Real-Time-Evolution of Heavy-Quarkonium Bound States

Alexander Lehmann
Department of Mathematics and Physics
University of Stavanger
Institute for Theoretical Physics
Heidelberg University

work in collaboration with A. Rothkopf
Heavy Quarkonia in Heavy Ion Collisions

- Initial state
- Pre-equilibrium
- QGP and hydrodynamic expansion
- Hadronization
- Hadronic phase and freeze-out

- $M_B$ = 4.183 GeV [PDG 2017]
- $v_B$ $\approx$ 0.1
- $M \sqrt{\alpha_{QCD}}$ is the momentum scale below which gluons strongly interact.

Illustration by L. McLerran
Heavy Quarkonia in Heavy Ion Collisions

- Initial state
- Pre-equilibrium
- Hydrodynamic expansion
- Hadronization

- $m_{	ext{Bottom}} = 4.18^{+3}_{-2}$ GeV [PDG 2017]
- $\mathcal{M}_v$... typical momentum
- $\mathcal{V}_{\text{Bottom}} = 0.1$... relative velocity
- $\mathcal{M}_v^2$... typical kinetic or potential energy

Illustration by L. McLerran
Heavy Quarkonia (Charmonium, Bottomonium) are well controlled experimental and theoretical probes for the quark-gluon-plasma.

Phenomenological models describe quarkonium supression via a Schrödinger equation + assumption of early formation of bound states.
Heavy Quarkonia in Heavy Ion Collisions

- Heavy Quarkonia (Charmonium, Bottomonium) are well controlled experimental and theoretical probes for the quark-gluon-plasma
- Phenomenological models describe quarkonium supression via a Schrödinger equation + assumption of early formation of bound states
- Characterized by a separation of scale:
  \[ M_Q \gg M_Q v \gg M_Q v^2 \gg \Lambda_{\text{QCD}} \]
- Very heavy states, e.g. \( \Upsilon(1S) \), already bound Coulombically

- \( M_Q \) ... heavy quark mass \((m_{\text{Bottom}} = 4.18(3)\text{GeV} \ [\text{PDG 2017}]\))
- \( v \) ... relative velocity in centre of mass frame \((v_{\text{Bottom}}^2 \approx 0.1)\)
- \( \Lambda_{\text{QCD}} \) ... momentum scale below which gluons strongly interacting
- \( M_v \) ... typical momentum
- \( M_v^2 \) ... typical kinetic or potential energy
Heavy Quarkonia in Early Stages in HICs

Early dynamics of heavy quarkonium in HIC largely unexplored

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Rule of thumb via uncertainty relation: $\tau_{form} \sim 1/E_{bind} \approx 0.2 \ldots 0.4 \text{fm/c}$

Can we find hints for heavy-quarkonium formation in the glasma?
Real-Time Evolution of the Gauge Fields

- Vital insight into glasma dynamics via classical statistical simulations of gauge fields in expanding geometry.
Real-Time Evolution of the Gauge Fields

- Vital insight into glasma dynamics via **classical statistical simulations** of gauge fields in expanding geometry

- In this study **Hamiltonian evolution** in axial gauge, formulated in spatial links and electric fields (Leapfrog) in a non-expanding box

\[ \partial_t U_j(x, t) = i E_j^a U_j(x, t) \quad \partial_t E_j^a(x, t) = -2 \text{Im} \text{Tr} \left\{ T^a \sum_{j \neq k} [U_{ij}(t, x) + U_i(-j)(t, x)] \right\} \]

\[ U_j(x) = \exp(i a_j A_j^a T^a) \quad E_j^a = F_{0j}^a = a_0 a_j 2 \text{Im} \text{Tr}[T^a U_{0j}] \]
Real-Time Evolution of the Gauge Fields

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U_j(x) &= \exp(i a_j A^a_j T^a) \\
E^a_j &= F^a_{0j} = a_0 a_j 2 \text{ImTr}[T^a U_{0j}] 
\end{align*}
\]

- Initial conditions drawn from a statistical ensemble
The real-time NRQCD setup

\[ a_s 2 M_Q = \frac{\pi}{a_s} \]

\[ \frac{g^2}{2} \quad (g=10^{-3}) \]

\[ |\vec{p}| \quad [a_s^{-1}] \]
The real-time NRQCD setup

\[ a_s 2M_Q = \pi / a_s \]

Gluon Occupation number (Coulomb gauge)

\[ \frac{\tilde{g}^2}{2} \quad (g=10^{-3}) \]

\[ g = 10^{-3} \]

\[ \frac{2M_Q}{a_s} \]

\[ g^2 \]

\[ \alpha_s Q = 1 \]

\[ t/a_s = 0 \]

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The real-time NRQCD setup

\[ a_s 2M_Q = \pi / a_s \]

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Gluon Occupation number (Coulomb gauge)

\( t/a_s = 0 \)
\( t/a_s = 100 \)
\( t/a_s = 200 \)
Real-time Lattice NRQCD

- Effective **non-relativistic formulation** of heavy quarks from systematic expansion of QCD action in quark velocity $v$ for 2-component **Pauli spinors** $\psi, \chi$

- Hamiltonian to order $O(v^3)$ with leading order Wilson coefficients $c_i=1$

$$H\psi = -\frac{\Delta^2}{2M} - c_1 \frac{g}{2M} \vec{\sigma} \cdot \vec{B} - c_2 \frac{g}{8M^2} \Delta \cdot \vec{E} - c_3 \frac{ig}{8M^2} \vec{\sigma} \cdot (\Delta \times \vec{E} - \vec{E} \times \vec{\Delta})$$

$$D_i\psi(x) = (U_{i,x} \psi_{x+1} - U_{i,x-1}^\dagger \psi_{x-1})/2a_i + O(a_i^2)$$
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- **Real-time** quarkonium current **correlator** \( D^\dagger \) from heavy quark propagator \( G \)

\[
D^\dagger_V (x_2, x_1) \sim i < J^i_{\text{NRQCD}}(x_2) J^\dagger_{i,\text{NRQCD}}(x_1) >
\]
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- **Real-time** quarkonium current **correlator** \( D^> \) from heavy quark propagator \( G \)

\[
D^>(x_2, x_1) \sim i Tr \int DU \, G^\psi(x_2, x_1) \sigma^i G^\chi(x_1, x_2) \sigma_i \, e^{iS[U]}
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**NRQCD-Dirac-Op. hermitian**

\[
G^c_\chi(x_1, x_2) \quad \text{NRQCD-Dirac-Op. hermitian} \quad G^c_\psi(x_2, x_1)
\]

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- Heavy quark equation of motion:

\[ G^\psi[U]^{t+\Delta t}_{x_2, x_1} = \exp[-i\Delta t H^\psi[U]] \cdot G^\psi[U]^t_{x_2, x_1} \]
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  $$G(t + a_t) = (1 - i a_t H[U(t)]) \cdot G(t)$$

Often via forward Euler: cheap but 1st order in $dt$, inherently unstable (Courant), range of validity of NRQCD mixed with breakdown of discretization
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$$D^\geq_v (x_2, x_1) \sim iTr \int DU \, G^\psi (x_2, x_1) \sigma^i G^\chi (x_2, x_1)^+ \sigma_i \, e^{iS[U]}$$

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$$G(t + a_t) = \left( 1 + \frac{ia_t}{2} H[U(t)] \right)^{-1} \cdot \left( 1 - \frac{ia_t}{2} H[U(t)] \right) \cdot G(t)$$

Optimal rational approximation of exp (Crank-Nicholson, $O(dt^2)$): **unconditionally stable**, no mixing of range of validity. (No operator splitting via MPI PETSc)
Wigner Coordinates for Non-Equilibrium

- **No time translational invariance**: need to correctly account for relative and central time coordinate in 2pt functions:

\[ t = \frac{t_2 + t_1}{2} \quad s = t_2 - t_1 \]
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- Spectral function from Fourier transform over **finite** temporal extent in \( s \)

\[
\rho(t, \omega, p = 0) = 2 \text{Im} \left[ \int_0^{s_{\text{max}}} D^{>}(t + \frac{s}{2}, t - \frac{s}{2}, p = 0)e^{-i\omega s} \, ds \right]
\]

\text{RT evolution of quarks & links}

Measurement of \( D^{>}(t_2, t_1) \)

\( s > 0 \)
Wigner Coordinates for Non-Equilibrium

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\]

- Spectral function has **explicit t dependence**, signaling real-time evolution of gauge fields
Free theory sanity check

- **Real-time** correlation function is **complex** – finite volume effects as recurrence
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Quarkonium in the Glasma (I)

- **Low energy gluons** do not significantly impact quarks at early times.
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Bulk glue effects manifest in the intermediate $(s,t)$ time physics of heavy quarks.
Quarkonium in the Glasma (I)

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**Diagram:**
- Two graphs showing vector channel, color singlet behavior with center time and relative time $s [a_s]$.
- Preliminary results for $\rho_{NRQCD}(\omega)$ with center time and $\omega [a_s^{-1}]$.

**Equation:**
$$V=64^3 \quad Q=1 \quad n=1 \quad g=10^{-3}$$
Quarkonium in the Glasma (I)

- **Low energy gluons** do not significantly impact quarks at early times.
- Bulk glue effects manifest in the **intermediate (s,t) time physics** of heavy quarks.
- At the parameters used here, **no signs for binding** into clear resonances.
Quarkonium in the Glasma (II)

- Reduction of singlet amplitude and broadening understood from **gluon absorption**
- **Octet enhancement** from interaction with low energy gluonic bulk

\[ \rho_{NRQCD}(\omega) \]

\[ V=64^3 \quad Q=1 \quad n=1 \quad g=10^{-3} \]
Classical Thermal Equilibrium

- Statistical operator: $\rho = e^{-\beta H}$
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- Initialisation method (coupling to a heat bath)
  1. Draw normal distributed random E-field with standard deviation $\sigma(\beta)$
  2. Restore Gauss law
  3. N update steps for links and E-field
  4. Repeat 1-3 until thermalized

A. Akamatsu, A. Rothkopf, N. Yamamoto, JHEP 1603 (2016) 210
Classical Thermal Equilibrium

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\[ \beta = 16 \]

\[ \rho = e^{-\beta H} \]

A. Akamatsu, A. Rothkopf, N. Yamamoto, JHEP 1603 (2016) 210
Static Potential in the Classical Equilibrium

- Consider static quarks via the equilibrium real-time Wilson loop $W(t,x)$
- Attempt to extract effective real-time potential via Wilson loop spectral function $\text{Re}[V]$ from position of lowest lying peak, $\text{Im}[V]$ from width, see Y. Burnier, A. Rothkopf PRD86 (2012) 051503

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  see Y. Burnier, A. Rothkopf PRD86 (2012) 051503

### Graphs

- **Left graph**: $\text{Re}[W(t,R)]$ vs $t[a_s]$ for different $R/a_s$ values.
- **Right graph**: $\rho_w(\omega,R)$ vs $\omega[a_s^{-1}]$ for different $R$ values.

$\beta = 16$
Static Potential in the Classical Equilibrium

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Same result as found in the literature: no real-part of the potential emerges see M. Laine et.al. JHEP 0709 (2007) 066
Finite Mass
Wilson Loop

- Replacing temporal Wilson lines by fermion and anti-fermion propagators
Finite Mass Wilson Loop

- Replacing temporal Wilson lines by fermion and anti-fermion propagators.

\[
W(x_R, x_R; t_1, t_2) \\
W(x_0, x_0; t_1, t_1) \\
W(x_R, x_0; t_2, t_1) \\
W(x_0, x_R; t_2, t_2)
\]
Finite Mass Wilson Loop

- Replacing temporal Wilson lines by fermion and anti-fermion propagators
Finite Mass Wilson Loop

- Replacing temporal Wilson lines by fermion and anti-fermion propagators

Finite mass Wilson loop (FM) reduces to Wilson loop for $M_Q \to \infty$:

- Axial gauge: $G_\psi(x, x_0, t) \equiv 1 \cdot \delta_{x,x_0}$ and $G_\chi(x, x_R, t) \equiv 1 \cdot \delta_{x,x_R}$

- General: Propagator collects temporal gauge links only, i.e.
  $G_\psi(x, x_0, t) \equiv \prod_{i=0}^{N_t-1} U_0(x, t_i) \cdot \delta_{x,x_0}$ and $G_\chi(x, x_R, t) \equiv \prod_{i=N_t-1}^{0} U_0(x, t_i) \cdot \delta_{x,x_R}$
Heavy Quarkonium Potential

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Heavy Quarkonium Potential

\[ \omega[\alpha_s^{-1}] \]

\[ \Re[V] > 0 \]
\[ \Im[V] > 0 \]

\[ \rho_{WF}\_FM(\omega, R) \]

\[ \beta = 16 \]
\[ \alpha_s M_Q = 10 \frac{\pi}{2} \]
**Heavy Quarkonium Potential**

\[
\psi [a_s^{-1}] = \rho_{WF} (\omega, R) \\
\beta = 16 \\
\alpha_s M_Q = 10 \frac{\pi}{2}
\]

\[
H = \frac{p^2}{M_Q} + V(R)
\]

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Heavy Quarkonium Potential

\[ Re[W_{FM}(t,R)] \]

\[ Im[W_{FM}(t,R)] \]

\[ \beta = 16 \]
\[ a_s M_Q = 10 \frac{\pi}{2} \]

\[ H = \frac{p^2}{M_Q} + V(R) \]
**Heavy Quarkonium Potential**

\[ \omega [a_s^{-1}] \]

\[ H = \frac{p^2}{M_Q} + V(R) \]

\[ \rightarrow \text{No real part of the potential even for finite mass} \]

\[ \beta = 16 \]

\[ a_s M_Q = 10 \frac{\pi}{2} \]
Real-Time-Evolution of Heavy-Quarkonium Bound States

Quarkonium Spectrum in the Class. Therm. Eq.

- Very similar result compared to the non-equilibrium at $t=100a_s$
- No sign of binding also in the classical thermal equilibrium
Quarkonium Spectrum in the Class. Therm. Eq.

- Very similar result compared to the non-equilibrium at $t=100a_s$
- **No sign of binding** also in the classical thermal equilibrium
Understanding the absence of binding

- Consider static quarks via the **non-equilibrium** real-time Wilson loop $W(t,s,r)$

![Graph showing the real part of the Wilson loop $W(t,s,r)$ for different values of $r/a_s$. The graph plots $Re[W(t,s,r)]$ against $s/a_s$ for $t/a_s=100$. The graph includes lines for $r/a_s=1, 2, 3, 4, 5$.](image-url)
Understanding the absence of binding

- Consider static quarks via the non-equilibrium real-time Wilson loop $W(t,s,r)$.

\[
Re[W(t,s,r)] = 0, \quad Im[V] > 0.
\]
Understanding the absence of binding

- Consider static quarks via the non-equilibrium real-time Wilson loop $W(t,s,r)$

- Similar to results in thermal equilibrium: no real-part of the potential emerges

- No indications of binding, not even Coulombic, found out of equilibrium so far
Summary

- Combination of real-time classical statistical simulations for gauge fields with novel stable lattice NRQCD solver

- Direct computation of non-equilibrium real-time quarkonium correlators and spectral functions in Wigner coordinates

- Enhancement in quarkonium colour octet channel and no signs of binding in the singlet channel

- Consistent with absence of a real-part in effective potential

- Late non-equilibrium results similar to classical thermal equilibrium

- Need further study at stronger couplings to confirm absence or presence of binding

Thank you for your attention - ご清聴ありがとうございました