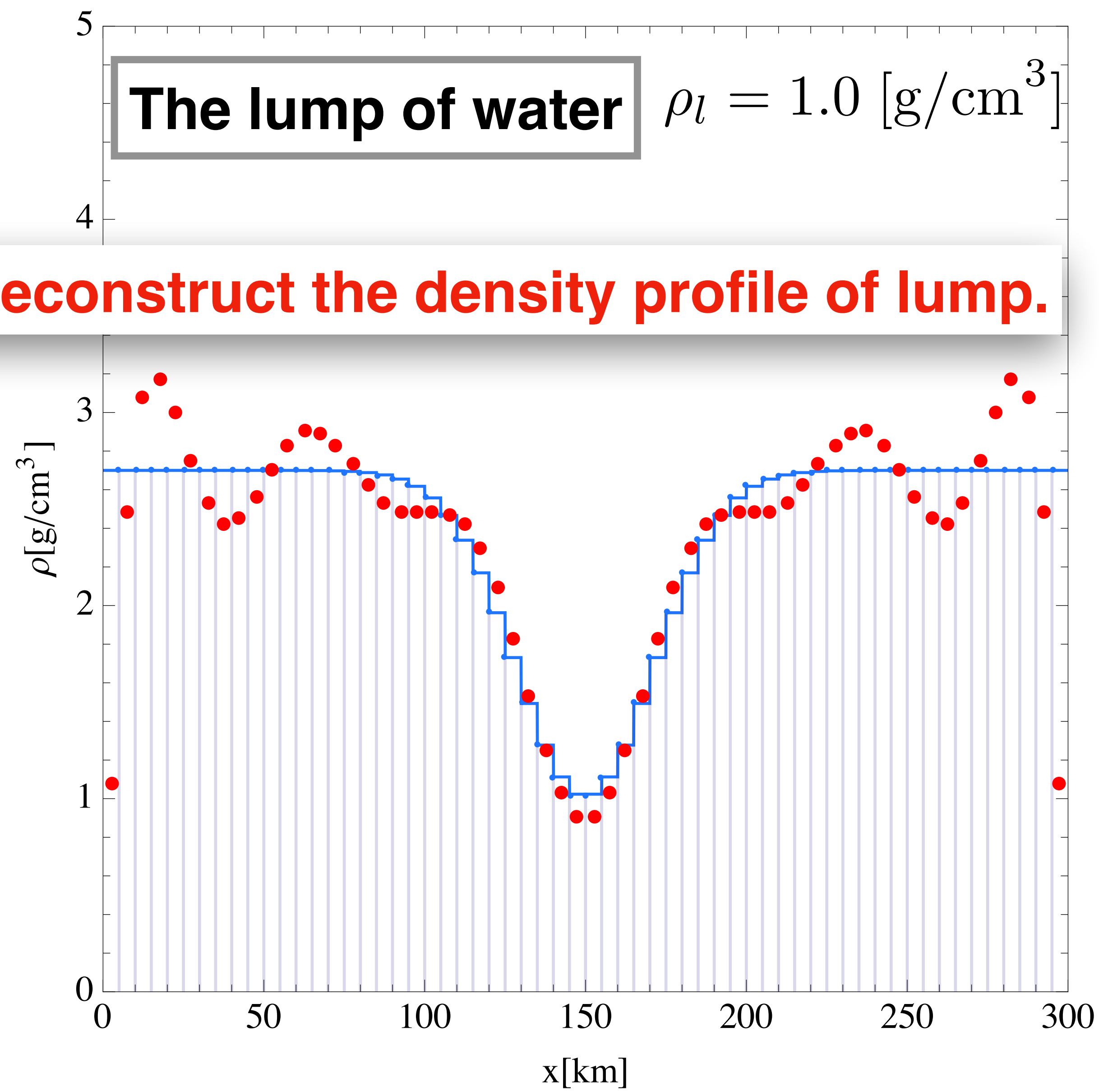
Reconstruction of 60 segment's densities

with 100 energy bins

Original density profile $\bar{\rho} = 2.7$ [g/cm^3]

$$\rho(x) = \bar{\rho} + (\rho_l - \bar{\rho}) \exp\left[-\frac{(x-\frac{L}{2})^2}{D_l^2}\right]$$





We can reconstruct the density profile of lump.

- : Original density profile

• : Reconstructed density profile

Result of the exotic density profile

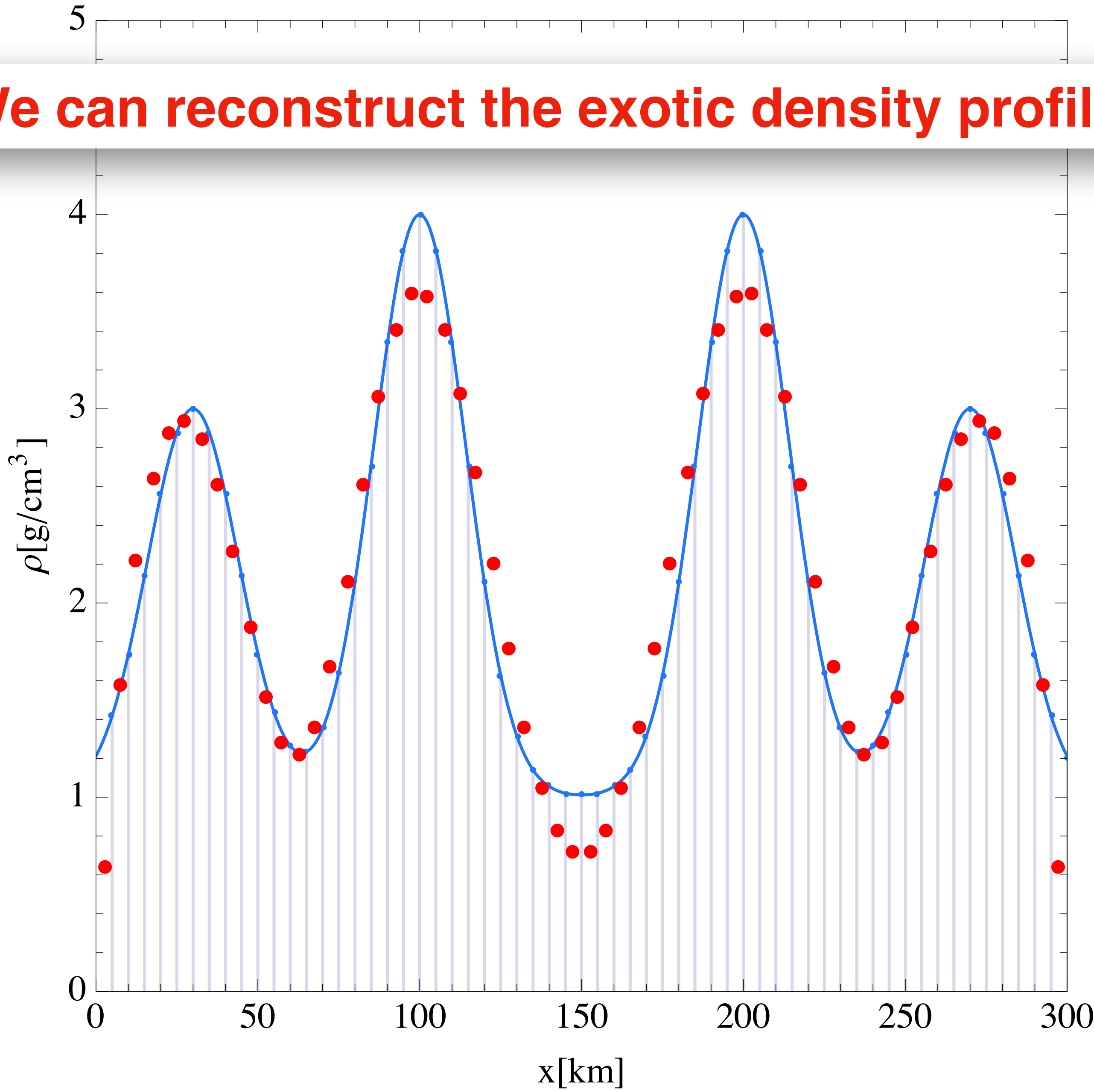
Reconstruction of 60 segment's densities

by 100 energy bin data

using the 2nd order formula

4 lump

We can reconstruct the exotic density profile.



- : Original density profile

• : Reconstructed density profile

Conclusions

Conclusions

We have investigated the neutrino oscillation tomography by the neutrino pair beam.

In this talk

- The neutrino pair beam is a powerful source to the probe of the Earth's interior, especially the structures inside the crust.
- The reconstruction method with the 2nd order perturbation formula is successful tool.
- We believe that these two ingredients give considerable progress toward the realization of the neutrino tomography.