Thermal Quarkonium Mass Shift from Euclidean Correlators

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Introduction

- Brambilla, Escobedo, Soto, Vairo have used potential non-relativistic QCD [1, 2]

$$\mathcal{L}_{\text{pNRQCD}} = \int d^3r\ \text{Tr}\left[\text{S}^\dagger(i\partial_0 - h_s)\text{S} + \text{O}^\dagger(iD_0 - h_o)\text{O}\right] + \text{Tr}\left[(\text{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\ \text{S} + \text{H.c.}) + \frac{1}{2}(\text{O}^\dagger \mathbf{r}\cdot g\mathbf{E}\ \text{O} + \text{c.c.})\right] + \mathcal{L}_{\text{light}}$$

at second order in the multipole expansion to build an eff. theory describing the evolution and breakup of quarkonium [3] in QGP for $m \gg 1/a_0 \gg T$, $a_0 \sim 1/(m\alpha_s)$ the Bohr radius

$$\frac{d\rho}{dt} = -i\left[H,\rho\right] + \sum_{n=0}^{1}\left(C_n\,\rho\,C_n^\dagger - \frac{1}{2}\left\{C_n^\dagger C_n, \rho\right\}\right),$$

$$\rho = \begin{pmatrix}\rho_s & 0\\ 0 & \rho_o\end{pmatrix} \qquad H = \begin{pmatrix}h_s & 0\\ 0 & h_o\end{pmatrix} + \frac{r^2\gamma(t)}{2}\begin{pmatrix}1 & 0\\ 0 & \frac{N_c^2-2}{2(N_c^2-1)}\end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{N_c^2-1}}\,r^i\begin{pmatrix}0 & 1\\ \sqrt{N_c^2-1} & 0\end{pmatrix} \qquad C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa(t)}{2(N_c^2-1)}}\,r^i\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}$$

- Their description depends on two (in principle nonperturbative) **key QCD parameters** κ, the well-known heavy-quark momentum-diffusion coefficient [4] and γ a mass shift in the heavy-quark bound states [5]

$$\kappa = \frac{g^2}{6N_c}\text{Re}\int_{-\infty}^{\infty} ds\,\langle \text{T}\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle\,,\quad \gamma = \frac{g^2}{6N_c}\text{Im}\int_{-\infty}^{\infty} ds\,\left\langle \text{T}E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\right\rangle, \quad (1)$$

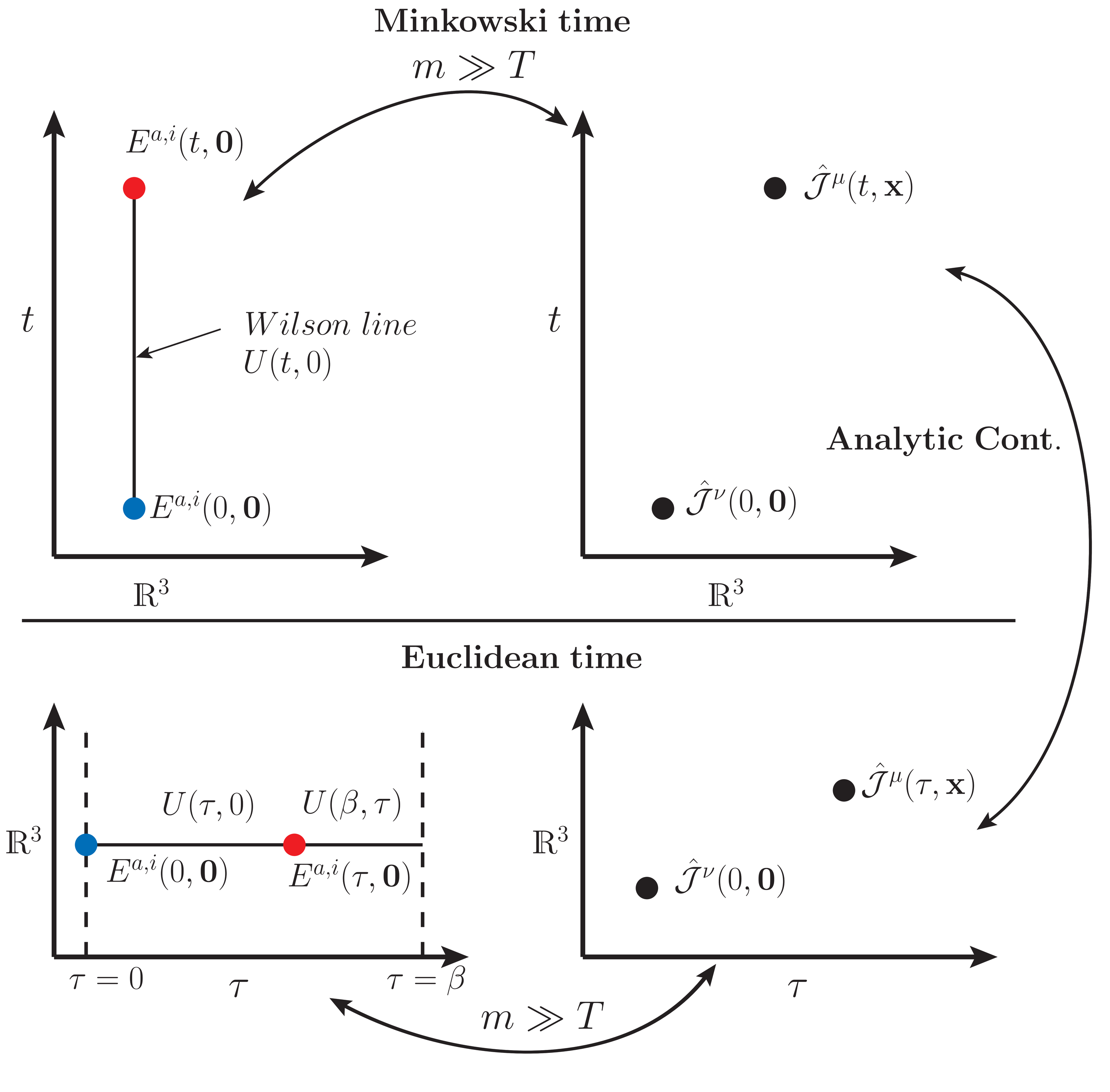
$E^{a,i}$ color electric field, a fund.-representation forward-time **Wilson line** from $-\infty$ to $+\infty$ is implicitly included in the time-ordered correlator.

Method

- The analytic continuation of Eq. (1) is **not trivial** due to the Wilson line.
- Eq. (1) originates from the heavy-quark number current-current correlator

$$\int_{-\infty}^{\infty} dt\ e^{i\omega t}\int d^3\mathbf{x}\ \langle[\hat{\mathcal{J}}^\mu(t,\mathbf{x}),\hat{\mathcal{J}}^\nu(0,\mathbf{0})]\rangle\,. \quad (2)$$

- κ is defined as the spectral function of Eq. (2) and its **analytic continuation** in the $M_{\text{kin}} \to \infty$ limit is derived in Ref. [6]



$$G_{\text{E}}^{\text{HQ}}(\tau) = -\sum_{i=1}^{3-2\epsilon}\frac{\langle\text{Re Tr}\left[U(\beta,\tau)g_{\text{B}}E_i(\tau,\mathbf{0})U(\tau,0)g_{\text{B}}E_i(0,\mathbf{0})\right]\rangle}{3\,\langle\text{Re Tr}\left[U(\beta,0)\right]\rangle}. \quad (3)$$

Result

- κ continues to [6]

$$\kappa = \lim_{\omega\to 0}\frac{2T}{\omega}\rho^{\text{HQ}}(\omega)$$

as a spectral function of $G_{\text{E}}^{\text{HQ}}(\tau) \Rightarrow$ Need continuation.
- However γ is [7]

$$\gamma = -\tilde{G}_{\text{E}}^{\text{HQ}}(\omega_n{=}0) = -\int_0^\beta d\tau\ G_{\text{E}}^{\text{HQ}}(\tau)\,, \quad (4)$$

and therefore directly a Euclidean quantity $\Rightarrow$ No continuation needed!

Cross Check using Perturbation Theory

- Eq. (3) was calculated in pert. theory up to next–to–leading order in Ref. [8]
- The integral expression of $G_{\text{E NLO}}^{\text{HQ}}(\tau)$ is

$$\gamma_{\text{LO}} = -\int_0^\beta d\tau\ G_{\text{E NLO}}^{\text{HQ}}(\tau) = \frac{g^4 C_F}{3}\left[N_c(\mathcal{I}_1 + 2(D-2)\mathcal{I}_2) - N_f(\tilde{\mathcal{I}}_1 + 4\tilde{\mathcal{I}}_2)\right],$$

where the sum integrals (fermionic denoted as $\tilde{\mathcal{I}}$) are defined as

$$\mathcal{I}_1\,,\tilde{\mathcal{I}}_1 = \int_k \oint_{Q,\{Q\}}\frac{1}{Q^2(K-Q)^2}\bigg|_{k_n=0}\,, \qquad \mathcal{I}_2\,,\tilde{\mathcal{I}}_2 = \int_k \oint_{Q,\{Q\}}\frac{q_n^2}{K^2Q^2(K-Q)^2}\bigg|_{k_n=0}\,,$$

$K^2 = k_n^2 + \mathbf{k}^2$, $\{Q\}$ represents the fermionic Matsubara frequencies $q_n = \pi T(2n+1)$.
- We obtain the known result from the real–time calculation of γ given in Ref. [5]

$$\gamma_{\text{LO}} = -2\alpha_s^2 T^3\zeta(3)C_F\left(\frac{4}{3}N_c + N_f\right).$$

- The only NLO contribution to γ to the best of your knowledge is

$$\gamma_{\text{NLO}} = \frac{g^2 C_F}{3}\int_k\frac{k^2}{k^2+m_{\text{D}}^2} = \frac{\alpha_s C_F m_{\text{D}}^3}{3},$$

where $m_{\text{D}}^2 = g^2T^2(N_c/3 + N_f/6)$ is the Debye mass.

Outlook

- γ is a **purely Euclidean quantity**, which can be determined nonpertubatively on the lattice.
- The computational cost calculating γ should be reasonable if smoothing techniques like gradient flow are used.
- After removing the vacuum contr. of γ, Eq. (4) should **not suffer from divergences**.
- Someone should do it!

References

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