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Axial kinetic theory and spin transport for massive fermions

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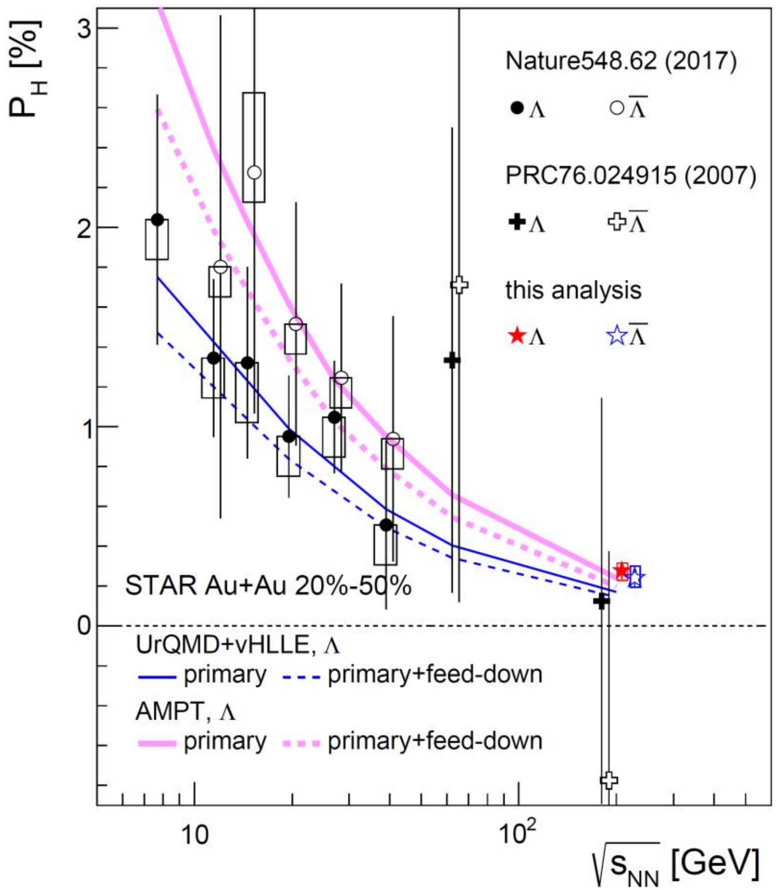
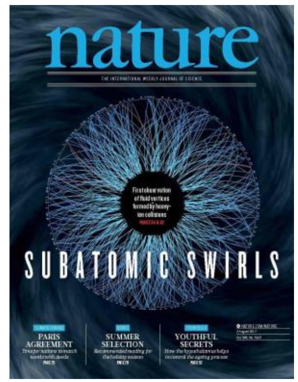
Reference :
Koichi Hattori (YITP) , Yoshimasa Hidaka (RIKEN), DY,
arXiv:1903.01653

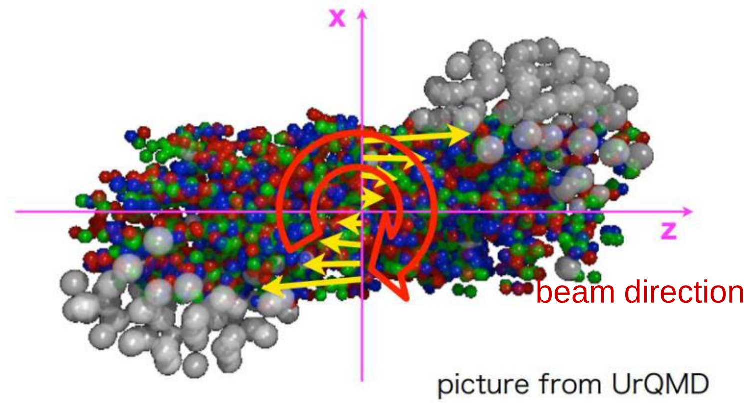
Rotating fluids with spins

- Global polarization of Λ hyperons : (see also Liao's talk)

STAR, Nature 548 (2017) 62-65







impact parameter

➢ self analyzing through the weak decay :

$$\Lambda \rightarrow p + \pi^-$$

(the momentum of daughter proton is preferable to align along the spin of Lambda)

STAR, PRC, 18

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Polarization led by magnetic/vortical fields

- Barnett effect : magnetization of an uncharged object with rotation

  S. J. Barnett, 1915

  $$M = \chi\omega/\gamma$$

  ω : angular velocity

  χ : magnetic susceptibility

  γ : gyromagnetic ratio

- Einstein-de Hass effect : change of the magnetic moment generates rotation   O. W. Richardson, 1908. A. Einstein, W. J. de Haas, 1905.

- chiral separation effect (CSE) and (axial-)chiral vortical effect (aCVE) : axial-charge (spin) currents led by magnetic/vortical fields for massless fermions.

CSE : $\mathbf{J}_5 = \frac{1}{2\pi^2}\mu_V \mathbf{B}$

aCVE : $\mathbf{J}_5 = \left(\frac{\mu_V^2 + \mu_5^2}{2\pi^2} + \frac{T^2}{6}\right)\boldsymbol{\omega}$

vorticity : $\boldsymbol{\omega} = \frac{1}{2}\nabla \times \boldsymbol{v}$

A. Vilenkin, 79, 80

K. Fukushima, D. Kharzeev, H. Warringa, 08

D. Kharzeev, L. McLerran, H. Warringa, 08

K. Landsteiner, E. Megias, F. Pena-Benitez, 11

❖ mass corrections on CSE/CVE : m_q ↗ $\sigma_{B/\omega}$ ↘

E. Gorbar, V. Miransky, I. Shovkovy, X. Wang, 13

S. Lin and L. Yang, 18

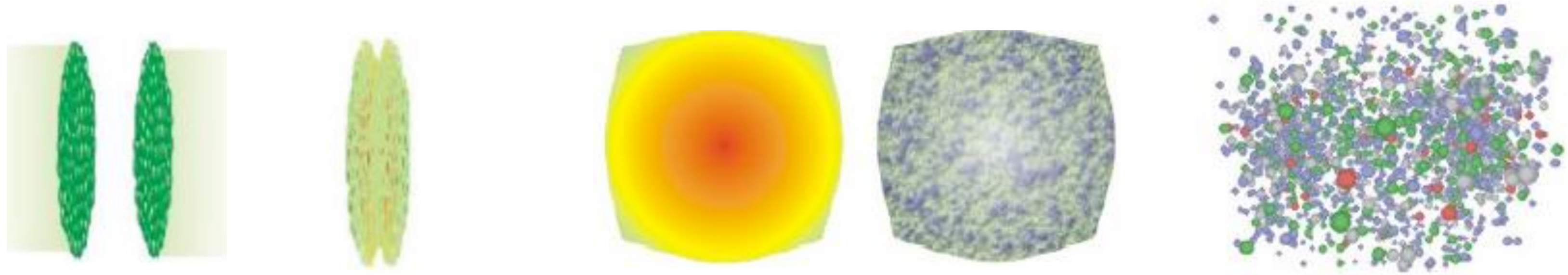
❖ non-equilibrium corrections

D. Kharzeev, et al., 17

Y. Hidaka, DY, 18

Evolution of the spin

- Previous studies in theory focused on the polarization of hadrons in thermal equilibrium. (no dynamics of polarization) e.g. F. Becattini, et al. 13 R. Fang, et al. 16
- How does the spin polarization of partons (s quark) evolve?
- Current theoretical studies :



Initial states | pre-equilibrium phase/thermaliation | QGP | hadronization/ freeze out | hadronic gas

Initial polarization : Hard scattering with $b \neq 0$
Z.-T. Liang, X.-N. Wang, 05

in between?

spin hydro.
(see e.g. Hongo's talk)

Polarization of hadrons in equilibrium : e.g. statistical model
F. Becattini, et al. 13

?

Final polarization : Observed in exp.

"Quantum kinetic theory (QKT) for spin transport" (microscopic theory, non-equilibrium, weak EM fields, weakly coupled)

K. Hattori, Y. Hidaka, DY, arXiv:1903.01653

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Quantum kinetic theory for fermions

- QKT for massless fermions : chiral kinetic theory (CKT)
- Modified Boltzmann (Vlasov) equation with the chiral anomaly
- Non-field theory construction : Berry phase { D. T. Son and N. Yamamoto, 12; M. Stephanov and Y. Yin, 12; J.-Y. Chen, et al. 14, 15 }
- QFT derivation : Wigner functions (WFs) { J.-W. Chen, S. Pu, Q. Wang, X.-N. Wang, 12; D. T. Son & N. Yamamoto, 12 }
- Covariant CKT in an arbitrary frame with collisions Hidaka, Pu, DY, 16, 17

- QKT for massive fermions ?
- Spin is no longer enslaved by chirality : a new dynamical dof
- To track both vector/axial charges and spin polarization
- To reproduce CKT in the massless limit
- Axial kinetic theory (AKT) : a scalar + an axial-vector equations

K. Hattori, Y. Hidaka, DY, arXiv:1903.01653 (in an arbitrary frame)

- Similar works : subject to the rest frame ⇨ become invalid with small mass

N. Weickgenannt, X. L. Sheng, E. Speranza, Q. Wang and D. H. Rischke, arXiv:1902.06513

J. H. Gao and Z. T. Liang, arXiv:1902.06510

Wigner functions (WFs)

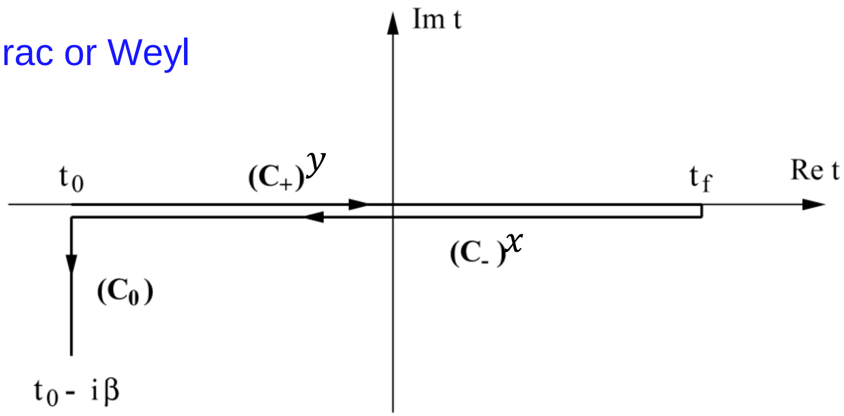
- lesser (greater) propagators :

Dirac or Weyl

$$S^{>}(x,y) = \langle \psi(x) \mathcal{P}\mathcal{U}^{\dagger}(A_\mu, x, y) \psi^{\dagger}(y) \rangle$$
$$S^{<}(x,y) = \langle \psi^{\dagger}(y) \mathcal{P}\mathcal{U}(A_\mu, x, y) \psi(x) \rangle$$

gauge link

$$X = \frac{x+y}{2},\ Y = x - y$$



review : J. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528

Wigner functions : $\grave{S}^{<(>)}(q,X) = \int d^4Y e^{\frac{iq\cdot Y}{\hbar}} S^{<(>)}\left(X + \frac{Y}{2}, X - \frac{Y}{2}\right)$

- Field-theory defined observables : $J^\mu_V = \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\left(\gamma^\mu \grave{S}^{<}\right),\ J^\mu_5 = \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}\left(\gamma^\mu \gamma^5 \grave{S}^{<}\right).$

- Kadanoff-Baym (KB) equations up to $\mathcal{O}(\hbar)$: ($q \gg \partial$: weak fields)

$$(\not{\Pi} - m)\grave{S}^{<} + \gamma^\mu i \frac{\hbar}{2} \nabla_\mu \grave{S}^{<} = \frac{i\hbar}{2}\left(\Sigma^{<}\grave{S}^{>} - \Sigma^{>}\grave{S}^{<}\right)$$

$$\nabla_\mu = \Delta_\mu + \mathcal{O}(\hbar^2),$$
$$\Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$$
$$\Pi^\mu = q^\mu + \mathcal{O}(\hbar^2)$$

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Vector/axial bases

- For simplicity, we focus on the collisionless case ($\Sigma^{<(>)}=0$).
- Decomposition : D. Vasak, M. Gyulassy, and H. T. Elze, 87

$$\grave{S}^{<}=\mathcal{S}+i\mathcal{P}\gamma^5+\mathcal{V}^{\mu}\gamma_{\mu}+\mathcal{A}^{\mu}\gamma^5\gamma_{\mu}+\frac{\mathcal{S}^{\mu\nu}}{2}\Sigma_{\mu\nu},\quad \Sigma_{\mu\nu}=\frac{i}{2}[\gamma_{\mu},\gamma_{\nu}].$$

(pseudo) scalar condensates — vector/axial-charge currents — magnetization

⇒ 10 vector/tensor equations with implicit redundancies

- Reducing redundant dof : replacing $\mathcal{S}$, $\mathcal{P}$, and $\mathcal{S}^{\mu\nu}$ in terms of $\mathcal{V}^{\mu}$ and $\mathcal{A}^{\mu}$.

e.g. $m\mathcal{P}=-\frac{\hbar}{2}\nabla_{\mu}\mathcal{A}^{\mu}$ ⇒ $\partial_{\mu}J^{\mu}_{5}=\frac{\hbar\mathbf{E}\cdot\mathbf{B}}{2\pi^2}+2im\bar{\psi}\gamma_5\psi$

- Master equations :

$$\Delta\cdot\mathcal{V}=0,\qquad q\cdot\mathcal{A}=0,$$

$$(q^2-m^2)\mathcal{V}_{\mu}=-\hbar\tilde{F}_{\mu\nu}\mathcal{A}^{\nu},\qquad (q^2-m^2)\mathcal{A}^{\mu}=\frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}q_{\sigma}\Delta_{\nu}\mathcal{V}_{\rho},$$

$$q_{\nu}\mathcal{V}_{\mu}-q_{\mu}\mathcal{V}_{\nu}=\frac{\hbar}{2}\epsilon_{\mu\nu\rho\sigma}\Delta^{\rho}\mathcal{A}^{\sigma},\qquad q\cdot\Delta\mathcal{A}^{\mu}+F^{\nu\mu}\mathcal{A}_{\nu}=\frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}(\partial_{\sigma}F_{\beta\nu})\partial^{\beta}_{q}\mathcal{V}_{\rho}$$

Leading-order kinetic equations

- Perturbative solution : $(\mathcal{V}/\mathcal{A})^\mu = (\mathcal{V}/\mathcal{A})^\mu_0 + \hbar(\mathcal{V}/\mathcal{A})^\mu_1$

- Leading order (LO) : $(\mathcal{V}_0/\mathcal{A}_0)^\mu = 2\pi(q/a)^\mu\delta(q^2-m^2)f_{V/A}$
  - Dynamical variables : $f_{V/A}(q,X)$ & $a^\mu(q,X)$
  - Spin four vector $a^\mu(q,X)$:

$$q\cdot\mathcal{A}=0 \Longrightarrow q\cdot a = q^2-m^2$$ (vanishes on-shell)

$$m=0 \Longrightarrow a^\mu = q^\mu$$ (spin enslavement)

- LO kinetic theory :

Vlasov Eq. : $$0=\delta(q^2-m^2)q\cdot\Delta f_V, \quad \Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$$

BMT Eq. : $$0=\delta(q^2-m^2)\Big(q\cdot\Delta(a^\mu f_A)+F^{\nu\mu}a_\nu f_A\Big)$$

Bargmann-Michel-Telegdi, 59

(off-shell, $g=2$)

$m=0$: BMT Eq. $\Longrightarrow$ $$0=\delta(q^2)q^\mu q\cdot\Delta f_A$$

Collisionless WFs for massive fermions

- WFs up to $\mathcal{O}(\hbar^1)$:

$$\mathcal{V}^\mu = 2\pi\left[\delta(q^2-m^2)\left(q^\mu f_V + \hbar G^\mu\right) + \hbar\tilde{F}^{\mu\nu}a_\nu\delta'(q^2-m^2)f_A\right],$$

$$\mathcal{A}^\mu = 2\pi\left[\delta(q^2-m^2)\left(a^\mu f_A + \hbar H^\mu\right) + \hbar\tilde{F}^{\mu\nu}q_\nu\delta'(q^2-m^2)f_V\right],$$

Magnetization currents (spin-orbit int.) :

$$G^\mu = \frac{\epsilon^{\mu\nu\rho\sigma}n_\nu}{2q\cdot n}[\Delta_\rho(a_\sigma f_A) + F_{\rho\sigma}f_A].$$

$$H^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2a\cdot n}\Delta_\nu f_V.$$

$\xrightarrow{m=0}$

Side-jump terms : for CVE

Chen et al. 14.
Hidaka, Pu, DY, 16

$$(G/H)^\mu = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}\Delta_\nu f_{V/A}$$

$$f_{V/A} = f_R \pm f_L$$

obtained from the wave functions for free Dirac spinors instead of KB equations

- The rest frame : $n^\mu = q^\mu/m$ ⟹ N. Weickgenannt, et al, arXiv:1902.06513; J. H. Gao and Z. T. Liang, arXiv:1902.06510
- WFs for Weyl fermions are reproduced in the massless limit

Hidaka, Pu, DY, 16, 17

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Axial kinetic theory

- AKT in an arbitrary spacetime-dep. frame $(n^\mu = n^\mu(X))$:
- Scalar kinetic equation (SKE): remaining in the massless limit $\xrightarrow{m=0}$ CKT

$$0 = \delta(q^2-m^2)\Big[q\cdot\Delta f_V + \hbar\Big(\frac{E_\mu S^{\mu\nu}_{a(n)}}{q\cdot n}\Delta_\nu + S^{\mu\nu}_{a(n)}(\partial_\mu F_{\rho\nu})\partial^\rho_q + (\partial_\mu S^{\mu\nu}_{a(n)})\Delta_\nu\Big)f_A\Big] - \frac{\delta'(q^2-m^2)}{q\cdot n}B^\mu\Box_{\mu\nu}\tilde{a}^\nu$$
$$+\frac{\hbar}{2}\delta(q^2-m^2)\epsilon^{\mu\nu\alpha\beta}\Big[\Delta_\mu\Big(\frac{n_\beta}{q\cdot n}\Big)\left[(\Delta_\nu a_\alpha)+F_{\nu\alpha}\right] + \frac{n_\beta}{q\cdot n}\Big((\partial_\mu F_{\rho\nu})(\partial^\rho_q a_\alpha) + \Big[(\Delta_\nu a_\alpha) - F_{\rho\nu}(\partial^\rho_q a_\alpha)\Big]\Delta_\mu\Big)\Big]f_A,$$

$$\Box_{\mu\nu}\tilde{a}^\nu = q\cdot\Delta\tilde{a}_\mu + F_{\nu\mu}\tilde{a}^\nu.$$

- Axial-vector kinetic equation (AKE) :

BMT Eq

remaining in the massless limit

$$0 = \delta(q^2-m^2)\Big(q\cdot\Delta(a^\mu f_A) + F^{\nu\mu}a_\nu f_A\Big) + \hbar q^\mu\Big\{\delta(q^2-m^2)\Big[(\partial_\alpha S^{\alpha\nu}_{m(n)})\Delta_\nu + \frac{S^{\alpha\nu}_{m(n)}E_\alpha\Delta_\nu}{q\cdot n+m} + S^{\rho\nu}_{m(n)}(\partial_\rho F_{\beta\nu})\partial^\beta_q\Big]$$
$$-\delta'(q^2-m^2)\frac{q\cdot B}{q\cdot n+m}q\cdot\Delta\Big\}f_V + \hbar m\Big\{\frac{\delta(q^2-m^2)\epsilon^{\mu\nu\alpha\beta}}{2(q\cdot n+m)}\Big[m(\partial_\alpha n_\beta)\Delta_\nu + (mn_\beta+q_\beta)\Big(\frac{(E_\alpha-\partial_\alpha(q\cdot n))}{q\cdot n+m}\Delta_\nu$$
$$-(\partial_\nu F_{\rho\alpha})\partial^\rho_q\Big)\Big] + \delta'(q^2-m^2)\frac{(mn_\beta+q_\beta)\tilde{F}^{\mu\beta}}{q\cdot n+m}q\cdot\Delta\Big\}f_V.$$

$\xrightarrow{m=0}$ q^μ CKT

spin enslavement by chirality & momentum

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Further comments on AKT

- WFs are "frame independent" though the wave-function parts and distribution functions therein are both frame dependent.

- Solving AKT for $f_{V/A}(q,X)$ & $a^{\mu}(q,X)$ with a proper choice of n^{μ}.

- Using the WFs to compute the field-theory defined observables :

vector/axial-charge currents :

(anti-)symmetric energy-momentum tensors :

$$J^{\mu}_{V/5} = 4\int_q (\mathcal{V}/\mathcal{A})^{\mu}, \qquad T^{\mu\nu}_{S/A} = 2\int_q \left(\mathcal{V}^{\mu} q^{\nu} \pm \mathcal{V}^{\nu} q^{\mu}\right), \qquad \int_q \equiv \int d^4q/(2\pi)^4.$$

- The anti-symmetric EM tensor is responsible for angular-momentum transfer (via spin-orbit coupling) :

$$\partial_{\lambda} M_C^{\lambda\mu\nu} = 0. \Rightarrow \underbrace{-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_{\lambda} J_{5\rho}}_{\text{spin}} + \underbrace{2T^{\mu\nu}_{A}}_{\text{orbit}} = 0$$

(AM conservation)

(see also DY,18 for the analysis with $m = 0$)

already captured by one of master Eqs., $q_{\nu}\mathcal{V}_{\mu} - q_{\mu}\mathcal{V}_{\nu} = \frac{\hbar}{2}\epsilon_{\mu\nu\rho\sigma}\Delta^{\rho}\mathcal{A}^{\sigma}$

Conclusions & outlook

- We have presented the AKT for massive fermions with EM fields, which can track the dynamics of vector/axial charges and spin polarization.

- The AKT incorporates the quantum corrections such as spin-orbit interaction and chiral anomaly.

- The AKT reproduce the CKT in the massless limit with the manifestation of spin enslavement.
- Extensions and applications :
  - Spin transport for strange quark in HIC : collisions have to be involved in AKT (where QCD enters)

    ⇒ track the evolution of the polarization for Λ hyperons
  - It is not guaranteed that the polarization should reach thermal equilibrium in the hadronic phase (chemical freeze-out≠polarization freeze-out).

Thank you!

Theoretical models for spin polarization

- Statistical model/Wigner-function approach :
- The present studies of Λ polarization assume thermal equilibrium of Λ at freeze-out, where the polarization is mostly proportional to thermal vorticity.

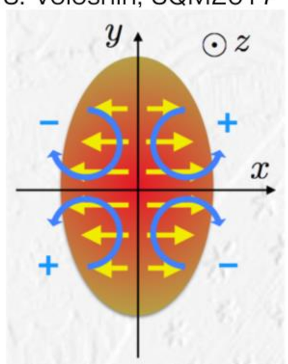
F. Becattini, et.al. 13   R. Fang, L.-G. Pang, Q. Wang, and X.-N. Wang, 16

$$\mathcal{P}^{\mu}(q) \approx \frac{1}{8m}\epsilon^{\sigma\mu\nu\rho} q_{\sigma} \frac{\int d\Sigma \cdot q\,\omega_{\nu\rho} f_q^{(0)}(1-f_q^{(0)})}{\int d\Sigma \cdot q f_q^{(0)}}, \qquad \omega_{\nu\rho} = \frac{1}{2}\left(\partial_{\rho}(u_{\nu}/T) - \partial_{\nu}(u_{\rho}/T)\right).$$

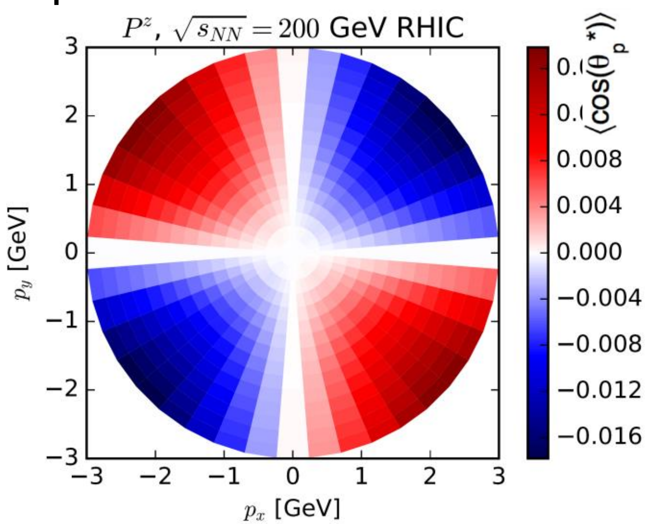
- Sign problem for local polarization :

Longitudinal Polarization (P^z):

S. Voloshin, SQM2017

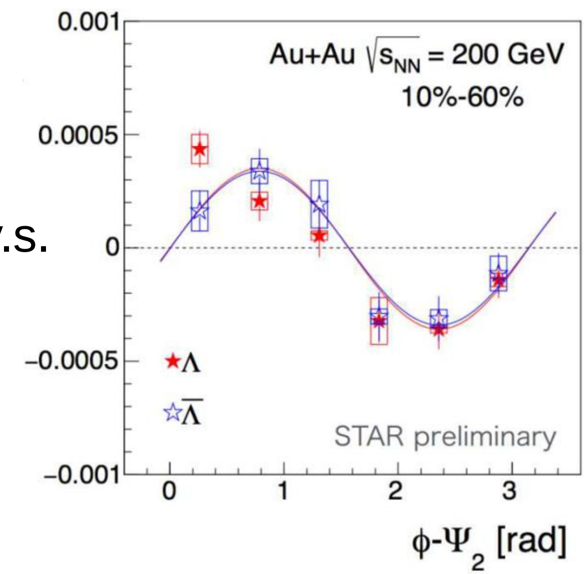


Niida, Quark matter 2018.



F. Becattini, I. Karpenko, 17

V.S.



(same structure, opposite signs!)

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AM conservation in global equilibrium

- Global equilibrium (no collisions $\omega^\mu \neq 0,\ B^\mu \neq 0.$) : $\partial_\mu J^\mu_{V/5} = \partial_\mu T^{\mu\nu} = 0,$
- Conservation of canonical EM & AM tensors :

$$\partial_\mu \bar{T}^{\mu\nu} = 0, \quad \partial_\lambda M_C^{\lambda\mu\nu} = 0.$$

⇒ spin $-\frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\partial_\lambda J_{5\rho}$ $+ 2T_A^{\mu\nu}$ orbit $= 0$

- Weyl fermions : DY, 18

$$T_A^{\mu\nu} = \frac{\hbar}{2}N_A\left(\omega^\mu u^\nu - \omega^\nu u^\mu\right)$$

from side-jumps

$$M^{\lambda\mu\nu}_{\rm spin}(X) = \frac{\hbar}{2}\epsilon^{\lambda\mu\nu\rho}\left(N_A u_\rho + \hbar\sigma_{BA}B_\rho + \hbar\sigma_{\omega A}\omega_\rho\right)$$

CSE & CVE

- $\mathcal{O}(\hbar)$: spin-orbit cancellation
- Higher orders : we need higher-order WFs.
- Near local equilibrium :

local torque even without EM fields

$$\partial_\lambda M_C^{\lambda\mu\nu} = X^{[\mu}F^{\nu]\rho}J_{V\rho} - \frac{u_\rho}{\tau_R}X^{[\mu}\delta T^{\rho\nu]} - \frac{\hbar}{4}\partial_\lambda\left(X^{[\mu}\epsilon^{\nu]\lambda\alpha\beta}\frac{u_\alpha \delta J_{5\perp\beta}}{\tau_R}\right)$$

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WFs from free Dirac fields

- Construction from wave functions :

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_{\mathbf{p}}}}\sum_s u^s(p)e^{-ip\cdot x}a^s_{\mathbf{p}}, \quad u^s(p) = (\sqrt{p\cdot\sigma}\xi^s, \sqrt{p\cdot\bar{\sigma}}\xi^s)^T$$

- Lesser propagator :

$$S^<(x,y) = \int_{\mathrm{inv}} dpdp' \sum_{s,s'}\left(u^s(p)\bar{u}^{s'}(p')\langle a^{s'\dagger}_{\mathbf{p}'}a^s_{\mathbf{p}}\rangle e^{ip_-\cdot X - \frac{i}{2}p_+\cdot Y}\right), \quad p^\mu_\pm = (p\pm p')^\mu$$

- Parameterizing the density operators :

$$\int \frac{d^3p_-}{(2\pi)^3}\left\langle a^{s'\dagger}_{\mathbf{p}'}a^s_{\mathbf{p}}\right\rangle e^{-ip_-\cdot X} = \delta_{ss'}\tilde{f}_V + \mathcal{A}_{ss'}\tilde{f}_A, \quad \mathcal{A}_{ss'}(X,q) \neq 0 \text{ when } s \neq s'$$

parameterization :

$$\textstyle\sum_s \xi_s\xi_s^\dagger = n\cdot\bar{\sigma} = I$$

$$\textstyle\sum_{s,s'} \xi_s\mathcal{A}_{ss'}\xi^\dagger_{s'} = \hat{S}\cdot\bar{\sigma} \Rightarrow \hat{S}\cdot n = 0$$

- Performing p_- expansion ($\hbar$ expansion) ⇨ WFs without EM fields

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Magnetization currents

- Re-parameterization :

$$f_V = \tilde{f}_V - \frac{\hbar S^{\mu\nu}_{m(n)}}{q\cdot n}\hat{S}_\mu \partial_\nu \tilde{f}_A, \quad f_A = \tilde{f}_A, \quad S^{\mu\nu}_{m(n)} = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2(q\cdot n + m)} = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2a\cdot n}.$$

$$a\cdot n = -\hat{S}\cdot q, \quad a_{\perp\mu} = \frac{-\hat{S}\cdot q}{q\cdot n + m} q_{\perp\mu} + m\hat{S}_\mu, \quad v^\mu_\perp \equiv v^\mu - (v\cdot n)n^\mu$$

- Free WFs up to $\mathcal{O}(\hbar^1)$:

$$\mathcal{V}^\mu = 2\pi\delta(q^2 - m^2)\Big[q^\mu f_V + \hbar\frac{\epsilon^{\mu\nu\alpha\beta}n_\beta}{2q\cdot n}\partial_\nu(a_\alpha f_A)\Big],$$

$$\mathcal{A}^\mu = 2\pi\delta(q^2 - m^2)\Big[a^\mu f_A + \hbar S^{\mu\nu}_{m(n)}\partial_\nu f_V\Big]. \Longrightarrow H^\mu = S^{\mu\nu}_{m(n)}\Delta_\nu f_V$$

generalization

- Freedom for redefining a^μ : $\bar{a}^\mu f_A \equiv a^\mu f_A + \hbar H^\mu$

non-uniqueness of magnetization-current terms