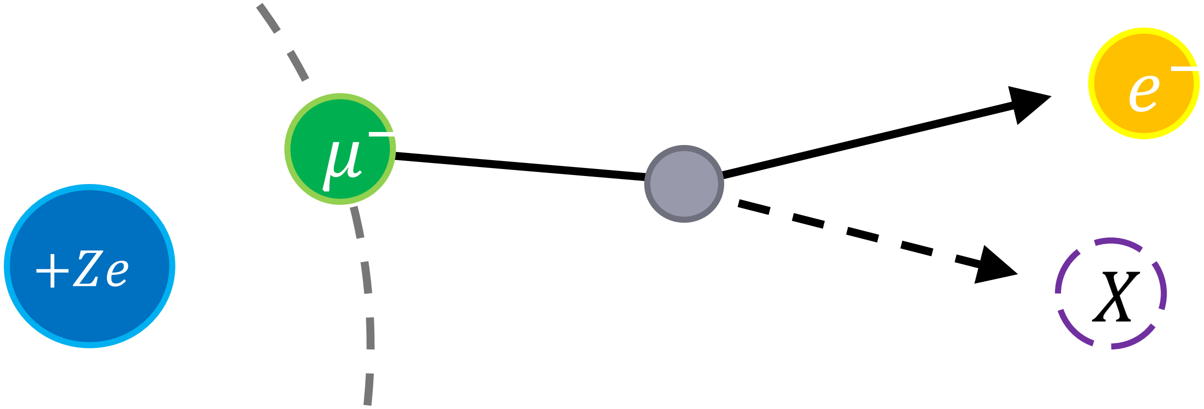
6th Dec. 2018 @ KEK-PH

Muon decay into an electron and a light boson in a muonic atom



Yuichi Uesaka (Saitama U.)

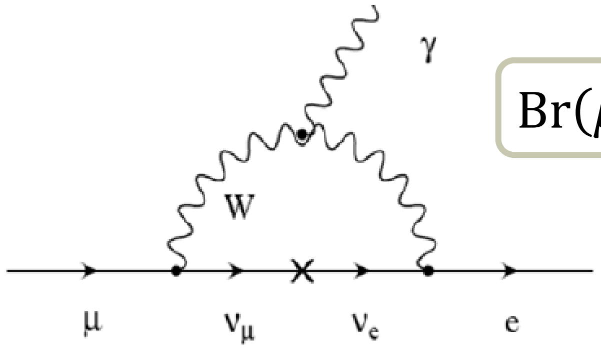
Charged Lepton Flavor Violation (CLFV)

- A probe for new physics -

◆ lepton flavor violation for charged lepton = CLFV

e.g. $\mu^+ \to e^+\gamma,\ \mu^+ \to e^+e^-e^+,\ \mu^-N \to e^-N,\ \tau^+ \to \mu^+\gamma$, etc.

- forbidden in SM
- contribution of neutrino mixing ⟶ very small

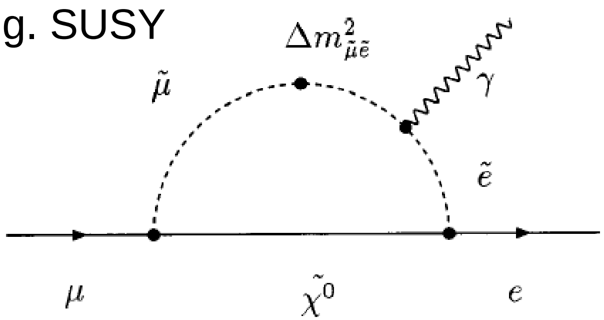


$$\mathrm{Br}(\mu \to e\gamma) \lesssim 10^{-54}$$

✓ cannot be observed by current technology

- enhanced in many theories beyond SM

e.g. SUSY



✓ Searches for CLFV can access new physics with little SM backgrounds.

CLFV searches in muon rare decay

Advantages of muon

1. high intensity
2. long lifetime

- Current bounds

L. Calibbi & G. Signorelli, arXiv:1709.00294 [hep-ph].

Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006

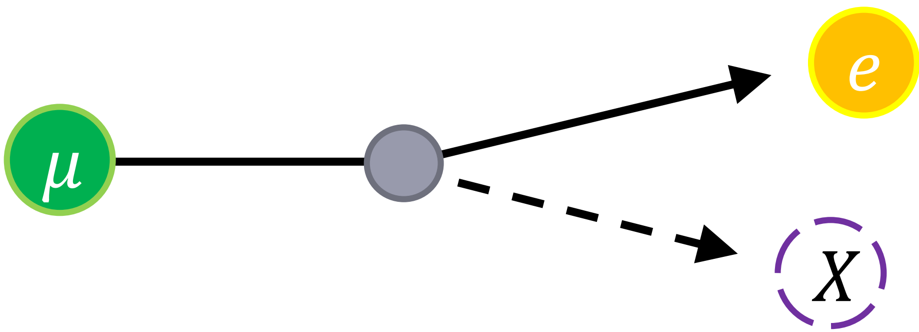
- Planned experiments

Reaction	Expected limit	Experiment
$\mu^+ \to e^+\gamma$	5×10^{-14}	MEG II
$\mu^+ \to e^+e^-e^+$	10^{-16}	Mu3e
$\mu^-\mathrm{Al} \to e^-\mathrm{Al}^{\,(a)}$	10^{-17}	Mu2e, COMET
$\mu^-\mathrm{Si/C} \to e^-\mathrm{Si/C}^{\,(a)}$	5×10^{-14}	DeeMe

CLFV with a light invisible

- If a light boson X ($m_X < m_\mu$) & X-μ-e coupling exist,

  the process where on-shell X is emitted should happen.

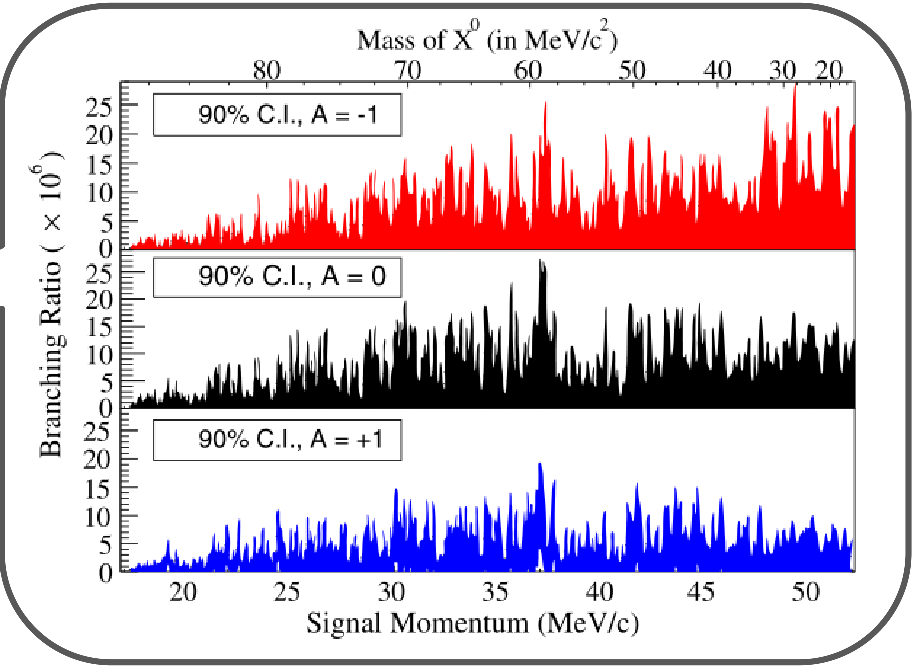


- Theoretical examples of X
  - light (pseudo-)scalar : majoron, familon, axion(-like) particle, …
  - light gauge boson

some of them can also be **dark matter** candidates

$\mu^+ \to e^+ X$ searches $\quad m_X < m_\mu$

- A. Jodidio *et al.* PRD **34**, 1967 (1986).
  - 1.8×10^7 μ^+ that was highly polarized
  - search for e^+ emitted in opposite direction for μ^+ polarization
  - $\mathrm{Br}(\mu^+ \to e^+ X) < 2.6 \times 10^{-6}$ for $m_X = 0$
- TWIST Collab. PRD **91**, 052020 (2015).
  - 5.8×10^8 μ^+
  - for various m_X & various angular property ($d\Gamma/d\cos\theta \propto 1 - AP_\mu \cos\theta$)
  - $\mathrm{Br} < 2.1 \times 10^{-5}$ ($m_X = 0, A = 0$)



- Mu3e Collab. A. Schöning, Talk at Flavour and Dark Matter Workshop, Heidelberg, September 28 (2017).
  - $\mathrm{Br} < 10^{-8}$ (for $25\mathrm{MeV} < m_X < 95\mathrm{MeV}$)

$\mu^- \to e^- X$ in a muonic atom

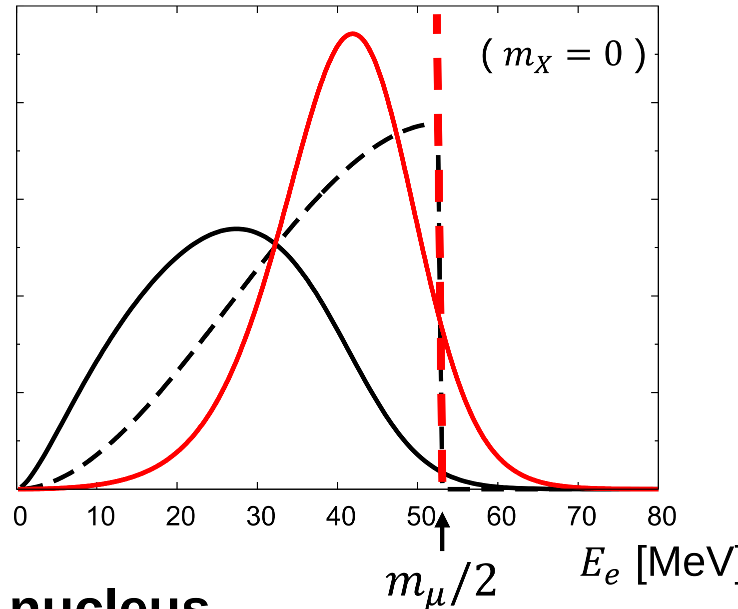
cf. X. G. i Tormo *et al*., PRD **84**, 113010 (2011).
& H. Natori, Talk at 73th JPS meeting (2018).

Advantage over free muon decay

1. less background

- - - : $\mu^+ \to e^+ X$ (free)
- - - : $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$ (free)
—— : $\mu^- \to e^- X$ (μ-gold)
—— : $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ (μ-gold)

shape of electron spectrum



- different peak positions of signal & BG

2. also sensitive to contact reactions with nucleus

3. more information : "spectrum" & "dependence on nucleus"

Disadvantage

✓ non-monochromatic signal

✓ shorter life time of muonic atom

Models

1. yukawa coupling (e.g. majoron incuded by R-parity violation, …)

already analyzed by X. G. i Tormo *et al*., PRD **84**, 113010 (2011).

$$\mathcal{L}_Y = g_Y(\overline{e}\mu)X + [H.c.]$$

2. derivative coupling (e.g. majoron, familon, axion, …)

$$\mathcal{L}_D = \frac{g_D}{\Lambda_D}(\overline{e}\gamma_\alpha\mu)\partial^\alpha X + [H.c.]$$

3. vector

$$\mathcal{L}_V = g_V(\overline{e}\gamma_\alpha\mu)X^\alpha + [H.c.]$$

4. contact coupling with nucleus

$$\mathcal{L}_N = \frac{g_N}{\Lambda_N^3}(\overline{e}\mu)\left(\overline{N}N\right)X + [H.c.]$$

N : nucleon

Formulation for decay rate

$$\Gamma = \int \frac{d^3 p_e}{(2\pi)^3 2E_e} \frac{d^3 p_X}{(2\pi)^3 2E_X} (2\pi)\delta\left(m_\mu - E_e - E_X\right)$$
$$\times \sum_{spins} \left|\left\langle \psi_e^{s_e}(\boldsymbol{p}_e)\phi_X^{s_X}(\boldsymbol{p}_X)\middle|\mathcal{L}\middle|\psi_\mu^{s_\mu}(1S)\right\rangle\right|^2$$

partial wave expansion for the electron in the final state

$$\psi_e^{p,s} = \sum_{\kappa,\mu,m} 4\pi\, i^{l_\kappa}(l_\kappa, m, 1/2, s|j_\kappa, \mu) Y_{l_\kappa,m}^*(\hat{p}) e^{-i\delta_\kappa} \psi_p^{\kappa,\mu}$$

Dirac eq. for radial wave function

$$\frac{dg_\kappa(r)}{dr} + \frac{1+\kappa}{r} g_\kappa(r) - \left(E + m + e\phi(r)\right) f_\kappa(r) = 0$$
$$\frac{df_\kappa(r)}{dr} + \frac{1-\kappa}{r} f_\kappa(r) + \left(E - m + e\phi(r)\right) g_\kappa(r) = 0$$

$$\psi_p^{\kappa,\mu}(\boldsymbol{r}) = \begin{pmatrix} g_\kappa(r)\chi_\kappa^\mu(\hat{r}) \\ i f_\kappa(r)\chi_{-\kappa}^\mu(\hat{r}) \end{pmatrix}$$

ϕ : nuclear Coulomb potential

Electron spectrum (yukawa)

1. yukawa coupling

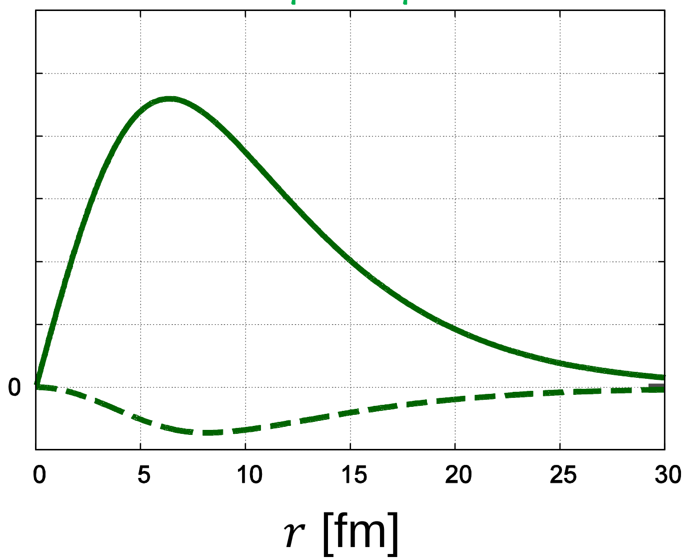
κ : angular momentum of e^-

l_X : orbital angular momentum of X

$$\frac{d\Gamma}{dE_e} = \frac{g_Y^2}{4\pi^2} p_e p_X \sum_{\kappa} (2j_\kappa + 1) \sum_{l_X = |j_\kappa - 1/2|}^{j_\kappa + 1/2} \left| S_{\kappa, l_X} \right|^2$$

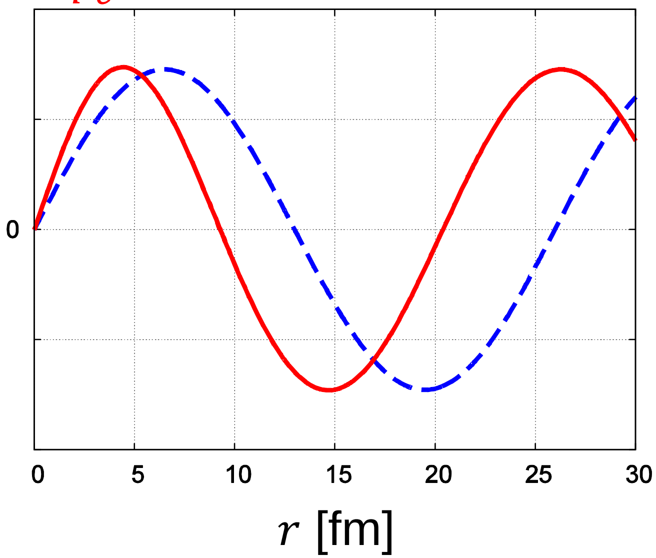
$$S_{\kappa, l_X} = \int_0^\infty dr r^2 j_{l_X}(p_X r) \left\{ g_{p_e}^\kappa(r) g_\mu^{1s}(r) - f_{p_e}^\kappa(r) f_\mu^{1s}(r) \right\} \delta_{l_\kappa, l_X}$$

$r g_\mu^{1s}, r f_\mu^{1s}$



$r g_{p_e = 48\mathrm{MeV}}^{\kappa=-1}, r j_0(48\mathrm{MeV} \times r)$

^{208}Pb



Electron spectrum (derivative & vector)

2. derivative coupling

$$\frac{d\Gamma}{dE_e} = \frac{g_D^2}{4\pi^2\Lambda_{\mathrm{D}}^2} p_e p_X \sum_\kappa (2j_\kappa + 1) \sum_{l_X=|j_\kappa-1/2|}^{j_\kappa+1/2} \left| E_X S^0_{\kappa,l_X} - p_X \frac{\sqrt{l_X+1}S^1_{\kappa,l_X+1,l_X} + \sqrt{l_X}S^1_{\kappa,l_X-1,l_X}}{\sqrt{2l_X+1}} \right|^2$$

$$S^0_{\kappa,l_X} = \int_0^\infty dr\, r^2 j_{l_X}(p_X r)\left\{g^\kappa_{p_e}(r) g^{1s}_\mu(r) + f^\kappa_{p_e}(r) f^{1s}_\mu(r)\right\}\delta_{l_\kappa,l_X}$$

$$S^1_{\kappa,l_X,J} = \int_0^\infty dr\, r^2 j_{l_X}(p_X r)\left\{g^\kappa_{p_e}(r) f^{1s}_\mu(r) V^{\kappa,-1}_{l_X,J} - f^\kappa_{p_e}(r) g^{1s}_\mu(r) V^{\kappa,+1}_{l_X,J}\right\}\delta_{l_{-\kappa},l_X}$$

$$V^{\kappa,\kappa_\mu}_{l_X,J} = \begin{cases} (J-\kappa_\mu-\kappa)/\sqrt{J(2J+1)}\ (J = l_X+1) \\ (\kappa_\mu-\kappa)/\sqrt{J(J+1)}\ (J = l_X) \\ -(J+1+\kappa_\mu+\kappa)/\sqrt{(J+1)(2J+1)}\ (J = l_X-1) \end{cases}$$

3. vector

$$\frac{d\Gamma}{dE_e} = \frac{g_V^2}{4\pi^2} p_e p_X \sum_\kappa (2j_\kappa + 1)$$

$$\times \sum_{l_X=|j_\kappa-1/2|}^{j_\kappa+1/2} \left\{ \sum_{L=|l_X-1|}^{l_X+1} \left|S^1_{\kappa,L,l_X}\right|^2 - \left|S^0_{\kappa,l_X}\right|^2 + \frac{1}{m_X^2}\left| E_X S^0_{\kappa,l_X} - p_X \frac{\sqrt{l_X+1}S^1_{\kappa,l_X+1,l_X} + \sqrt{l_X}S^1_{\kappa,l_X-1,l_X}}{\sqrt{2l_X+1}} \right|^2 \right\}$$

same as “derivative coupling” ↓

Contact process with nucleus

4. contact coupling with nucleus

$$\mathcal{L}_N = \frac{g_N}{\Lambda_N^3}(\overline{e}\mu)(\overline{N}N)X + [H.c.]$$

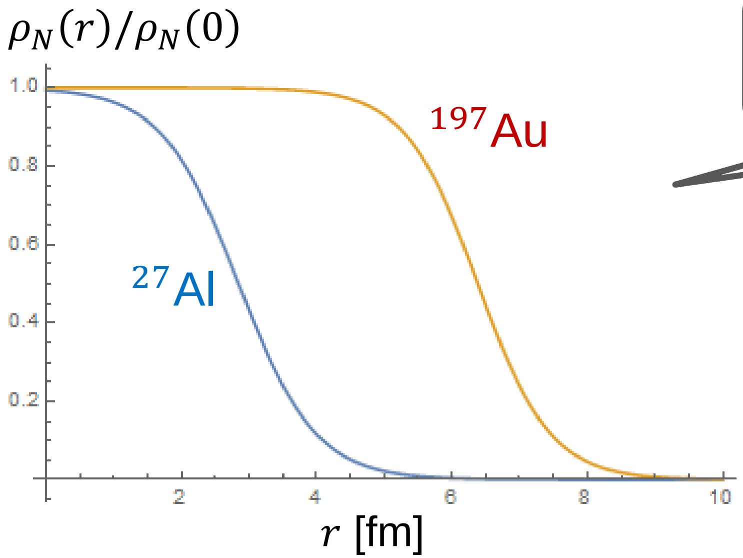
$\langle N|\overline{N}N|N\rangle \sim \rho_N(r)$

rewrite the overlap integral

$$\int_0^\infty dr r^2 j_{l_X}(p_X r) g_{p_e}^{\kappa}(r) g_\mu^{1s}(r) \Rightarrow \frac{1}{\Lambda_N^3}\int_0^\infty dr r^2 j_{l_X}(p_X r) g_{p_e}^{\kappa}(r) g_\mu^{1s}(r)\, \rho_N(r)$$

$\rho_N(r)$: nucleon density of nucleus

($\int d^3 r \rho_N(r) = A$)



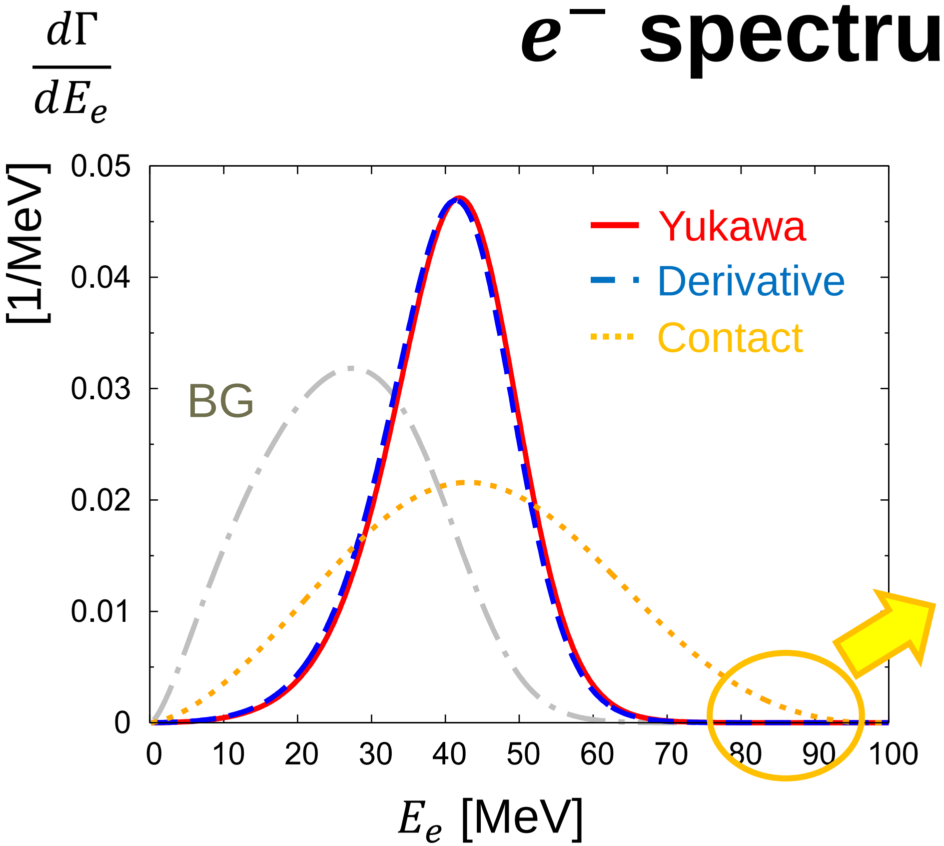
$\rho_N(r)$ is more compact than $g_\mu^{1s}(r)$

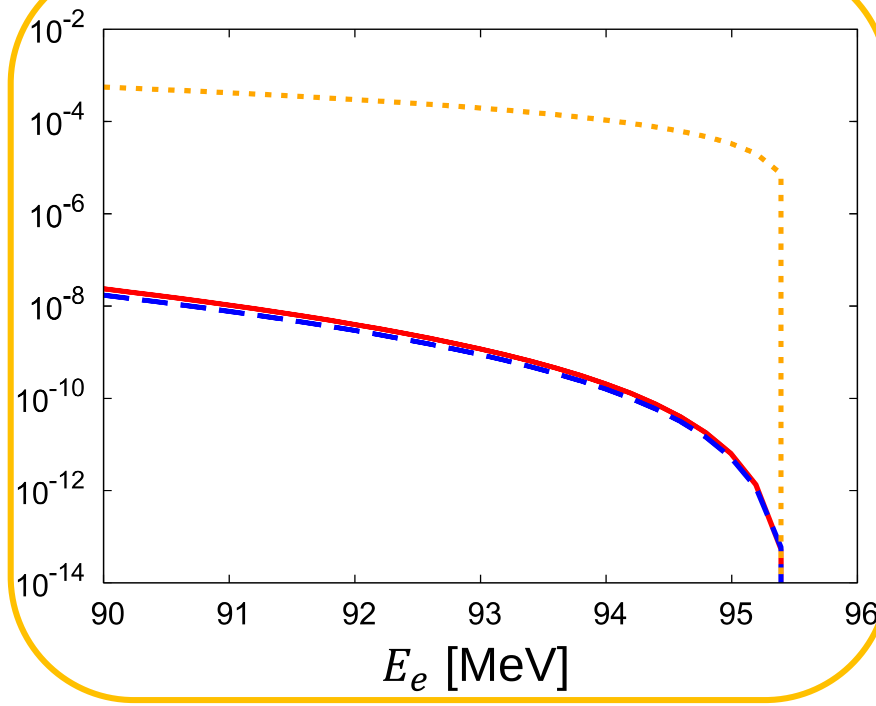
a wider width of spectrum

than CLFV decay in orbit

e^{-} spectrum ($m_X = 0$)

^{197}Au



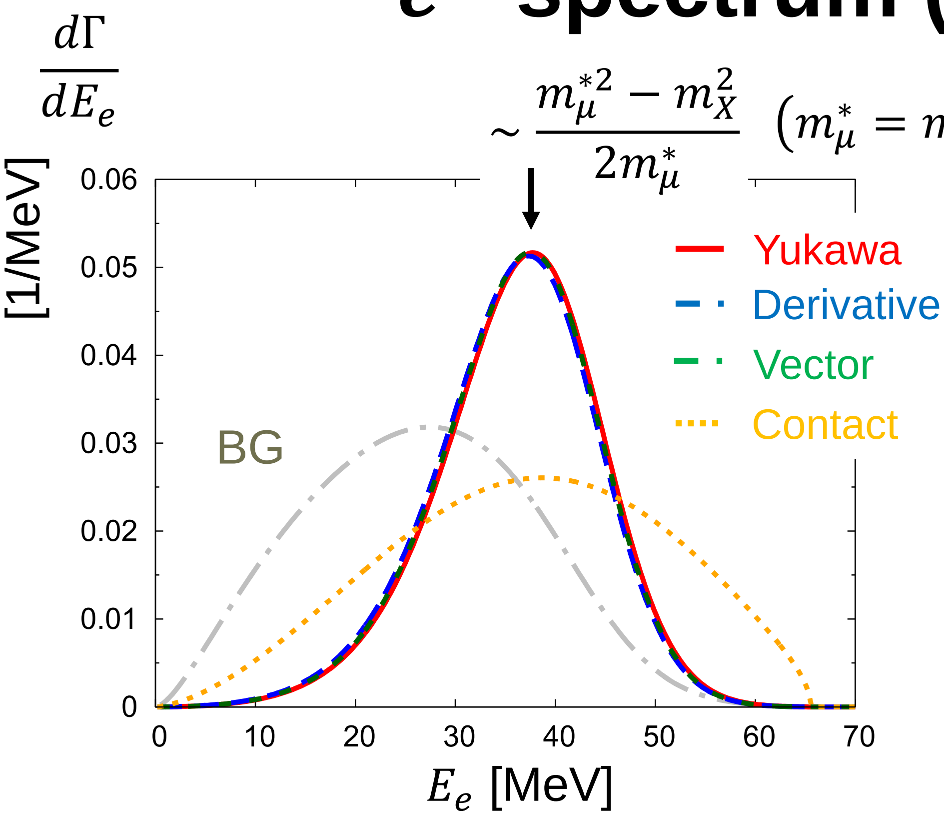


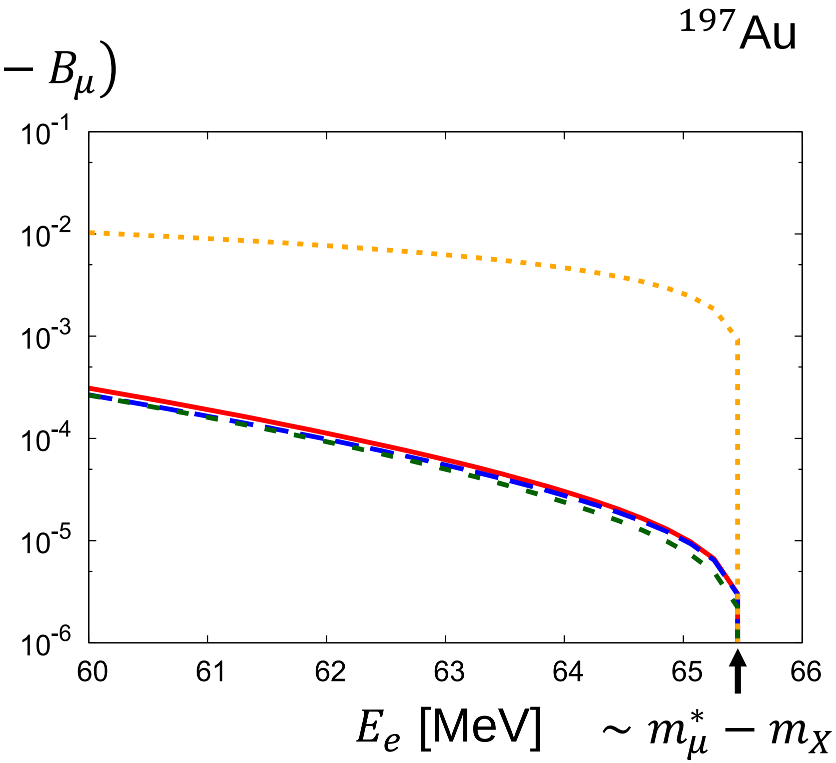
rate of high energy electron

$$f(E_{Low}) = \frac{1}{\Gamma}\int_{E_{Low}}^{E_{EndPoint}} dE_e \frac{d\Gamma}{dE_e}$$

Model	$f(70\text{MeV})$	$f(80\text{MeV})$	$f(90\text{MeV})$
Yukawa	4.8×10^{-4}	1.0×10^{-5}	2.6×10^{-8}
Derivative	3.7×10^{-4}	7.2×10^{-6}	1.9×10^{-8}
Contact	7.4×10^{-2}	1.8×10^{-2}	1.3×10^{-3}

e^- spectrum ($m_X = 30\text{MeV}$)



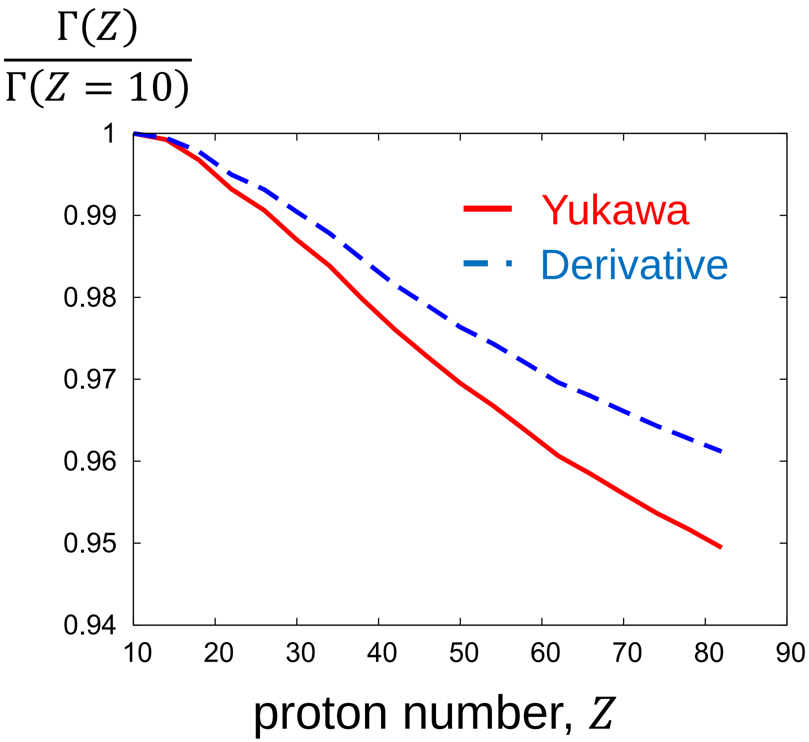


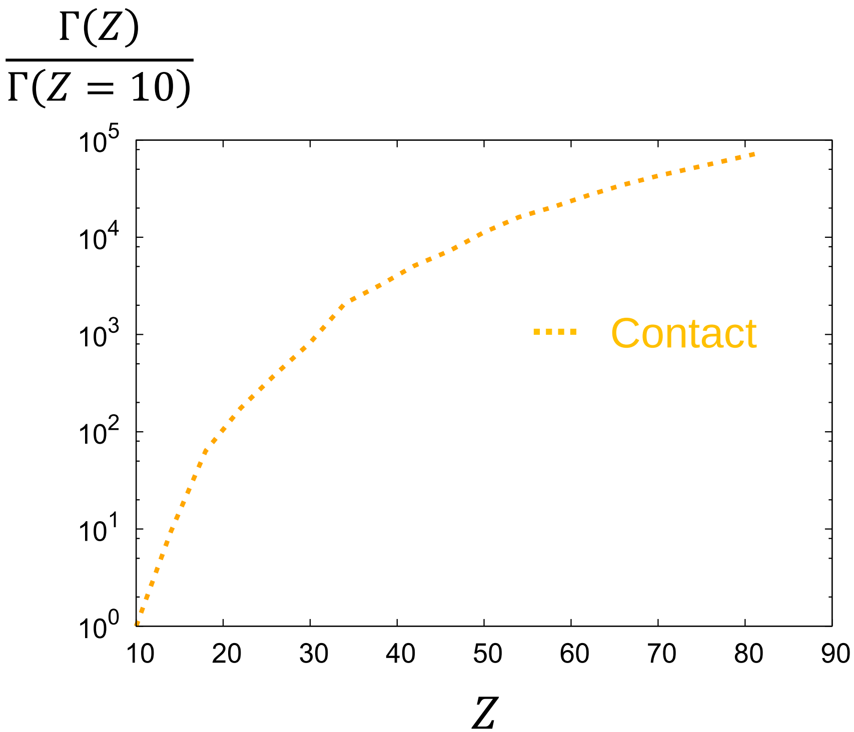
rate of high energy electron

$$f(E_{Low}) = \frac{1}{\Gamma}\int_{E_{Low}}^{E_{EndPoint}} dE_e \frac{d\Gamma}{dE_e}$$

Model	$f(50\text{MeV})$	$f(60\text{MeV})$
Yukawa	3.4×10^{-2}	5.5×10^{-4}
Derivative	3.0×10^{-2}	4.8×10^{-4}
Vector	3.1×10^{-2}	4.6×10^{-4}
Contact	2.0×10^{-1}	3.5×10^{-2}

Nuclear dependence ($m_X = 0$)





- small Z-dependence
  for CLFV decay in orbit

- strong Z-dependence

$$\Gamma \propto Z^3 A^2$$

$\psi_\mu(0) \propto Z^{3/2}$ for small Z

$\rho_N \propto A$

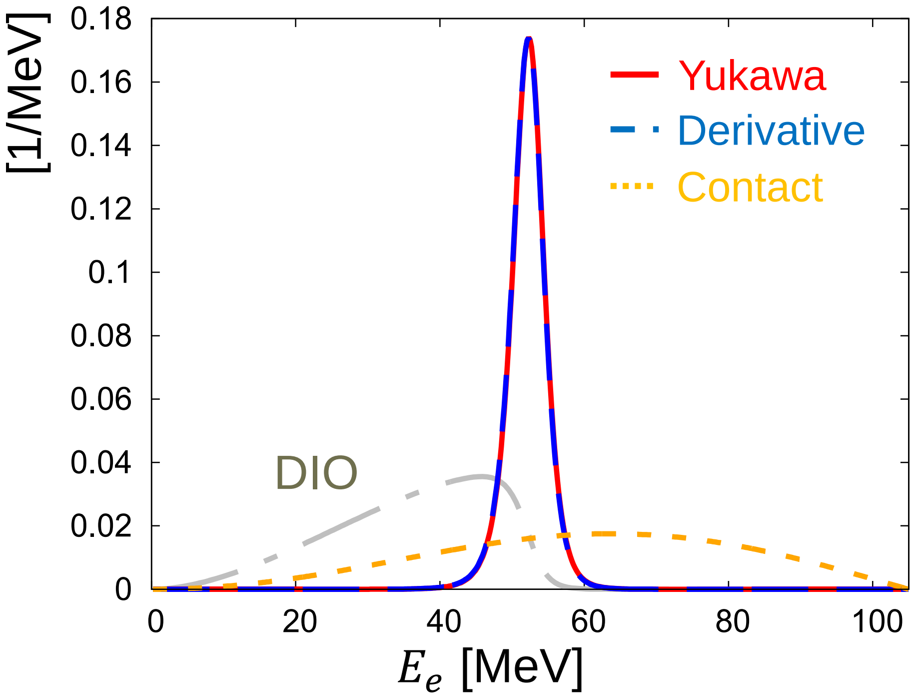
Summary

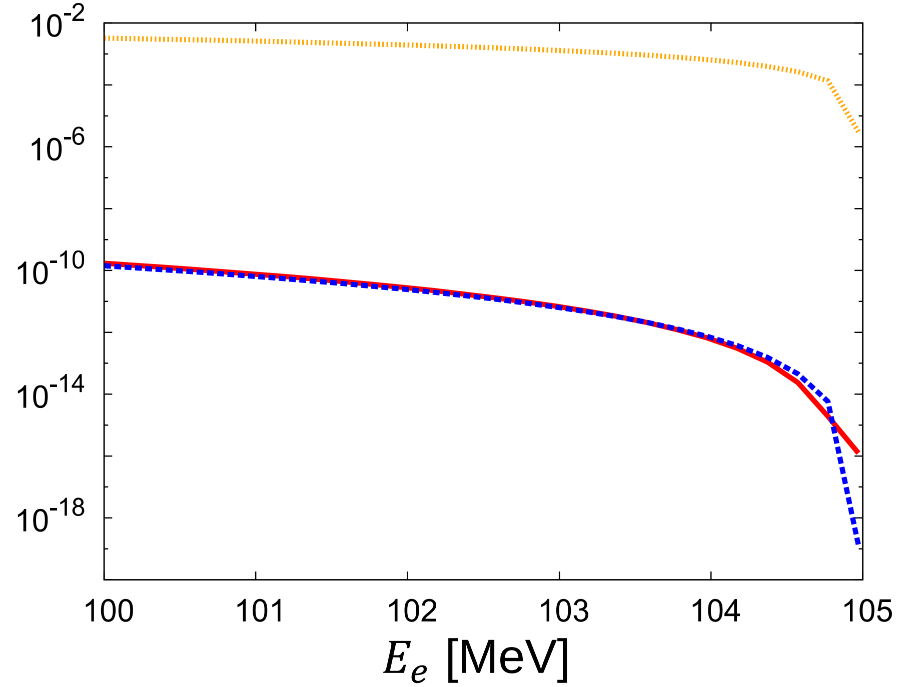
- $\mu^- \to e^- X$ in a muonic atom
  - ✓ a promising search for a light neutral boson with CLFV
  - ✓ Expected advantages over free muon
    - less background
    - also sensitive to contact process with nucleus
    - more information (Z-dependence, spectrum, …)
  - ✓ Our finding
    - the rough shape of spectrum is determined by overlap integral
    - clear differences in experimental observables
      between CLFV decay in orbit & contact process with nucleus
  - ✓ Experimental simulation is in progress with members of COMET

Backups

e^{-} spectrum ($m_X = 0$)

^{27}Al





rate of high energy electron

$$f(E_{Low}) = \frac{1}{\Gamma}\int_{E_{Low}}^{E_{EndPoint}} dE_e \frac{d\Gamma}{dE_e}$$

Model	f(80MeV)	f(90MeV)	f(100MeV)
Yukawa	1.7×10^{-6}	5.0×10^{-8}	1.9×10^{-10}
Derivative	1.4×10^{-6}	3.9×10^{-8}	1.6×10^{-10}
Contact	1.9×10^{-1}	7.2×10^{-2}	8.1×10^{-3}

Lepton Flavors

◆ three lepton flavor #'s, L_e, L_μ, L_τ

	e^-	μ^-	τ^-	ν_e	ν_μ	ν_τ	e^+	μ^+	τ^+	$\overline{\nu_e}$	$\overline{\nu_\mu}$	$\overline{\nu_\tau}$	others
L_e	+1	0	0	+1	0	0	-1	0	0	-1	0	0	0
L_μ	0	+1	0	0	+1	0	0	-1	0	0	-1	0	0
L_τ	0	0	+1	0	0	+1	0	0	-1	0	0	-1	0

- each lepton flavor # is conserved in SM (where neutrinos are massless)

e.g.

$$\mu^- \rightarrow e^- \nu_\mu \overline{\nu}_e$$

$$L_e \quad 0 \ = \ +1 + 0 - 1$$

$$L_\mu \quad +1 = +0 + 1 + 0$$

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$L_\mu \quad 0 \ = \ -1 + 1$$

- neutrino oscillation violates the conservation

$$\nu_\mu \rightarrow \nu_\tau$$

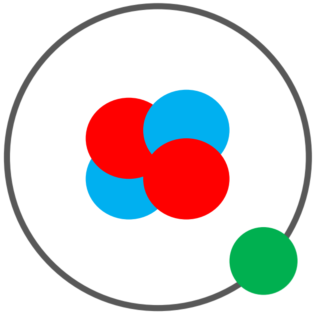
$$L_\mu \quad +1 \neq 0$$

$$L_\tau \quad 0 \neq +1$$

Lepton Flavor Violation
in "neutral lepton sector"

✓ How about charged lepton sector ?

Muonic atom

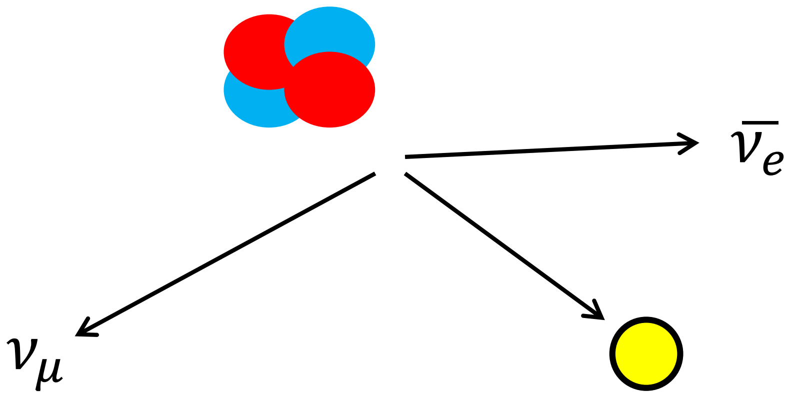


- Muon Bohr radius : 200 times smaller than electron's

- Two standard decays

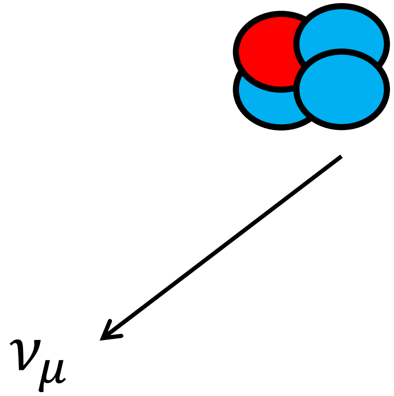
"Decay in orbit (DIO)"

($\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$)



"Nuclear capture"

($\mu^- p \rightarrow \nu_\mu n$)



- Lifetime

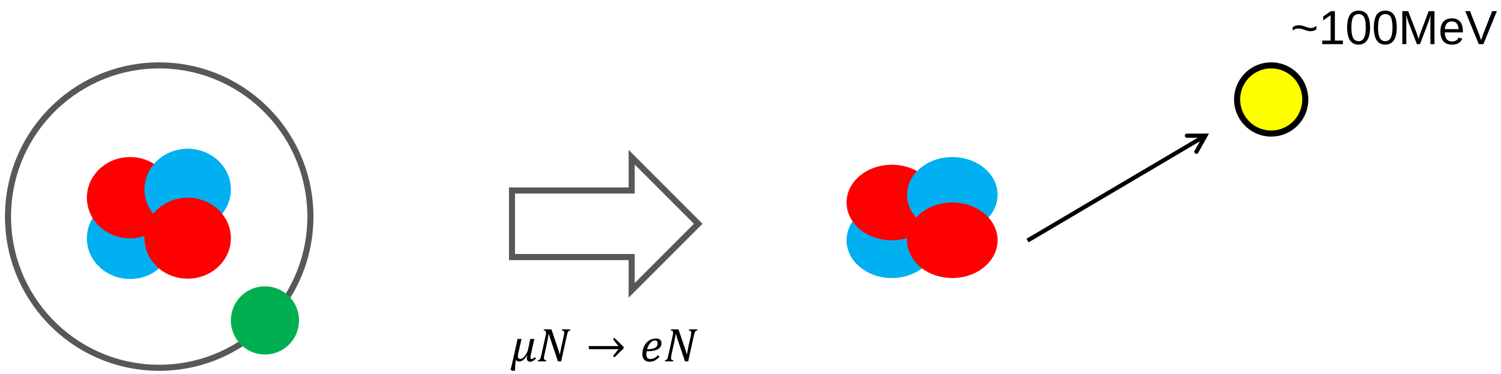
  $\sim 2.2\mu s$ for hydrogen ($Z = 1$)

  $\sim 900ns$ for aluminum ($Z = 13$)

  $\sim 80ns$ for lead ($Z = 82$)

$\mu^- \to e^-$ conversion

- CLFV process between a nucleus & a muon



clear signal : one e^- with $E_e = m_\mu - B_\mu \sim 100\text{MeV}$

- Planned experiments (10^{10-11} μ^--atom / s)

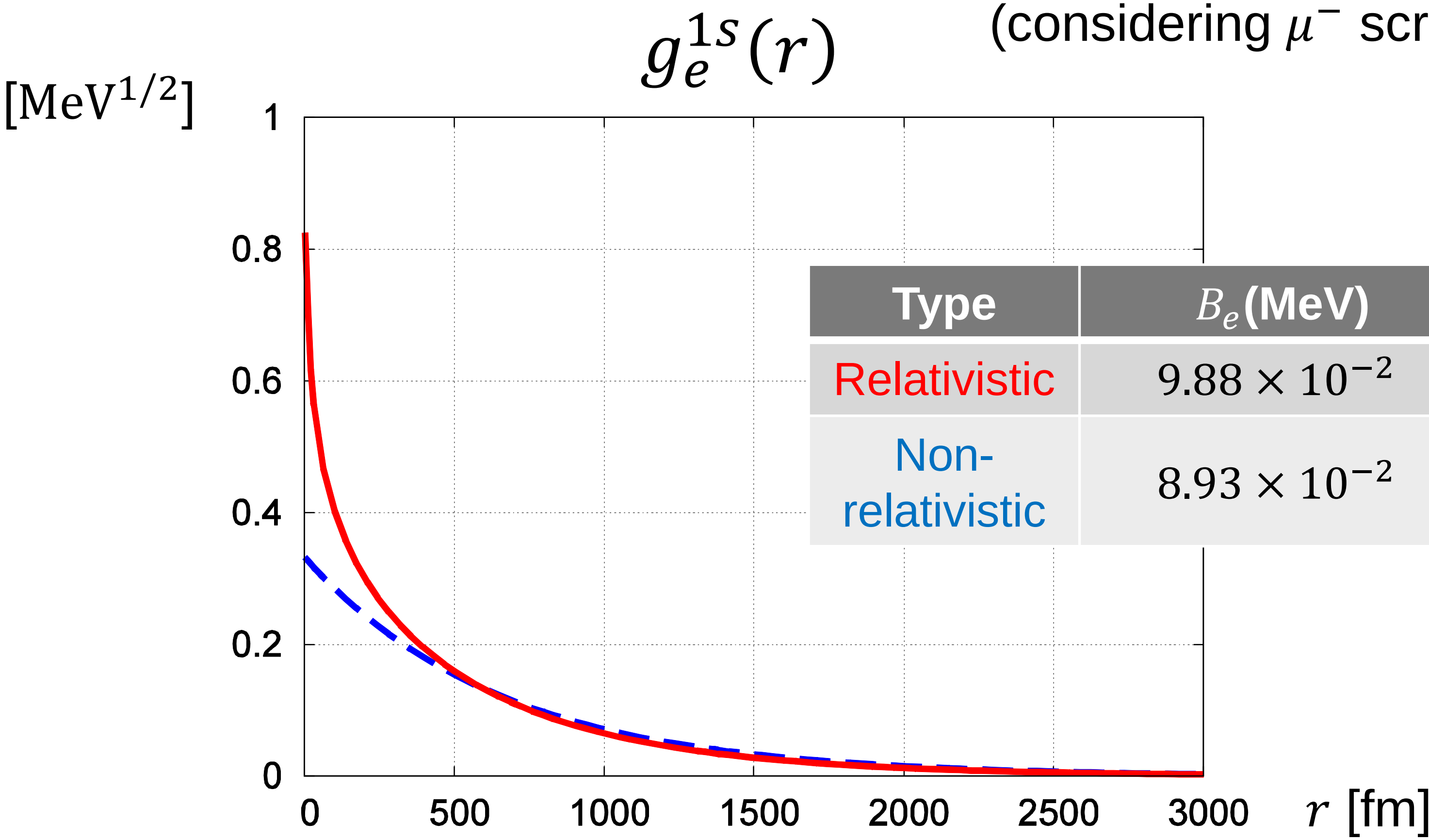
  COMET, DeeMe @ J-PARC

  Mu2e @ Fermilab

Radial wave function (bound e^-)

^{208}Pb case $Z = 81$

(considering μ^- screening)



Type	B_e(MeV)
Relativistic	9.88×10^{-2}
Non-relativistic	8.93×10^{-2}

Relativity enhances the value near the origin.

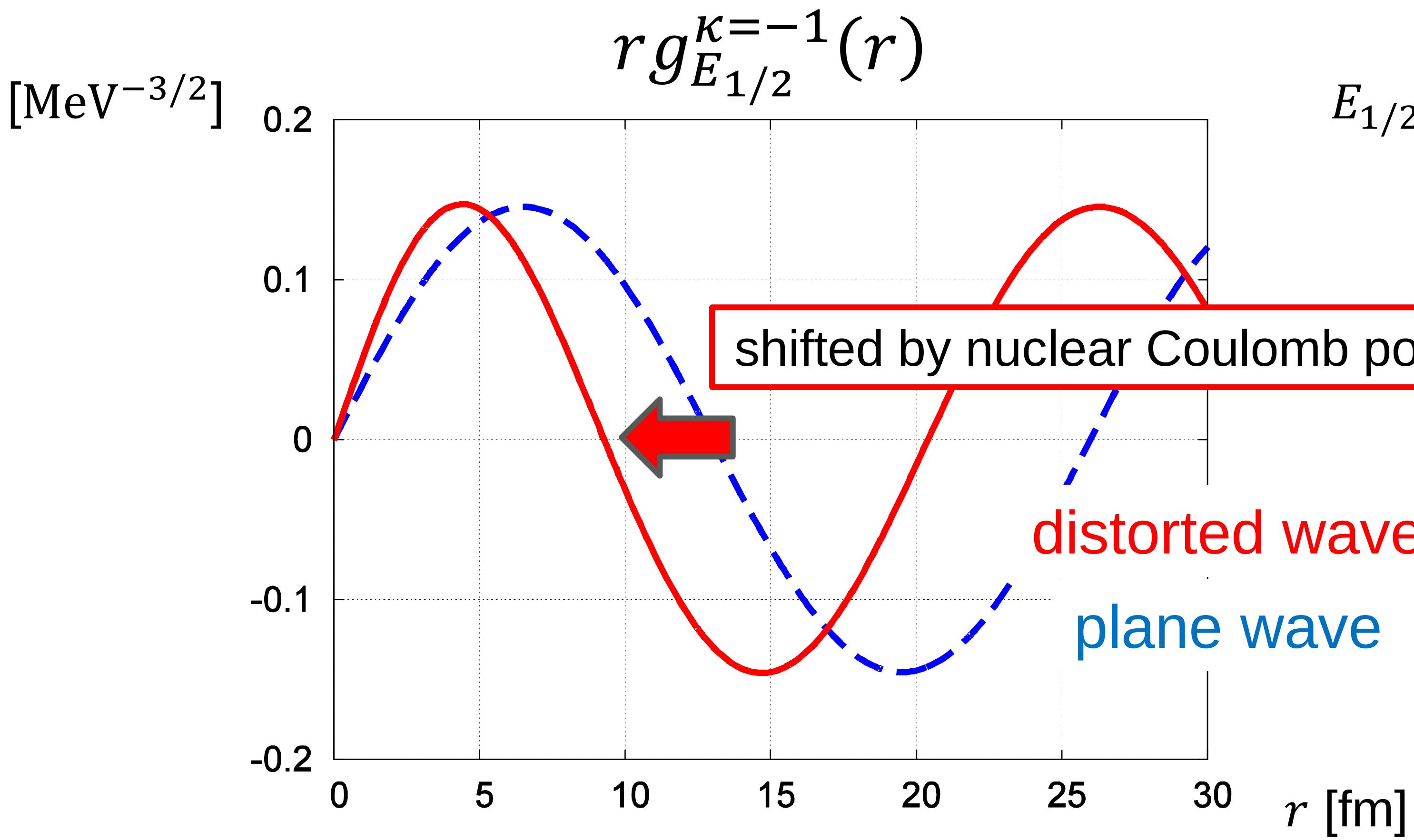
Radial wave function (scattering e^-)

e.g. $\kappa = -1$ partial wave

^{208}Pb case

$Z = 82$

$E_{1/2} \approx 48$MeV



① enhanced value near the origin

② local momentum increased effectively

Example : Majoron

- Singlet Majoron model

Y. Chikashige, R.N. Mohapatra, & R.D. Peccei, PLB**98** (1981) 265.

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + i\overline{N}_R \gamma^\mu \partial_\mu N_R + \left(\partial_\mu \sigma\right)^\dagger (\partial^\mu \sigma) - V(\sigma)$$
$$- \left(\overline{L} y N_R H + \frac{1}{2} \overline{N}_R^c \lambda N_R \sigma + \mathrm{h.c.} \right)$$

N_R : right-handed neutrino
σ : scalar with $L = -2$

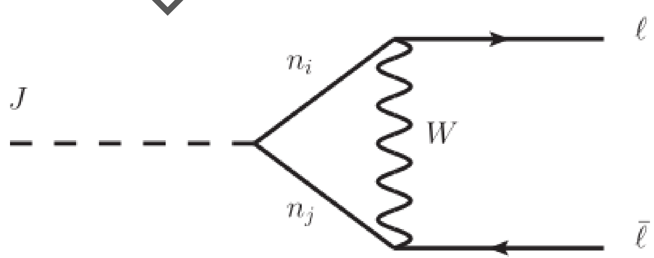
SSB of lepton #

$$\sigma(x) = f + \rho(x) + iJ(x)$$

J : majoron
(NG boson of lepton #)

- **majorana mass of neutrino**
- **interaction of majoron with neutrino**

- **CLFV coupling at one loop !**



$$K \equiv \frac{m_D m_D^\dagger}{vf}$$

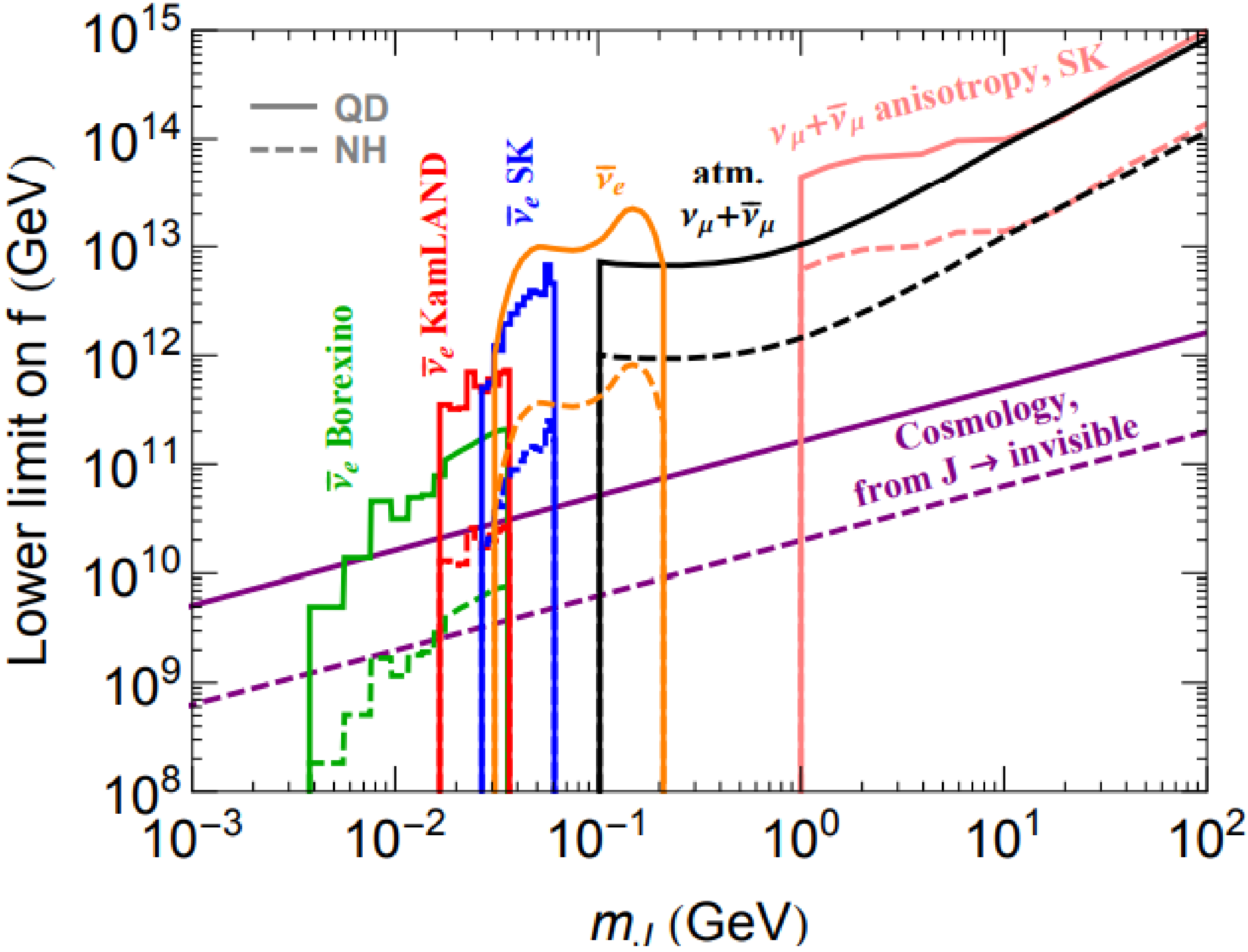
$$\mathcal{L}_{J\ell'\ell} \simeq \frac{i m_\ell}{8\pi v} K_{\ell'\ell} J \overline{\ell'} P_R \ell$$

for $m_\ell \gg m_{\ell'}$ ($\ell \neq \ell'$)

n_i : Majorana neutrino mass eigenstate

Constraint for Majoron parameter

C. Garcia-Cely & J. Heeck, JHEP 05 (2017) 102.



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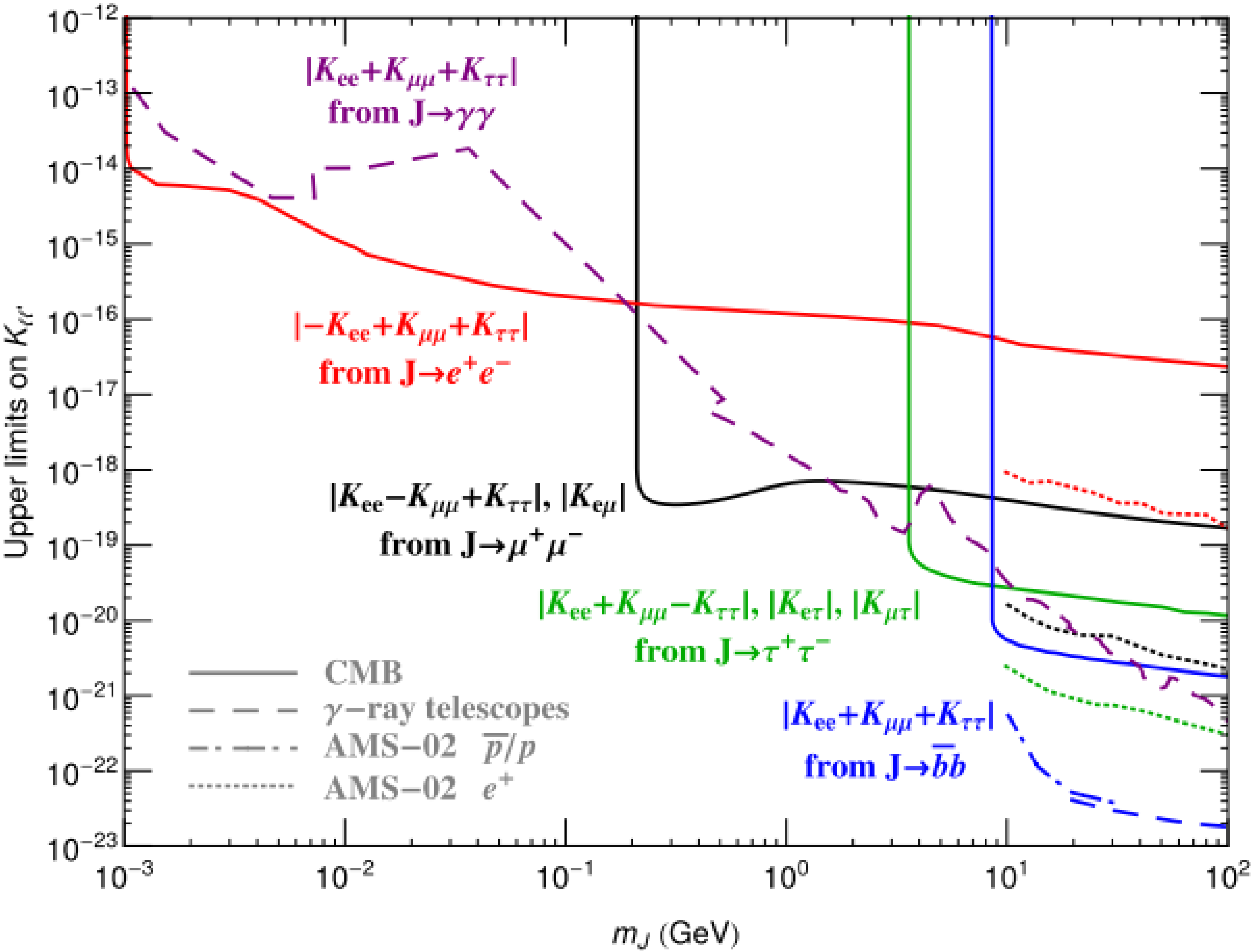
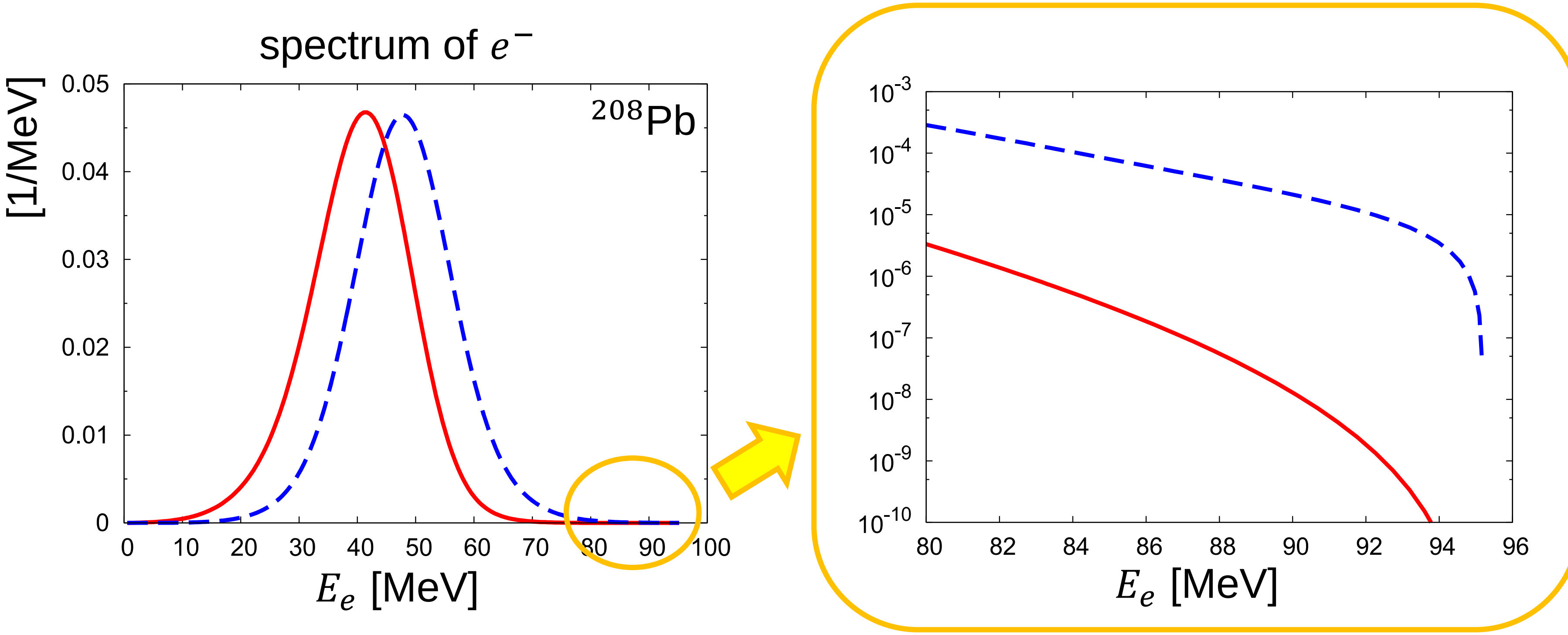


Effect of distortion

electron wave function : plane wave or distorted wave



high energy electron is suppressed, so the spectrum shifts to low energy

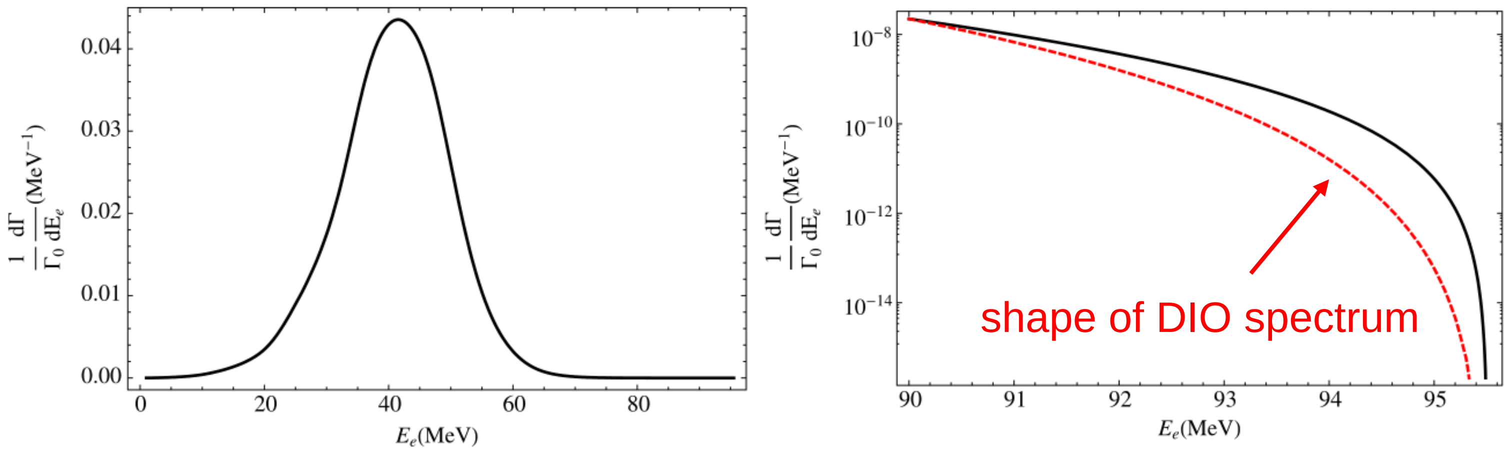
(similar effect is also known in beta decay)

Previous work X. G. i Tormo *et al*., PRD **84**, 113010 (2011).

- assuming that massless X has yukawa-type CLFV interaction

$$\mathcal{L}_I = g(\overline{\mu}e)X \qquad (\ g : \text{coupling}\ )$$

electron spectrum of $\mu^- \to e^- X$ (gold)



- ✓ result of the past μ-e conv. can be convert to $\mathrm{Br}(\mu \to eX) < 3 \times 10^{-3}$
- ✓ COMET & Mu2e could search at $\mathrm{Br}(\mu \to eX) \sim 2 \times 10^{-5}$

⇧

same level as the corrent limit of free μ^+ search ($\sim 10^{-5}$)

Experimental search

(cf : X. G. i Tormo *et al*., PRD **84**, 113010 (2011).)

1. corrent limit ($m_X = 0$)

$$\mathrm{Br}(\mu^- \mathrm{Au} \to e^- \mathrm{Au}) < 7 \times 10^{-13}$$

⇨

Model	Upper limit of $\mathrm{Br}(\mu^+ \to e^+ X)$
Yukawa	3×10^{-3}
Derivative	3×10^{-3}

cf. limit from free μ^+ decay

$$\mathrm{Br}(\mu^+ \to e^+ X) < 2.1 \times 10^{-5}$$

2. future limit

$\mathrm{Br}(\mu^- \mathrm{Al} \to e^- \mathrm{Al}) < 10^{-16}$ (COMET, Mu2e)

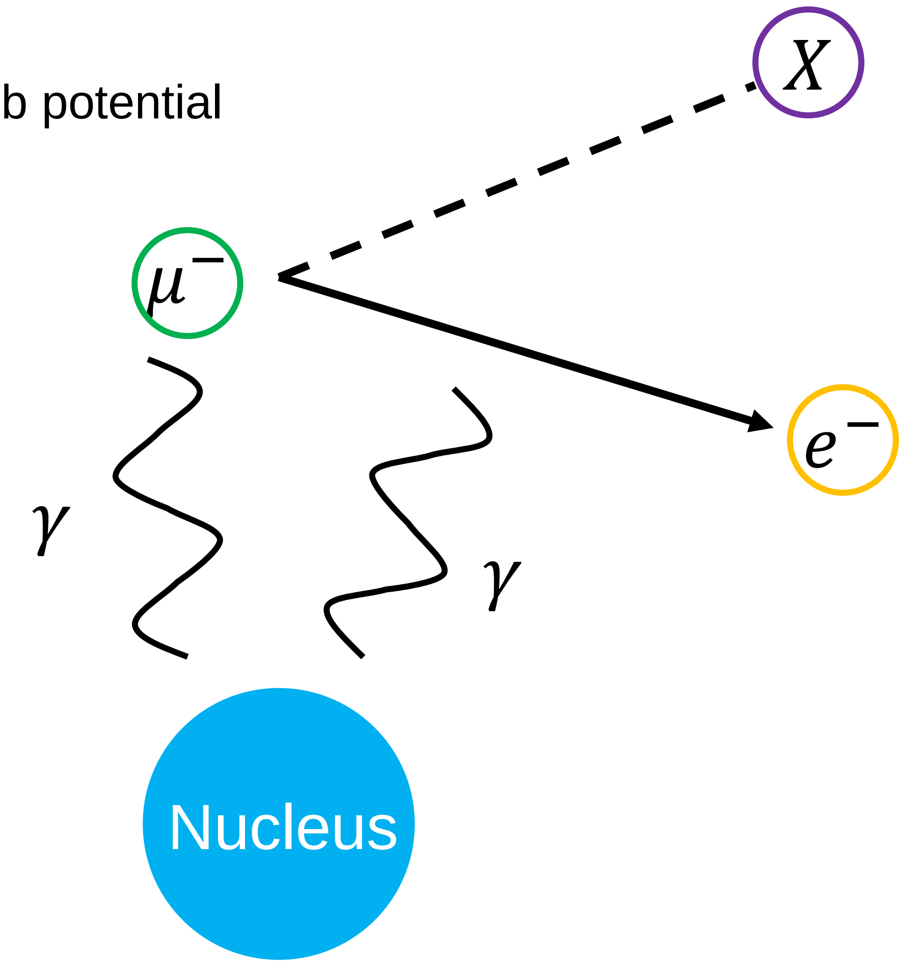
⇨

Model	Upper limit of $\mathrm{Br}(\mu^+ \to e^+ X)$
Yukawa	2×10^{-5}
Derivative	3×10^{-5}

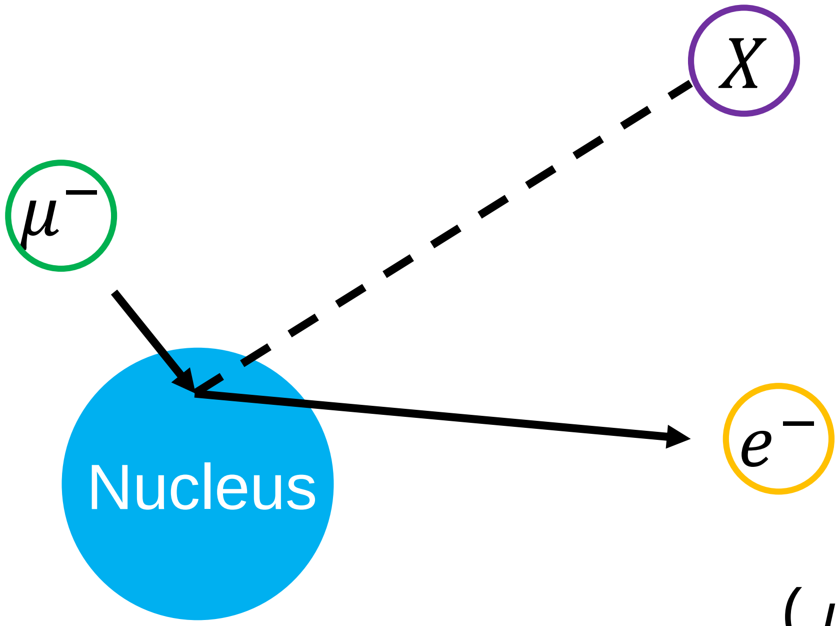
- detailed estimation is in progress with members of COMET

CLFV decay in orbit

$\mu^- \to e^- X$ in nuclear Coulomb potential



Contact process with nucleus

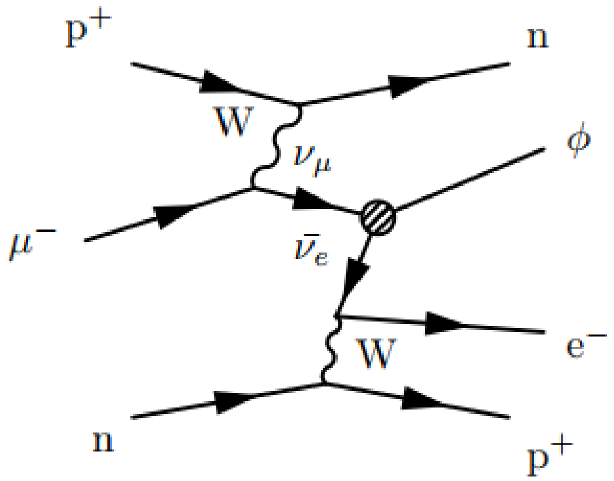


$$(\ \mu^- N \to e^- X N\ )$$

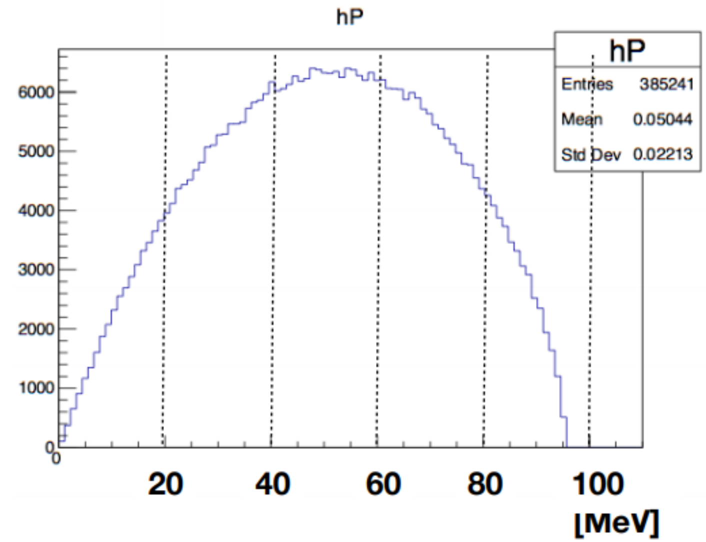
Assuming that the state of the nucleus does not change

Contact process with nucleus

Example of process requiring nucleus :



e⁻ spectrum of signal (Mϕ=10MeV)



名取寛顕, Talk at 日本物理学会 第73回 年次大会,
東京理科大 野田キャンパス, 3月22日 (2018).

In this talk, let us assume

$$\mathcal{L}_N = \frac{g_N}{\Lambda_N^3}(\overline{e}\mu)\left(\overline{N}N\right)X + [H.c.]$$

(N : nucleon)