The Kondo effect in dense QCD

In collaboration with Xu-Guang Huang (Fudan U.) and Rob Pisarski (BNL)

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-- Dense quark matter with heavy-flavor impurities

KH, K. Itakura, S. Ozaki, S. Yasui, **arXiv:1504.07619 [hep-ph]**

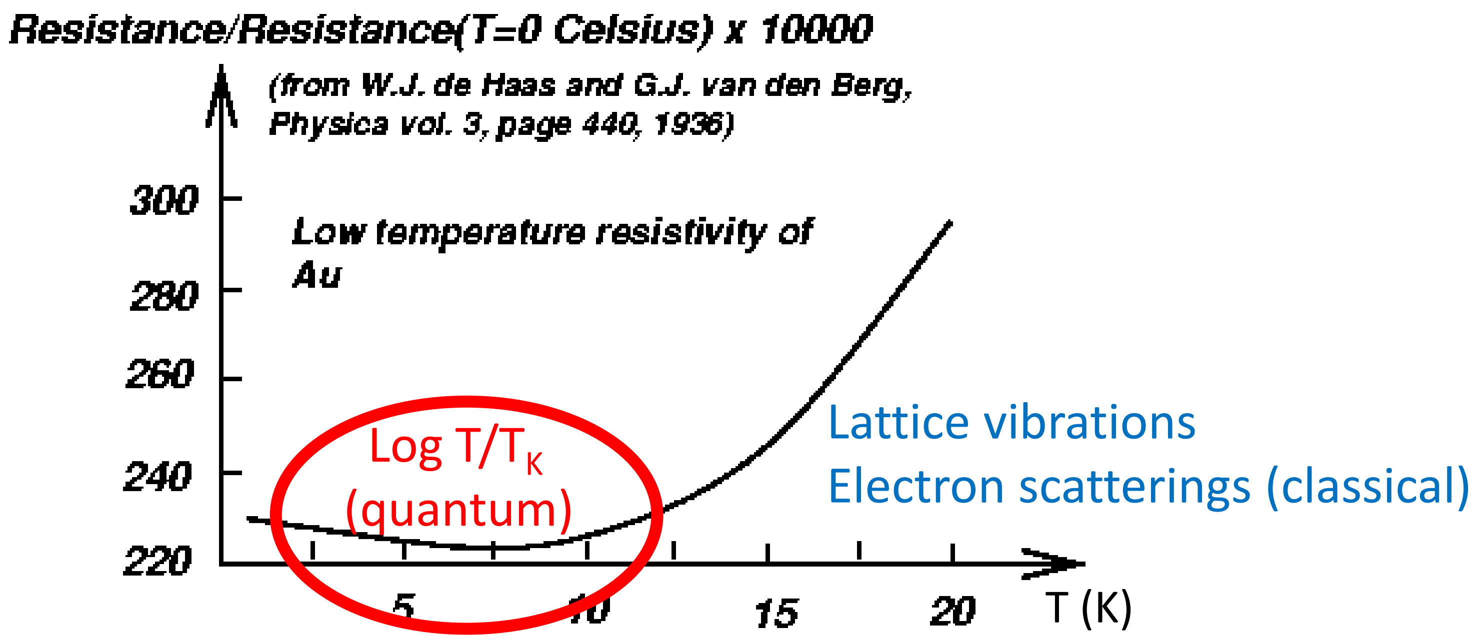
+ Impurity (heavy quark) scattering
+ Role of dimensional reduction in dense systems
+ Non-Abelian interaction in QCD

2 The Kondo effect in two-flavor superconducting phase

KH, X.-G. Huang, R. Pisarski, **arXiv:1903.10953 [hep-ph]**

The Kondo effect in cond. matt.

Measurement of the resistance of alloy (with impurities)



T_K: Kondo Temp. (Location of the minima)

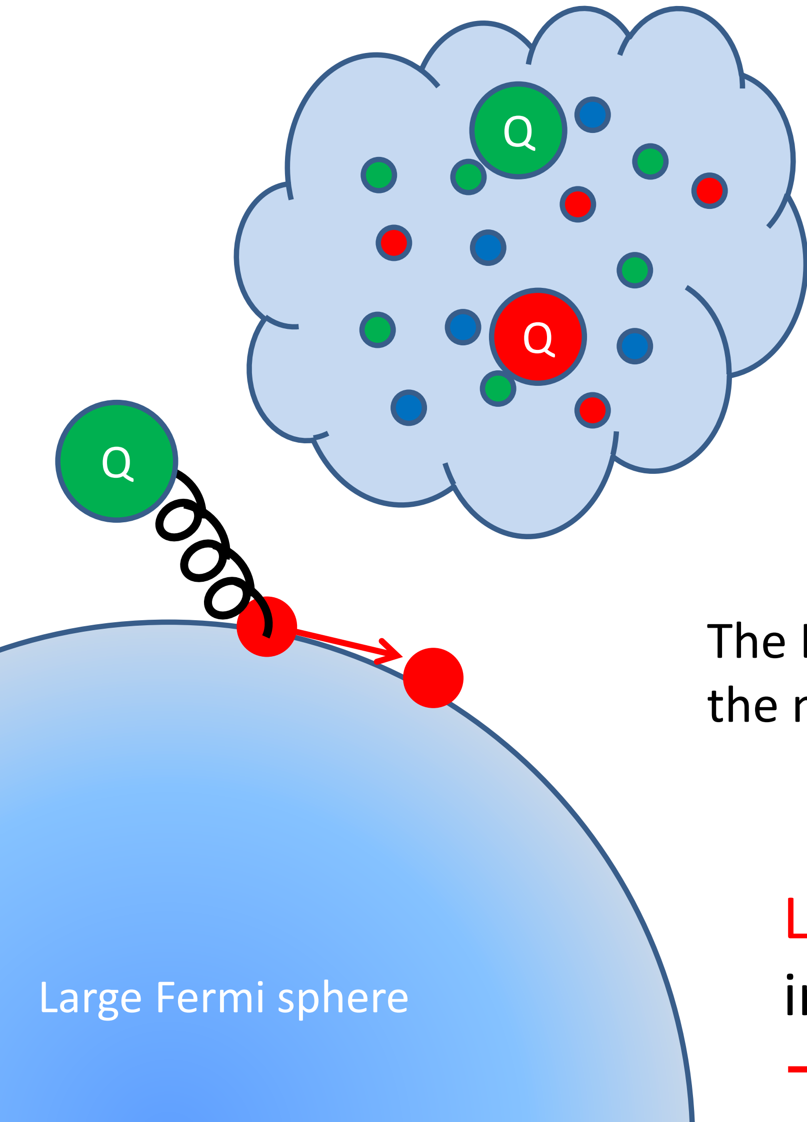
Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

Impurity scatterings near a Fermi surface

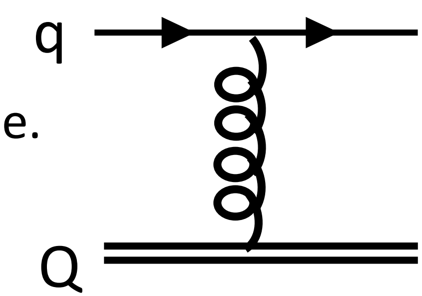
Heavy-quark impurity in light-quark matter



$$G(\Lambda)(\bar{\psi}\psi)(\bar{\Psi}\Psi)$$

How does the coupling evolve in the IR regime, Λ --> 0?

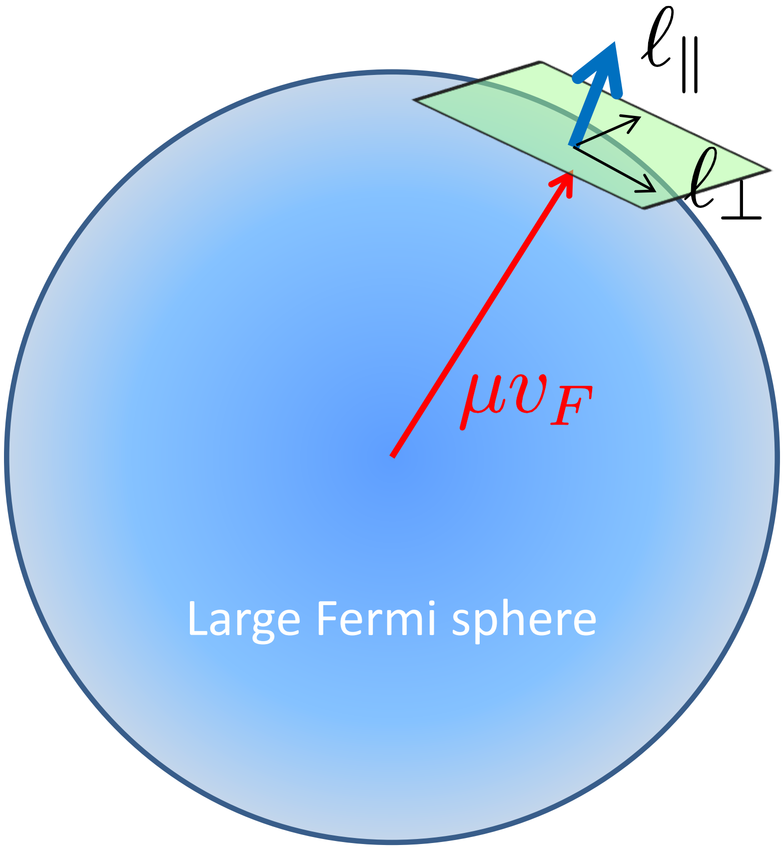
The LO does not explain
the minimum of the resistance.



Logarithmic quantum corrections arise in special kinematics and circumstances.

→ Kondo effect

"Dimensional reduction" in dense systems
-- (1+1)-dimensional low-energy effective theory



+ Low energy excitation along radius [(1+1) D]

$$\epsilon = \pm \ell_{\parallel} \quad (\ell_{\parallel} \ll \mu)$$

+ Degenerated states in the tangential plane [2D]

Phase space volume ~ p^{d-1} dp (No suppression for d=1).
Enhanced IR dynamics induces nonperturbative physics.

SC and Kondo effect occur for the dimensional reason,
and no matter how weak the attraction is.

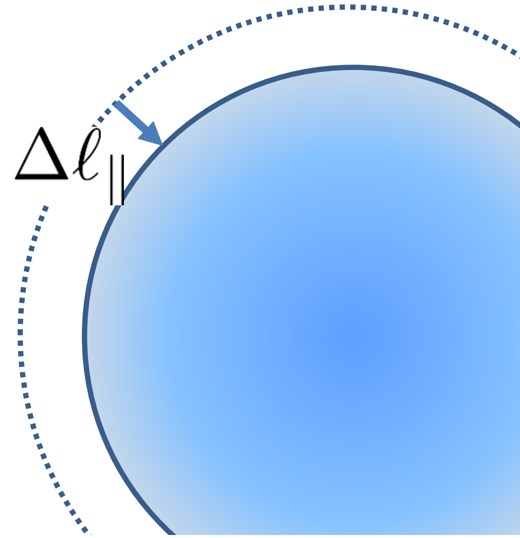
Scaling argument

Scaling dimensions in the IR

Evolution from UV to IR: $\epsilon \to \epsilon - \Delta\epsilon$

$$\ell_{||} \to \ell_{||} - \Delta\ell_{||}$$
$$\ell_{\perp} = \ell_{\perp}$$

$\ell_{\perp}$: Label of the degenerated states (Does not scale)



Scaling dimension of ψ is determined from the kinetic term.

$$\mathcal{S}^{\rm kin} = \int dt \sum_{\boldsymbol{v}_F} \int \frac{d^2\ell_{\perp} d\ell_{||}}{(2\pi)^3} \bar{\psi}_+ (i\partial_t - \ell_{||})\gamma^0 \psi_+ + \mathcal{O}(1/\mu)$$

$$0 = \underset{\bar{\psi}\cdot\psi}{\boxed{2d_\psi}} + \underset{dt}{\boxed{(-1)}} + \underset{d\ell_{||}}{\boxed{1}} + \underset{\partial_t}{\boxed{1}} \quad \Rightarrow \quad \boxed{d_\psi = -\frac{1}{2}}$$

Spatial dimension = 1

IR scaling dimension for the Kondo effect

Heavy-light 4-Fermi operator

Light quark: $d_\psi = -1/2$
Heavy quark: $d_\Psi = 0$

$$S^{\rm int}_{\rm H-L} = \int dt \left[\int \frac{d^2\ell_\perp d\ell_\parallel}{(2\pi)^3} \right]^2 G[\bar{\psi}^{(3)}_+ t^a \psi^{(1)}_+][\bar{\Psi}^{(4)}_+ t^a \Psi^{(2)}_+]$$

Heavy-quark field (impurity) is a scattering center for light quarks (No scaling).

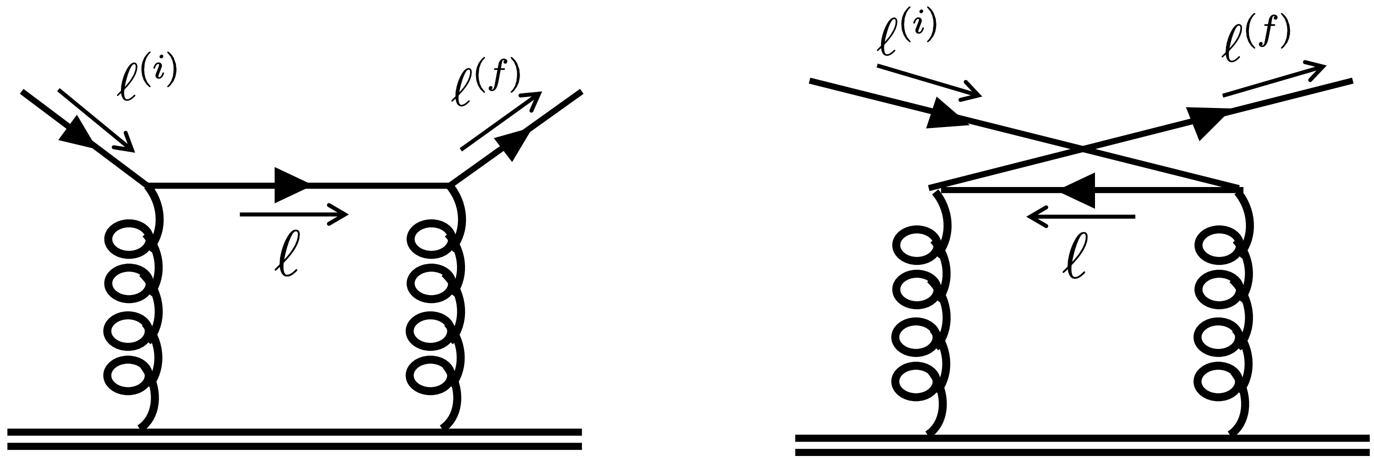
$$d_{(\psi\Psi)^2} = (-1) + 2(1 + d_\psi) + 2d_\Psi = 0$$

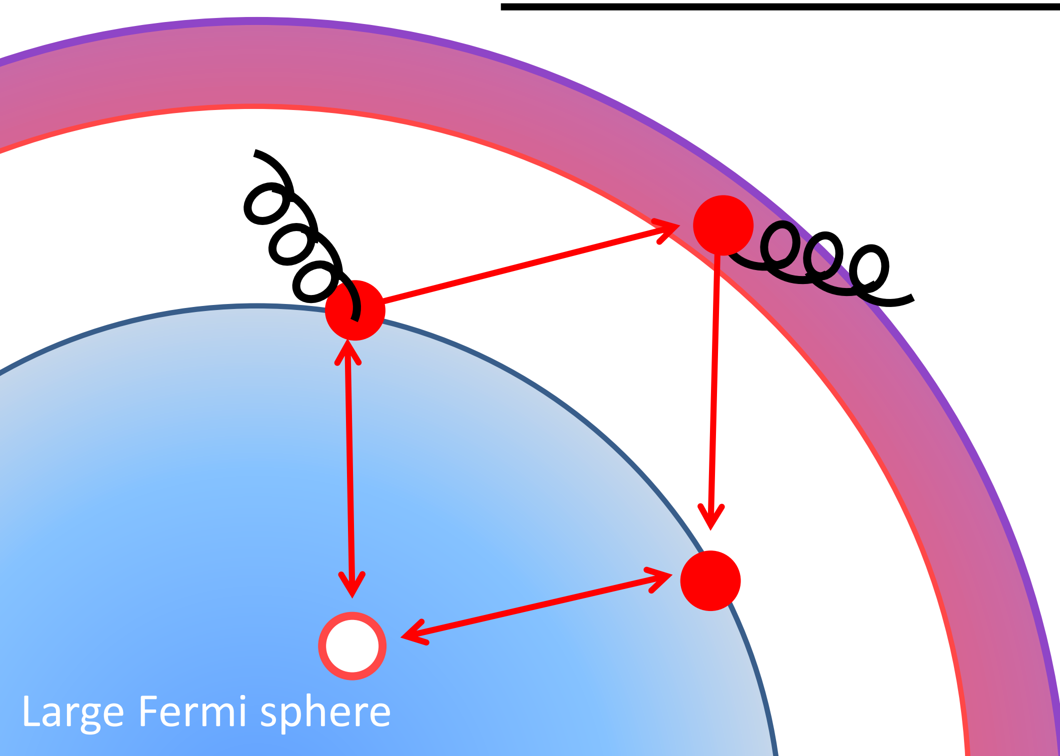
Marginal !! Let us proceed to diagrams.

Logarithms from the NLO diagrams

The NLO scattering amplitudes
-- Renormalization in the low energy dynamics

$\mathcal{M} =$



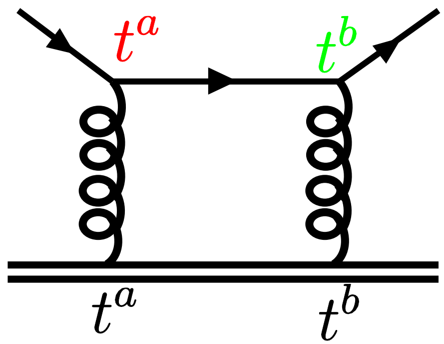


$$\int \frac{d\ell_{\parallel}}{\ell_{\parallel}} \sim \log \Lambda$$

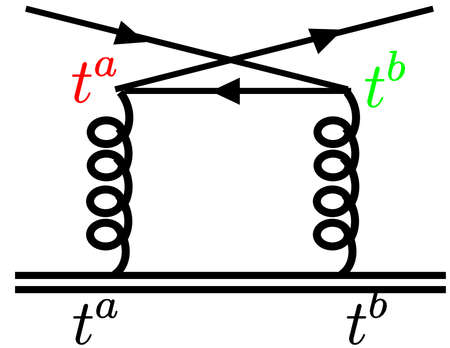
$$\int d^2\ell_{\perp} \sim \rho_F = \frac{\mu^2}{2\pi^2}$$

Density of states

Log correction and color-matrix structures



$$\bar{u}^k (t^b)^{kj} (t^a)^{ji} u^i$$



$$\bar{u}^k (t^a)^{kj} (t^b)^{ji} u^i$$

Particle contribution

$$\int_{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim + \log \frac{\Lambda}{\Lambda - d\Lambda}$$

Hole contribution

$$\int^{\epsilon_F} \frac{d\epsilon}{\epsilon} \sim - \log \frac{\Lambda}{\Lambda - d\Lambda}$$

Logs corrections cancel each other in an Abelian theory (No net effect).

✔ *Incomplete cancellation* due to the color matrices

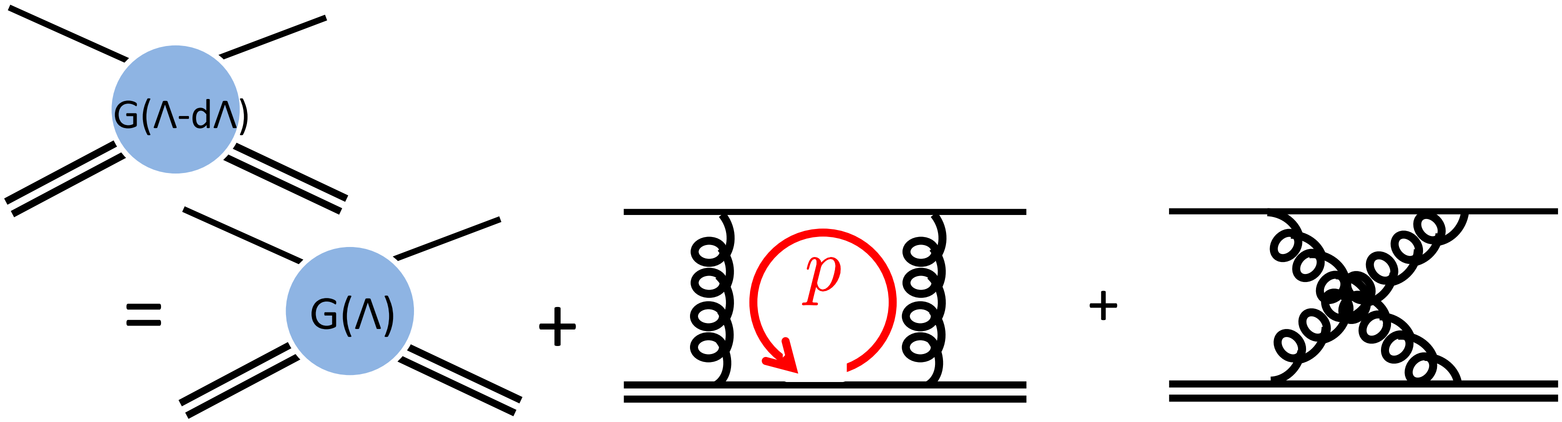
Particle contribution

$$[t^a t^b]_{ij} [t^a t^b]_{k\ell} = c\delta_{ij}\delta_{k\ell} - \frac{1}{n} t^c_{ij} t^c_{k\ell}$$

Hole contribution

$$[t^a t^b]_{ij} [t^b t^a]_{k\ell} = c\delta_{ij}\delta_{k\ell} - \frac{1}{n} t^c_{ij} t^c_{k\ell} + \frac{n}{2} t^c_{ij} t^c_{k\ell}$$

RG analysis for “the QCD Kondo effect”

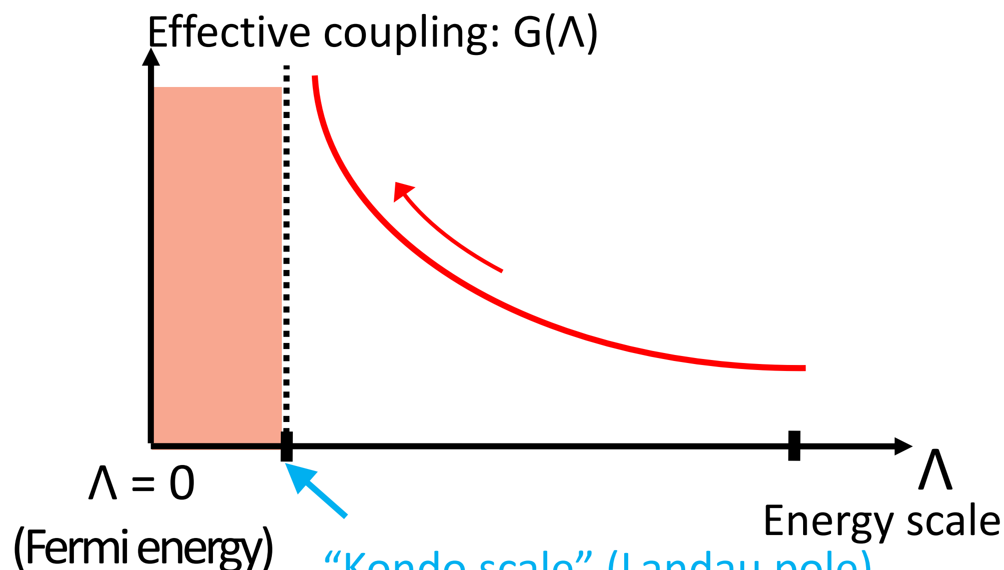


$\int dp_{\|}/p_{\|} \sim \log\Lambda$

$\int d^2p_{\perp} \sim \rho_F = \frac{\mu^2}{2\pi^2}$ (Density of states)

RG equation

$$\Lambda \frac{dG(\Lambda)}{d\Lambda} = -cN_c \cdot \rho_F \cdot G^2(\Lambda)$$



Landau pole in the asymptotic-free solution

$$\Lambda_K \sim \mu \exp\left(-\frac{c}{N_c g^2}\right)$$

Depends on the interactions.
(Debye screening mass for $A^0 \rightarrow g^2$ dep.)

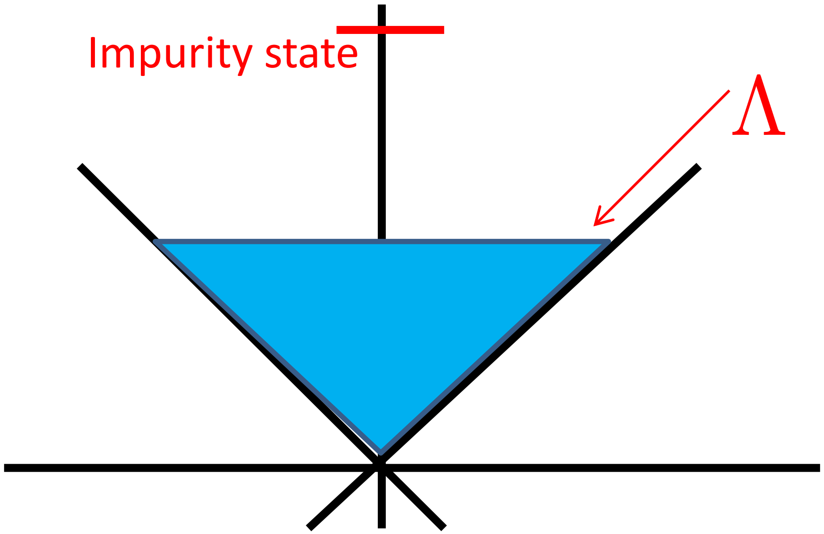
Resistivity is enhanced in the strong-coupling regime.

Short summary for the Kondo effect in quark matter

1. Non-Ablelian interaction

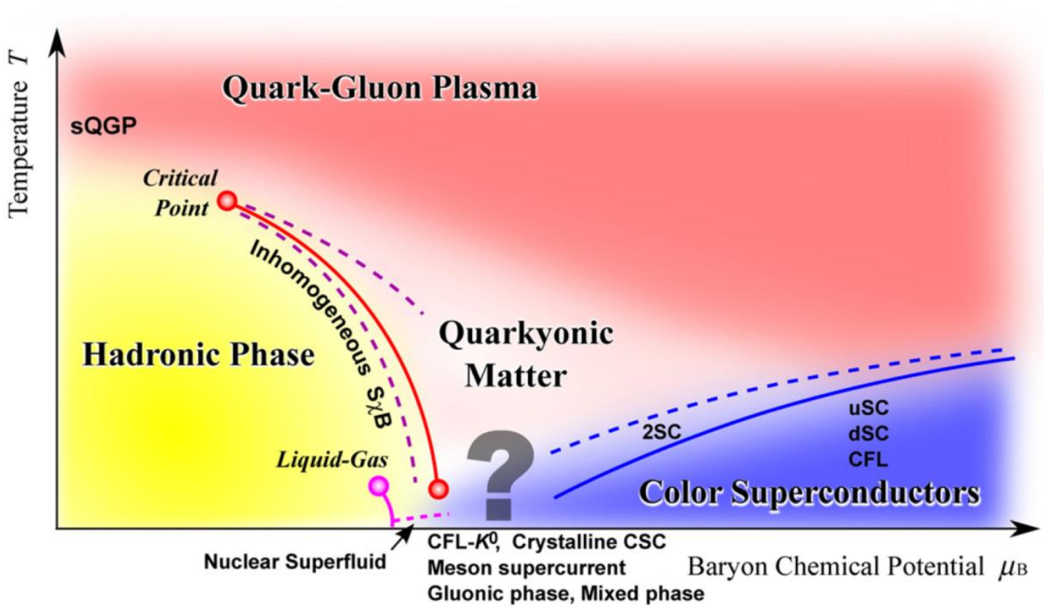
2. Dimensional reduction near the Fermi surface

3. Continuous spectra near the Fermi surface,
   and heavy impurities (gapped spectra).

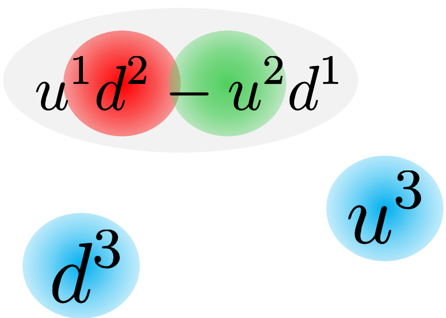


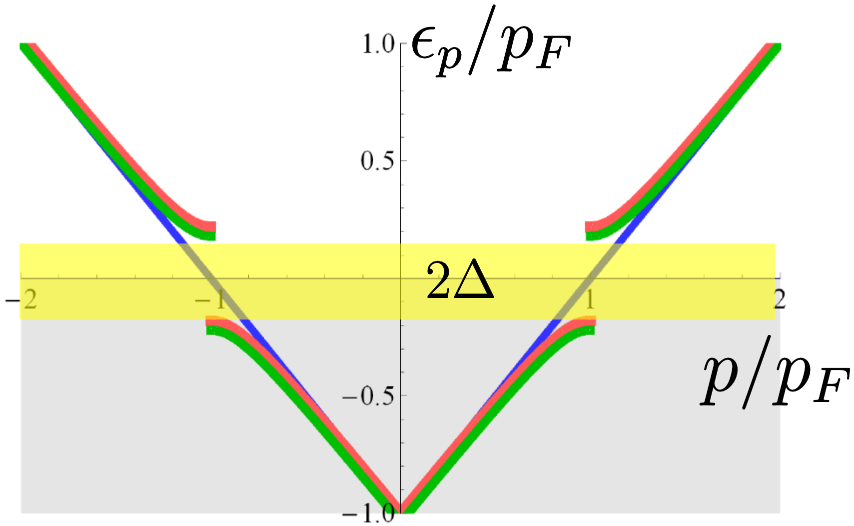
The Kondo effect in 2SC phase

“Gapped” and “ungapped” quarks in 2SC phase



Attraction in color $\overline{3}$
S-wave
Spin-0
Flavor antisymmetric





$$\epsilon_p = \sqrt{(|\boldsymbol{p}| - \mu)^2 + \Delta^2}$$

$$\epsilon_p = ||\boldsymbol{p}| - \mu|$$

Gluons in the 2SC phase

Gapped - Gapped

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Ungapped - Gapped

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Pure gluodynamics in the unbroken sector
Rischke, Son, Stephanov

Gluons in the broken sector are all gapped by the Debye and Meissner masses.

Gluon color a	$-\Pi^{00}_{aa}(0)$ $T=0$	$T \geqslant T_c$	$\Pi^{ii}_{aa}(0)$ $T=0$	$T \geqslant T_c$
1–3	0	$3\, m_g^2$	0	0
4–7	$\frac{3}{2} m_g^2$	$3\, m_g^2$	$\frac{1}{2}\, m_g^2$	0
8	$3\, m_g^2$	$3\, m_g^2$	$\frac{1}{3}\, m_g^2$	0

$$m_g^2 = \frac{(g\mu)^2}{6\pi^2}$$

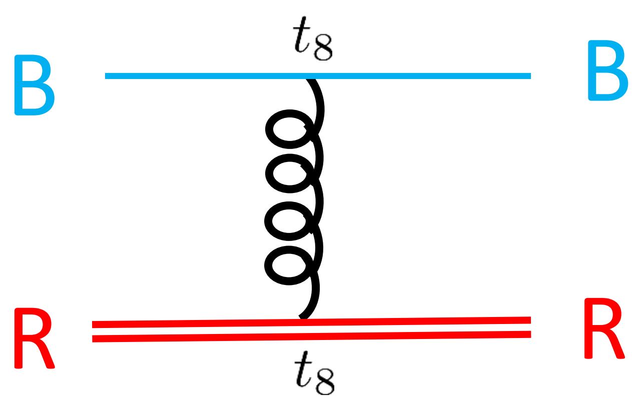
D. Rischke

Scattering btw the red (gapped) and blue (ungapped).

-- Gluons 4, 5, 8 are coupled to R and B.

LO diagrams

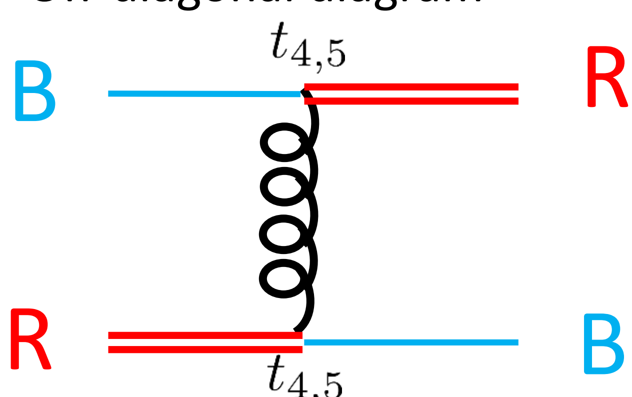
"Diagonal diagram"



$$\mathcal{M}_0 \sim \bar{G}[\bar{u}_B \gamma^0 \bar{u}_B][\bar{u}_R \gamma^0 \bar{u}_R]$$

$$\bar{G}(\Lambda_0) \sim -g^2/m_g^2 < 0$$

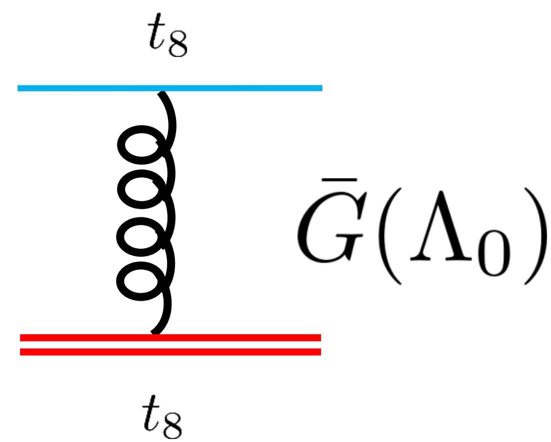
"Off-diagonal diagram"

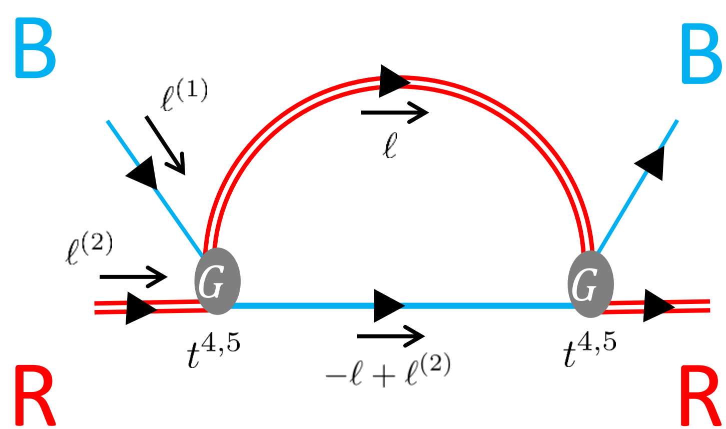


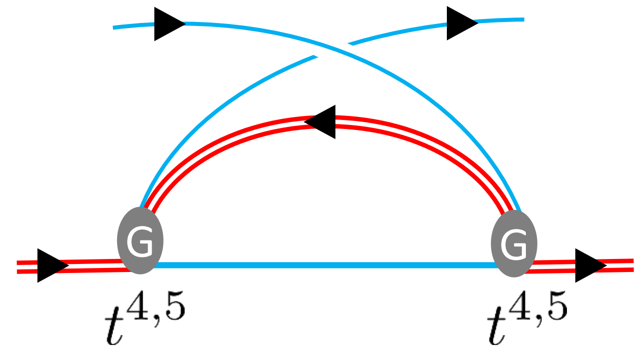
$$\mathcal{M}_0 \sim G[\bar{u}_R \gamma^0 \bar{u}_B][\bar{u}_B \gamma^0 \bar{u}_R]$$

$$G(\Lambda_0) \sim g^2/m_g^2 > 0$$

Log corrections to the “diagonal diagram”



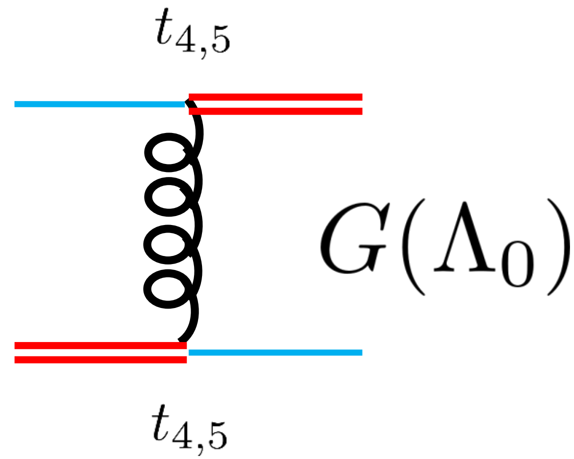


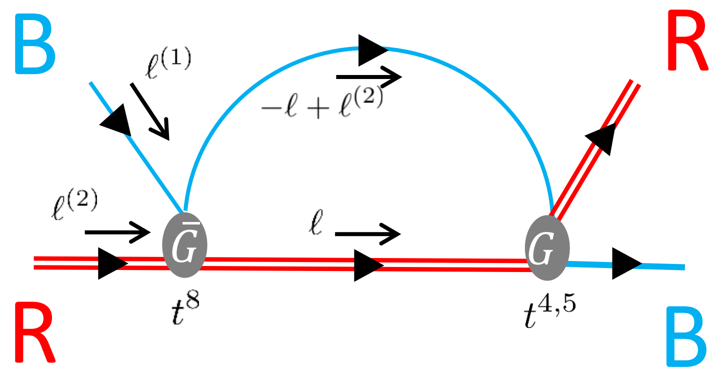


Diagrams with two diagonal matrices t^8 cancel each other (Abelian).

$$\mathcal{M} \sim G^2 \log \Lambda \qquad \Lambda \frac{d\bar{G}}{d\Lambda} = -\frac{3}{4} \rho_F \, G^2$$

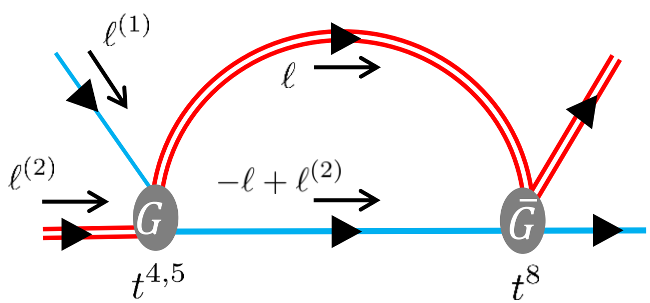
Log corrections to the "off-diagonal diagram"





+ Disconnected diagrams (cross channels)

→ Do not yield logs.



$$\mathcal{M} \sim G\bar{G}\log\Lambda \quad \Rightarrow \quad \Lambda\frac{dG}{d\Lambda} = -\frac{1}{6}\rho_F\, G\bar{G}$$

Evolution of the coupled RG equations

$$\begin{cases} \Lambda \dfrac{dG}{d\Lambda} = -\dfrac{1}{6}\rho_F \, G\bar{G} \\ \Lambda \dfrac{d\bar{G}}{d\Lambda} = -\dfrac{3}{4}\rho_F \, G^2 \end{cases}$$

Density of states:
$\rho_F = \frac{\mu^2}{2\pi^2}$

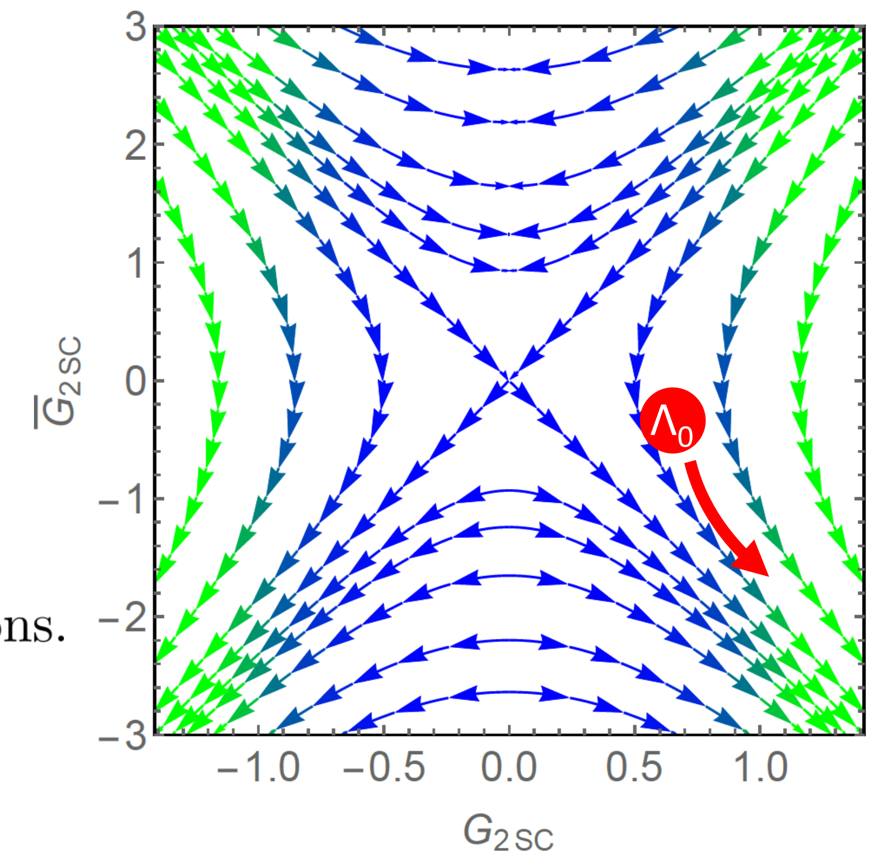
$$\Rightarrow \quad \frac{dG}{d\bar{G}} = \frac{2}{9} \cdot \frac{\bar{G}}{G}$$

RG evolution along the hyperbolic curves

$$(3G)^2 - 2(\bar{G})^2 = C$$

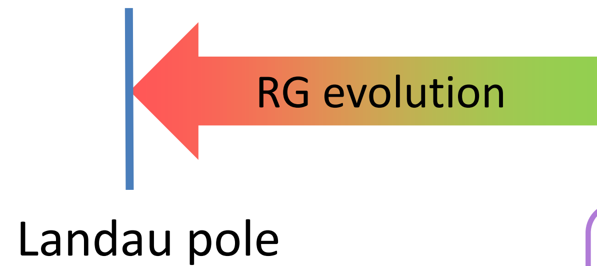
C is determined by the initial conditions.

$$G(\Lambda_0) > 0, \quad \bar{G}(\Lambda_0) < 0$$



Hierarchy in the 2SC phase

$$T_K \ll \Delta \ll \mu$$



Quark chemical potential μ

SC gap $\Delta \sim T_c \sim \mu \exp(-c' g^{-1})$

g^{-1} dep. from the unscreened magnetic gluon

Kondo temperature $T_K^{2\mathrm{SC}} \sim \mu \exp(-c g^{-2})$

g^{-2} dep. from the all screened gluons (Meissner mass)

Summary

The QCD Kondo effect occurs in various systems.

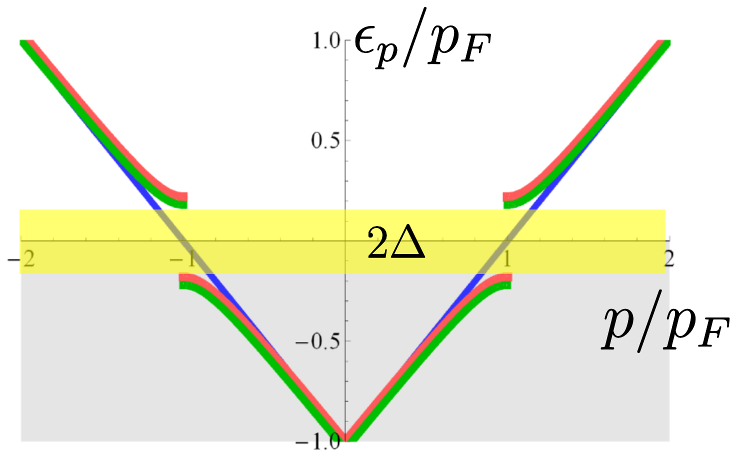
Necessary ingredients

1) Non-Abelian interactions (QCD)

2) Gapped and ungapped spectra near the Fermi surface

-- Heavy-quark impurities

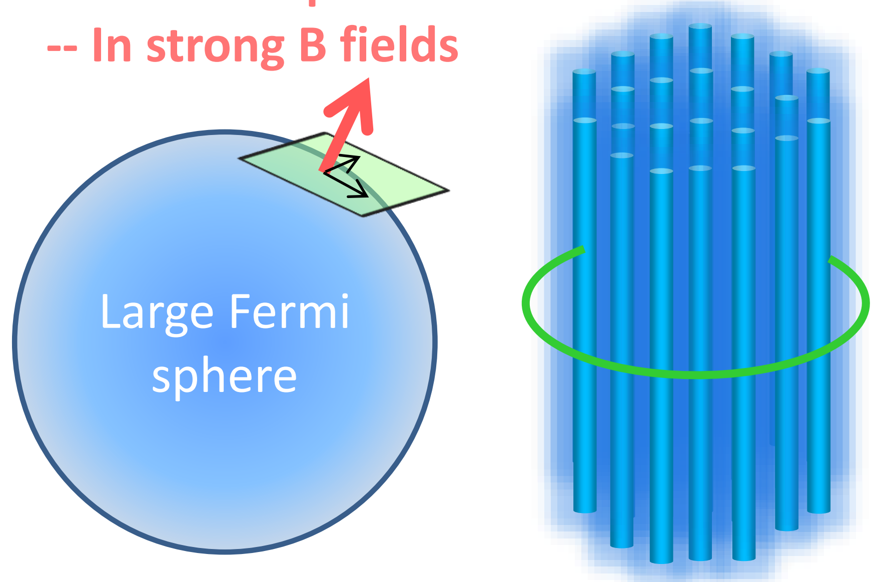
-- Gapped quarks in 2SC



3) Dimensional reductions

-- In dense quark matter

-- In strong B fields



Ozaki, K. Itakura, Y. Kuramoto

Prospects

--- Transport properties in neutron star physics

--- Realization with ultracold atoms

Back-up

IR scaling dimensions

$$\text{When } \epsilon \to s\epsilon,\ \ell_\parallel \to s\ell_\parallel. \quad (s<1)$$

Kinetic term

$$\mathcal{S}^{\mathrm{kin}} = \int dt \sum_{\boldsymbol{v}_F} \int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \bar{\psi}_+ (i\partial_t - \ell_\parallel)\gamma^0 \psi_+ + \mathcal{O}(1/\mu)$$

$$0 = \underbrace{2d_\psi}_{\bar{\psi}\cdot\psi} + \underbrace{(-1)}_{dt} + \underbrace{1}_{d\ell_\parallel} + \underbrace{1}_{\partial_t} \qquad \boxed{d_\psi = -\frac{1}{2}}$$

Four-Fermi operators for superconductivity

Polchinski (1992)

$$\mathcal{S}^{\mathrm{int}} = \int dt \left[\int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3} \right]^4 G[\bar{\psi}_+^{(4)} \hat{\gamma}_\parallel^\mu \psi_+^{(2)}][\bar{\psi}_+^{(3)} \hat{\gamma}_\mu^\parallel \psi_+^{(1)}]\delta^{(3)}(\boldsymbol{p}^{(1)} + \boldsymbol{p}^{(2)} - \boldsymbol{p}^{(3)} - \boldsymbol{p}^{(4)})$$

In general momentum config.

$$p^{(1)} + p^{(2)} \sim \mu \qquad d_{4-\mathrm{Fermi}} = \underbrace{(-1)}_{dt} + \underbrace{4(1-\frac{1}{2})}_{4(d\ell_\parallel + d_\psi)} = +1$$

In the BCS config.

$$p^{(1)} + p^{(2)} \sim \ell_\parallel \ll \mu \qquad d_{4-\mathrm{Fermi}} = (-1) + 4(1-\frac{1}{2}) \boxed{-1} = 0$$

IR scaling dimension for the Kondo effect

Heavy-quark Kinetic term

$$S_H^{\rm kin} = \int dt \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\, \Psi_+^\dagger(\boldsymbol{k})\, i\partial_t \Psi_+(\boldsymbol{k}) + \mathcal{O}(1/m_H)$$

$$d_\Psi = (-1) + 1 = 0$$

Heavy-light four-Fermi operator

$$S_{\rm H-L}^{\rm int} = \int dt \left[\int \frac{d^2\boldsymbol{\ell}_\perp d\ell_\parallel}{(2\pi)^3}\right]^2 \left[\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\right]^2 G[\bar{\psi}_+^{(3)} t^a \psi_+^{(1)}][\bar{\Psi}_+^{(4)} t^a \Psi_+^{(2)}]$$

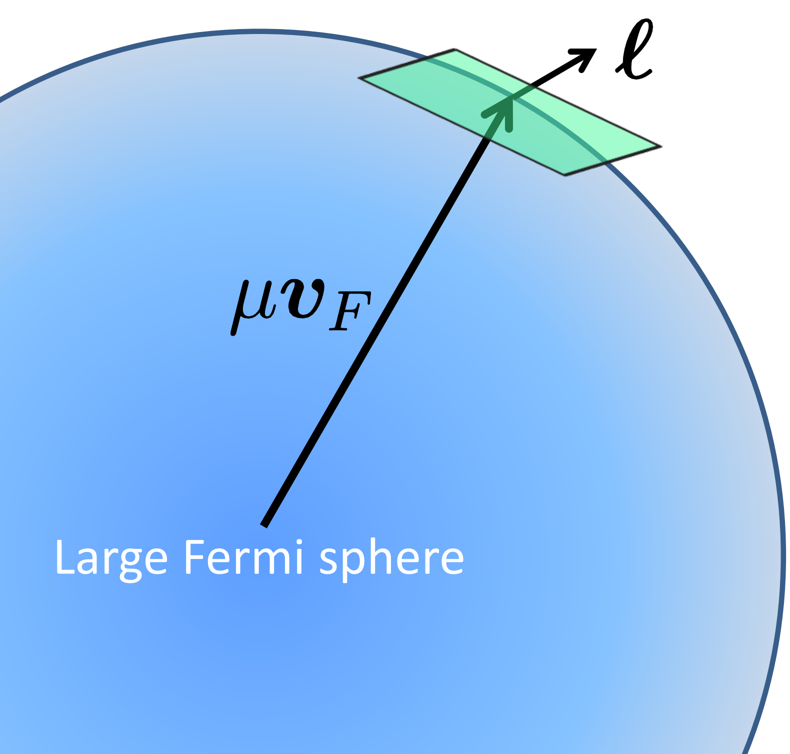
$$d_{\rm H-L} = (-1) + 2(1 + d_\psi) + 2d_\Psi = 0$$

Marginal !! Let us proceed to diagrams.

High-Density Effective Theory (LO)

Expansion around the large Fermi momentum

$$p^0 = \ell^0 , \quad \boldsymbol{p}^i = \mu \boldsymbol{v}_F^i + \boldsymbol{\ell}^i$$

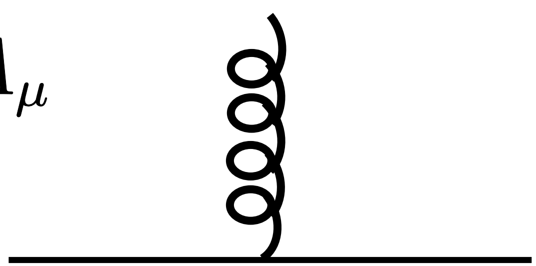


(1+1)-dimensional dispersion relation

$$\ell^0 = \boldsymbol{v}_F \cdot \boldsymbol{\ell} \equiv \ell_{\parallel}$$

Spin flip suppressed
when the mass is small m << μ.

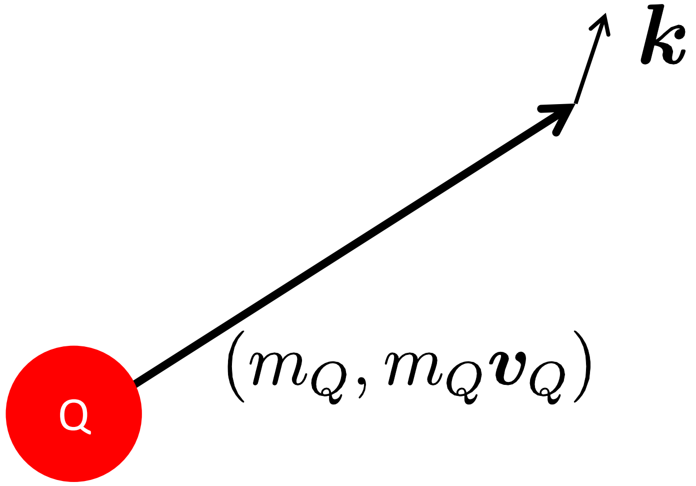
$$\gamma^\mu A_\mu \rightarrow \gamma^0 v_F^\mu A_\mu$$



Heavy-Quark Effective Theory (LO)

HQ-momentum decomposition

$$p^\mu = m_Q v_Q^\mu + k^\mu$$



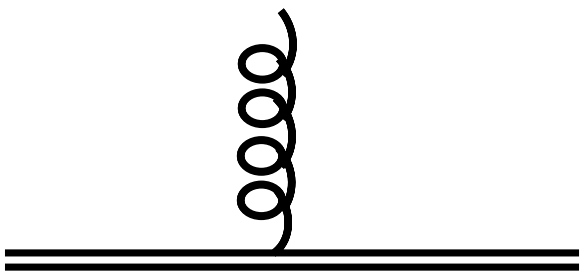
HQ velocity

$$v_Q^\mu = \frac{1}{m_Q} P^\mu \Big|_{P^2 = m_Q^2}$$

Nonrelativistic magnetic moment suppressed by $1/m_Q$

$$\gamma^\mu A_\mu \to v_Q^\mu A_\mu$$

$$\gamma^\mu A_\mu = A^0 \text{ when } \vec{\boldsymbol{v}}_Q = 0.$$



Gluon propagator in dense matter

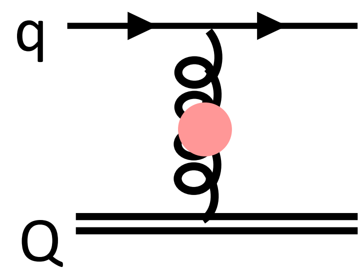
$$D^{\mu\nu}(k) = \frac{P_L^{\mu\nu}}{k^2 - \Pi_L} + \frac{P_T^{\mu\nu}}{k^2 - \Pi_T} - \xi \frac{k^\mu k^\nu}{k^4}$$

$$P_T^{\mu\nu} = \delta^{\mu i}\delta^{\nu j}\left(\delta^{ij} - \frac{k^i k^j}{|k|^2}\right)$$

$$P_L^{\mu\nu} = -\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) - P_T^{\mu\nu}$$

Screening of the <A^0A^0> from the HDL

$$\Pi_L \sim m_{\mathrm{Debye}}^2 \sim (g\mu)^2$$



Cf., Son, Schaefer, Wilczek, Hsu, Schwetz, Pisarski, Rischke, ……,
showed that unscreened magnetic gluons play a role in the cooper paring.

Propagator for the gapped quasiparticles and quasiholes

$$G(p) = i\frac{p^0 - (\mu - \boldsymbol{p})}{(p^0)^2 - \epsilon_p^2 + i\epsilon} P_+ \gamma^0 \qquad \epsilon_p = \sqrt{(|\boldsymbol{p}| - \mu)^2 + \Delta^2}$$

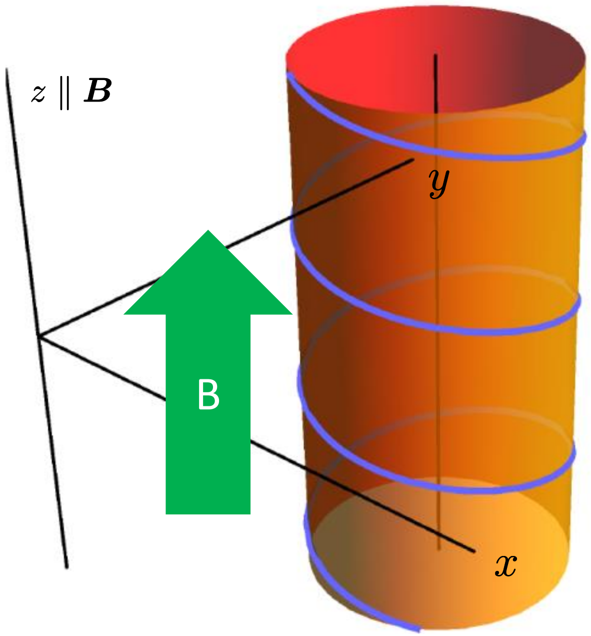
Rischke, Pisarski, ...

The LO expansion by 1/μ

$$p^0 = \ell^0 \,, \quad \boldsymbol{p}^i = \mu \boldsymbol{v}_F^i + \boldsymbol{\ell}^i$$

$$G(p) = i\frac{\ell^0 + \ell_{\parallel}}{(\ell^0 - \epsilon_\ell + i\varepsilon)(\ell^0 + \epsilon_\ell - i\varepsilon)} P_+ \gamma^0$$

Landau level discretization due to the cyclotron motion



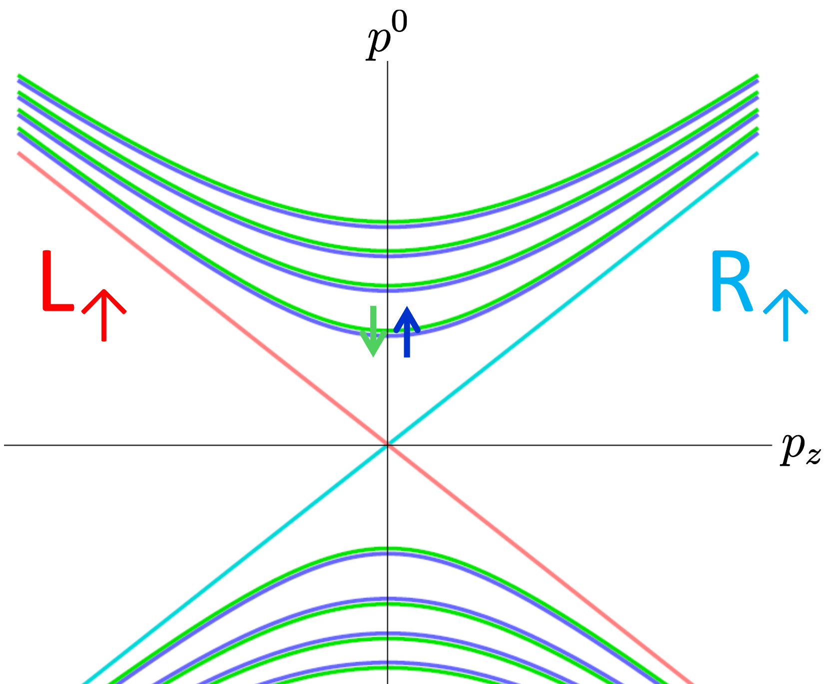
"Harmonic oscillator" in the transverse plane

Nonrelativistic: $\epsilon_n = \dfrac{p_z^2}{2m} + (n + \dfrac{1}{2})\boxed{\dfrac{eB}{m}}$

Cyclotron frequency

Relativistic: $\epsilon_n = \sqrt{p_z^2 + (2n+1)eB + m^2}$

In addition, there is the Zeeman effect.



Scaling dimensions in the LLL

When $\epsilon_{\rm LLL} \to s\epsilon_{\rm LLL}$, $p_z \to sp_z$. ($\boldsymbol{p}_\perp$ does not scale.)

(1+1)-D dispersion relation → d_ψ = - 1/2

Four-light-Fermi operator

$$\mathcal{S}^{\rm int} = \int dt \left[\int \frac{dp_z}{2\pi} \right]^4 G[\bar{\psi}^{(4)}_{\rm LLL} \hat{\gamma}^{\mu}_{\parallel} \psi^{(2)}_{\rm LLL}][\bar{\psi}^{(3)}_{\rm LLL} \hat{\gamma}^{\parallel}_{\mu} \psi^{(1)}_{\rm LLL}] \delta(p_z^{(1)} + p_z^{(2)} - p_z^{(3)} - p_z^{(4)})$$

Always marginal thanks to the dimensional reduction in the LLL.

→ Magnetic catalysis of chiral condensate.

Chiral symmetry breaking occurs even in QED.

Gusynin, Miransky, and Shovkovy. Lattice QCD data also available (Bali et al.).

Heavy-light four-Fermi operator

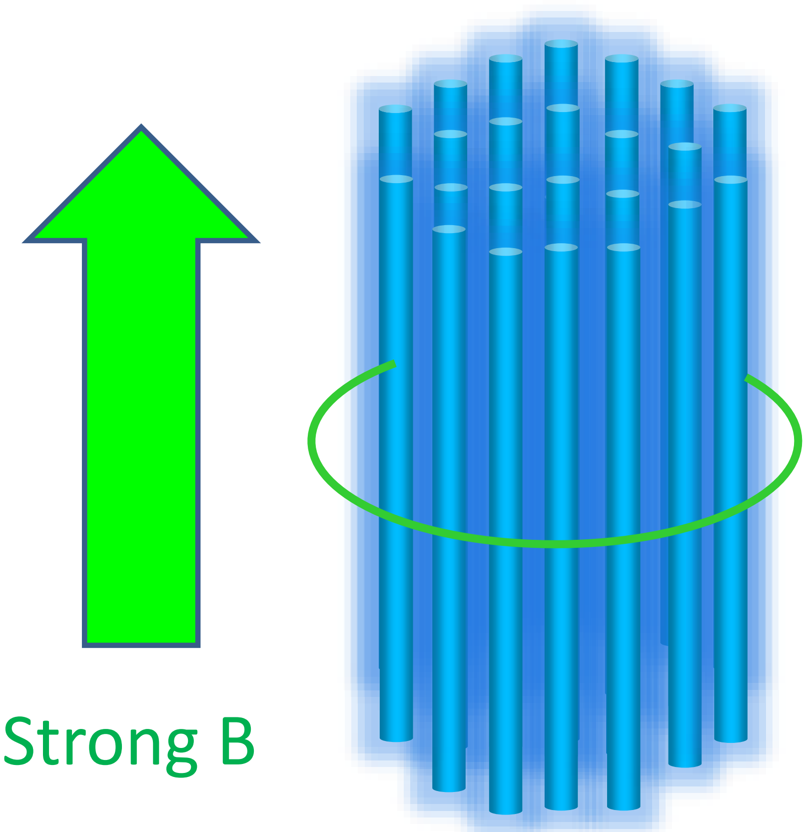
$$S^{\rm int}_{\rm H-L} = \int dt \left[\int \frac{dp_z}{2\pi} \right]^2 \left[\int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \right]^2 G[\bar{\psi}^{(3)}_{\rm LLL} t^a \psi^{(1)}_{\rm LLL}][\bar{\Psi}^{(4)}_{+} t^a \Psi^{(2)}_{+}]$$

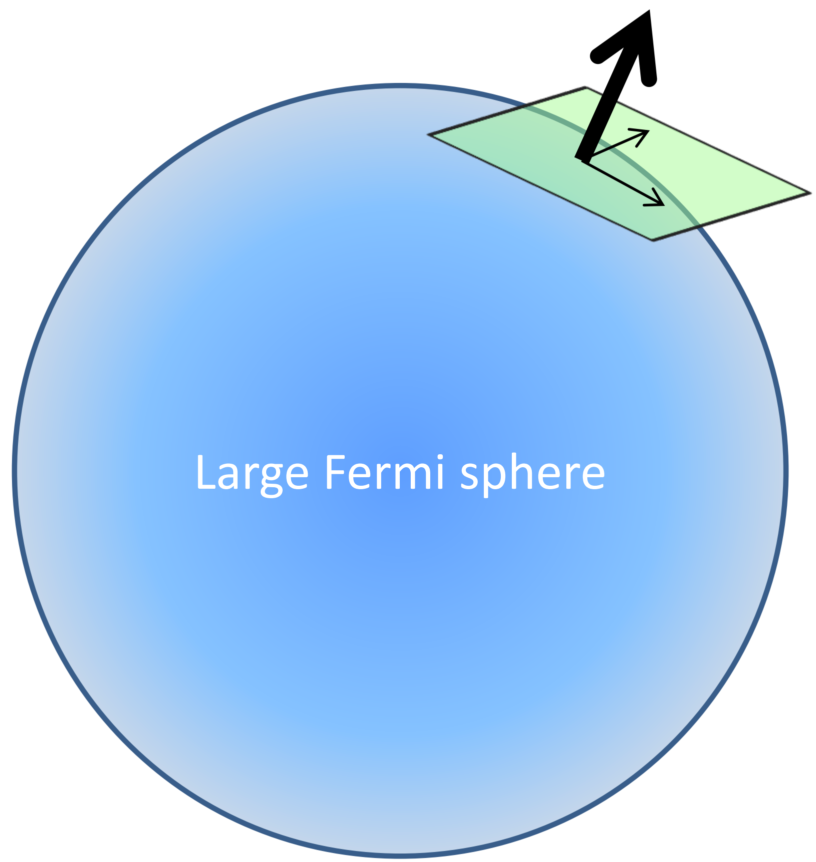
Marginal !! Just the same as in dense matter.

Analogy btw the dimensional reduction in a large B and μ

(1+1)-D dispersion relations

$$\varepsilon = \pm p_z \qquad\qquad \epsilon = \pm \ell_{\parallel} \quad (\ell_{\parallel} \ll \mu)$$





2D density of states

$$\rho = \frac{N_{\mathrm{state}}}{S} = \frac{eB}{2\pi} \qquad\qquad \rho_F \sim \mu^2$$