**Introduction**

- Brambilla, Escobedo, Soto, and Vairo have used potential non-relativistic QCD [1, 2]
  \[ \mathcal{L}_{\text{NRQCD}} = \int d^4x \left( \mathcal{L}_{\text{potential}} + \mathcal{L}_{\text{kinematics}} + \mathcal{L}_{\text{interactions}} \right) \]
  \[ + \int d^4x \left( \mathcal{L}_{\text{kinematics}} + \mathcal{L}_{\text{interactions}} \right) \]
  \[ + \int d^4x \left( \mathcal{L}_{\text{kinematics}} + \mathcal{L}_{\text{interactions}} \right) \]

- The analytic continuation of Eq. (1) is
  \[ \kappa \rightarrow \tau \]

- Eq. (1) originates from the heavy-quark number current-current correlator \( \langle \bar{Q}Q \rangle \).
- \( \kappa \) represents the well-known heavy-quark momentum-diffusion coefficient [4] and \( \gamma \) a mass shift in the heavy-quark bound states [5].
- Their description depends on two (in principle nonperturbative) key QCD parameters \( \kappa \), the well-known heavy-quark momentum-diffusion coefficient [4] and \( \gamma \) a mass shift in the heavy-quark bound states [5].

\[ \kappa = \frac{g^2}{\delta N} \int d\tau E^{-\gamma}(\tau) = \gamma \int d\tau E^{-\gamma}(\tau) \]

- \( \kappa \) is defined as the spectral function of Eq. (1) and its analytic continuation in the \( M_{QCD} \rightarrow \infty \) limit is derived in Ref. [6].

**Method**

- The analytic continuation of Eq. (1) is not trivial due to the Wilson line.
- Eq. (1) originates from the heavy-quark number-current-correlator
  \[ \int dN d\tau e^{-\tau} \int d^4x \left( \mathcal{J}(\tau, x), \mathcal{J}(\tau, 0) \right) \]

- \( \kappa \) is defined as the spectral function of Eq. (2) and its analytic continuation in the \( M_{QCD} \rightarrow \infty \) limit is derived in Ref. [6].

**Result**

- \( \kappa \) continues to [6]

\[ \kappa = \lim_{\omega \rightarrow \infty} \left[ \frac{2\pi}{\omega} T \mathcal{D}_{\text{NRQCD}}(\omega) \right] \]

as a spectral function of \( \mathcal{D}_{\text{NRQCD}}(\tau) \) \( \rightarrow \) Need continuation.
- However \( \gamma \) is [7]

\[ \gamma = \frac{2\pi}{\omega} \int d\tau \mathcal{D}_{\text{NRQCD}}(\tau), \]

and therefore directly a Euclidean quantity \( \rightarrow \) No continuation needed!

**Cross Check using Perturbation Theory**

- Eq. (3) was calculated in pert. theory up to next-to-leading order in Ref. [8]

\[ \mathcal{Y}_{\text{LO}} = \int_0^\infty d\tau \mathcal{D}_{\text{NRQCD}}(\tau) \]

- The integral expression of \( \mathcal{D}_{\text{NRQCD}}(\tau) \) is

\[ \gamma_{\text{LO}} = \int_0^\infty d\tau \mathcal{D}_{\text{NRQCD}}(\tau) \]

- The sum integrals (fermionic denoted as \( \bar{T} \)) are defined as

\[ I_1, I_2 = \int_0^\infty \left( \mathcal{J}(\tau), \mathcal{J}(\tau) \right) \]

\[ I_3, I_4 = \int_0^\infty \left( \mathcal{J}(\tau), \mathcal{J}(\tau) \right) \]

- The only NLO contribution to \( \gamma \) to the best of your knowledge is

\[ \gamma_{\text{NLO}} = \frac{g^2}{2} \int_1^\infty \frac{k^2}{k^2 + m_0^2} \left[ \mathcal{J}(\tau), \mathcal{J}(\tau) \right] \]

**References**