QCD Topology to High Temperatures via Reweighting

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Introduction

- The QCD axion is a hypothetical particle, predicted in models which solve the strong CP problem via the Peccei-Quinn mechanism [1] and is also a dark matter candidate [2].
- There is currently lots of experimental effort to detect axions and theoretical effort to determine its properties, especially its mass.
- From a theoretical point of view, the axion properties depend on the **QCD topological susceptibility**

$$\chi_{\text{top}} = \frac{1}{\mathcal{V}}\langle Q^2\rangle\,, \qquad Q = \frac{1}{32\pi^2}\int \mathrm{d}^4x\, F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \in \mathbb{Z}\,, \qquad \tilde{F}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^a_{\alpha\beta}$$

up to temperatures of $\sim 7\,T_c$ [3] with $\mathcal{V}$ the four-dimensional volume, Q the **topological charge**, and $\tilde{F}_{\mu\nu}$ the **dual field-strength tensor**.

- Topologically non-trivial fields are **instantons/calorons** with weight

$$\exp(-S) = \exp\left(-\frac{8\pi^2|Q|}{g^2}\right) \to 0 \qquad \text{as} \qquad T\to\infty\ (g\to 0)\,.$$

- Since topological phenomena are inherently non-perturbative, lattice QCD is the method of choice. However, at high temperatures calorons are badly suppressed and it becomes impossible to measure fluctuations of Q with standard lattice techniques.

Reweighting Method

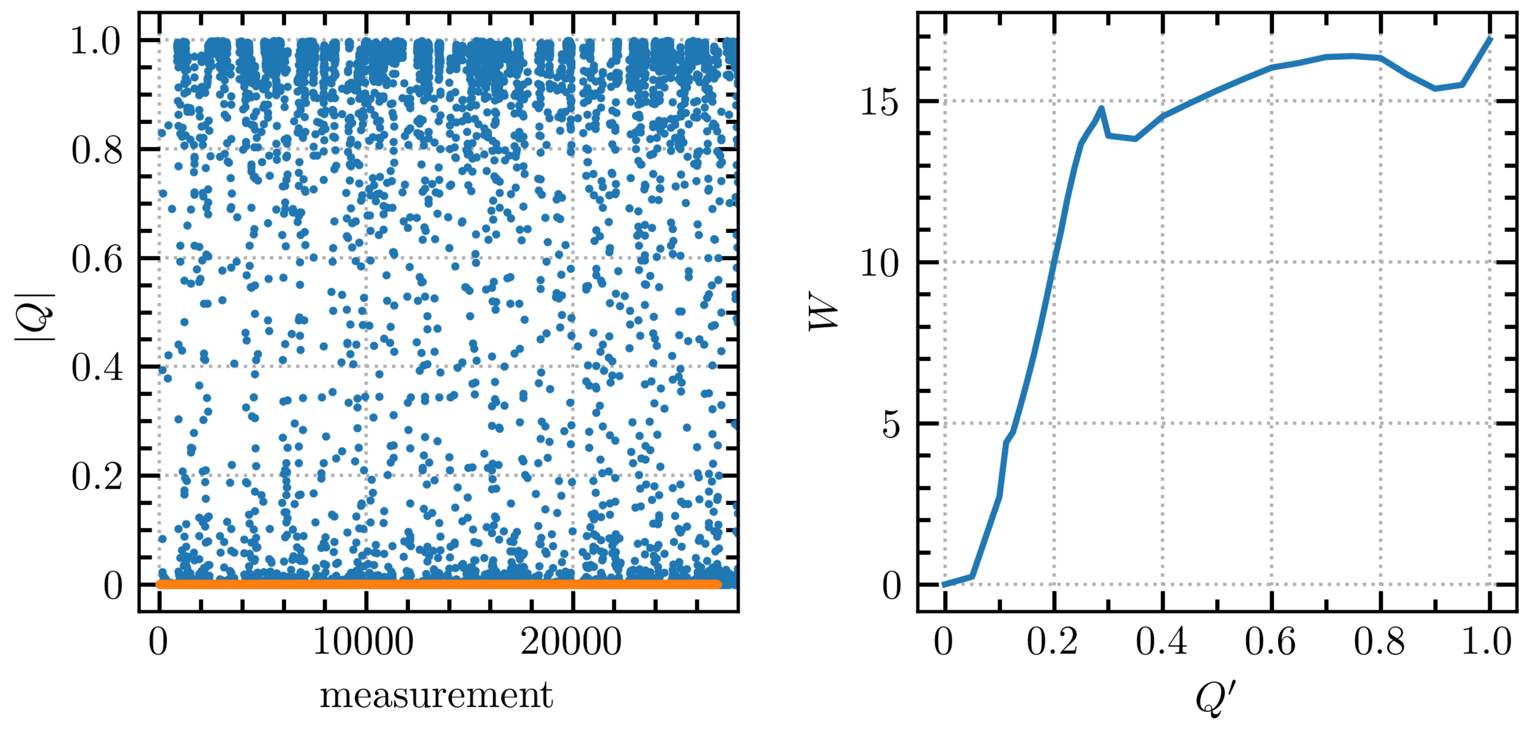
- In order to artificially enhance the number of caloron configurations, we developed a technique based on **reweighting** [4].
- Instead of determining the expectation value via standard importance sampling

$$\langle Q^2\rangle = \frac{\int \mathcal{D}U\, Q^2\, \mathrm{e}^{-S[U]}}{\int \mathcal{D}U\, \mathrm{e}^{-S[U]}} \qquad \Longrightarrow \qquad \langle Q^2\rangle \simeq \frac{1}{N}\sum_{i=1}^{N} Q_i^2$$

we apply importance sampling with a modified weight by introducing the **reweighting function** $W(Q')$:

$$\langle Q^2\rangle = \frac{\int \mathcal{D}U\, Q^2\, \mathrm{e}^{-S[U]+W(Q')}\,\mathrm{e}^{-W(Q')}}{\int \mathcal{D}U\, \mathrm{e}^{-S[U]+W(Q')}\,\mathrm{e}^{-W(Q')}} \qquad \Longrightarrow \qquad \langle Q^2\rangle \simeq \frac{\sum_{i=1}^{N} Q_i^2\, \mathrm{e}^{-W(Q_i')}}{\sum_{i=1}^{N} \mathrm{e}^{-W(Q_i')}}$$

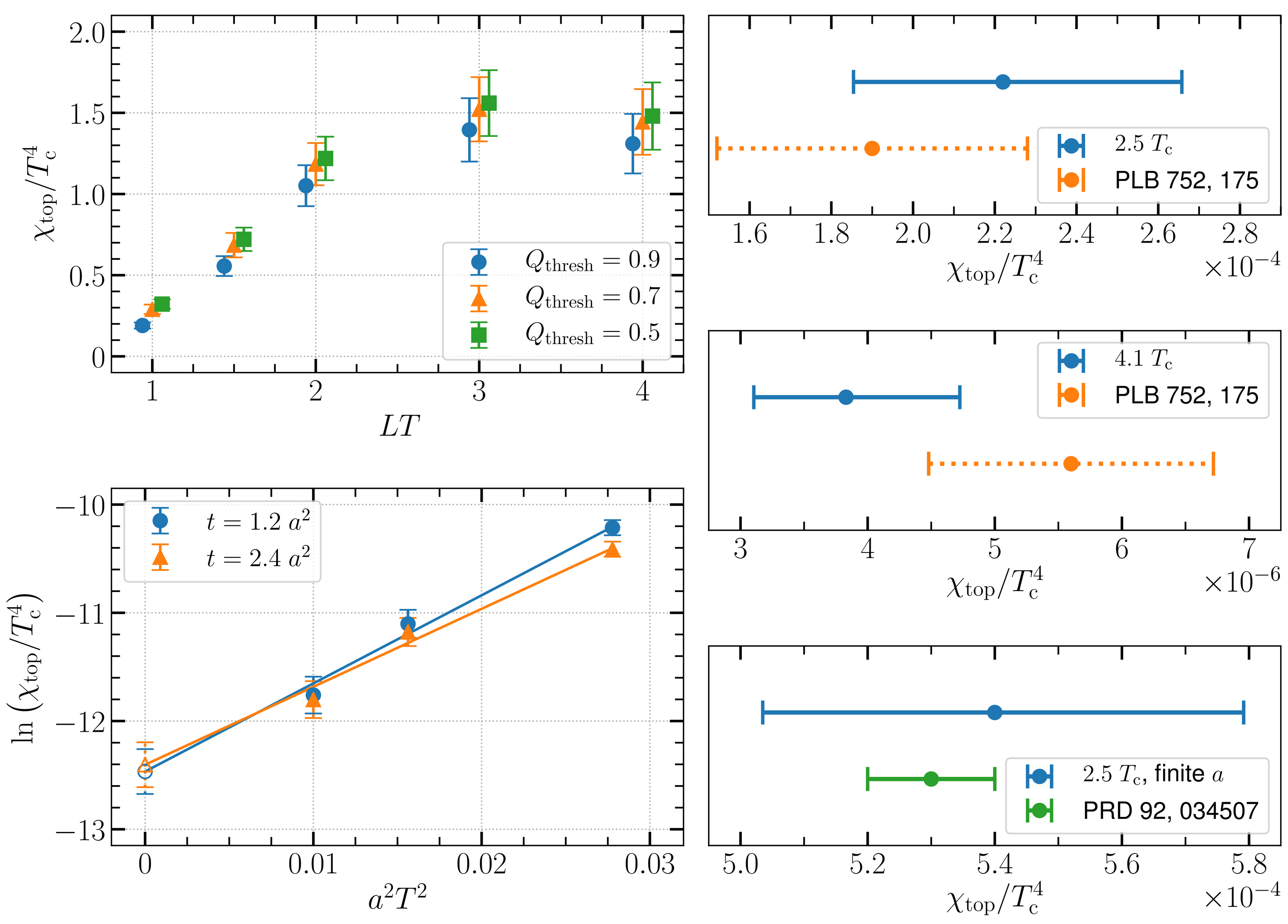
- Since the topological charge is badly contaminated by UV fluctuations, we apply **gradient flow** to both the observable Q (large amount) and the reweighting variable Q' (small amount).
- The reweighting technique is implemented via an additional Metropolis step.
- Using reweighting, the number of caloron configurations can be significantly enhanced if the reweighting function is chosen correctly. We developed an automated way to build the reweighting function:
  - perform a separate Monte Carlo simulation only for building W
  - constrain to the interval $|Q'| \in [0,1]$ which is sufficient for high T
  - start with a flat function $W(Q') = 0$
  - at each Monte Carlo step measure Q' and lower W at the measured value
  - make the procedure converge by reducing the amount of lowering W after the whole range was explored



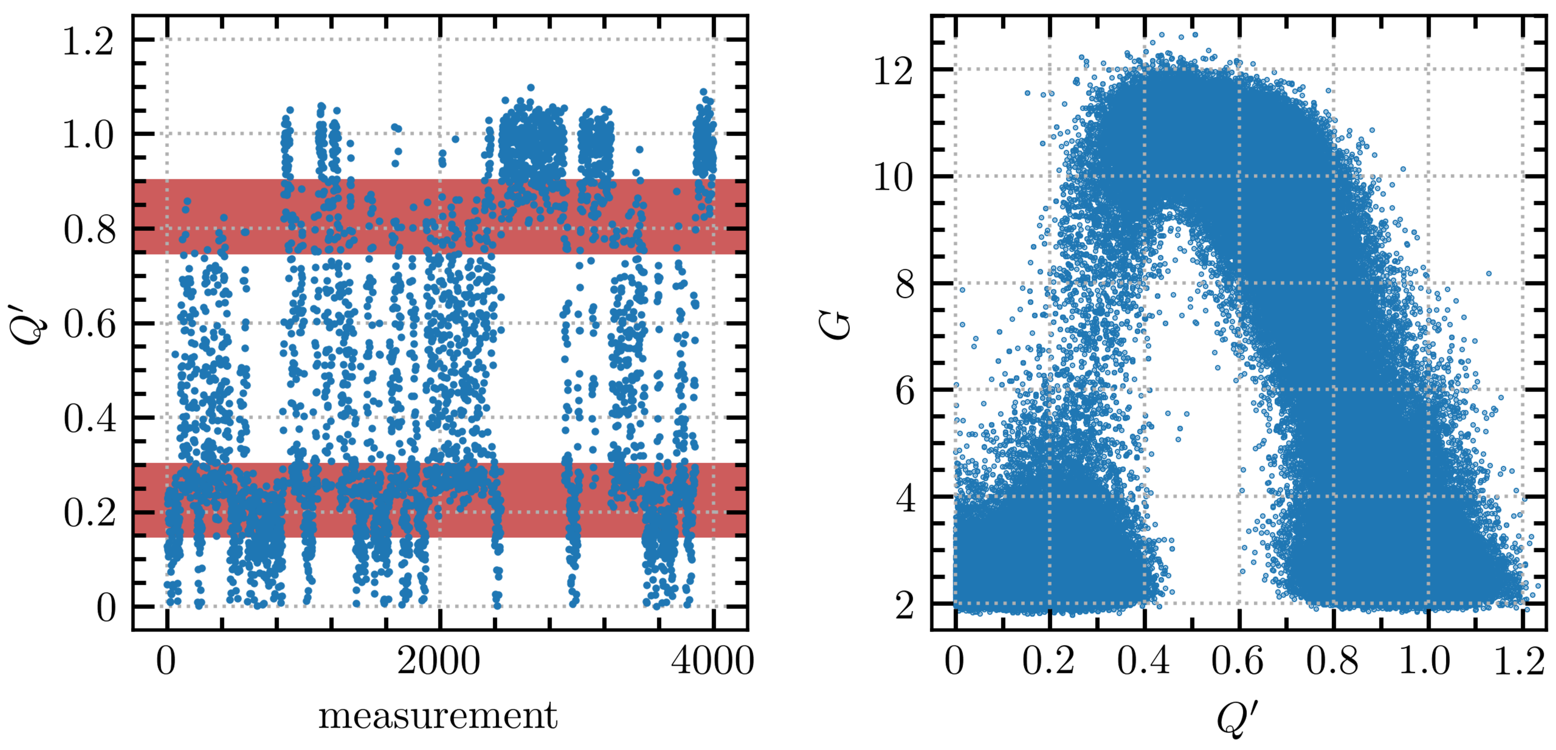
Results

- Continuum extrapolated quenched results at 2.5 T_c and 4.1 T_c
- Thresholded, flowed topological charge as observable:

$$\langle Q^2\rangle = \frac{\sum_i \mathrm{e}^{-W(Q_i')}\,\Theta\!\left(Q_i^2(t) - Q^2_{\text{thresh}}\right)}{\sum_i \mathrm{e}^{-W(Q_i')}}$$



Improvement of the Technique



- Reweighting in terms of the topological charge only is incomplete; Q' is not homogeneously distributed but has "barriers"
- Algorithm has problems to move between trivial topology, "dislocations" (very small calorons, lattice artifacts), and genuine calorons
- **Peak action density** G is able to distinguish trivial topology and dislocations $\to$ additional reweighting in terms of G to overcome the lower barrier
- High barrier is mildened by using larger HMC steps and no reweighting at all
- A middle region becomes necessary to connect both regions
- **ONGOING:** Continuum extrapolated determination of the topological susceptibility up to 7 T_c and extension to full QCD

References

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