Complex poles and spectral function of the Landau gauge gluon propagator: effects of quark flavors

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Introduction

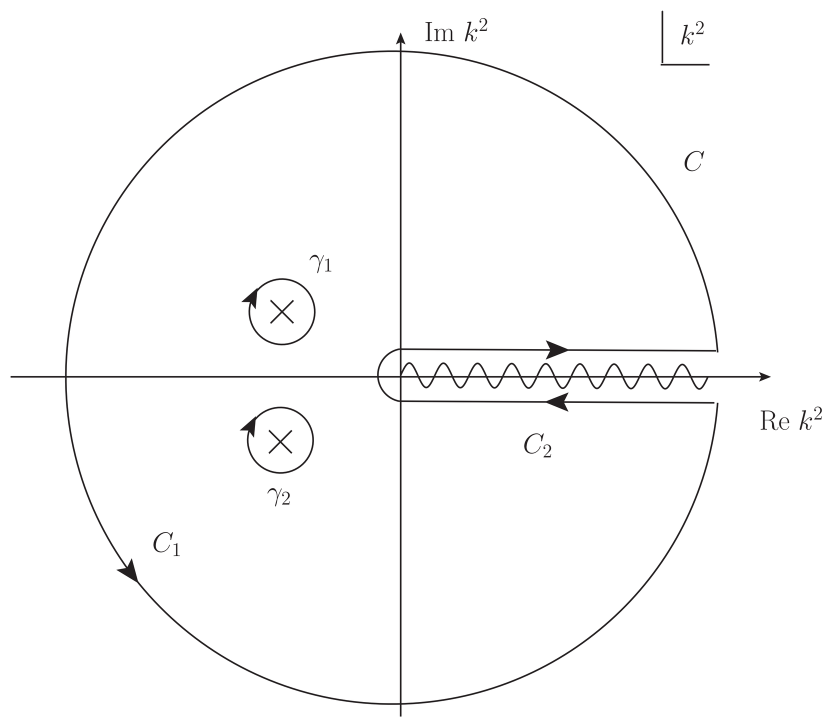
- The analytic structure of a propagator contains information on the spectrum
  - Källén-Lehmann spectral representation: the spectral function is the time-like singularities
  - A confined particle can have other analytic structure.
    e.g. (refined-)Gribov-Zwanziger theory (obtained by improving the gauge-fixing of the Landau gauge) predicts the gluon propagator with a pair of complex conjugate poles.
- Massive Yang-Mills model: an effective model of the Landau-gauge pure Yang-Mills theory [Tissier and Wschebor 2011] or QCD [Peláez et al. 2014].

$\rightarrow$ We study the analytic structure of the gluon propagator in various circumstances in the massive Yang-Mills model.

Winding number and complex poles

- A generalization of the spectral representation from analyticity

1. $D(z)$ is holomorphic except for singularities on the positive real axis and and a finite number of simple poles.
2. $D(z) \to 0$ as $|z| \to \infty$.
3. $D(z)$ is real on the negative real axis.



$$\Rightarrow D(k^2) = \frac{1}{2\pi i}\oint_{C\cup\{\gamma_\ell\}} d\zeta \frac{D(\zeta)}{\zeta - k^2},$$
$$= \int_0^\infty d\sigma^2 \frac{\rho(\sigma^2)}{\sigma^2 - k^2} + \sum_{\ell=1}^{n} \frac{Z_\ell}{z_\ell - k^2},$$
$$\rho(\sigma^2) := \frac{1}{\pi}\text{Im } D(\sigma^2 + i\epsilon),$$
$$Z_\ell := \oint_{\gamma_\ell} \frac{d\zeta}{2\pi i} D(\zeta).$$

- Argument principle

$$N_W(C) := \frac{1}{2\pi i}\oint_C dk^2 \frac{D'(k^2)}{D(k^2)} = N_Z - N_P.$$

- Positive spectral function
  1. $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z) > 0$ for large $|z|$.
  2. $\rho(\sigma^2) > 0$, *i.e.*, $\text{Im } D(\sigma^2 + i\epsilon) > 0$ for $\sigma^2 > 0$.
  3. $D(-\epsilon) > 0$ for sufficiently small $\epsilon > 0$.

$$\Rightarrow N_W(C) = N_W(C_1) + N_W(C_2) = -1 + 1 = 0 \Rightarrow N_P = N_Z.$$

- Negative spectral function
  1. $D(z) \sim -\frac{1}{z}\tilde{D}(z)$ as $|z| \to \infty$, where $\tilde{D}(z) > 0$ for large $|z|$.
  2. $\rho(\sigma^2) < 0$, *i.e.*, $\text{Im } D(\sigma^2 + i\epsilon) < 0$ for $\sigma^2 > 0$.
  3. $D(k^2 = 0) > 0$.

$$\Rightarrow N_W(C) = N_W(C_1) + N_W(C_2) = -1 - 1 = -2 \Rightarrow N_P = 2 + N_Z.$$

In particular, negativity of a spectral function in a weak sense leads to the existence of complex poles.

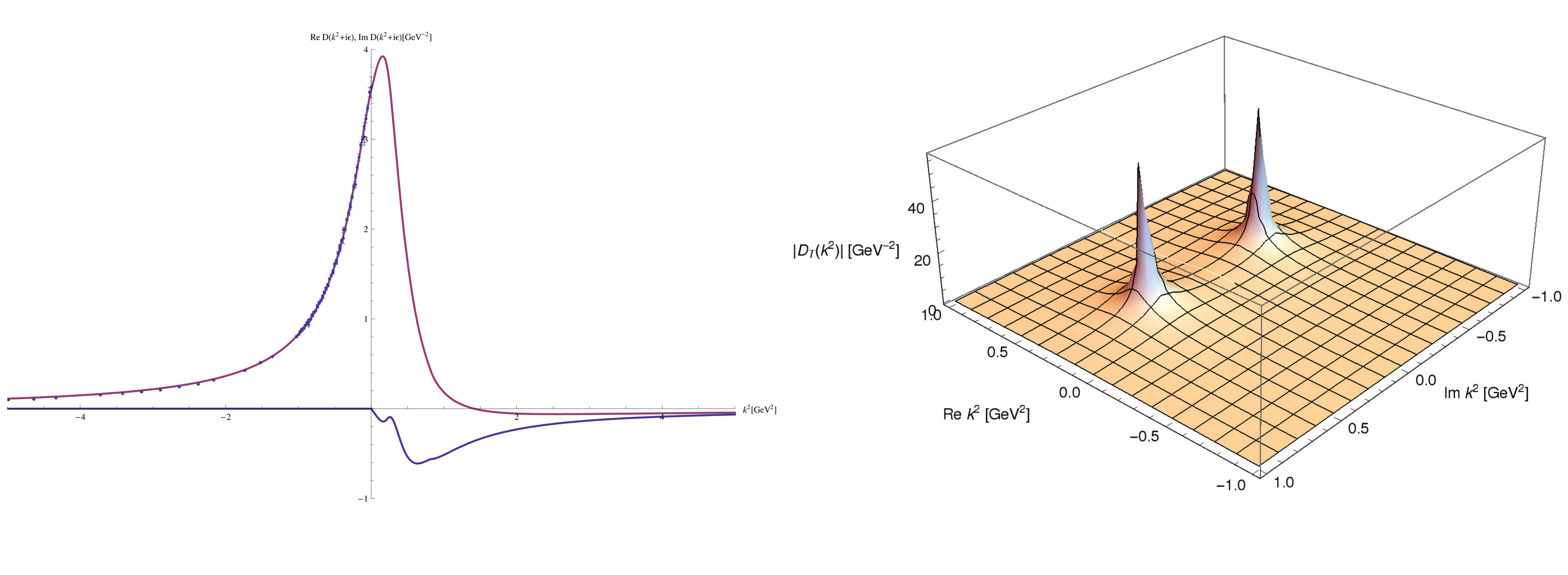
Massive Yang-Mills model

The Landau gauge Yang-Mills theory $(\alpha \to 0)$ + gluon mass term

$$\mathcal{L}_{mYM} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{2\alpha}(\partial^\mu A^A_\mu)^2 + i\bar{c}^A\partial^\mu \mathcal{D}_\mu[A]^{AB}c^B + \frac{1}{2}M^2 A^A_\mu A^{A\mu}$$

- reproduces the decoupling solution in a good accordance with the lattice results even in the strict one-loop level.
- For some renormalization conditions and parameters, the running coupling has **no Landau pole in all scales**. [Tissier and Wschebor 2011]
- A simple modification of this model predicts a sensible deconfinement temperature. [Reinosa et al. 2014]

In this model, we find the gluon propagator has a negative spectral function and therefore two complex poles for any parameters (g^2, M^2).

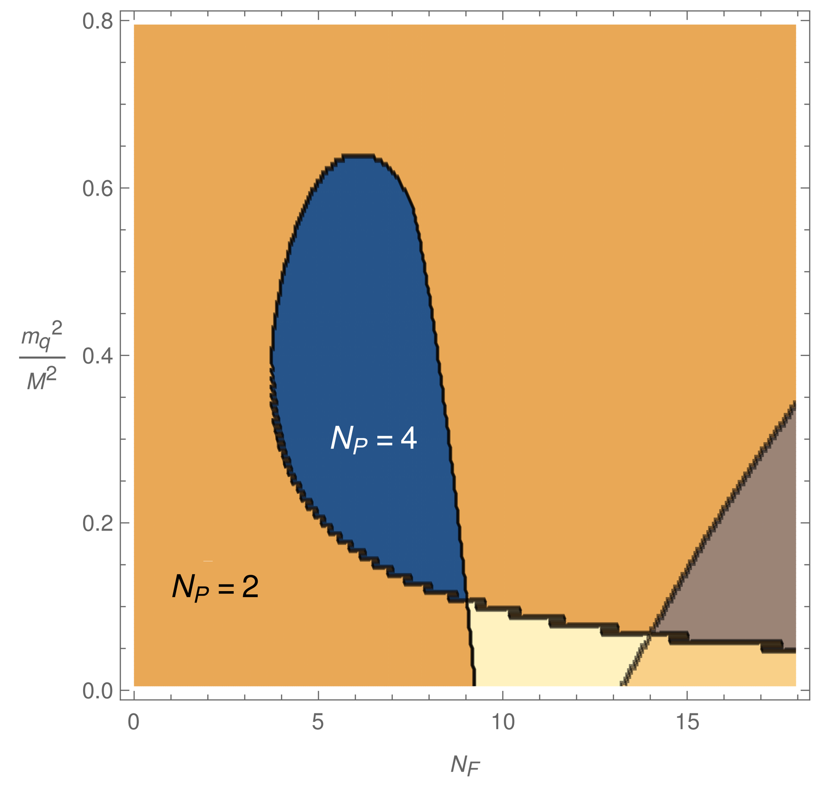


The gluon propagator at $g = 4.1$, $M = 0.45$ GeV, $G = SU(3)$ in a suitable renormalization condition (used in [Tissier and Wschebor 2011]). It has one pair of complex poles at $k^2 = 0.23 \pm 0.42i$ GeV^2

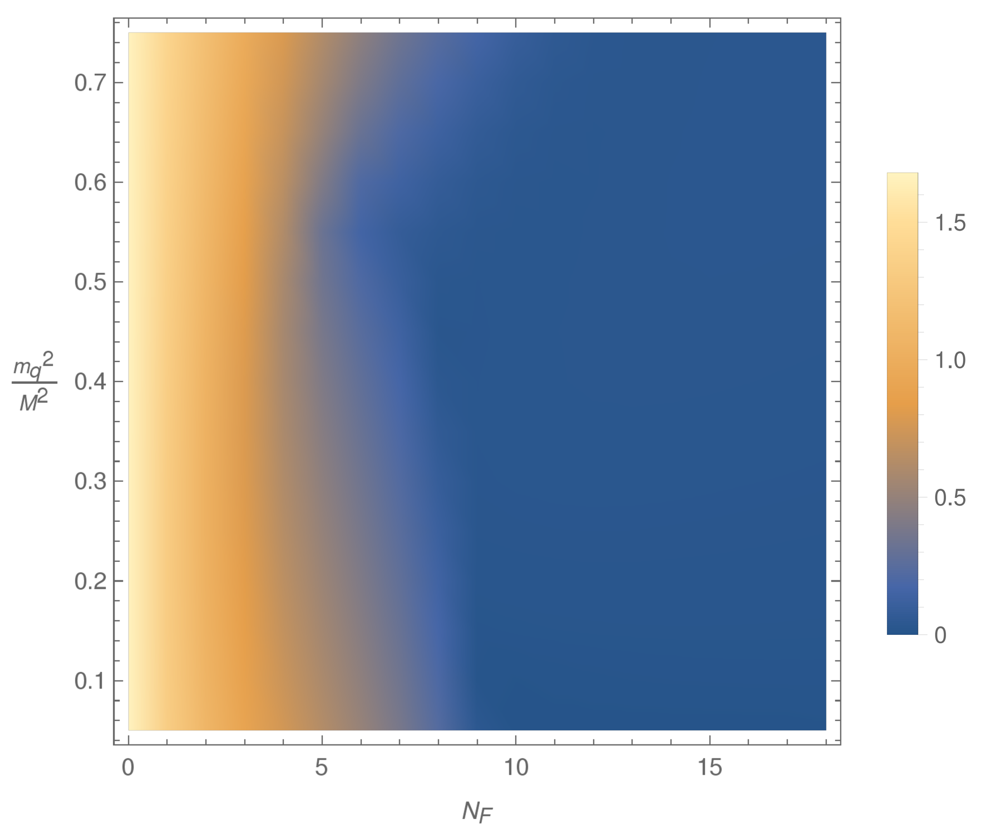
Massive Yang-Mills model with many quarks

We investigate the analytic structure of the gluon propagator in the massive Yang-Mills model with N_F quarks of mass m_q.

- There is a transition between a phase with $N_P = 2$ (one pair of complex conjugate poles) and the other phase with $N_P = 4$ (two pairs); on the boundary a real pole appears.
- Complex poles are related to confinement mechanism $\rightarrow$ The confinement mechanism may depend on N_F.
- Pole location: $k^2 = v \pm iw,\ w \geq 0$
  $\max w/v$ decreases as N_F increases: "particle-like" gluon for large N_F



The number of complex poles N_P computed from counting $N_W(C)$.



Larger value of the ratio w/v: $\max w/v$

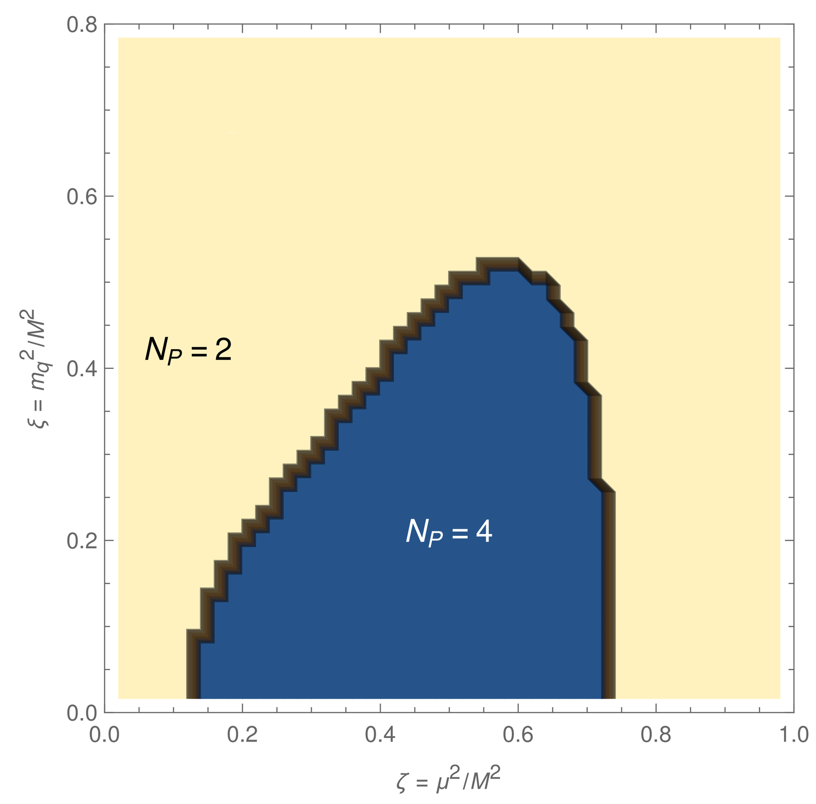
In a suitable renormalization condition (used in [Tissier and Wschebor 2011]), $G = SU(3)$, $g = 4$, and $M^2 = 0.2$ GeV^2.

For $N_F \geq 10$, the results is not reliable due to one-loop artifacts.

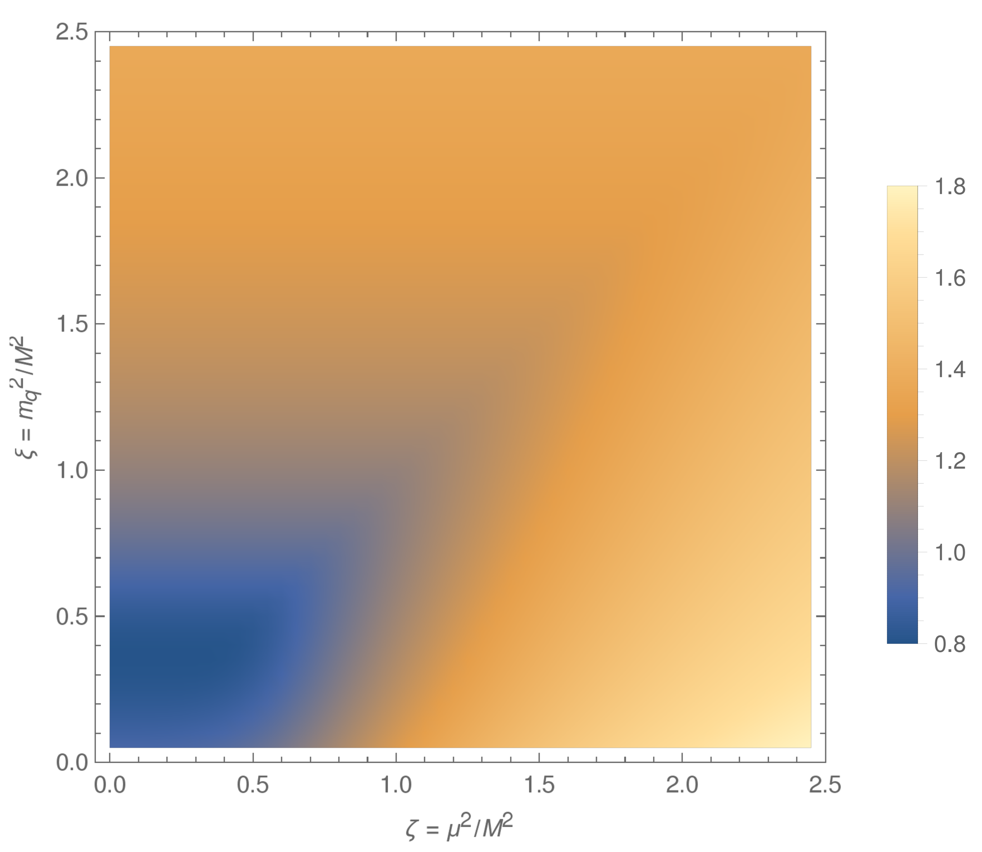
Gluon in the cold quark matter: massive YM at finite μ

We investigate the analytic structure of $D(k_0, \vec{k} = 0)$ at $T = 0$, $\mu \neq 0$ and $N_F = 3$ quarks of mass m_q. We focus on the pole locations $k_0^2 = v \pm iw,\ w \geq 0$.

- The gluon exhibits "confinement-like" behavior in the quark medium; $\max w/v$ increases as μ^2.
- In the $N_P = 4$ phase, almost real poles $(w/v \ll 1)$ appear at $v \approx (2\mu)^2$ from $\mu^2 \gtrsim m_q^2$ until $\mu^2 \lesssim 0.7M^2$ in addition to the original two complex poles.
  - 2μ is the least energy to interact with the on-shell quarks at vanishing momentum. The quark medium may largely affect the gluon in this scale.
  - In IR region, the light quarks dominate due to the "decoupling" of gluon: similar to QED $\rightarrow$ the pair of almost real complex poles appears as a remnant of the QED quasiparticle.
  - In UV region, the gluodynamics wins against the quark loop, e.g., $\rho < 0$ in UV for $N_F < 10$. $\rightarrow$ "confinement-like" behavior.



The number of complex poles N_P computed from counting $N_W(C)$.



Larger value of the ratio w/v: $\max w/v$

With the renormalization condition (used in [Tissier and Wschebor 2011]), $G = SU(3)$, $g = 4$, and $M^2 = 0.2$ GeV^2

Summary

- Relations between the number of complex poles and the sign of a spectral function are derived from applying **the argument principle** to the propagator.
- In the massive Yang-Mills model, an effective theory of the Landau gauge Yang-Mills theory, **the gluon propagator has a negative spectral function and one pair of complex conjugate poles** in the one-loop level.
- The gluon propagator can have **two pairs** of complex conjugate poles depending on quark flavors: confinement mechanism depending on N_F?
- The light quarks induce almost real complex poles in the small $\mu \gtrsim m_q$, similar to the QED quasi-particle. On the other hand, the gluon presents more "confinement-like" behavior in the high-density quark matter.