Conductivity of quark-gluon plasma in the presence of external magnetic field


XQCD 2019
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Heavy ion collisions — large $eB$

In non-central heavy ion collisions very strong magnetic field may emerge: $|e\vec{B}| \sim (3 - 10) m_\pi^2$
Chiral magnetic effect (CME)


“A system with a nonzero chirality responds to a magnetic field by inducing a current along the magnetic field. This is the Chiral Magnetic Effect.”

- macroscopic effect of microscopic dynamics of QCD
- allows probing the topological structure of $SU(3)$ gauge field
- non-dissipative, topologically protected
Parallel $\vec{E}$ and $\vec{B}$ — topologically non-trivial EM–field (non-zero winding number), Adler-Bell-Jackiw chiral anomaly generates topological density:

$$\frac{d\rho_5}{dt} = \frac{q^2}{2\pi^2} \vec{E} \cdot \vec{B}$$

Nielsen and Ninomiya energy argument:

$$\vec{j} \cdot \vec{E} = \mu_5 \frac{d\rho_5}{dt} = \frac{q^2 \mu_5}{2\pi^2} \vec{E} \cdot \vec{B} \implies$$

$$\vec{j} = \frac{q^2 \mu_5}{2\pi^2} \vec{B}$$

The expression for $j$ can be also calculated microscopically and is independent on the model.
From $\rho_5$ to $\mu_5$

Chirality-changing processes:

$$\frac{d\rho_5}{dt} = -\frac{\rho_5}{\tau} + \frac{q^2}{2\pi^2} \vec{E} \cdot \vec{B} \implies \rho_5 = \frac{q^2}{2\pi^2} \vec{E} \cdot \vec{B}\tau$$

At small $\mu_5 \ll T$, $\mu_5 \ll \sqrt{qB}$, $\rho_5 = \chi(B, T)\mu_5$

1. $T \gg \sqrt{qB}$, temperature dominates: $\chi(B, T) = T^2/3$,

2. $T \ll \sqrt{qB}$, 1st Landau level degeneracy:
$$\chi(B, T) = |qB|/2\pi^2$$

Linear response theory:

$$j^{i}_{\text{CME}} = \sigma^{ij}_{\text{CME}} E^j, \quad \sigma^{ij}_{\text{CME}} = \frac{q^4}{8\pi^4} \frac{\tau}{\chi(T, B)} B^i B^j$$
CME observation: QCD

- CME current forms dipole in the QGP fireball that affects hadron production at freeze-out

\[
\frac{dN_\pm}{d\phi} \propto 1 + 2v_1 \cos \phi + 2v_2 \cos 2\phi + \ldots + 2a_\pm \sin \phi + \ldots,
\]

where \(a_\pm = \pm \mu_5 |\vec{B}|\)

- However, \(\mu_5\) sign is event-dependent — can not observe \(\mathcal{P}\)-odd \(a_\pm\) directly (this would mean global \(\mathcal{P}\)-symmetry violation in QCD)

More complicated observables yet do not allow to 100%–confirm the existence of CME, but the data favors the existence of CME in QGP (see also the talk by Jinfeng Liao on Tuesday)
CME observation: Dirac semimetals

- Experimental: Q. Li et al., *Observation of the chiral magnetic effect in ZrTe$_5$*, Nature Physics 12, 550 – 554 (2016)

![Graphs showing experimental results and theoretical predictions for ZrTe$_5$](image)

experiment with ZrTe$_5$  

σ$_{\text{CME}}$ within QMC
Conductivity in external magnetic field

\[ \vec{E} \parallel \vec{B} \]

\[ \dot{\rho}_5 = \frac{q^2}{4\pi^2} (\vec{E}, \vec{B}) - \rho_5 / \tau, \]
\[ \tau \] — chirality-changing scattering time

\[ \rho_5 = \frac{q^2}{4\pi^2} (\vec{E}, \vec{B}) \text{ for } \dot{\rho}_5 = 0 \]

\[ \vec{J}_{\text{CME}} = \frac{q^2}{2\pi^2} \mu_5 \vec{B} \]

\[ \vec{J} = \sigma \vec{E} + \frac{q^2}{2\pi^2} \vec{B} \times \mu_5 \left( \rho_5 \sim \tau(\vec{E}, \vec{B}) \right) \]

Large magnetoconductivity \( \sigma_\parallel \)

Classically \( \delta \sigma_\parallel = 0 \)

Observed in experiment (Weyl semimetals):
H. Li et al., Nat. Comm. 7, 10301 (2016)

What happens in QCD?
Lattice details

- $N_f = 2 + 1$, physical quark masses
- Staggered fermions with improved action
- $T = 125$ MeV, 200 MeV, 250 MeV
- Lattice sizes and steps:

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<th>$N_t$</th>
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- Integral Kubo equation

\[
C(\tau_i) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau_i, \omega) \rho(\omega), \quad K(\tau_i, \omega) = \frac{\cosh \omega (\beta - \tau_i / 2)}{\sinh \omega \beta / 2} \omega
\]

- Conductivity ($C_{em} = q_u^2 + q_d^2 + q_s^2$):

\[
\frac{\sigma}{TC_{em}} = \frac{1}{6C_{em}} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}
\]
The Backus-Gilbert method

- The method is designed for solving linear ill-defined problems with controllable regularization and systematic uncertainty.

- Define the (normalized) resolution function \( \delta \) as the linear combination of adjustable coefficients \( q(\bar{\omega}) \):

\[
\tilde{\rho}(\bar{\omega}) = \int d\omega \delta(\bar{\omega}, \omega) \rho(\omega),
\]

\[
\delta(\bar{\omega}, \omega) = \sum_i q_i(\bar{\omega}) K(\tau_i, \omega),
\]

- Minimize the BG–functional:

\[
\mathcal{H}(\rho) = \lambda A(\rho) + (1 - \lambda) B(\rho),
\]

\[
A(\rho) = \int d\omega \delta(\bar{\omega}, \omega)(\omega - \bar{\omega})^2, \quad B(\rho) = \text{Var}[\rho] = q^T C q.
\]

The \( A \) part is the width of the resolution function (2nd moment to make \( q_i \) easy to find), \( B(\rho) \) — make less dependent on data (regularize). The method provides \( \rho(\omega) \) and \( \delta(\bar{\omega}, \omega) \) as the output!
Rescaling and resolution function

Rescaling of the kernel $K(\tau, \omega) \rightarrow f(\omega)K(\tau, \omega)$ leads to reconstruction of $\rho(\omega)/f(\omega)$ instead of $\rho(\omega)$. For conductivity we take $f(\omega) = \omega$.

Figure: Sample resolution function peaked at $\bar{\omega} = 0$ for rescaling $f(\omega) = \omega$.

The width is of order $\leq 3.5T$ (not enough $N_\tau$).
Ultraviolet contamination

Ultraviolet shape of the spectral function in the LO on the lattice:

\[
\rho_{UV}(\omega) = C_{e/o} \frac{3}{4\pi^2} \omega^2 \tanh \left( \frac{\omega \beta}{4} \right) \frac{\rho_{\text{lat}}(\omega)}{\rho_{\text{cont}}(\omega)}
\]

In the free case \( C_{\text{even}} = 1/2, \ C_{\text{odd}} = 3/2 \)
Staggered fermions and two branches

The staggered $\langle jj \rangle$ correlator has the oscillating structure:

\[ C(\tau) = A(\tau) + (-1)^\tau B(\tau) \]

\( \Delta \sigma(0) = A \int_{\omega_0}^{\infty} d\omega \frac{\rho_{UV}^e(\omega) + \rho_{UV}^o(\omega)}{2} \delta(0, \omega) \)  \( (1) \)
UV contribution estimation

- It is hard to do it model-independently.
- We assume that spectral function approximately reads (QCD sum rules):

\[
\rho(\omega) \approx (B\omega)_{\text{small } \omega} \theta(\omega_0 - \omega) + (A\rho_{\text{UV}}(\omega))_{\text{large } \omega} \theta(\omega - \omega_0).
\]

- The factor \( A \approx 1 \) accounts for radiative corrections, \( \omega_0 \) — threshold frequency.
- Fit in B. Brandt et al. [1512.07249], A. Amato et al. [1307.6763]: \( A \approx 1, \omega_0 \approx 7T, \chi^2/\text{ndof} \sim 1 \).
- Take \( f(\omega) = \rho_{\text{UV}}(\omega) \), expect that

\[
\lim_{\omega \to \infty} \frac{\hat{\rho}(\omega)}{f(\omega)} = A.
\]
Ultraviolet reconstruction for $N_t = 96, eB = 0$

- In the free case $1/2$ and $3/2$ coefficients are obtained easily.
- Interaction noticeably shifts $C_e/0$, but the sum is almost constant, $(C_e + C_0)/2 \approx 1$.
Ultraviolet reconstruction for $N_t = 96$, finite $eB$

Free case with $eB$: asymptotic region is shifted to higher $\omega$

Interaction noticeably shifts $C_e/\rho$, but the sum is almost constant, $(C_e + C_0)/2 \approx 1$
Check at $eB = 0$ and $eB > 0$

$N_t = 10$ vs $N_t = 16$

- Our results are consistent for two different time extensions both at zero and finite $eB$
- Good agreement with previous studies at zero $eB$
Results at $eB = 0$

- At $T = 200$ MeV flat spectral function $\rightarrow$ good analysis
- At $T = 250$ MeV B. Brandt et al. report the rise of peak at zero $\rightarrow$ possible underestimation
Conductivity at finite magnetic field

Idea: consider difference $C(t, eB) - C(t, eB = 0)$ to possibly avoid UV contamination, also $\delta$ becomes narrower

- The peak grows around $\omega = 0$, UV behavior is indeed small
- Correction due to the intermediate region is hard to estimate
Conductivity at finite magnetic field

- Linear growth is observed in $\sigma_\parallel$ at $eB \gg T^2$
- The $\sigma_\perp$ decay results from the Lorentz force acting on charged particles moving in the direction of $\vec{E} \perp \vec{B}$
- Estimation for chirality-changing scattering time from the slope of $\sigma_\parallel(eB)$ at $\sqrt{eB} \gg T$:
  - $\tau = 0.54(14) \text{ fm/c at } T = 200 \text{ MeV}$
  - $\tau = 0.62(12) \text{ fm/c at } T = 250 \text{ MeV}$