QCD Topology to High Temperatures via Reweighting

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Introduction
• The QCD axion is a hypothetical particle, predicted in models which solve the strong CP problem via the Peccei-Quinn mechanism [1] and is also a dark matter candidate [2].
• There is currently less of experimental effort to detect axions and theoretical effort to determine its properties, especially its mass.
• From a theoretical point of view, the axion properties depend on the QCD topological susceptibility

\[ \chi_{\text{top}} = \frac{1}{2} \left( \frac{d}{d\mu} \left( F_\mu \right) \right) \]

up to temperatures of \( T_\chi \approx 7 T_c \) [3] with \( \chi \) the four-dimensional volume, \( Q \) the topological charge, and \( F_\mu \) the dual field-strength tensor.

• Topologically non-trivial fields are instantons/calorons with weight

\[ \exp(-\chi) = \exp\left(-\frac{8\pi^2 |Q|^2}{g^2}\right) \to 0 \quad \text{as} \quad T \to \infty \quad (g \to 0). \]

• Since topological phenomena are inherently non-perturbative, lattice QCD is the method of choice. However, at high temperatures calorons are badly suppressed and it becomes impossible to measure fluctuations of \( Q \) with standard lattice techniques.

Reweighting Method
• In order to artificially enhance the number of caloron configurations, we developed a technique based on reweighting [4].

\[ \langle q^2 \rangle = \frac{\sum q^2 \exp(-\chi)}{\sum \exp(-\chi)} \Rightarrow \langle q^2 \rangle \approx q^2 \sum Q^2 \exp(-\chi) \]

we apply importance sampling with a modified weight by introducing the reweighting function \( W(Q) \):

\[ \langle q^2 \rangle = \frac{\int \sum q^2 \exp(-\chi) \exp(W)}{\int \sum \exp(-\chi) \exp(W)} \Rightarrow \langle q^2 \rangle \approx \frac{\sum Q^2 \exp(W)}{\sum Q_0 \exp(W)} \]

• Since the topological charge is badly contaminated by UV fluctuations, we apply gradient flow to both observable \( Q \) (large amount) and the reweighting variable \( Q' \) (small amount).

• The reweighting technique is implemented via an additional Metropolis step.

• Using reweighting, the number of caloron configurations can be significantly enhanced if the reweighting function is chosen correctly. We developed an automated way to build the reweighting function:
  - perform a separate Monte Carlo simulation only for building \( W \)
  - constrain to the interval \([0,1]\) which is sufficient for high \( T \)
  - start with a flat function \( W(Q') = 0 \)
  - at each Monte Carlo step measure \( Q' \) and lower \( W \) at the measured value
  - make the procedure converge by reducing the amount of lowering \( W \) after the whole range was explored

References