Relation between chirality imbalance and fermion pair-production under the parallel electromagnetic field

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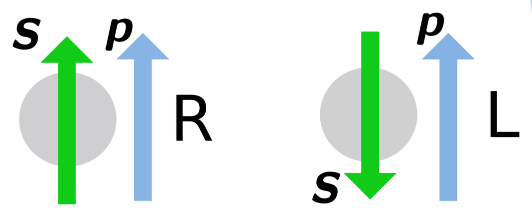
Chirality imbalance

The number difference between right- and left-handed fermions

$$n_5 = \langle \bar{\psi}\gamma^0\gamma^5\psi \rangle = (n_R - \bar{n}_R) - (n_L - \bar{n}_L)$$



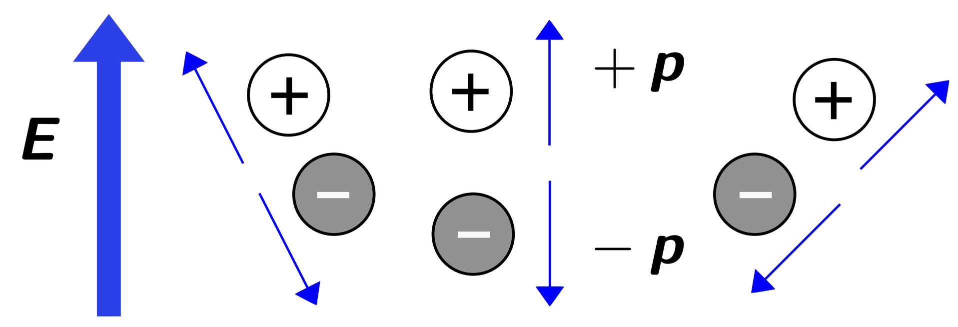
- dynamically induced by **quantum anomaly**

$$\frac{\partial}{\partial t}\int \mathrm{d}^3x \, \langle \bar{\psi}\gamma^0\gamma^5\psi \rangle = 2m \int \mathrm{d}^3x \, \langle \bar{\psi} i\gamma^5\psi \rangle + \frac{2\alpha}{\pi}\int \mathrm{d}^3x E \cdot B$$

- the source of anomalous transportation in QGP, Neutron stars, etc
  ex. **Chiral magnetic effect**, Chiral vortival effect

Fermion pair-production

The real fermion/anti-fermion pair-production from the vacuum by the strong electric field $\quad (E_c \simeq 10^{18}\mathrm{V/m})$



the dacay rate of vacuum in constant electric field

$$\Gamma \sim \exp\left(-\pi\frac{m^2}{|eE|}\right)$$

- occur **non-perturbatively**
- strongly **mass dependent**

How is the relation between chirality imbalance and pair-production in electromagnetic field?

Assumptions

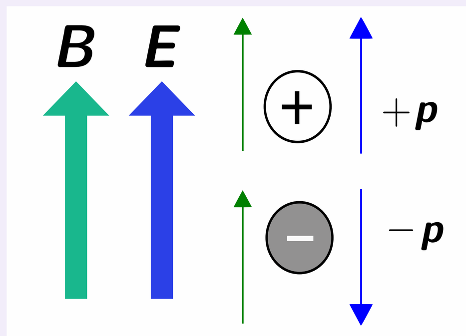
- ***parallel electromagnetic field***
- massive charged fermion
- zero-temperature and zero-density

B
- align the spin
- 1-dimensionize the momentum

E produce the fermion/anti-fermion pair whose momentum align in direction of E

Expected Result

The appearance of Chirality imbalance



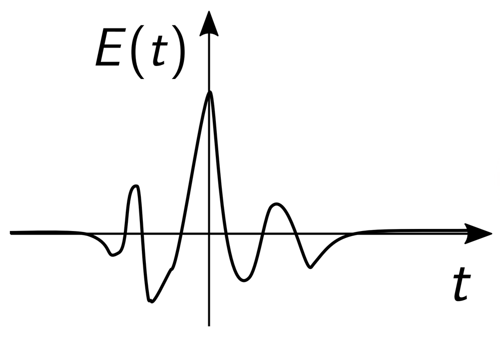
Formalism

Parallel electromagnetic field

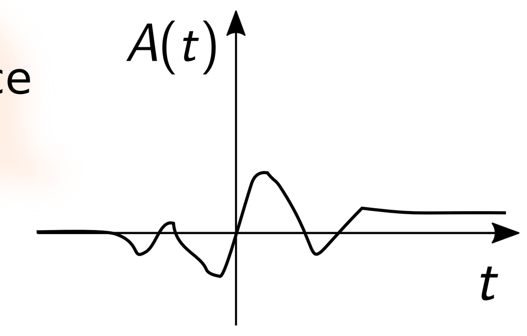
$$B = (0,0,B), \quad E = (0,0,E(t)) \quad \Leftarrow \quad A^\mu = (0,0,Bx,A(t))$$

Boundary condition about time

$$\lim_{t\to\pm\infty} E(t) = 0 \quad \Leftarrow \quad \lim_{t\to-\infty} A(t) = 0, \; \lim_{t\to\infty} A(t) = \mathrm{const.}$$



intermediate time-dependence is ***arbitrary***



- To define the vacuum state in $t \to \pm\infty$ called *in-out state*
- To extract the properties of chirality imbalance independent of the intermediate time dependence

solve the Dirac equation $\quad i\partial_t\psi = H_D\psi$

Field operator & vacuum state

$$\hat{\psi}(x) = \sum_{n=0}^{\infty}\int \frac{\mathrm{d}p_y}{\sqrt{2\pi}}\int \frac{\mathrm{d}p_z}{\sqrt{2\pi}}\sum_{s=R,L}\left(\hat{b}_{s,p}\psi_p^{(+,s)}(x) + \hat{d}^\dagger_{s,-p}\psi_p^{(-,s)}(x)\right)$$

- p_y, p_z : canonical momentum , $\quad n$: Landau level
- $\hat{b}_{s,p}\,|0\rangle = 0, \; \hat{d}_{s,p}\,|0\rangle = 0$ (for all s, p); $\; \langle 0|0\rangle = 1$
- $\{\psi_p^{(\pm,s)}(x)\}$: Eigenfunction of H_D in $t \to -\infty$

$|0\rangle$ is interpreted as the vacuum state of in $\; t \to -\infty$

calculate $\; \langle \bar{\psi}\Gamma\psi \rangle = \frac{1}{2}\langle 0|\, \hat{\bar{\psi}}\Gamma\hat{\psi} - \Gamma\hat{\psi}\hat{\bar{\psi}}\,|0\rangle$

Vacuum expectation values

$$\langle \mathcal{H} \rangle = -\frac{|eB|}{2\pi}\sum_{n=0}^{\infty}\alpha_n\int \frac{\mathrm{d}p_z}{2\pi}\left[\Pi\left(|\phi^{(+)}_{n,p_z}|^2 - |\phi^{(-)}_{n,p_z}|^2\right) + 2M_n \mathrm{Re}[\phi^{(+)}_{n,p_z}\phi^{(-)}_{n,p_z}]\right]$$

$$n_5(t) = \frac{eB}{2\pi}\int \frac{\mathrm{d}p_z}{2\pi}\left[|\phi^{(+)}_{0,p_z}|^2 - |\phi^{(-)}_{0,p_z}|^2\right]$$

$$\Pi(t) = p_z + eA(t)$$
$$M_n = \sqrt{m^2 + 2|eB|n}$$
$$\alpha_n = \begin{cases} 1 & (n=0) \\ 2 & (n = 1,2,3,\cdots) \end{cases}$$

Only the lowest Landau level *contribute to chirality imbalance*

The time-dependent part $\{\phi^{(\pm)}_{n,p_z}\}$ follows the equation:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\phi^{(+)}_{n,p_z} \\ \phi^{*(-)}_{n,p_z}\end{pmatrix} = \begin{pmatrix}\Pi & M_n \\ M_n & -\Pi\end{pmatrix}\begin{pmatrix}\phi^{(+)}_{n,p_z} \\ \phi^{*(-)}_{n,p_z}\end{pmatrix}$$

analogues of time dependent two-state system

Results & Discussion

The asymptotic solution & fermion pair-production

The asymptotic solution of $\{\phi^{(\pm)}_{n,p_z}\}$ in $t\to+\infty$:

$$\begin{pmatrix}\phi^{(+)}_{n,p_z} \\ \phi^{*(-)}_{n,p_z}\end{pmatrix} = \begin{pmatrix}\cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}\alpha_{n,p_z}\mathrm{e}^{-i\omega_n t} \\ \beta_{n,p_z}\mathrm{e}^{+i\omega_n t}\end{pmatrix}$$

$$\omega_n = \sqrt{p_z^2 + m^2 + 2|eB|n}, \quad \theta = \arctan(p_z/\omega_n)$$

$\alpha_{n,p_z}, \beta_{n,p_z}$:

- time-independent constant determined by solving the equation of $\{\phi^{(\pm)}_{n,p_z}\}$
- satisfy the relation: $\; |\alpha_{n,p_z}|^2 + |\beta_{n,p_z}|^2 = 1$

calculate the VEV of energy in $t\to+\infty$

$$\langle \mathcal{H} \rangle \xrightarrow[t\to+\infty]{} \frac{|eB|}{2\pi}\sum_{n=0}^{\infty}\alpha_n\int \frac{\mathrm{d}p_z}{2\pi}\left(-\omega_n + 2|\beta_{n,p_z}|^2\omega_n\right)$$

the energy of vacuum

contribution to the total energy as plus

||

interpreted as the energy distribution of pair-created fermions

Chirality imbalance in the remote future

$$n_5(t) \xrightarrow[t\to+\infty]{} \frac{eB}{2\pi}\int \frac{\mathrm{d}p_z}{2\pi}\left[-2\frac{p_z}{\omega_0}|\beta_{0,p_z}|^2 + 2\frac{m}{\omega_0}\mathrm{Re}[\alpha_{0,p_z}\beta^*_{0,p_z}\mathrm{e}^{2i\omega_0 t}]\right]$$

Time-independent term

$$\left[\frac{p_z}{\omega_0}\;\middle|\; \textit{the relativistic velocity of pair-created fermion}\right] \times \left[|\beta_{0,p_z}|^2 \;\middle|\; \textit{the momentum distribution of pair-created fermion}\right]$$

In the lowest Landau level, fermions have spin up(down) only

Chirality imbalance is characterized by the relativistic velocity

Temporal oscillation term

- no oscillation in massless limit
- anologues of *two-state oscillation* such as neutrino oscillation

In the parallel electromagnetic field, the appearance of chirality imbalance can be described by the fermion pair-production in the lowest Landau level.

[1] H. J. Warringa, Phys. Rev. D **86**, 085029 (2012)
[2] P. Copinger, K. Fukushima and S. Pu, Phys. Rev. Lett. **121**, no. 26, 261602 (2018)