Conductivity of quark-gluon plasma in the presence of external magnetic field

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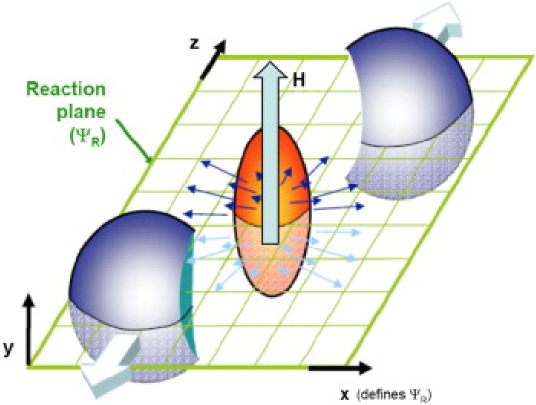
Heavy ion collisions — large eB

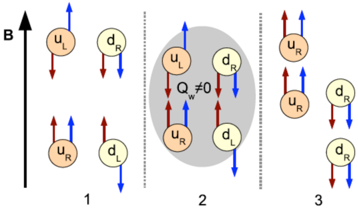
In non-central heavy ion collisions very strong magnetic field may emerge: $|e\vec{B}| \sim (3-10)\, m_\pi^2$



Chiral magnetic effect (CME)

K. Fukushima, D. Kharzeev, H.J. Warringa, PRD78 (2008) 074033

"A system with a nonzero chirality responds to a magnetic field by inducing a current along the magnetic field. This is the Chiral Magnetic Effect."



- macroscopic effect of microscopic dynamics of QCD
- allows probing the topological structure of $SU(3)$ gauge field
- non-dissipative, topologically protected

CME current

Parallel $\vec{E}$ and $\vec{B}$ — topologically non-trivial EM–field (non-zero winding number), Adler-Bell-Jackiw chiral anomaly generates topological density:

$$\frac{d\rho_5}{dt} = \frac{q^2}{2\pi^2}\vec{E}\cdot\vec{B}$$

Nielsen and Ninomiya energy argument:

$$\vec{j}\cdot\vec{E} = \mu_5\frac{d\rho_5}{dt} = \frac{q^2\mu_5}{2\pi^2}\vec{E}\cdot\vec{B} \implies$$

$$\boxed{\vec{j} = \frac{q^2\mu_5}{2\pi^2}\vec{B}}$$

The expression for j can be also calculated microscopically and is *independent on the model.*

From ρ_5 to μ_5

Chirality-changing processes:

$$\frac{d\rho_5}{dt} = -\rho_5/\tau + \frac{q^2}{2\pi^2}\vec{E}\cdot\vec{B} \implies \rho_5 = \frac{q^2}{2\pi^2}\vec{E}\cdot\vec{B}\tau$$

At small $\mu_5 \ll T$, $\mu_5 \ll \sqrt{qB}$, $\rho_5 = \chi(B,T)\mu_5$

1. $T \gg \sqrt{qB}$, temperature dominates: $\chi(B,T) = T^2/3$,
2. $T \ll \sqrt{qB}$, 1st Landau level degeneracy:
   $\chi(B,T) = |qB|/2\pi^2$

Linear response theory:

$$j^i_{\text{CME}} = \sigma^{ij}_{\text{CME}} E^j, \qquad \boxed{\sigma^{ij}_{\text{CME}} = \frac{q^4}{8\pi^4}\frac{\tau}{\chi(T,B)}B^i B^j}$$

CME observation: QCD

- CME current forms dipole in the QGP fireball that affects hadron production at freeze-out

$$\frac{dN_\pm}{d\phi} \propto 1 + 2v_1 \cos\phi + 2v_2 \cos 2\phi + \ldots + \boxed{2a_\pm \sin\phi} + \ldots,$$

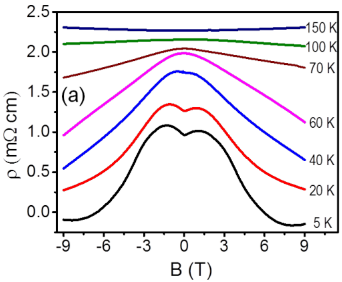
  where $a_\pm = \pm\mu_5|\vec{B}|$

- However, μ_5 sign is event-dependent — can not observe $\mathcal{P}$–odd $a_\pm$ directly (this would mean global $\mathcal{P}$–symmetry violation in QCD)

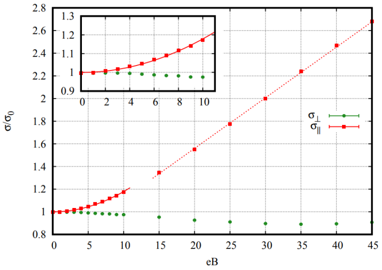
More complicated observables *yet* do not allow to 100%–confirm the existence of CME, but the data favors the existence of CME in QGP (see also the talk by Jinfeng Liao on Tuesday)

CME observation: Dirac semimetals

- Experimental: Q. Li *et al.*, *Observation of the chiral magnetic effect in* $ZrTe_5$, Nature Physics 12, 550 – 554 (2016)
- QMC: D. Boyda, V. Braguta, M. Katsnelson, A. Kotov, *Lattice quantum Monte Carlo study of chiral magnetic effect in Dirac semimetals*, Annals of Physics (2018), arXiv:1707.09810



experiment with $ZrTe_5$



σ_{CME} within QMC

Conductivity in external magnetic field

- $\vec{E} \parallel \vec{B}$
- $\dot{\rho}_5 = \frac{q^2}{4\pi^2}(\vec{E}, \vec{B}) - \rho_5/\tau$,
  τ — chirality-changing scattering time
- $\rho_5 = \frac{q^2\tau}{4\pi^2}(\vec{E}, \vec{B})$ for $\dot{\rho}_5 = 0$
- $\vec{J}_{\mathrm{CME}} = \frac{q^2}{2\pi^2}\mu_5 \vec{B}$
- $\vec{J} = \sigma\vec{E} + \frac{q^2}{2\pi^2}\vec{B} \times \mu_5 \left(\rho_5 \sim \tau(\vec{E}, \vec{B})\right)$
- Large magnetoconductivity $\sigma_{\parallel}$
- Classically $\delta\sigma_{\parallel} = 0$
- Observed in experiment (Weyl semimetals):
  Q. Li *et al.*, Nature Phys. 12 (2016) 550-554
  H. Li *et al.*, Nat. Comm. 7, 10301 (2016)

What happens in QCD?

Lattice details

- $N_f = 2+1$, physical quark masses
- Staggered fermions with improved action
- T =125 MeV, 200 MeV, 250 MeV
- Lattice sizes and steps:

a, fm	L_s	N_t
0.988	48	10
0.0618	64	16
0.0989	48	16
0.0493	64	16

- Integral Kubo equation

$$C(\tau_i) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau_i, \omega)\rho(\omega), \quad K(\tau_i, \omega) = \frac{\cosh \omega(\beta - \tau_i/2)}{\sinh \omega\beta/2}\omega$$

- Conductivity ($C_{em} = q_u^2 + q_d^2 + q_s^2$):

$$\frac{\sigma}{TC\text{em}} = \frac{1}{6C\text{em}} \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

The Backus-Gilbert method

- The method is designed for solving linear ill-defined problems with controllable regularization and systematic uncertainty.
- define the (normalized) resolution function δ as the linear combination of adjustable coefficients $q(\bar{\omega})$:

$$\tilde{\rho}(\bar{\omega}) = \int d\omega \delta(\bar{\omega}, \omega) \rho(\omega),$$

$$\delta(\bar{\omega}, \omega) = \sum_i q_i(\bar{\omega}) K(\tau_i, \omega),$$

- minimize the BG–functional:

$$\mathcal{H}(\rho) = \lambda \mathcal{A}(\rho) + (1 - \lambda) \mathcal{B}(\rho),$$

$$\mathcal{A}(\rho) = \int d\omega \delta(\bar{\omega}, \omega)(\omega - \bar{\omega})^2, \; \mathcal{B}(\rho) = \mathrm{Var}[\rho] = q^T C q.$$

The $\mathcal{A}$ part is the width of the resolution function (2nd moment to make q_i easy to find), $\mathcal{B}(\rho)$ — make less dependent on data (regularize).
The method provides $\rho(\omega)$ and $\delta(\bar{\omega}, \omega)$ as the output!

Rescaling and resolution function

Rescaling of the kernel $K(\tau,\omega) \to f(\omega)K(\tau,\omega)$ leads to reconstruction of $\rho(\omega)/f(\omega)$ instead of $\rho(\omega)$. For conductivity we take $f(\omega) = \omega$.

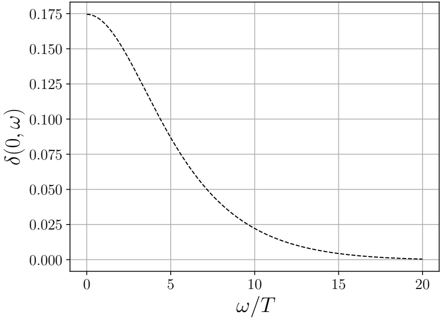


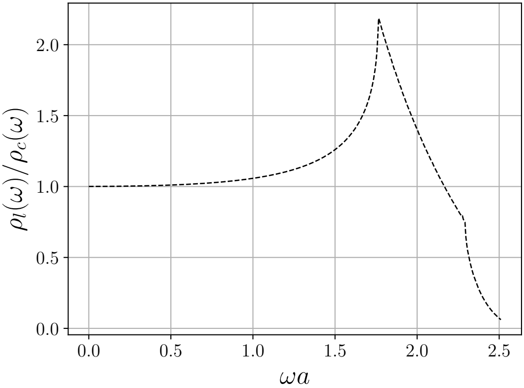
Figure: Sample resolution function peaked at $\bar{\omega} = 0$ for rescaling $f(\omega) = \omega$.

The width is of order $\leqslant 3.5T$ (not enough N_τ).

Ultraviolet contamination

Ultraviolet shape of the spectral function in the LO on the lattice:

$$\rho_{\mathrm{UV}}(\omega) = C_{\mathrm{e/o}} \frac{3}{4\pi^2} \omega^2 \tanh\left(\frac{\omega\beta}{4}\right) \frac{\rho_{\mathrm{lat}}(\omega)}{\rho_{\mathrm{cont}}(\omega)}$$

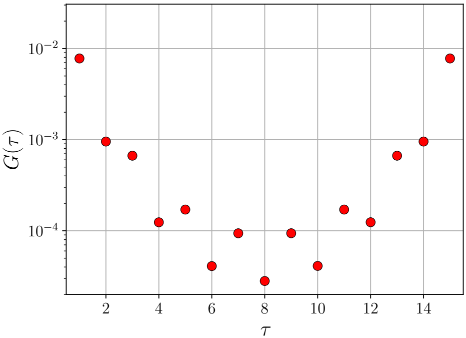


In the free case $C_{\mathrm{even}} = 1/2$, $C_{\mathrm{odd}} = 3/2$

Staggered fermions and two branches

The staggered $\langle jj \rangle$ correlator has the oscillating structure:

$$C(\tau) = A(\tau) + (-1)^\tau B(\tau)$$



$$\Delta\sigma(0) = A \int_{\omega_0}^{\infty} d\omega \, \frac{\rho^e_{\rm UV}(\omega) + \rho^o_{\rm UV}(\omega)}{2} \delta(0, \omega) \tag{1}$$

UV contribution estimation

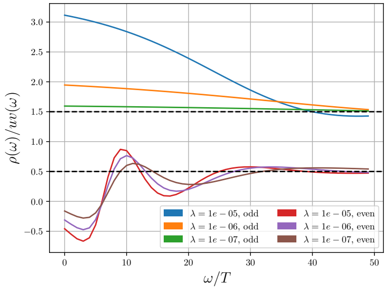
- It is hard to do it model-independently
- We assume that spectral function approximately reads (QCD sum rules):

$$\boxed{\rho(\omega) \approx (B\omega)_{\text{small } \omega}\, \theta(\omega_0 - \omega) + (A\rho_{\text{UV}}(\omega))_{\text{large } \omega}\, \theta(\omega - \omega_0).}$$

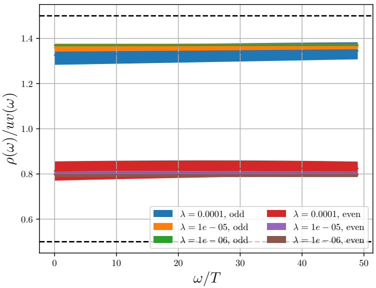
- The factor $A \approx 1$ accounts for radiative corrections, ω_0 — threshold frequency.
- Fit in B. Brandt *et al.* [1512.07249], A. Amato *et al.* [1307.6763]: $A \approx 1$, $\omega_0 \approx 7T$, $\chi^2/\text{ndof} \sim 1$.
- Take $f(\omega) = \rho_{\text{UV}}(\omega)$, expect that

$$\lim_{\omega\to\infty} \tilde{\rho}(\omega)/f(\omega) = A.$$

Ultraviolet reconstruction for $N_t = 96,\ eB = 0$



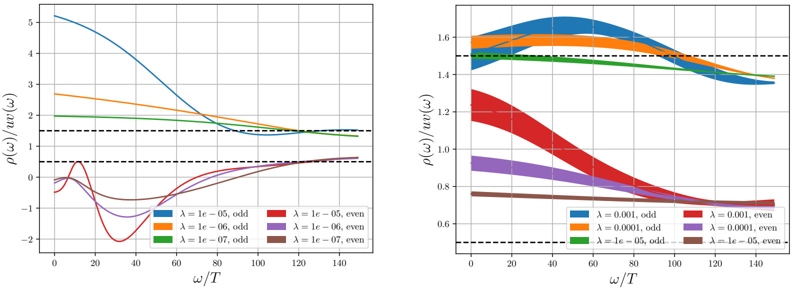
free



interacting

- In the free case 1/2 and 3/2 coefficients are obtained easily
- Interaction noticeably shifts $C_{\mathrm{e/o}}$, but the sum is almost constant, $(C_{\mathrm{e}} + C_{\mathrm{o}})/2 \approx 1$

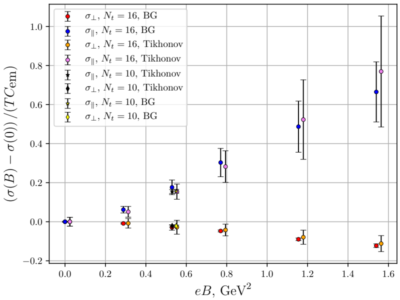
Ultraviolet reconstruction for $N_t = 96$, finite eB



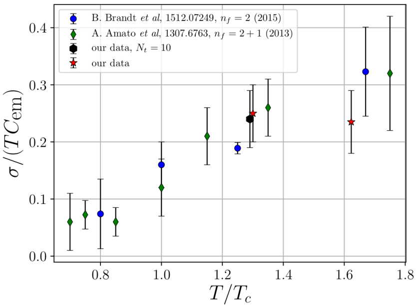
free　　　　interacting

- Free case with eB: asymptotic region is shifted to higher ω
- Interaction noticeably shifts $C_{\mathrm{e/o}}$, but the sum is almost constant, $(C_{\mathrm{e}} + C_{\mathrm{o}})/2 \approx 1$

Check at $eB = 0$ and $eB > 0$



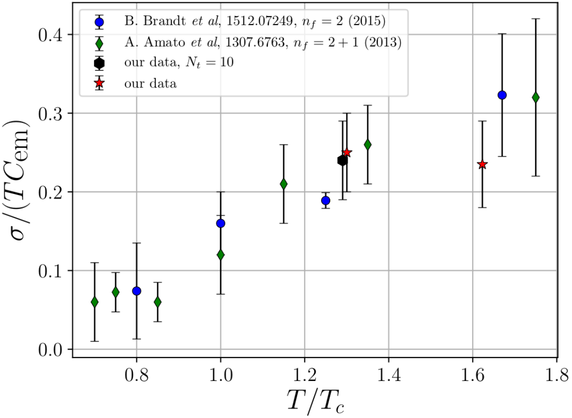
$N_t = 10$ vs $N_t = 16$



Comparison at $eB = 0$

- Our results are consistent for two different time extensions both at zero and finite eB
- Good agreement with previous studies at zero eB

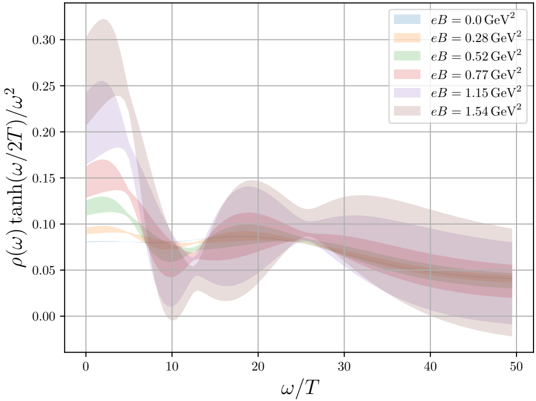
Results at $eB = 0$



- At $T = 200$ MeV flat spectral function → good analysis
- At $T = 250$ MeV B. Brandt *et al.* report the rise of peak at zero → possible underestimation

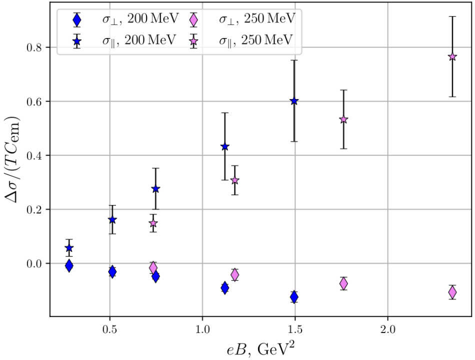
Conductivity at finite magnetic field

Idea: consider difference $C(t, eB) - C(t, eB = 0)$ to possibly avoid UV contamination, also δ becomes narrower



- The peak grows around $\omega = 0$, UV behavior is indeed small
- Correction due to the intermediate region is hard to estimate

Conductivity at finite magnetic field



- Linear growth is observed in $\sigma_\parallel$ at $eB \gg T^2$
- The $\sigma_\perp$ decay results from the Lorentz force acting on charged particles moving in the direction of $\vec{E} \perp \vec{B}$
- Estimation for chirality-changing scattering time from the slope of $\sigma_\parallel(eB)$ at $\sqrt{eB} \gg T$:
  - $\tau = 0.54(14)$ fm/c at $T = 200$ MeV
  - $\tau = 0.62(12)$ fm/c at $T = 250$ MeV