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Real-Time-Evolution of Heavy-Quarkonium Bound States

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Heavy Quarkonia in Heavy Ion Collisions

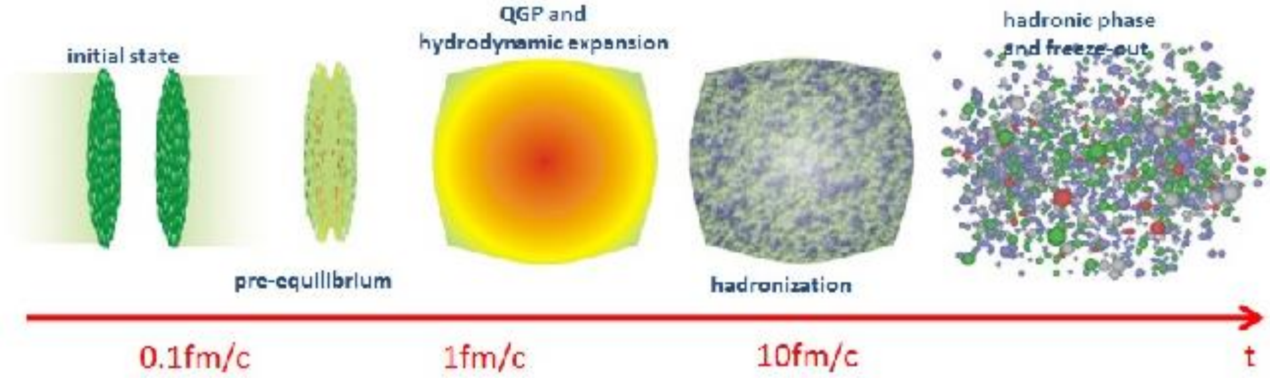

illustration by
L. McLerran

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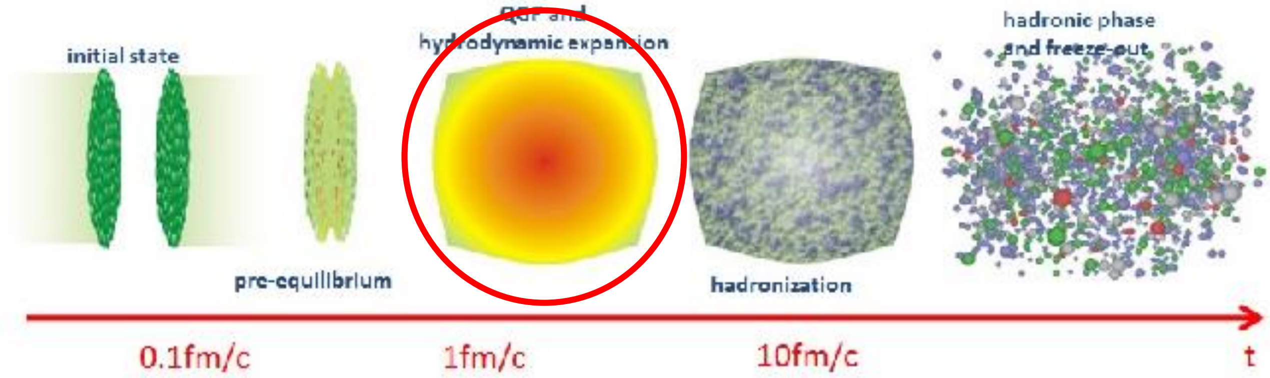

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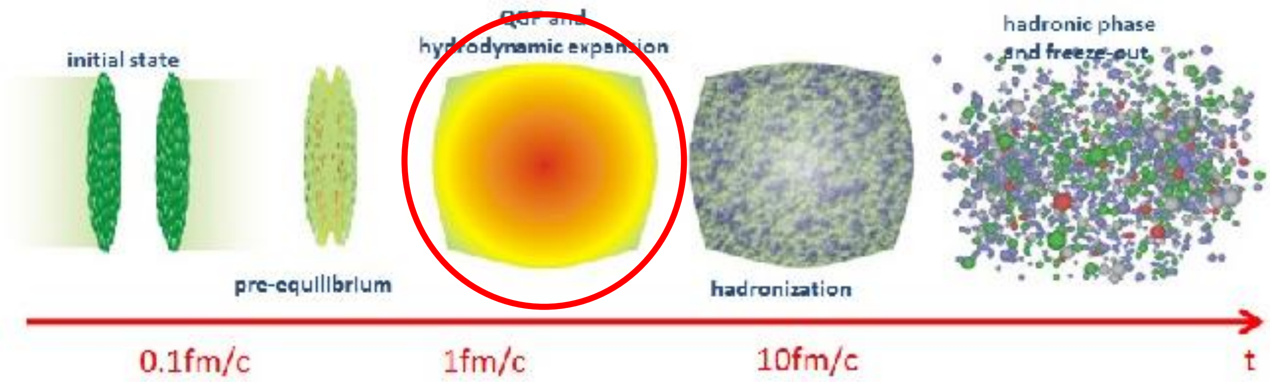

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- Heavy Quarkonia (Charmonium, Bottomonium) are **well controlled** experimental and theoretical **probes** for the **quark-gluon-plasma**
- Phenomenological models describe quarkonium supression via a Schrödinger equation + **assumption** of **early formation** of bound states

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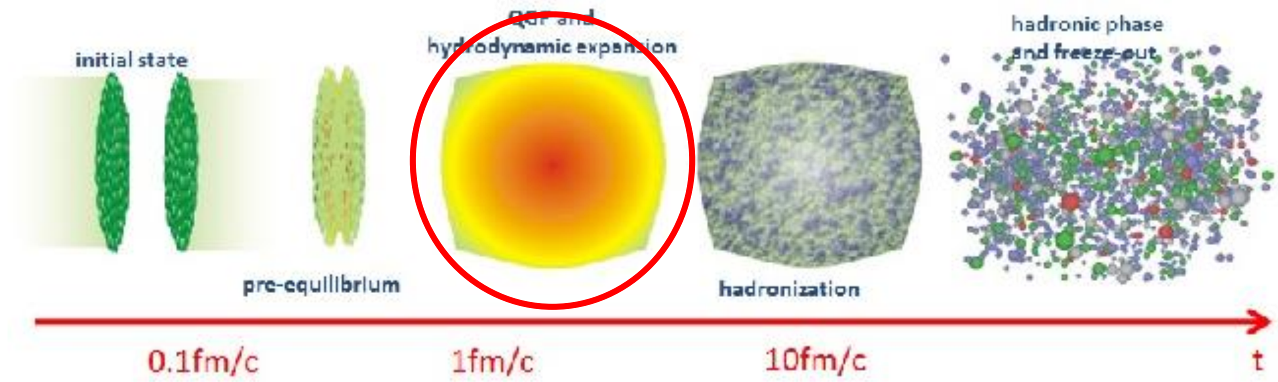

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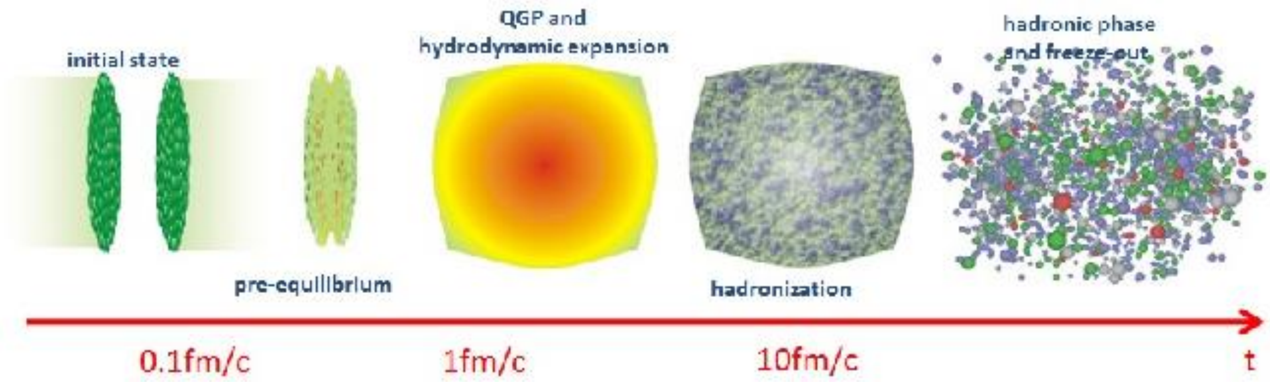
- Heavy Quarkonia (Charmonium, Bottomonium) are **well controlled** experimental and theoretical **probes** for the **quark-gluon-plasma**
- Phenomenological models describe quarkonium supression via a Schrödinger equation + **assumption** of **early formation** of bound states
- Characterized by a **separation of scale**:

$$M_Q \gg M_Q v \gg M_Q v^2 \gg \Lambda_{\mathrm{QCD}}$$

- Very heavy states, e.g. Υ(1S), already bound **Coulombically**

M_Q … heavy quark mass ($m_{Bottom} = 4.18(3)$GeV [PDG 2017])
v … relative velocity in centre of mass frame ($v_{Bottom}^2 \approx 0.1$)
Λ_{QCD} … momentum scale below which gluons strongly interacting
Mv … typical momentum
Mv^2 … typical kinetic or potential energy

Heavy Quarkonia in Early Stages in HICs



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Early dynamics of heavy quarkonium in HIC largely **unexplored**

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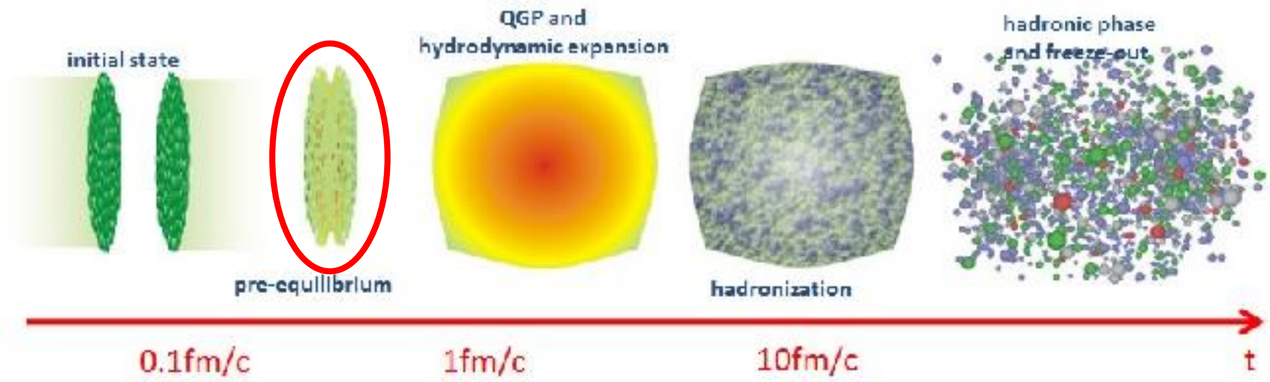

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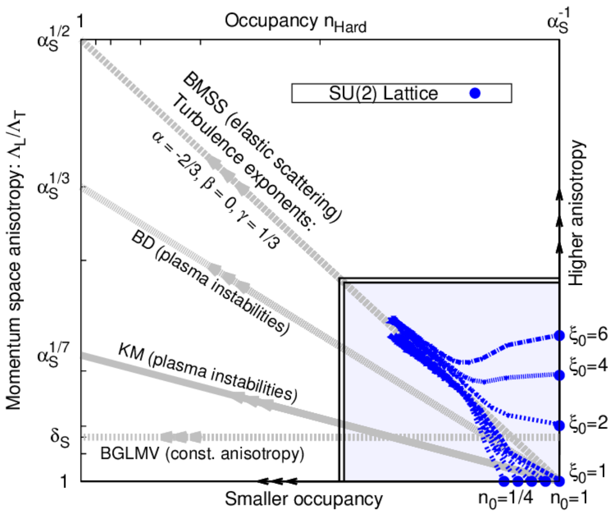
Early dynamics of heavy quarkonium in HIC largely **unexplored**

Rule of thumb via uncertainty relation: $\tau_{form} \sim 1/E_{\text{bind}} \approx 0.2 \ldots 0.4\text{fm/c}$

Can we find hints for heavy-quarkonium formation in the glasma?

Real-Time Evolution of the Gauge Fields

- Vital insight into glasma dynamics via **classical statistical simulations** of gauge fields in expanding geometry



J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan, PRL. 114 (2015) 061601

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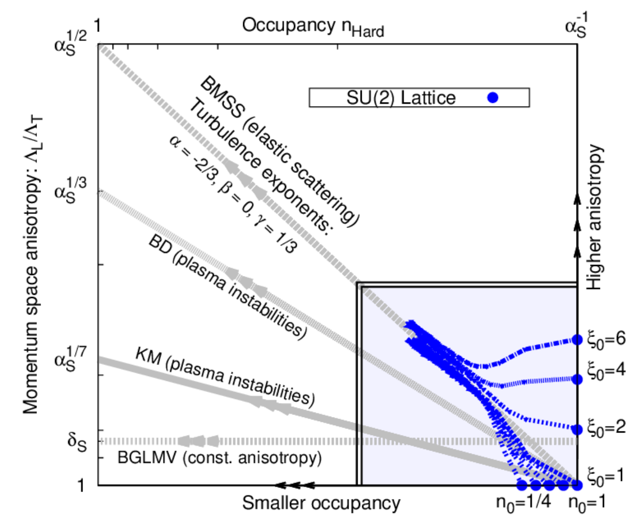

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- In this study **Hamiltonian evolution** in axial gauge, formulated in spatial links and electric fields (Leapfrog) in a non-expanding box

$$\partial_t U_j(x,t) = iE_j^a U_j(x,t) \qquad \partial_t E_j^a(x,t) = -2\mathrm{ImTr}\left\{T^a \sum_{j\neq k}\left[U_{ij}(t,x) + U_{i(-j)}(t,x)\right]\right\}$$

$$U_j(x) = \exp\left(ia_j A_j^a T^a\right) \quad E_j^a = F_{0j}^a = a_0 a_j 2ImTr[T^a U_{0j}]$$

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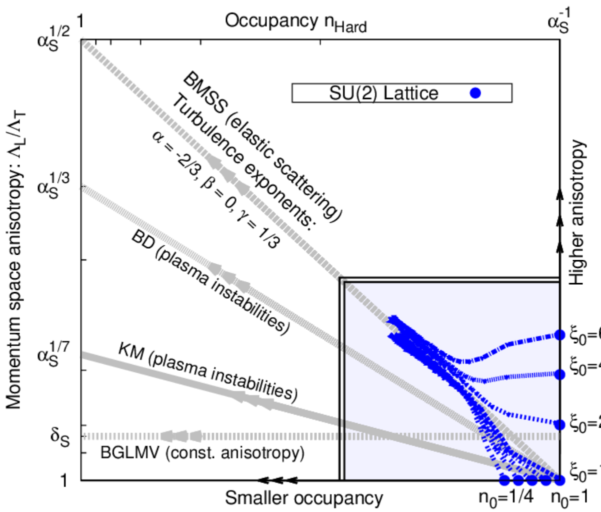

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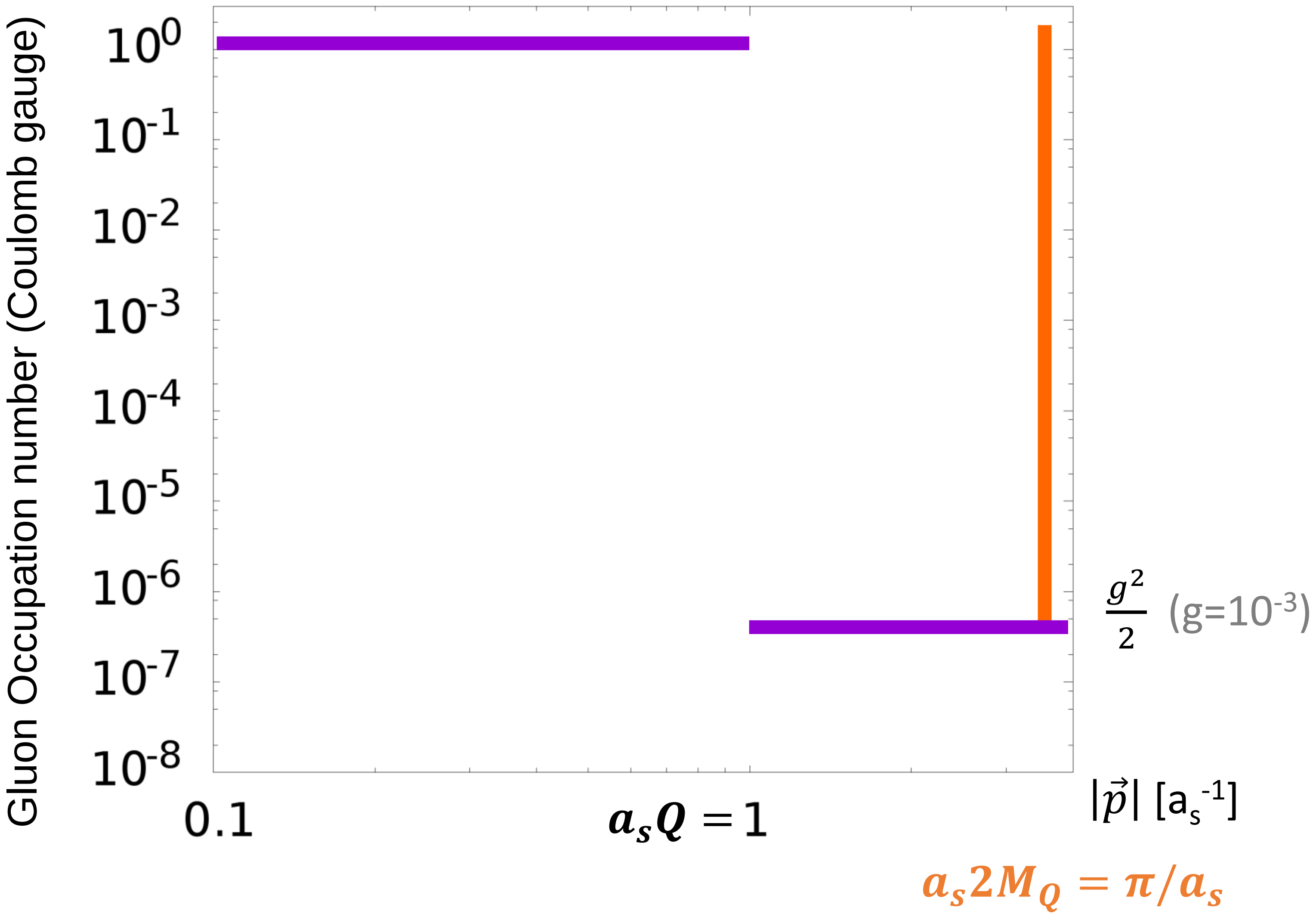
- Initial conditions drawn from a statistical ensemble

The real-time NRQCD setup



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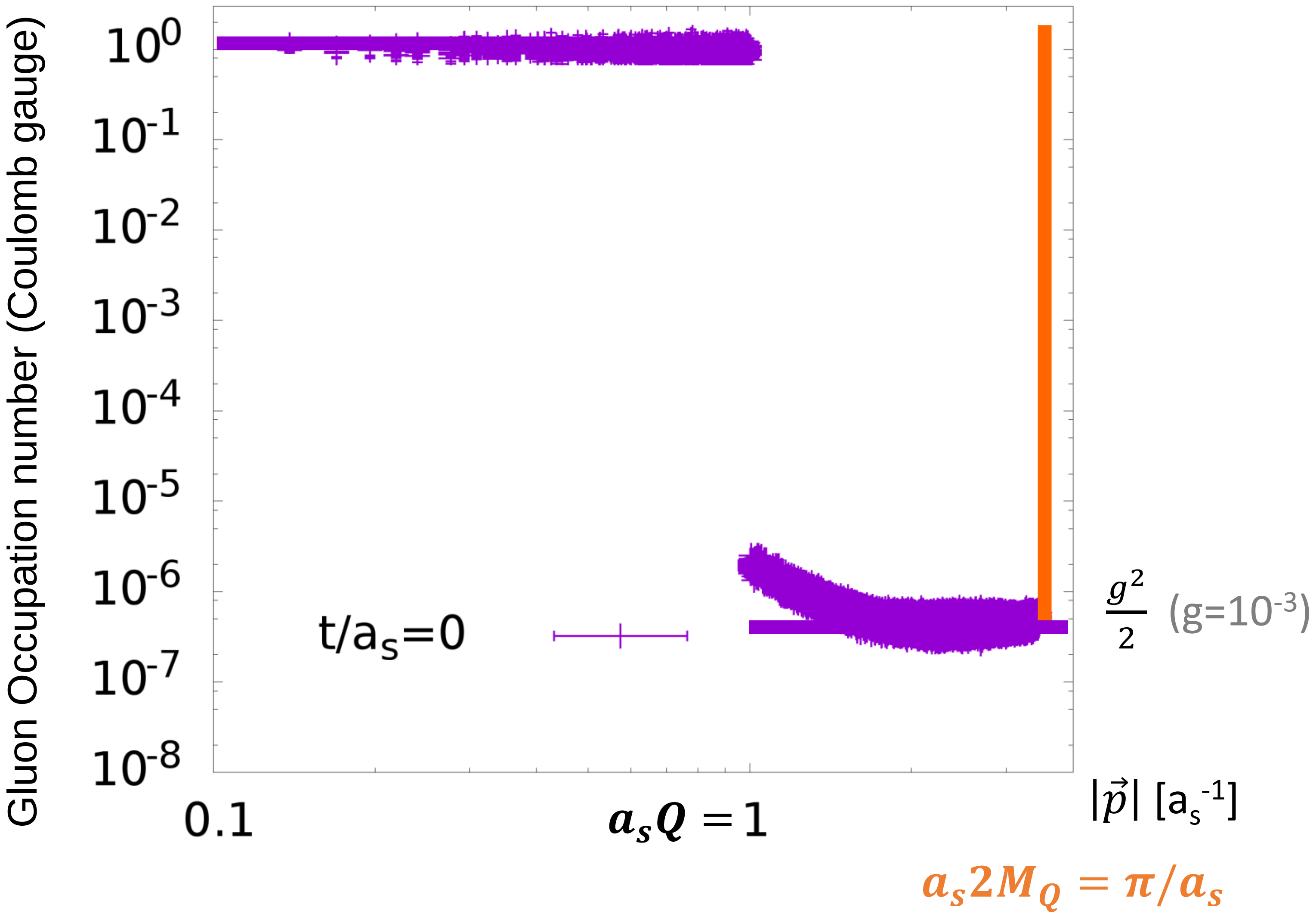

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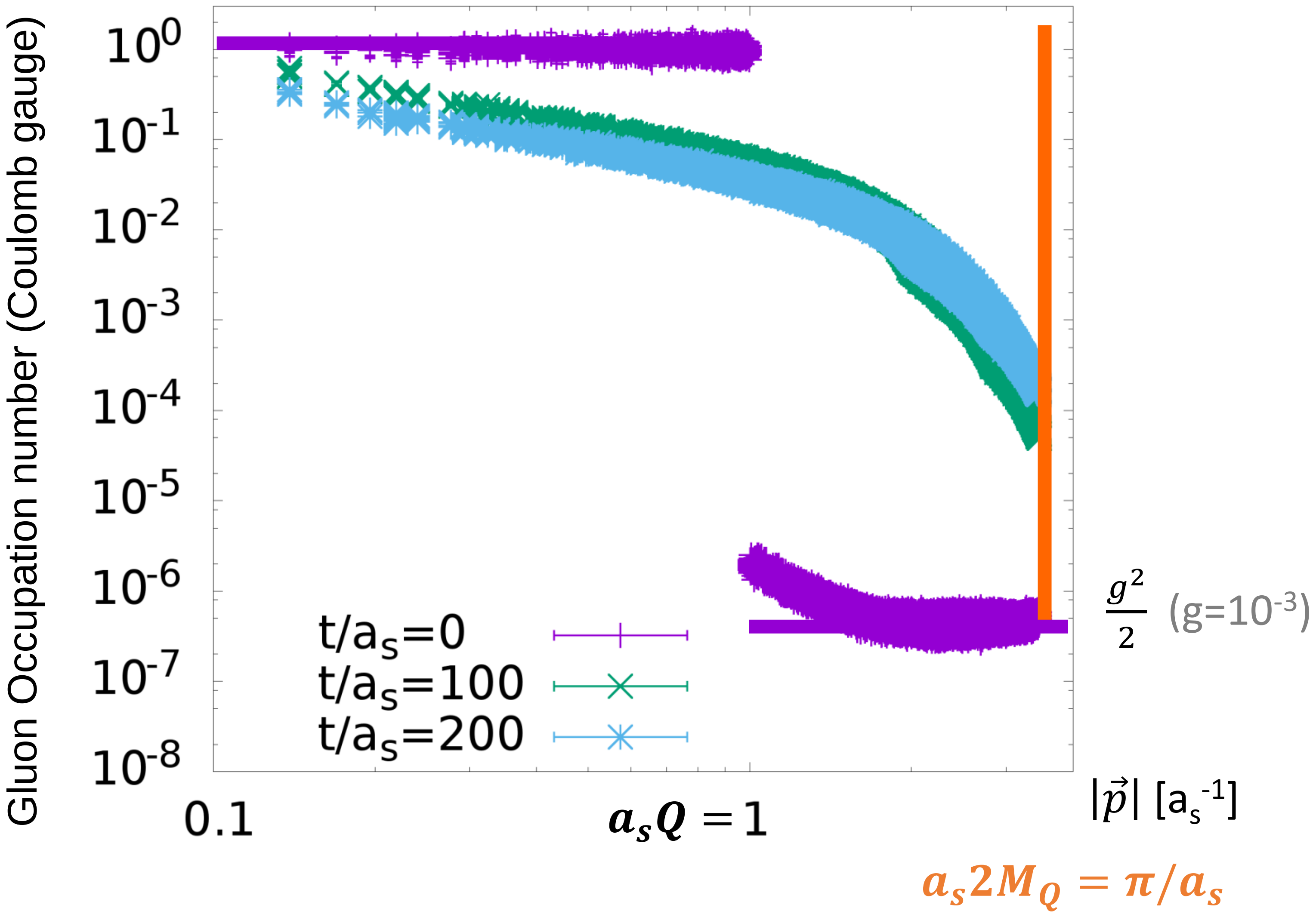

Real-time Lattice NRQCD

- Effective **non-relativistic formulation** of heavy quarks from systematic expansion of QCD action in quark velocity v for 2-component **pauli spinors** $\boldsymbol{\psi, \chi}$
- Hamiltonian to order $O(v^3)$ with leading order Wilson coefficients c_i=1

$$H^{\psi} = -\frac{\vec{D}^2}{2M} - c_1 \frac{g}{2M} \vec{\sigma} \cdot \vec{B} - c_2 \frac{g}{8M^2} \vec{D} \cdot \vec{E} - c_3 \frac{ig}{8M^2} \vec{\sigma} \cdot \left(\vec{D} \times \vec{E} - \vec{E} \times \vec{D} \right)$$

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- **Real-time** quarkonium current **correlator** $D^{>}$ from heavy quark propagator G

$$D_V^{>}(x_2, x_1) \sim i < J^{i}_{\mathrm{NRQCD}}(x_2) J^{+}_{i,\mathrm{NRQCD}}(x_1) >$$

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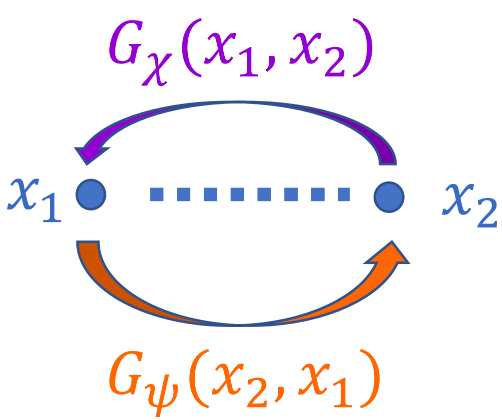

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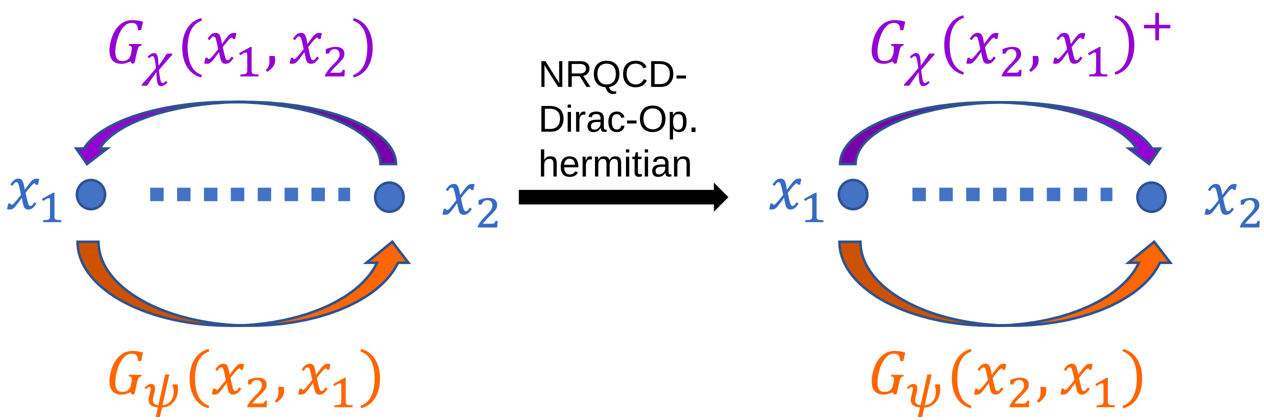

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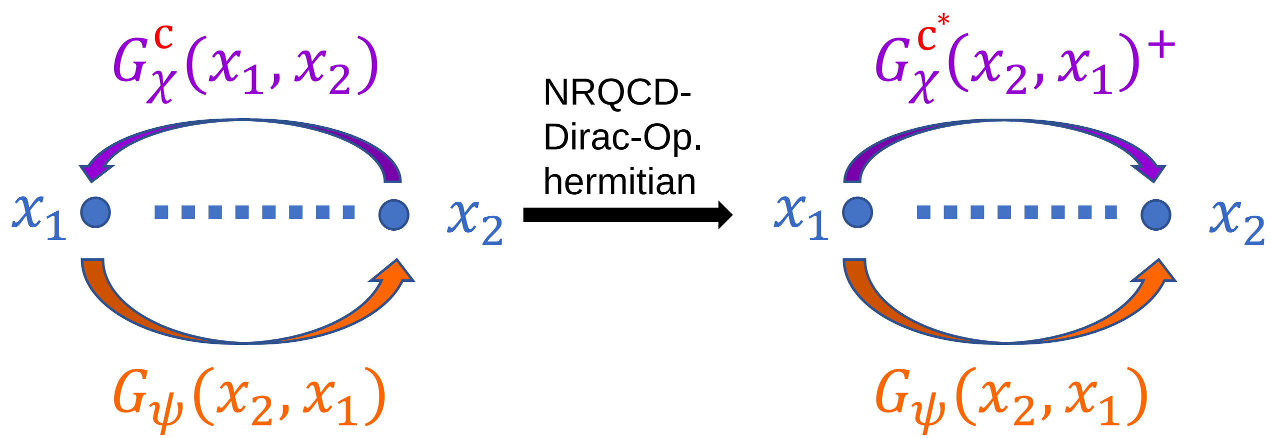

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- Heavy quark equation of motion: $G^{\psi}[U]^{t+\Delta t}{}_{x_2,x_1} = \mathbf{exp}[-\boldsymbol{i\Delta t} \, \boldsymbol{H}^{\psi}[\boldsymbol{U}]] \cdot G^{\psi}[U]^{t}{}_{x_2,x_1}$

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$$G(t + a_t) = (1 - i a_t H[U(t)]) \cdot G(t)$$

Often via forward Euler: cheap but 1st order in dt, inherently unstable (Courant), range of validity of NRQCD mixed with breakdown of discretization

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$$G(t + a_t) = \left(1 + \frac{ia_t}{2} H[U(t)]\right)^{-1} \cdot \left(1 - \frac{ia_t}{2} H[U(t)]\right) \cdot G(t)$$

Optimal rational approximation of exp (Crank-Nicholson, $O(dt^2)$): **unconditionally stable**, no mixing of range of validity. (No operator splitting via MPI PETSc)

Wigner Coordinates for Non-Equilibrium

- **No time translational invariance**: need to correctly account for relative and central time coordinate in 2pt functions:

$$t = \frac{t_2 + t_1}{2} \quad s = t_2 - t_1$$

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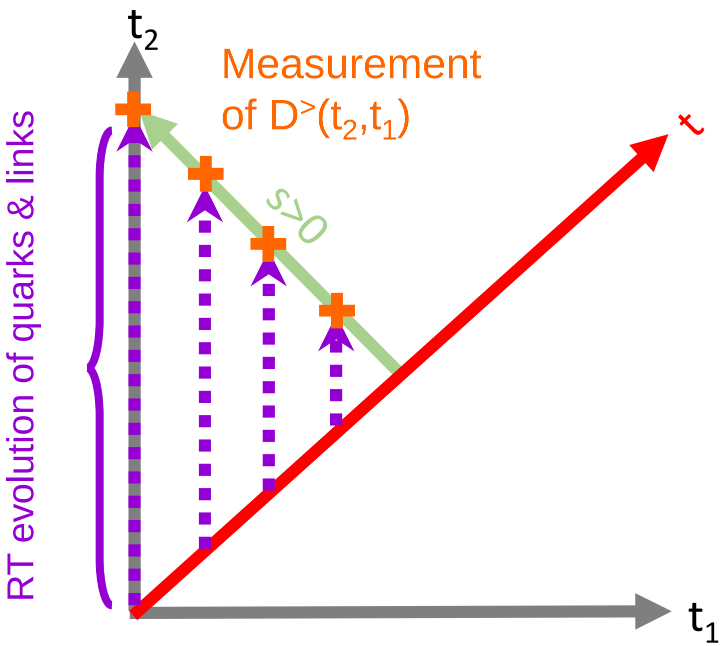
- Spectral function from Fourier transform over **finite** temporal extent in s

$$\rho(t, \omega, \boldsymbol{p} = 0)$$
$$= 2Im[\int_0^{s_{max}} D^> \left(t + \frac{s}{2}, t - \frac{s}{2}, \boldsymbol{p} = 0\right) e^{-i\omega s}\, ds]$$



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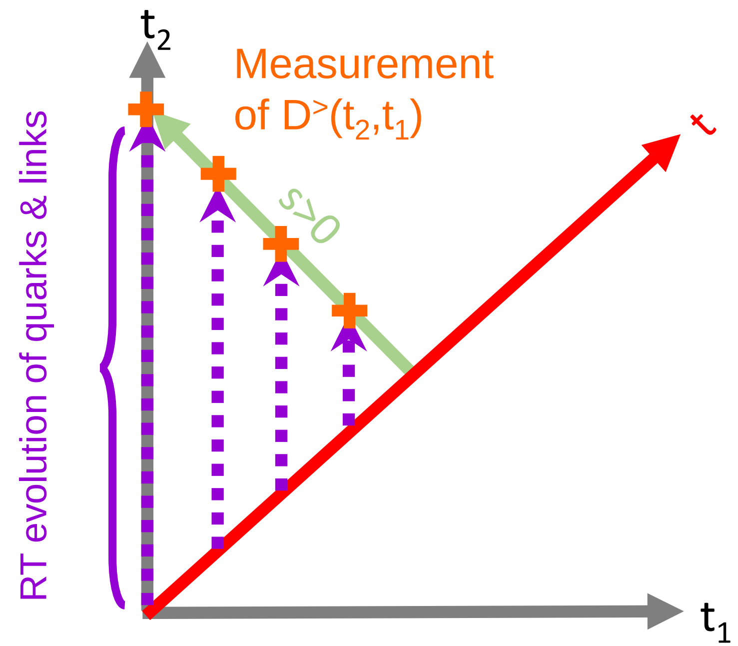
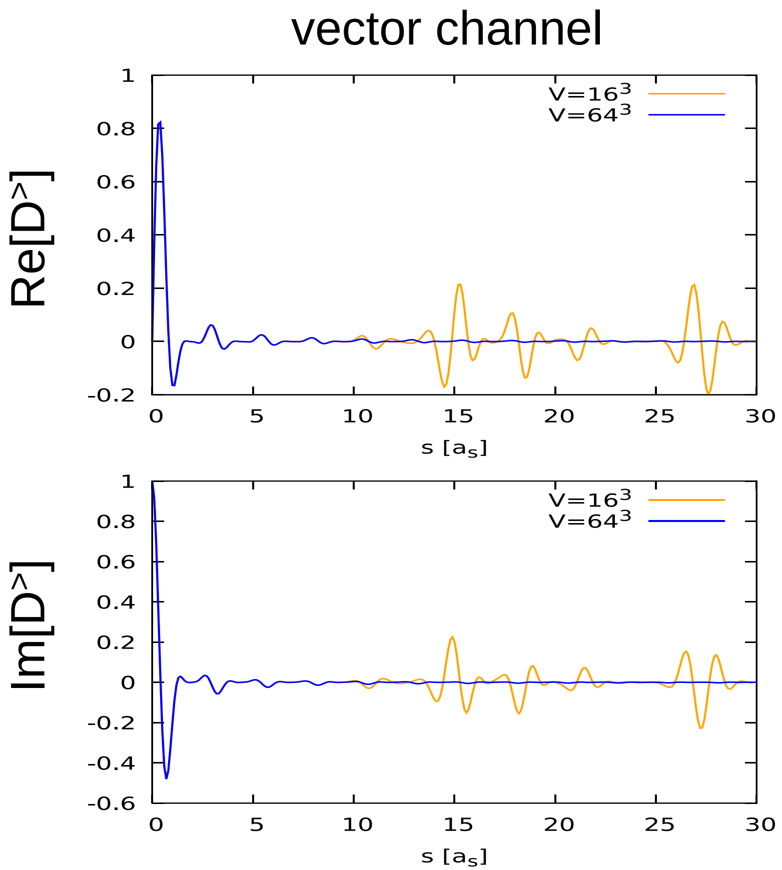


- Spectral function has **explicit t dependence**, signaling real-time evolution of gauge fields

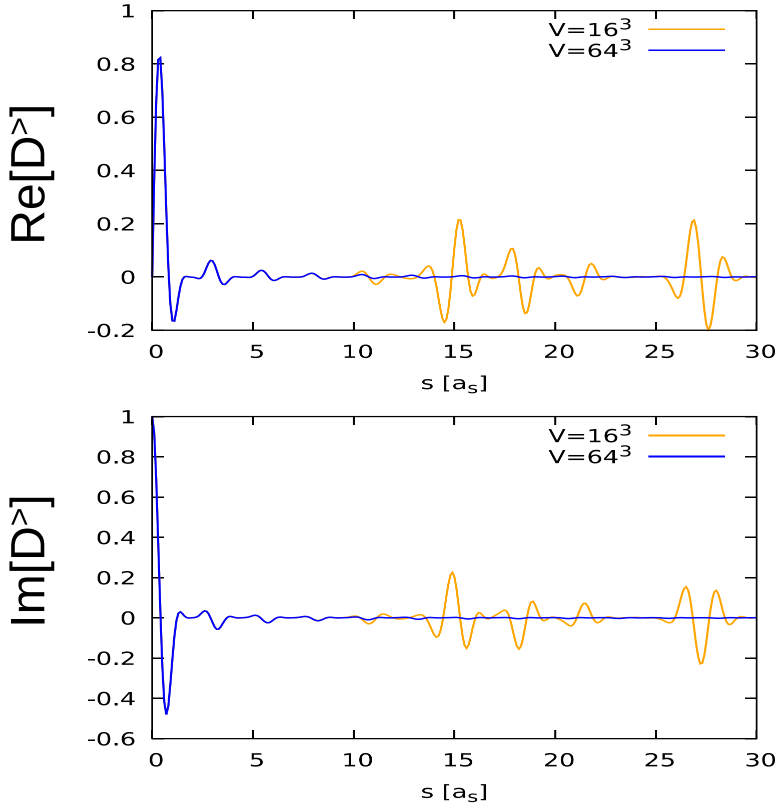
Free theory sanity check

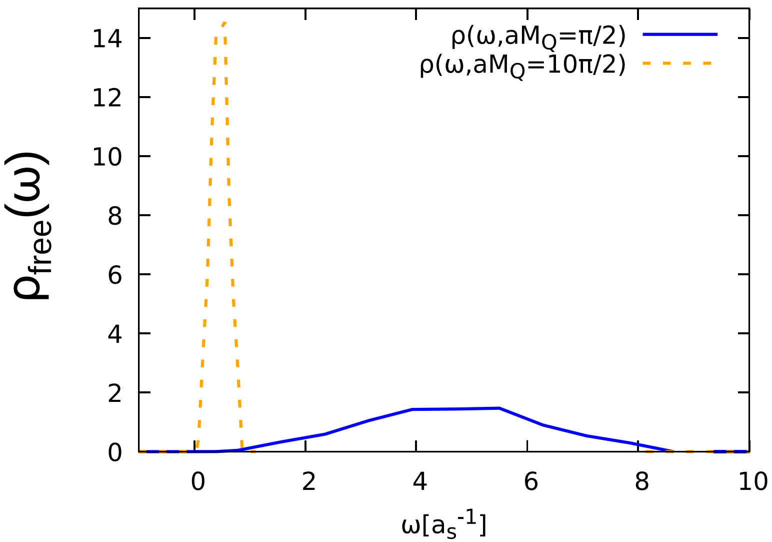


- **Real-time** correlation function is **complex** – finite volume effects as recurrence

Free theory sanity check

vector channel





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- Free spectral function reproduced – reducing mass does not lead to breakdown

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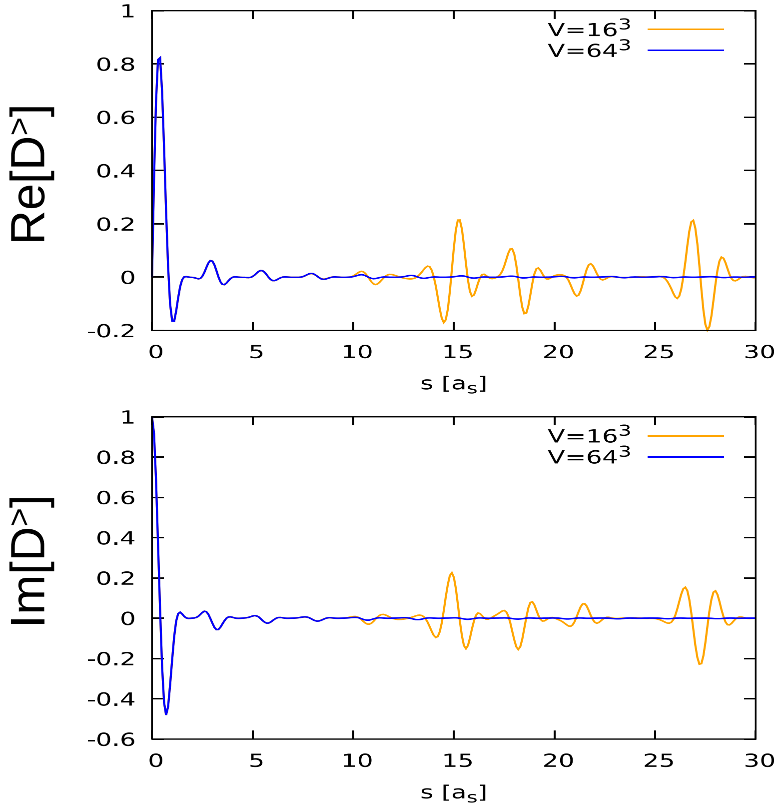

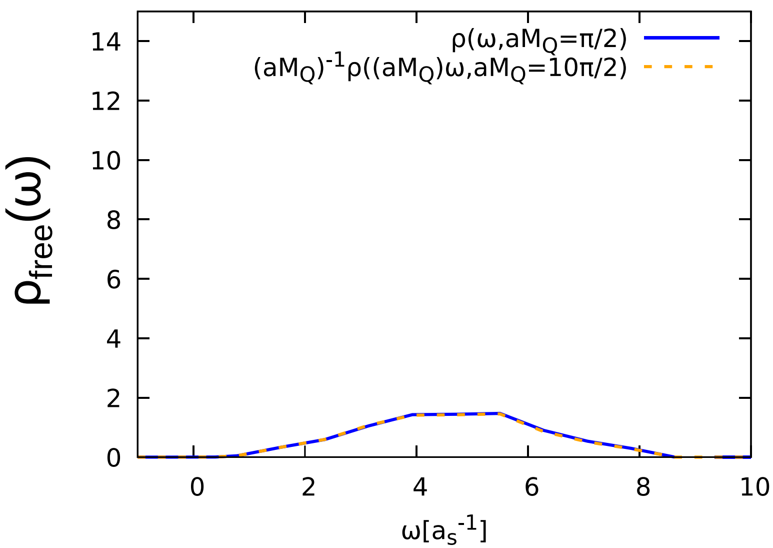

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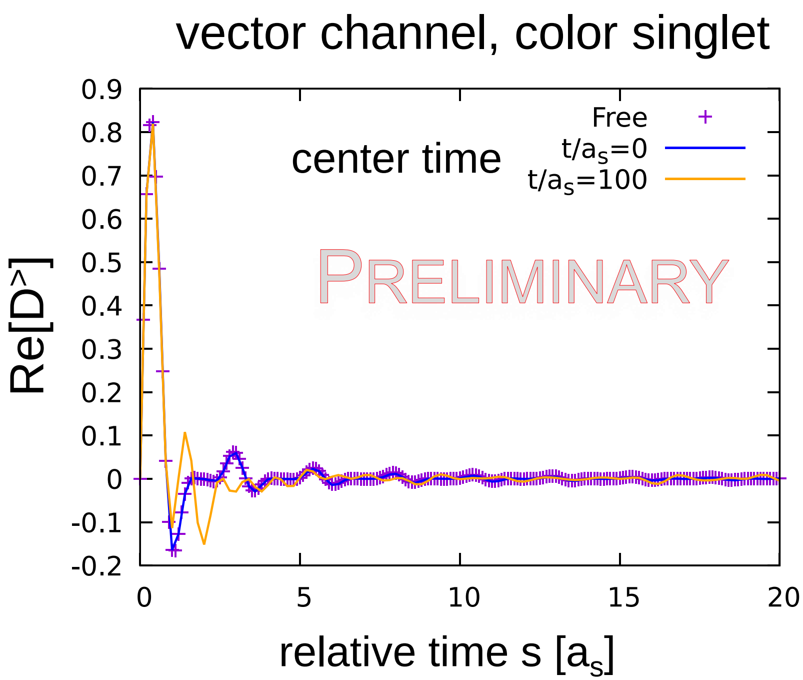
Quarkonium in the Glasma (I)



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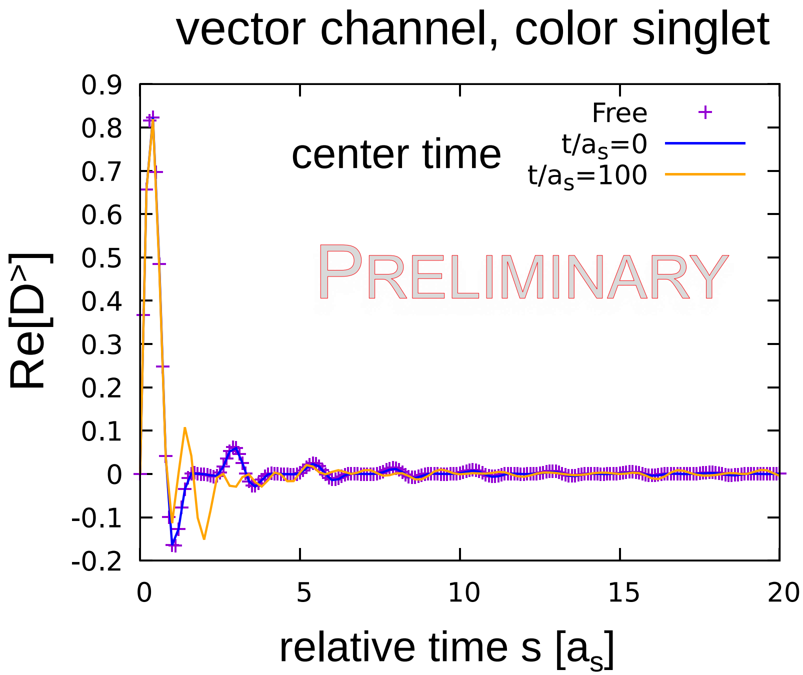

- **Low enegy gluons** do not significantly impact quarks at early times
- Bulk glue effects manifest in the **intermediate (s,t) time physics** of **heavy quarks**

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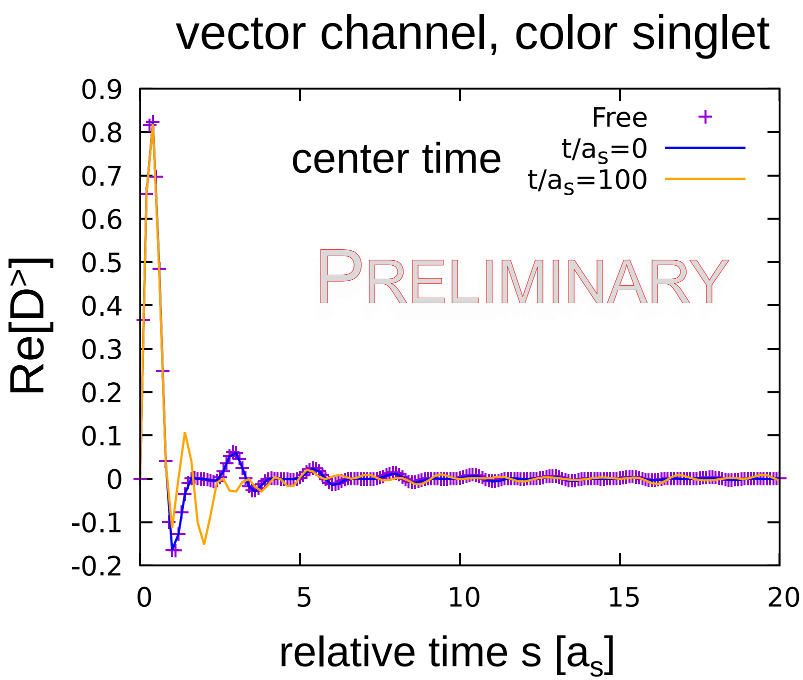

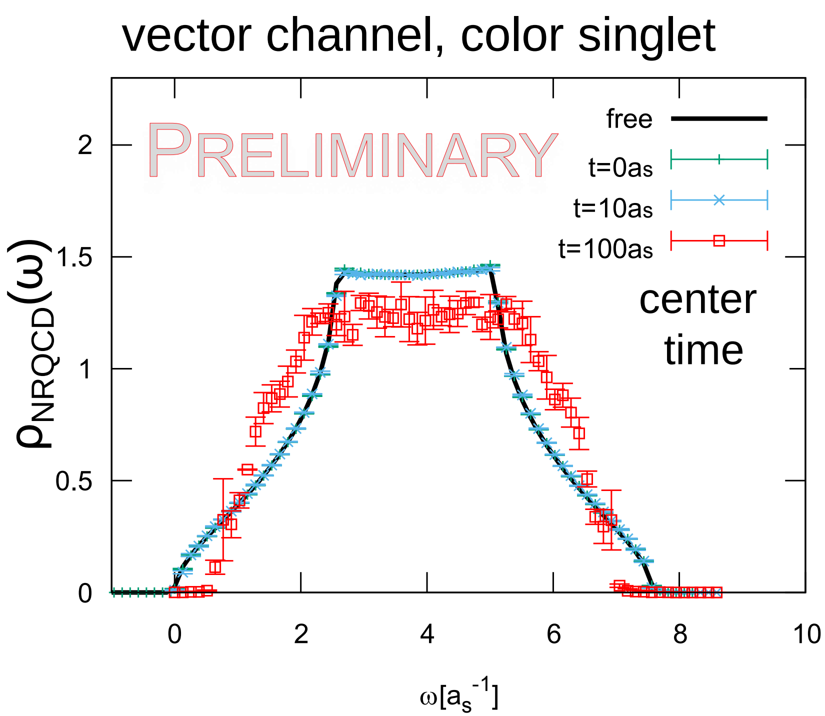

V=64^3
Q=1
n=1
g=10^{-3}

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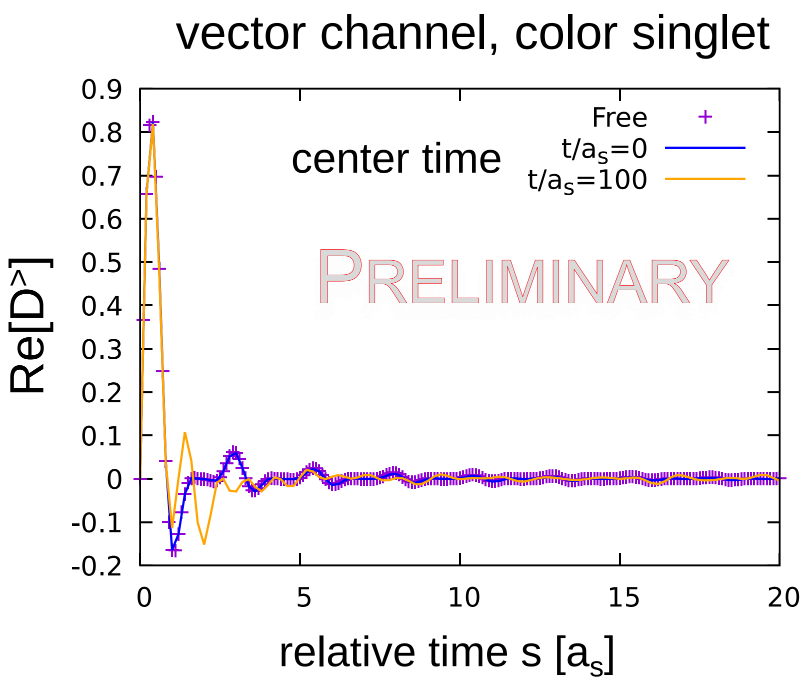
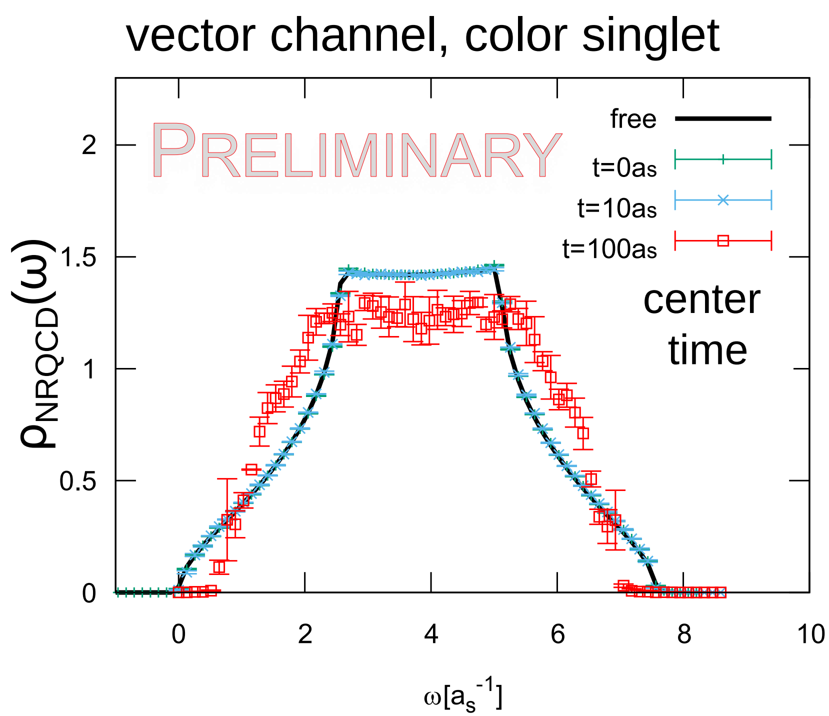
- Bulk glue effects manifest in the **intermediate (s,t) time physics** of **heavy quarks**

Quarkonium in the Glasma (I)





V=64^3
Q=1
n=1
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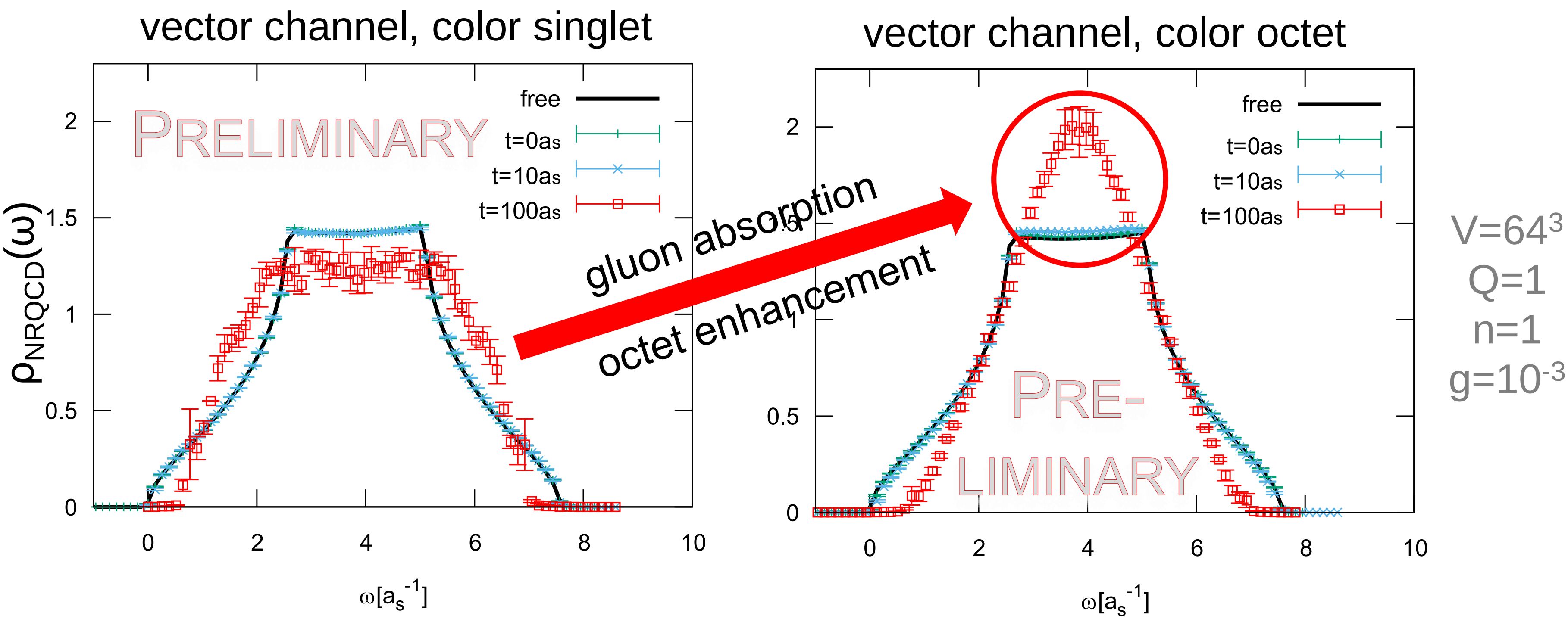
- **Low enegy gluons** do not significantly impact quarks at early times
- Bulk glue effects manifest in the **intermediate (s,t) time physics** of **heavy quarks**
- At the parameters used here, **no signs for binding** into clear resonances

Quarkonium in the Glasma (II)



- Reduction of singlet amplitude and broadening understood from **gluon absoprtion**
- **Octet enhancement** from interaction with low enegy gluonic bulk

Classical Thermal Equilibrium

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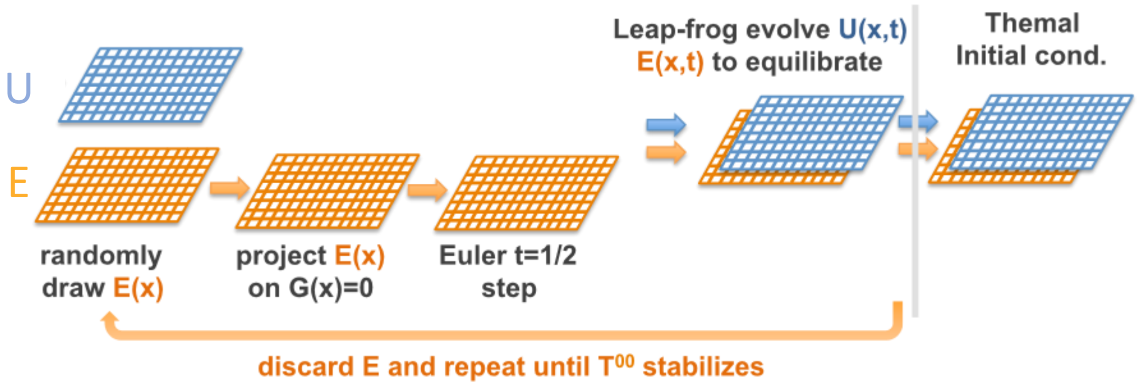

A. Akamatsu, A. Rothkopf, N. Yamamoto, JHEP 1603 (2016) 210

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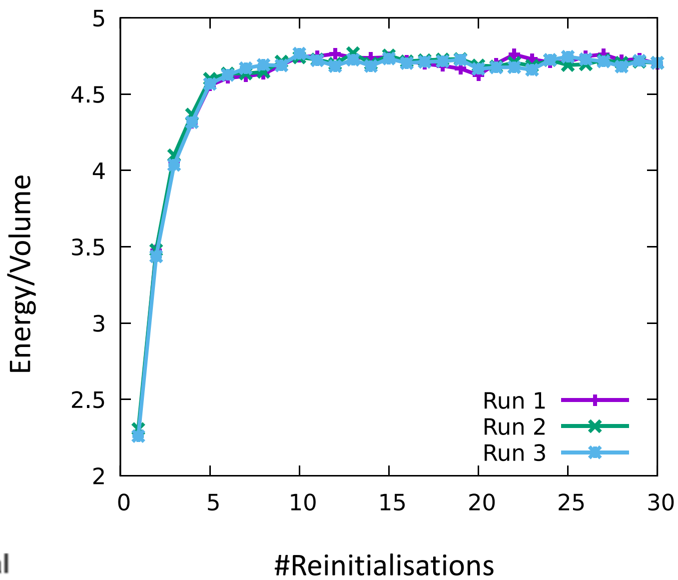

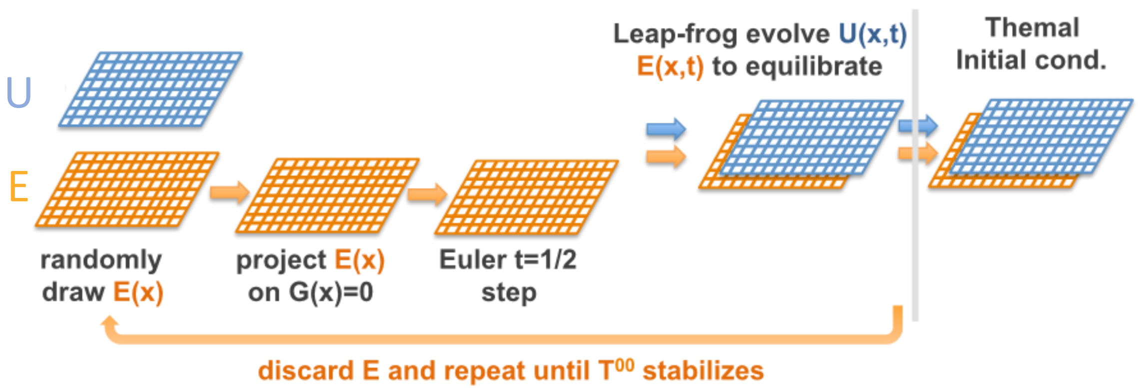

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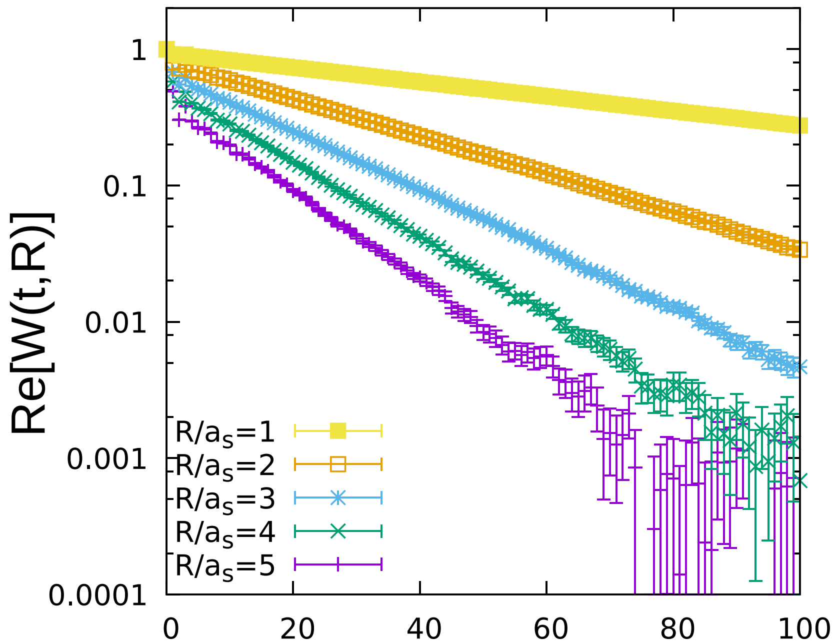
Static Potential in the Classical Equilibrium

- Consider static quarks via the **equilibrium real-time Wilson loop** W(t,x)
- Attempt to extract effective **real-time potential** via Wilson loop spectral function
  Re[V] from position of lowest lying peak, **Im[V] from width**, **see Y.Burnier, A. Rothkopf PRD86 (2012) 051503**



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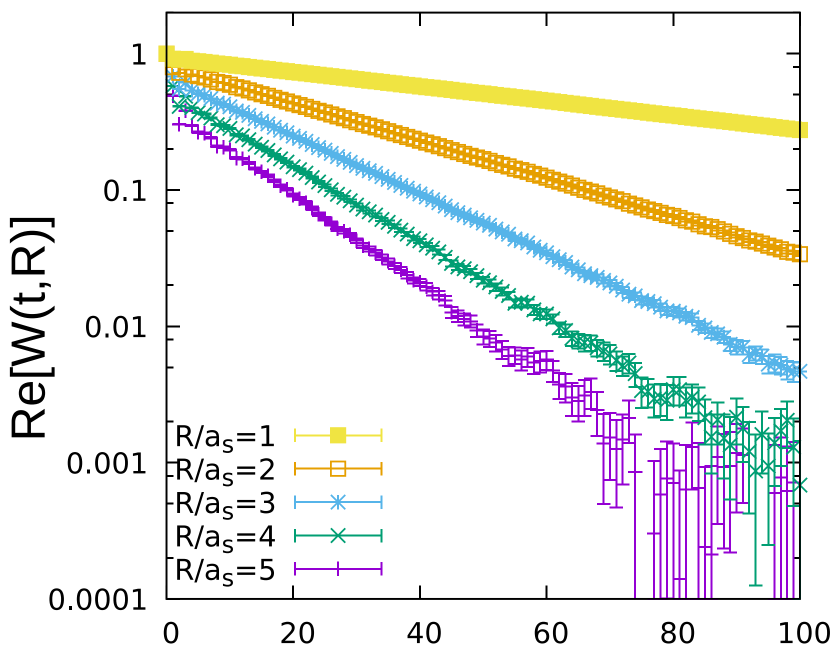

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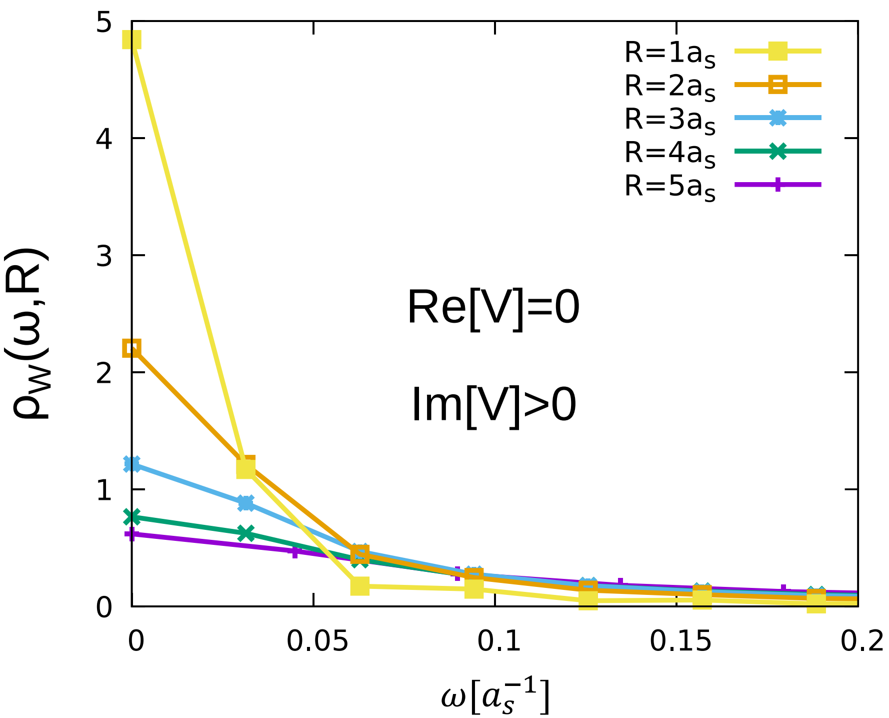

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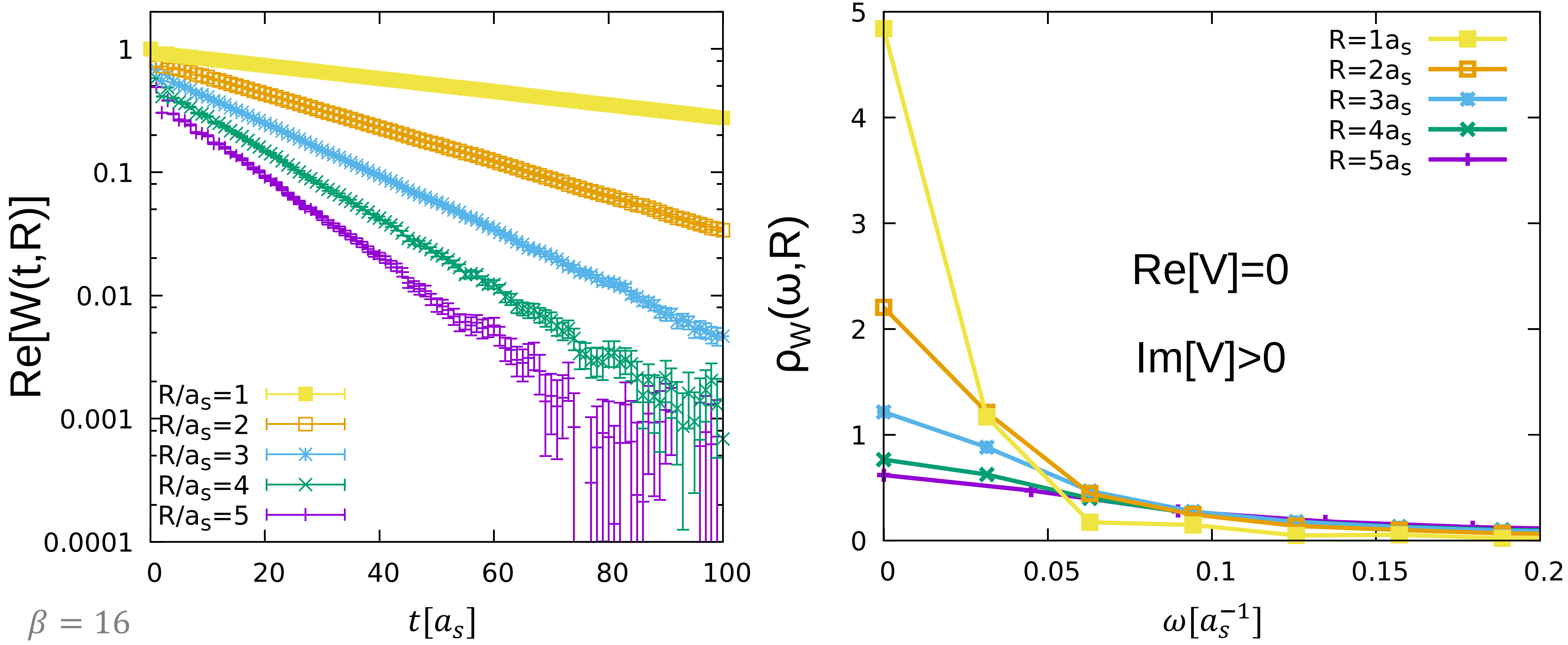

- Same result as found in the literature: **no real-part** of the potential emerges

see M. Laine et.al. JHEP 0709 (2007) 066

Finite Mass Wilson Loop

- Replacing temporal Wilson lines by fermion and anti-fermion propagators

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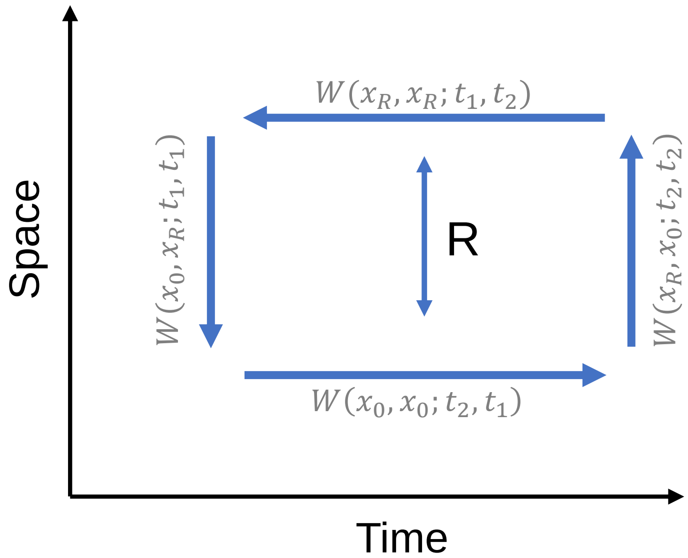

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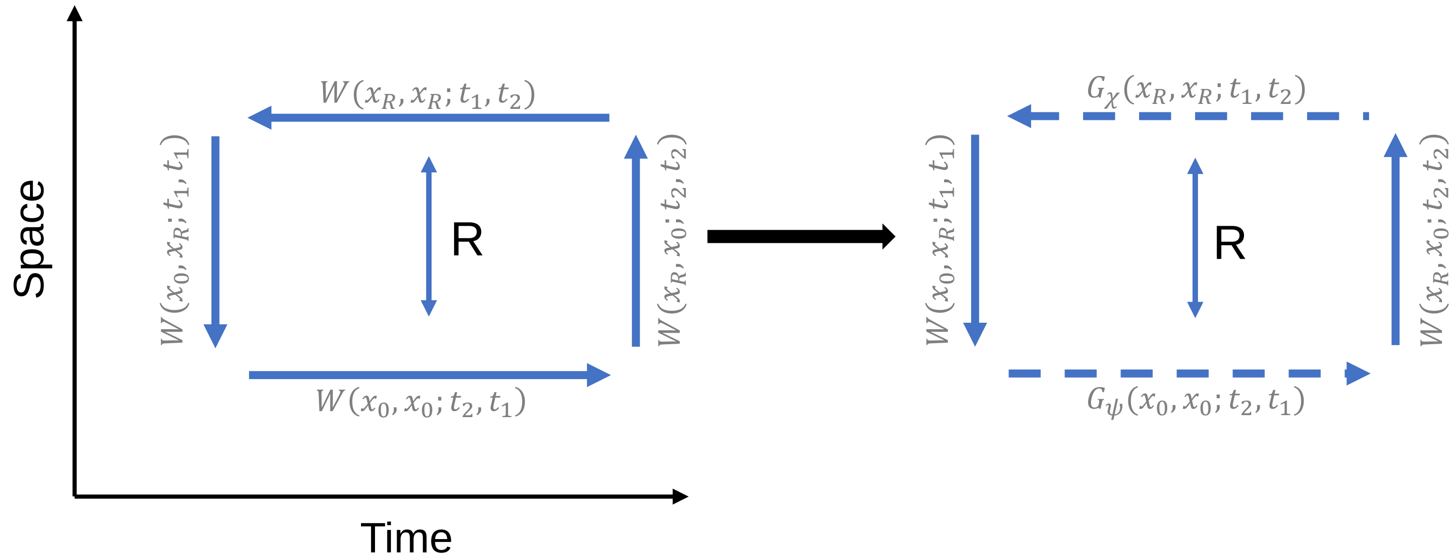

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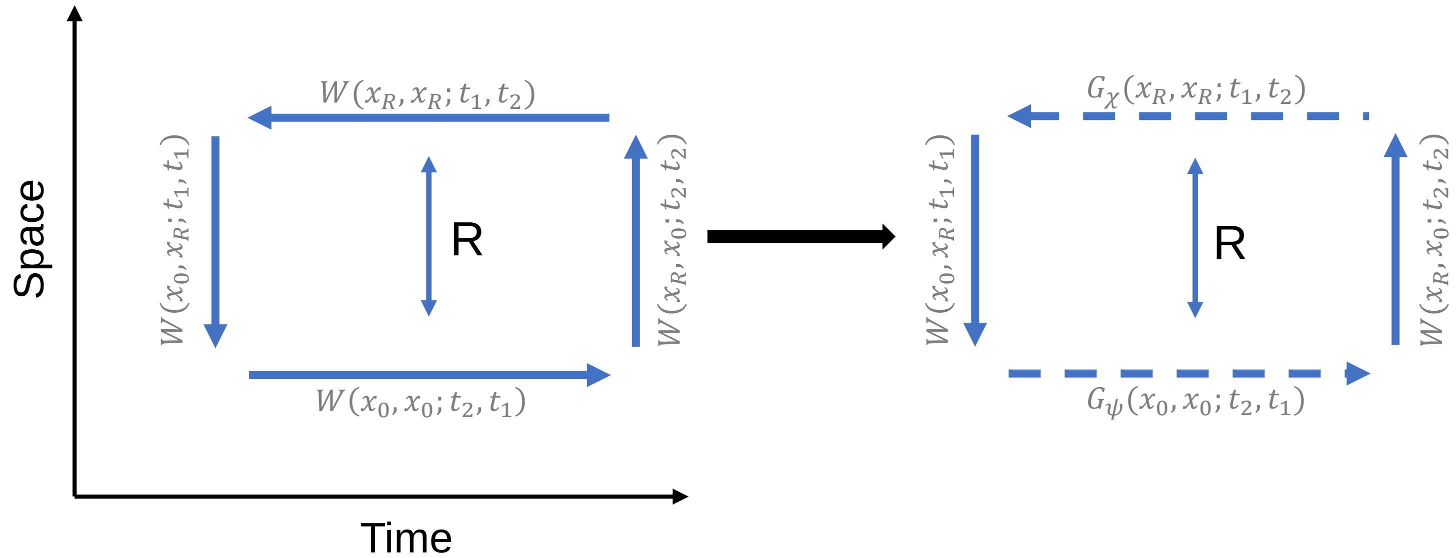
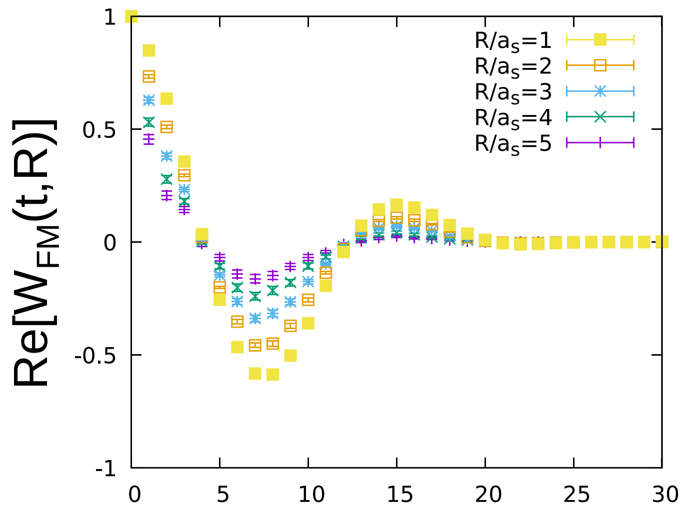
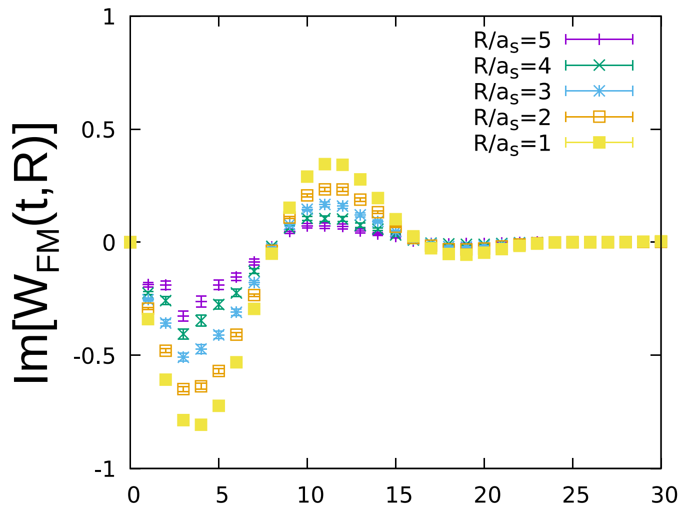


- Finite mass Wilson loop (FM) reduces to Wilson loop for $M_Q \to \infty$:
  - Axial gauge: $G_\psi(x, x_0, t) \equiv 1 \cdot \delta_{x,x_0}$ and $G_\chi(x, x_R, t) \equiv 1 \cdot \delta_{x,x_R}$
  - General: Propagator collects temporal gauge links only, i.e. $G_\psi(x, x_0, t) \equiv \prod_{i=0}^{N_t-1} U_0(x, t_i) \cdot \delta_{x,x_0}$ and $G_\chi(x, x_R, t) \equiv \prod_{i=N_t-1}^{0} U_0(x, t_i) \cdot \delta_{x,x_R}$

Heavy Quarkonium Potential





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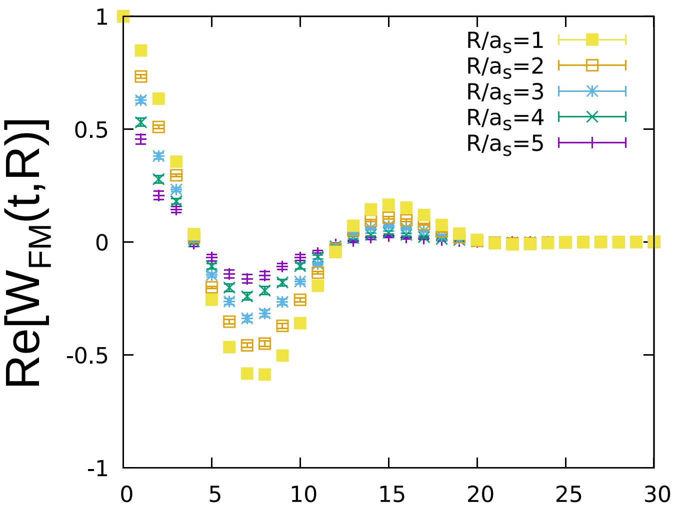

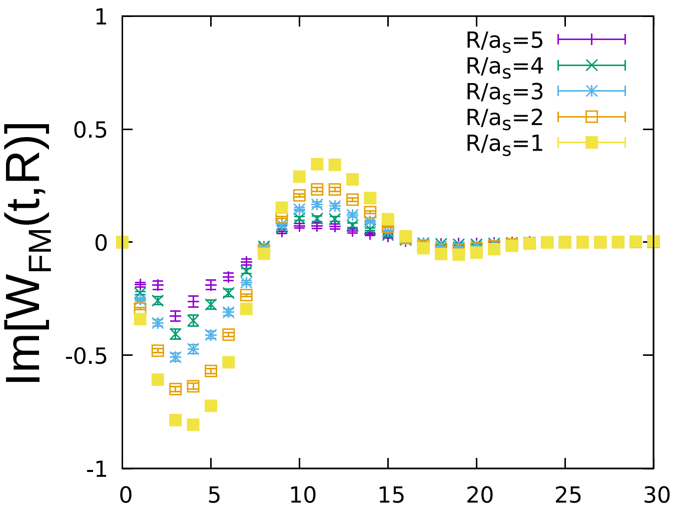

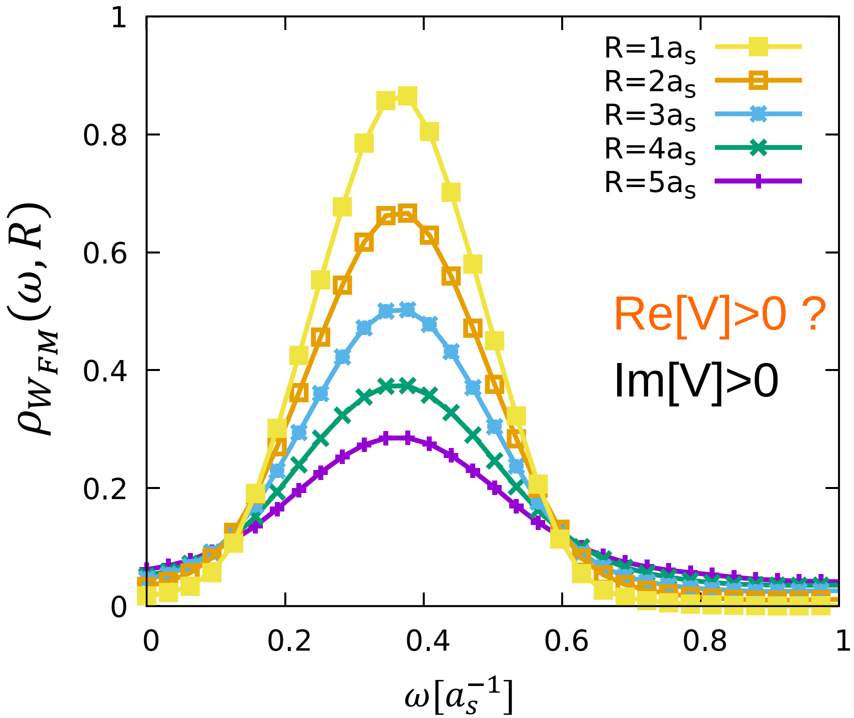

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$a_S M_Q = 10 \frac{\pi}{2}$

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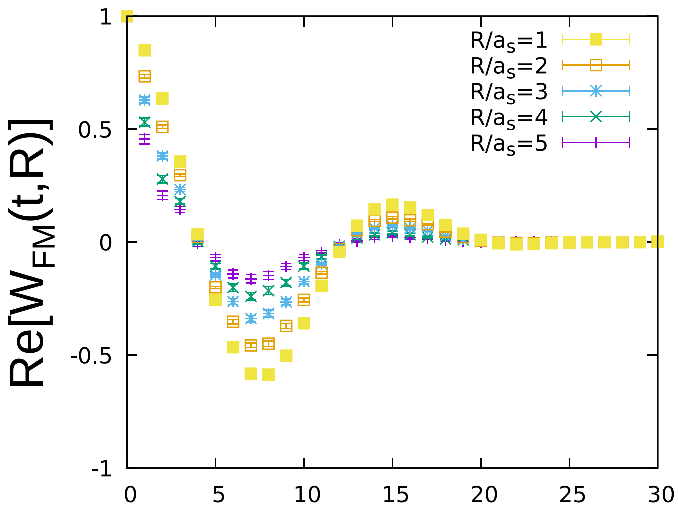

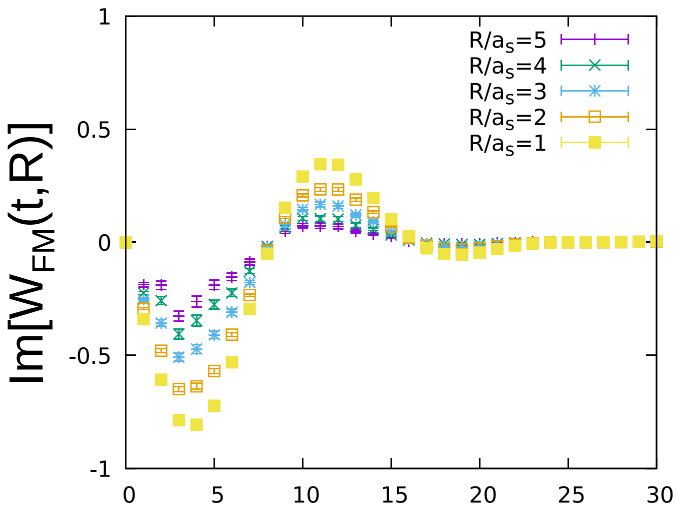

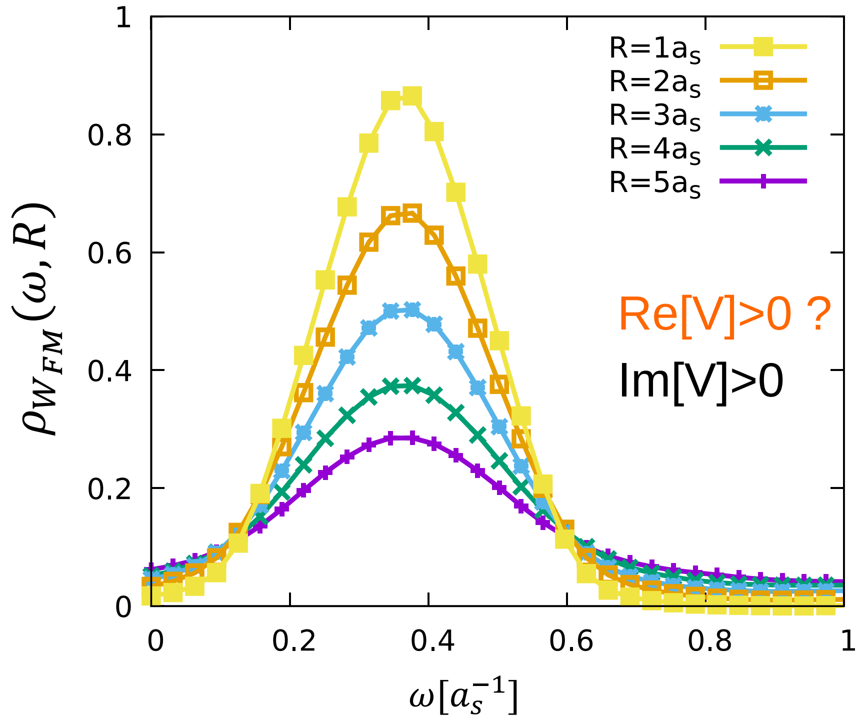

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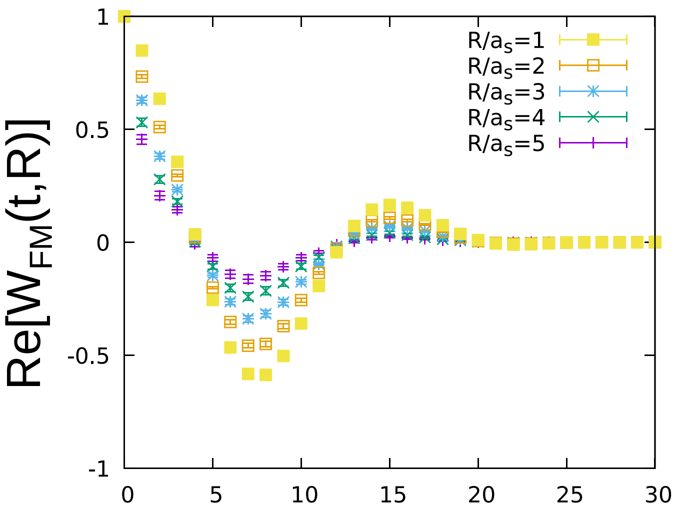
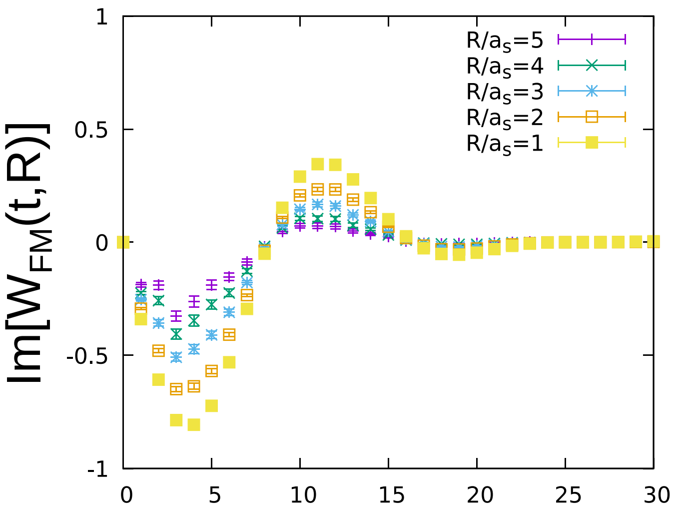
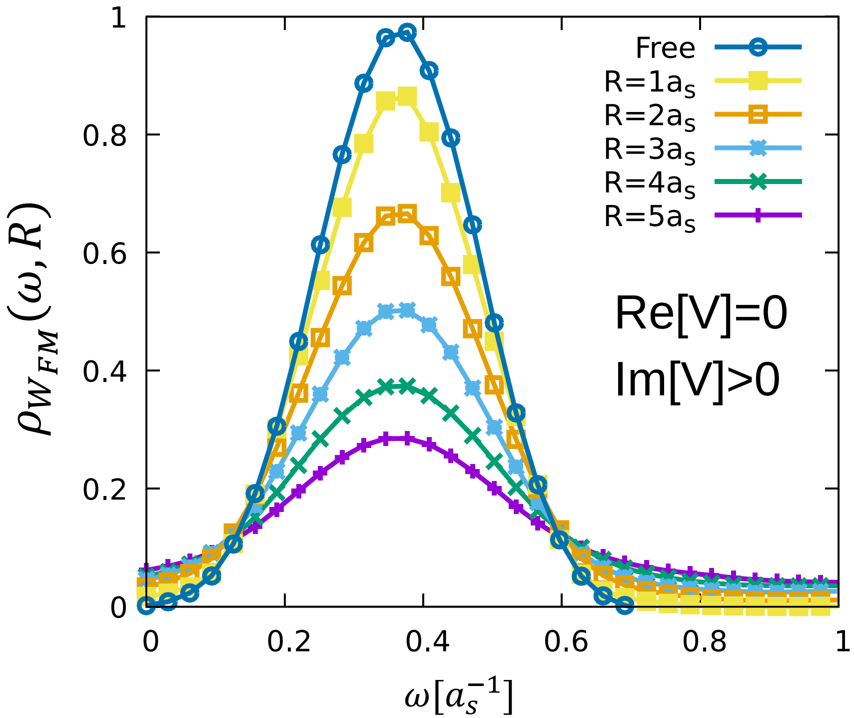
Heavy Quarkonium Potential







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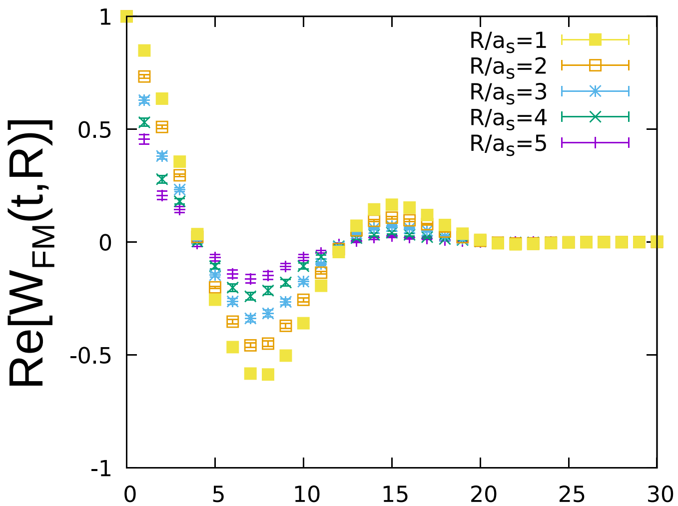

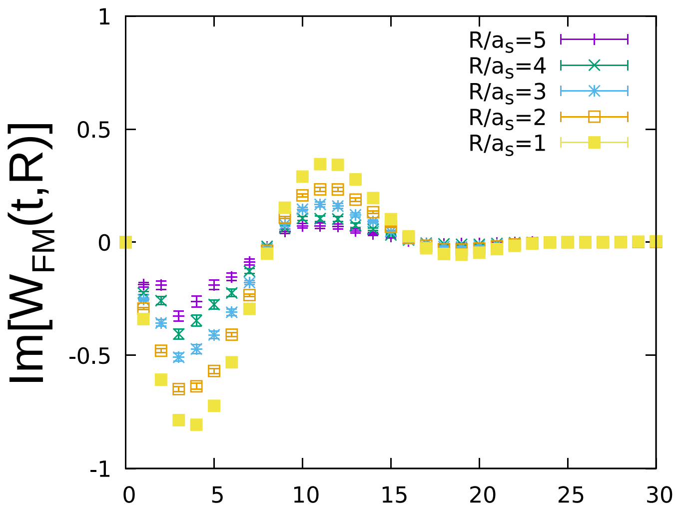

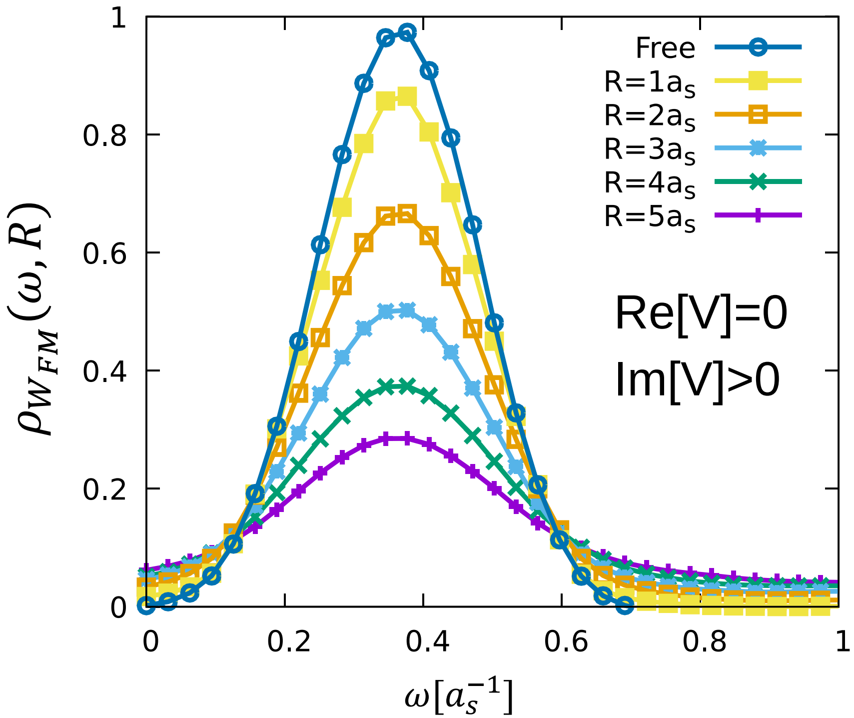

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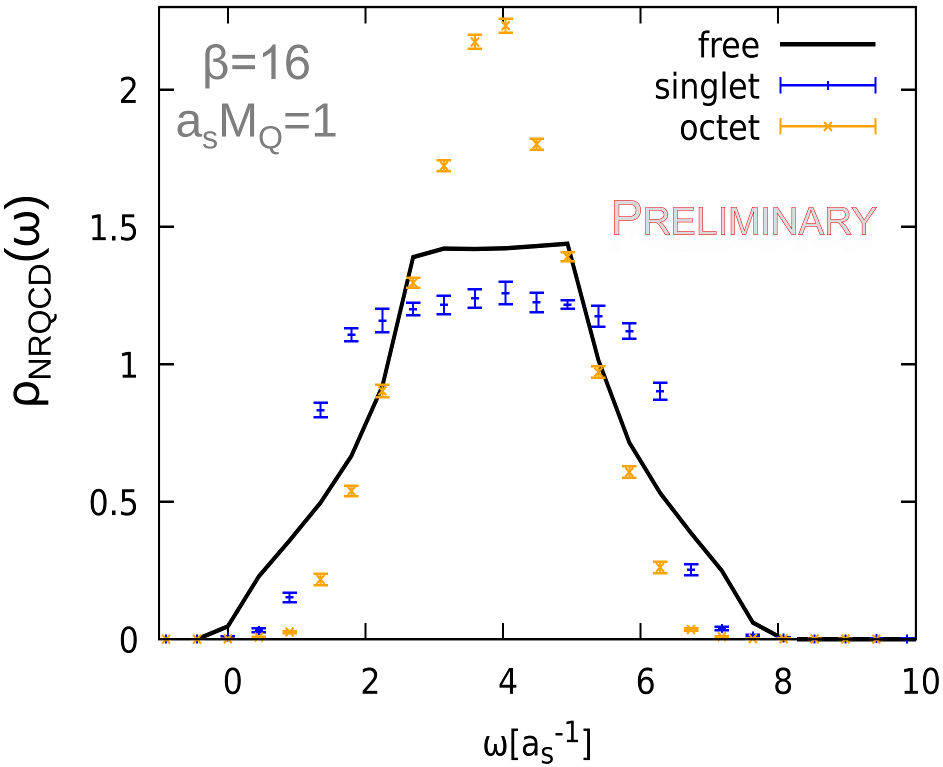
→ No real part of the potential even for finite mass

Quarkonium Spectrum in the Class. Therm. Eq.



- Very similar result compared to the non-equilibrium at $t=100a_s$
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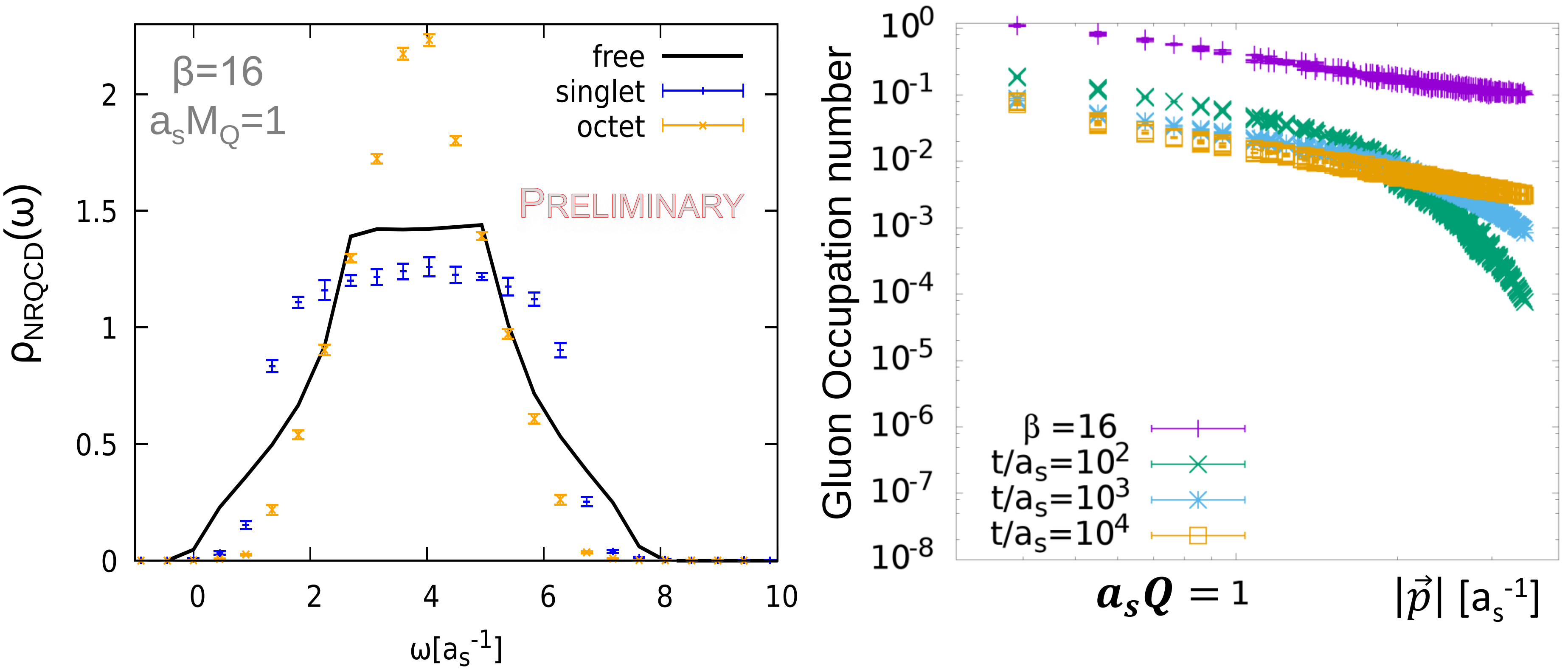

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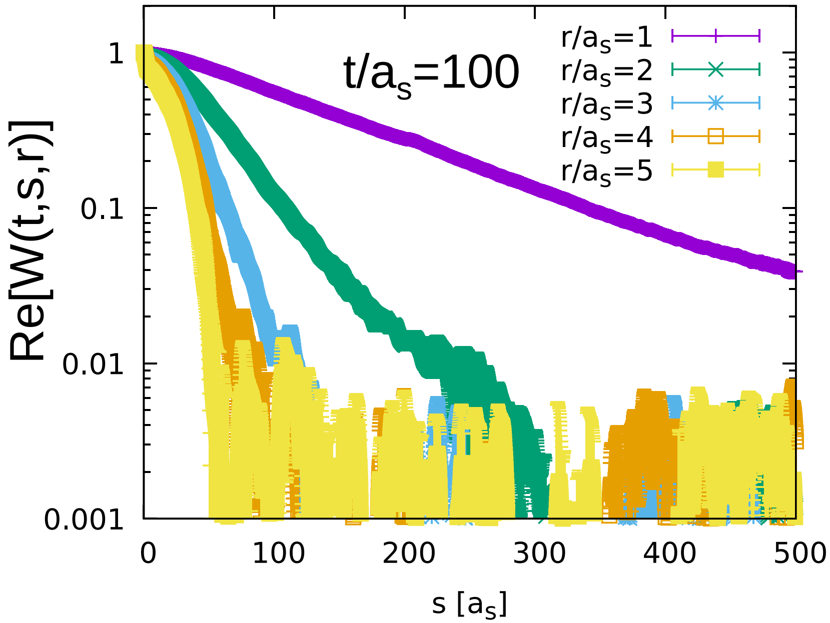
Understanding the absence of binding

- Consider static quarks via the **non-equilibrium** real-time Wilson loop W(t,s,r)



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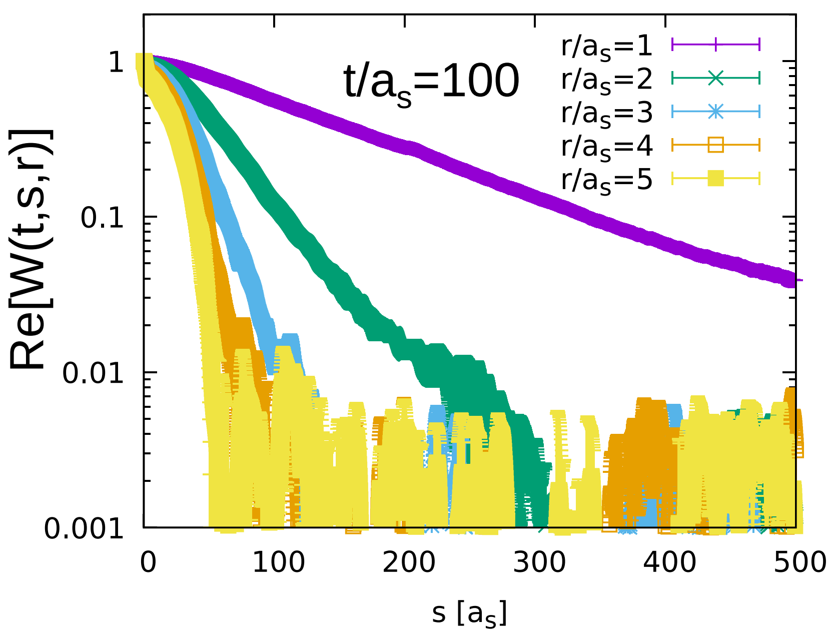

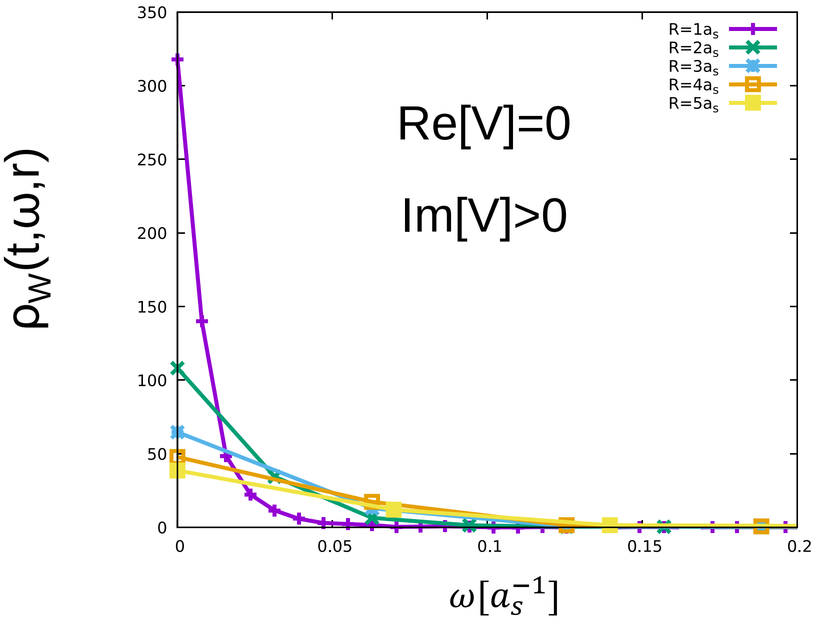

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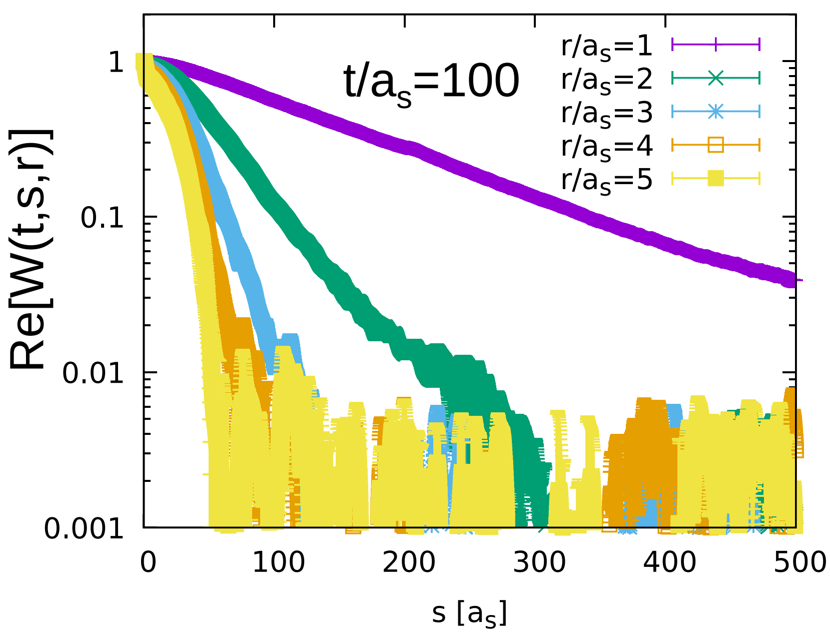

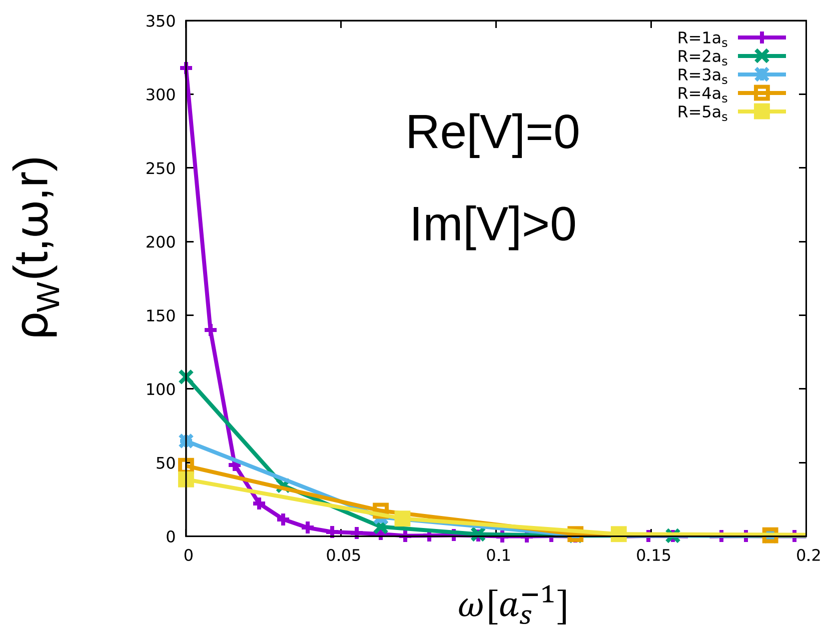

- Similar to results in thermal equilibrium: **no real-part** of the potential emerges
- **No indications of binding**, not even Coulombic, found out of equilibrium so far

Summary

- Combination of real-time classical statistical simulations for gauge fields with novel stable lattice NRQCD solver
- Direct computation of non-equilibrium real-time quarkonium correlators and spectral functions in Wigner coordinates
- Enhancement in quarkonium colour octet channel and no signs of binding in the singlet channel
- Consistent with absence of a real-part in effective potential
- Late non-equilibrium results similar to classical thermal equilibrium
- Need further study at stronger couplings to confirm absence or presence of binding

Thank you for your attention -

ご清聴ありがとうございました