Quantum dissipation of quarkonium in quark-gluon plasma: Lindblad equation approach

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Data from heavy ion collisions

Spectral for upsilon

Comparing pp collision data, yields are relatively suppressed

Data reflect interactions between a quarkonium and QGP

We need to understand the dynamics of a quarkonium in QGP

[CMS collaboration(19)]
Quarkonium in QGP

How to describe?

• Two interacting Brownian particles

  → Langevin eq. \[ \frac{dp}{dt} = -\eta p - \nabla V(x) + \xi(t) \]

  White noise

• Quantum mechanical bound state

  Quantum Brownian motion
Open quantum system

We would like to describe a quarkonium in quantum way

→ open quantum system approach

\[
\frac{d}{dt} \rho_{\text{total}} = -i [H_{\text{total}}, \rho_{\text{total}}]
\]

\[
H_{\text{total}} = H_{\text{QGP}} + H_{Q\bar{Q}} + H_{\text{int}}
\]

trace out QGP variables

\[
\rho_{Q\bar{Q}} = \text{Tr}_{\text{QGP}} \rho_{\text{total}}
\]

reduced density matrix

master eq. for only quarkonium

\[
\frac{d}{dt} \rho_{Q\bar{Q}} = \hat{\mathcal{L}} \rho_{Q\bar{Q}}
\]

Liouville operator

information of interactions
Lindblad master equation

- Positivity of density matrix

We would like to interpret quarkonium state as a mixed state

→ Is positivity satisfied? \( \forall |\alpha\rangle, \langle \alpha | \rho_{Q\bar{Q}} |\alpha\rangle \geq 0 \)

- Lindblad form \([\text{Lindblad}(76)]\)

\[
\frac{d}{dt} \rho_{Q\bar{Q}} = -i[H^\prime_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \int dk \{ 2L_k \rho_{Q\bar{Q}} L_k^\dagger - L_k^\dagger L_k \rho_{Q\bar{Q}} - \rho_{Q\bar{Q}} L_k^\dagger L_k \}
\]

\( L \): Lindblad operator

→ interacting forces

Important properties

\[
\text{Tr}[\rho_{Q\bar{Q}}] = 1
\]

\( \rho_{Q\bar{Q}} = \rho_{Q\bar{Q}}^\dagger \)

\( \forall |\alpha\rangle, \langle \alpha | \rho_{Q\bar{Q}} |\alpha\rangle \geq 0 \)
Quantum State Diffusion (QSD) method

- Stochastic unravelling

\[ \rho_{QQ}(x, y, t) = \langle \psi(x, t) \psi^*(y, t) \rangle \]

Lindblad master eq. via QSD equivalent nonlinear stochastic Schrödinger eq.

density matrix ensemble

\[ |d\psi\rangle = -iH'_{QQ} |\psi(t)\rangle \, dt + \int d\tilde{k} \left( 2\langle L_{\tilde{k}}^\dagger \rangle \psi L_{\tilde{k}} - L_{\tilde{k}}^\dagger L_{\tilde{k}} - \langle L_{\tilde{k}}^\dagger \rangle \psi \langle L_{\tilde{k}} \rangle \psi \right) |\psi(t)\rangle \, dt \]

\[ + \int d\tilde{k} \left( L_{\tilde{k}} - \langle L_{\tilde{k}} \rangle \psi \right) |\psi(t)\rangle \, d\xi_{\tilde{k}} \]

\[ \langle \ldots \rangle_{\psi} \quad \text{expectation value with respect to wave function} \]

\[ \rightarrow \text{nonlinearity} \]

Apply QSD method to Lindblad master equation
Caldeira Leggett model for quarkonium?

Caldeira Legette model  [Caldeira-Leggett(83)]

- Prototype of quantum Brownian particle with potential $V(x)$

- quantum Brownian particle
  $\leftarrow$ localized wave packet
  smaller than QGP correlation length

In our case, NOT the case  $\rightarrow$ improve the model based on QCD

[ Akamatsu(15) ]
Lindblad operator for quarkonium in QGP

Reduction to relative motion

\[ L_{k,a}^\text{relative} \]

for quarkonium \( (\vec{x}_Q, \vec{x}_{\bar{Q}}) \)

\[ H_{\text{relative}} \]

[Akamatsu(15)]

trace out center-of-mass motion under its constant momentum

Result (CM momentum=0)

\[ L_{k,a}^\text{relative} = \sqrt{\frac{D(k)}{2}} \left[ 1 - \frac{\vec{k} \cdot \hat{p}}{4MT} \right] e^{i\vec{k} \cdot \hat{r}/2} (t^a \otimes 1) - \sqrt{\frac{D(k)}{2}} \left[ 1 + \frac{\vec{k} \cdot \hat{p}}{4MT} \right] e^{-i\vec{k} \cdot \hat{r}/2} (1 \otimes t^{a*}) \]

\[ H_{\text{relative}} \in V(x) \]

Q part

fluctuation

\( \bar{Q} \) part

dissipative term

(heavy quark recoil)

momentum transfer

Solve Lindblad eq. for relative motion with this Lindblad operator (NOT model)

\[ \frac{d}{dt} \rho_{Q\bar{Q}} = -i[H'_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \int dk \{ 2L_k^\dagger \rho_{Q\bar{Q}} L_k^\dagger - L_k^\dagger L_k \rho_{Q\bar{Q}} - \rho_{Q\bar{Q}} L_k^\dagger L_k \} \]
NUMERICAL ANALYSIS
QSD simulation for quarkonium relative motion

For simplicity, in one spatial dimension, with heavy quark color ignored

Parameter setups in terms of heavy quark mass $M$

<table>
<thead>
<tr>
<th>$\Delta x$</th>
<th>$\Delta t$</th>
<th>$N_x$</th>
<th>$T$</th>
<th>$\gamma$</th>
<th>$l_{\text{corr}}$</th>
<th>$\alpha$</th>
<th>$m_D$</th>
<th>$r_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/M$</td>
<td>$0.1M(\Delta x)^2$</td>
<td>254</td>
<td>0.1$M$</td>
<td>$T/\pi$</td>
<td>1/$T$</td>
<td>0.3</td>
<td>$T$</td>
<td>1/$M$</td>
</tr>
</tbody>
</table>

Noise correlation function

$$D(r) = \gamma \exp\left(-\frac{r^2}{l_{\text{corr}}^2}\right)$$

- More realistic setup (Bjorken expanding QGP)

QGP temperature decreases in time

$$T(t) = T_0 \left(\frac{t_0}{t + t_0}\right)^{1/3} \quad T_0/M_b = 0.1, \ M_b t_0 = 20$$
QSD simulation for quarkonium relative motion

For simplicity, in one spatial dimension, with heavy quark color ignored

Outline of the numerical calculation

1. Initial Wave function
2. Time evolution event by event
3. Eigenstate projection
4. Occupations

QSD method

$$H = \frac{p^2}{M} - \frac{\alpha}{r} e^{-m_D r}$$

occupation

$$N_i = \int dx dy \, \phi_i^*(x) \rho(x, y) \phi_i(y)$$

More realistic setup (Bjorken expanding QGP)

- vacuum eigenstate
  $$\phi_i(x)$$

$$H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r \quad \sigma = 0.01 M_b^2$$
Results - equilibration

- Time evolution of occupation number of eigenstates

\[ H = \frac{p^2}{M} - \frac{\alpha}{r} e^{-m_D r} \]

Initial state
ground/1st excited state

Eigenstate occupation respectively approaches the static value

Independent of initial conditions
Results - equilibration

- Eigenstate steady distribution at Mt=4650

\[ H = \frac{p^2}{M} - \frac{\alpha}{r} e^{-m_D r} \]

Occupation at equilibrium

Temperature slope

\[ N_0(0)=1, \ T/M=0.1 \]
\[ 0.02 \times \exp(-E/T) \]

Eigenstate distribution approaches the Boltzmann distribution
Results - In Bjorken expanding QGP

- Time evolution of occupation number of eigenstates

\[ H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r \]

With dissipation, Bjorken, \( N_0(0) = 1 \), \( N_0(Mt) \)

\( T/M = 0.1 \), \( N_0(0) = 1 \), \( N_0(Mt) \)

Initial state

Ground state

Evolution becomes milder in Bjorken expanding QGP

Cornell

Debye screening

Evolution becomes milder in Bjorken expanding QGP
Results - Dissipative effect

- Time evolution of occupation number of eigenstates

\[ H = \frac{p^2}{M} - \frac{\alpha}{r}e^{-m_D r} \]

**Initial state**

QGP life time 10 fm

**Dissipation is effective and important to be considered in QGP life time**

Without dissipation occupations are underestimated

Dissipation is effective and important to be considered in QGP life time
Results - In Bjorken expanding QGP

- Time evolution of occupation number of eigenstates

\[ H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r \]

with dissipation, \(N_0(0)=1, N_0(Mt)\)
\(N_1(0)=1, N_0(Mt)\)

without dissipation \(N_0(0)=0, N_0(Mt)\)
\(N_1(0)=1, N_1(Mt)\)

Initial state
ground/1st excited state

Dissipative effect is not negligible in short lived QGP

Dissipative effect is not negligible in short lived QGP
Summary

- Quarkonium is treated as a open quantum system
- Simulation for Lindblad master equation in Quantum State Diffusion method
  - We numerically confirm a quarkonium is thermalized under dissipation
  - We simulate effects of dissipation
    Localized bound state is affected more, non negligible effect

Outlook

- 3D analysis
- Color effect
- Comparison with experimental results
BACK UP
Nonlinear stochastic Schrödinger equation

Nonlinear stochastic Schrödinger eq. in QSD

\[
d\psi(x) = -idt \left[ -\frac{\nabla^2}{M} + V(x) \right] \psi(x) - dt \left[ D(0) - D(x) \right] \psi(x) \\
+ \frac{2dt}{\langle \psi|\psi \rangle} \int dy \left[ D\left(\frac{x - y}{2}\right) - D\left(\frac{x + y}{2}\right) \right] [\psi^\dagger(y)\psi(y)] \psi(x) \\
+ \left[ d\xi\left(\frac{x}{2}\right) - d\xi\left(\frac{-x}{2}\right) \right] \psi(x) + \mathcal{O}(T/M),
\]

Noise correlation of complex noise

\[
\langle d\xi(x)d\xi^*(y) \rangle = D(x - y)dt, \quad \langle d\xi(x)d\xi(y) \rangle = 0,
\]

Density matrix

\[
\rho_{\mathcal{Q}\bar{\mathcal{Q}}}(x, y) = \frac{\langle \psi(x)\psi^*(y) \rangle}{\|\psi\|^2}
\]
Results - localized wave function

- Time evolution of wave function via QSD eq. in one sampling

\[ H = \frac{p^2}{M} - \frac{\alpha}{r} e^{-m_D r} \]

Initial condition

- Ground state

Wave function is typically solitonic from nonlinearity

Each sample shows similar behavior