1. INTRODUCTION

- Color superconductivity is the generalization of ordinary superconductivity of electrons to quarks with color charge.
- Because of the attractive interaction originating from gluon exchange, the Fermi surface of quarks gets unstable against pair creation, which condense at low temperature.
- The Ginzburg-Landau effective theory, i.e. scalar QCD in three dimensions, describes the system close to the critical temperature.
- Order of the color superconducting transition is predicted to be of first order by 1-loop calculations. Same argument fails for ordinary superconductors.
- Asymptotic freedom of QCD also means the absence of infrared (IR) stable fixed points, thus no 2nd order transition can occur in $d = 4 - \epsilon$ dimensions. Is $\epsilon = 1$ reliable? It is not in ordinary superconductivity.
- Goal: search for IR stable fixed points in the Ginzburg-Landau theory of color superconductivity directly in three dimensions.

2. METHOD

- The employed method is the Functional Renormalization Group (FRG).
- In the $\Gamma$ scale dependent quantum effective action all fluctuations are incorporated beyond momentum scale $k$.
- A regulator function $R_k(x)$ is attached to the classical Lagrangian, which suppresses low momentum modes.
- $\Gamma_k$ obeys the RG flow equation:
  \[
  \partial_k \Gamma_k = \frac{1}{2} \int_x \int_y \text{Tr} \left\{ \Gamma''_n + R_k \right\}^{-1}(x,y) \partial_k R_k(y,x) \]
- One-loop diagrams need to be evaluated!

3. SCALAR QCD

- The effective theory of color superconductivity:
  \[
  \mathcal{L} = \frac{1}{2} A^i_k \partial^a \phi^b_k \partial_a \phi^b_k + c^i \left( \partial^a \phi^b_k \partial_a \phi^b_k - g \Delta^i_k \right) + \frac{\phi^b_k}{2} \left( \partial^a \phi^b_k \partial_a \phi^b_k \right) + V(\phi) + \lambda \partial^2 \phi^b_k (\partial^a \phi^b_k)^2 \]
- $\Delta^i_k$ is an unspecified scalar potential

4. $\beta$-FUNCTION DEFINITIONS

- There are multiple ways to define the $\beta$-function of the gauge coupling, which agree due to the Slavnov-Taylor identities.
  - Choice I: scalar-gauge-gauge vertex
  - Choice II: scalar-gauge-gauge-gauge vertex
  - Choice III: three gluon vertex
  - Choice IV: four gluon vertex
- Advantage of choice V:
  - Generalization of the QED Ward identity is automatically satisfied
  - i.e. $Z_q - Z_\phi$ is matter independent (zero in QED, nonzero in QCD)
  - Corresponding diagrams are simple

5. GLUON-GHOST-ANTIQUARK VERTX

- Two diagrams need to be evaluated:

6. GHOST WAVEFUNCTION RENORMALIZATION

- Here we have only one diagram:

7. GLUON WAVEFUNCTION RENORMALIZATION

- Three diagrams need to be evaluated:

8. $\beta$-FUNCTION

- The $\beta$-function of the gauge coupling in $d = 3$ turns out to be:
  \[
  \beta(\phi)_{d=3} = -\phi_k - \frac{g_k^2}{2\pi^2} \left[ \frac{19}{3} + \frac{16}{15} \right] \frac{2}{5}
  \]
  \[
  \rightarrow \beta \text{ is nonpositive and the only fixed point is } g_k = 0, \text{ which is UV stable}
  \rightarrow \text{no chance to have IR stable fixed point for any } V(\Phi) \text{ potential}
  \text{(NO second order phase transition is found)}
  \rightarrow \text{vertex regularization is introducable but does not change the conclusion}

9. CONCLUSIONS

- Order of the color superconducting transition is investigated via the FRG
- Calculation of the flow of the gauge coupling directly in $d = 3$ shows that no IR stable fixed points exist $\Rightarrow$ 2nd order transition is not possible
- Conclusions are stable against introducing vertex regulators in the RG flow