The Schwinger mechanism with perturbative electric fields

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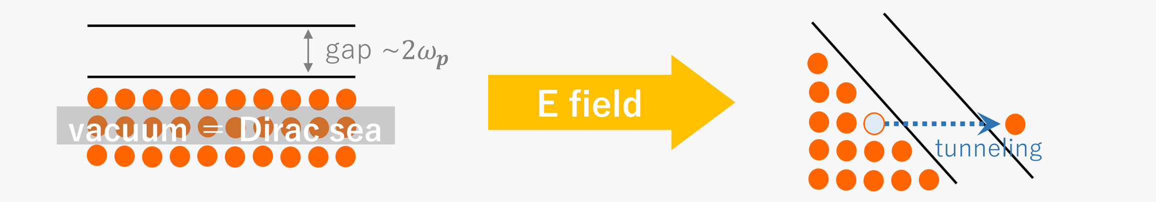
1. INTRODUCTION

1-1. The Schwinger mechanism

Particles are spontaneously created from the vacuum in a strong slow (non-perturbative) electric field.



- ✔ **a QED analog of electrical breakdown** of semi-conductors (Landau-Zener transition)
- ✔ phenomenological applications e.g.) heavy ion collisions, early Universe, laser
- ✔ **very well formulated for a static limit $E = \mathrm{const}$.**

$$n^{(\mp)}_{p,s} = \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+p_\perp^2)}{eE}\right]$$ [Schwinger 1951]

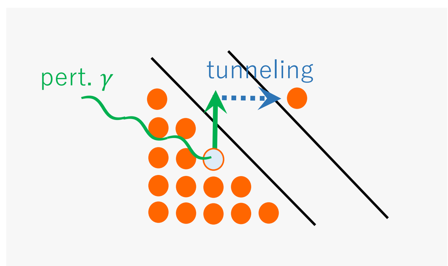
1-2. Time-dep. by a weak fast perturbation

What happens if one adds a weak fast (perturbative) time-depending electric field onto the slow strong field?

Known facts in high energy community since 2008~

Dynamically assisted Schwinger mechanism

[Schutzhold, Gies, Dunne 2008]

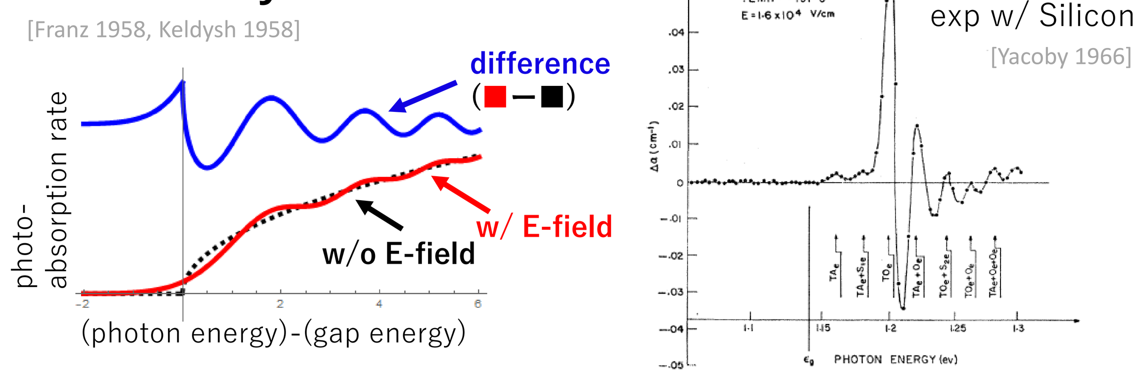


- ✔ tunneling length ~$2\omega_p$ is reduced by a perturbative kick by the fast weak E-field
- ✔ big **enhancement**
- ✔ not verified in experiments yet (hopefully in the near future)

Known facts in cond-mat community since 1958~

Franz-Keldysh effect

[Franz 1958, Keldysh 1958]



- ✔ the same setup in semi-conductor and measures photo-absorption rate w/ & w/o strong E-field
- ✔ **a long history (> 60 years) in both theory and experiment in cond-mat.**
- ✔ **not only enhancement, but also oscillation**

Q1: What happens if the electric field becomes time-dependent?

Q2: Is it OK to understand the dynamically assisted Schwinger mechanism as an analog of the Franz-Keldysh effect in QED? If yes, can we see the oscillation? How does it modify the momentum spectrum?

AIM: Answer Q1 and Q2 by deriving an analytical formula for the particle production from a strong slow E-field superimposed by a weak fast E-field based on the perturbation theory in the Furry picture

2. THEORY

2-1. Setup

Compute the production number analytically for

- ✔ QED in the presence of a static strong E-field superimposed by a weak time-depending E-field

$$\boldsymbol{E}(t) = \bar{\boldsymbol{E}} + \boldsymbol{\mathcal{E}}(t) \quad \text{where } |\bar{\boldsymbol{E}}| \gg |\boldsymbol{\mathcal{E}}|$$

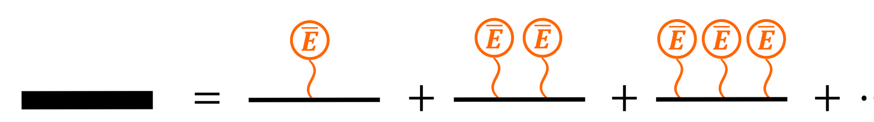
- ✔ Assume $\bar{\boldsymbol{E}} \parallel \boldsymbol{\mathcal{E}}(t)$ (for arbitrary direction, see [Huang, HT, 1904.08200])
- ✔ Assume spatial homogeneity
- ✔ Neglect backreaction (E-field is external)

2-2. Perturbation theory in the Furry picture

- ✔ **Treat $\bar{E}$ non-perturbatively, but $\mathcal{E}$ perturbatively**
- ✔ **Evaluate diagrams up to first order in $\mathcal{E}$ with a propagator fully dressed by $\bar{E}$ (Furry picture):** [Furry 1951]

$$n^{(\mp)}_{\pm p,s} = \sum_{s'}\int d^3\boldsymbol{p}'\left|\int d^3x\, {}_\pm\psi^{(0)\mathrm{out}\dagger}_{p,s}\, {}_\pm\psi^{(0)\mathrm{in}}_{p',s'} - \int d^4x\, {}_\pm\bar{\psi}^{(0)\mathrm{out}}_{ps}\, \not{\mathcal{A}}\, {}_\pm\psi^{(0)\mathrm{in}}_{p',s'}\right|^2$$

where



is the dressed propagator, and ${}_\pm\psi^{(0)\mathrm{in/out}}_{ps}$ is sol. of Dirac eq.

$$0 = [i\not\partial - e\bar{\not{A}} - m]\, {}_\pm\psi^{(0)\mathrm{in/out}}_{p,s}$$

with boundary conditions (plane wave at $t \to \pm\infty$)

$$\lim_{t\to -\infty/+\infty} {}_\pm\psi^{(0)\mathrm{in/out}}_{p,s} \propto \exp[\mp i\omega_p t + i\boldsymbol{p}\cdot\boldsymbol{x}]$$

2-3. Analytical formula

The number of produced particles is given by

$$n^{(\mp)}_{p,s} = \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+p_\perp^2)}{e\bar{E}}\right]\left| 1 + \frac{1}{2}\frac{m^2+p_\perp^2}{e\bar{E}}\int_0^\infty d\omega\,\frac{\tilde{\mathcal{E}}(\omega)}{\bar{E}}\exp\left[-\frac{i}{4}\frac{\omega^2+4\omega p_\parallel}{e\bar{E}}\right]{}_1F_1\left(1-\frac{i}{2}\frac{m^2+p_\perp^2}{e\bar{E}};2;\frac{i}{2}\frac{\omega^2}{e\bar{E}}\right)\right|^2$$

usual Schwinger formula — correction by perturbative $\mathcal{E}$

- ✔ **static limit $\omega/\sqrt{e\bar{E}} \ll 1$:** ■ dominates

$$n^{(\mp)}_{p,s} \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+p_\perp^2)}{e\bar{E}}\right]\left|1+\frac{\pi}{2}\frac{m^2+p_\perp^2}{e\bar{E}}\frac{\mathcal{E}}{\bar{E}}\right|^2 \sim \frac{V}{(2\pi)^3}\exp\left[-\frac{\pi(m^2+p_\perp^2)}{e(\bar{E}+\mathcal{E})}\right]$$

- ✔ **perturbative limit $\omega/\sqrt{e\bar{E}} \gg 1$:** ■ dominates

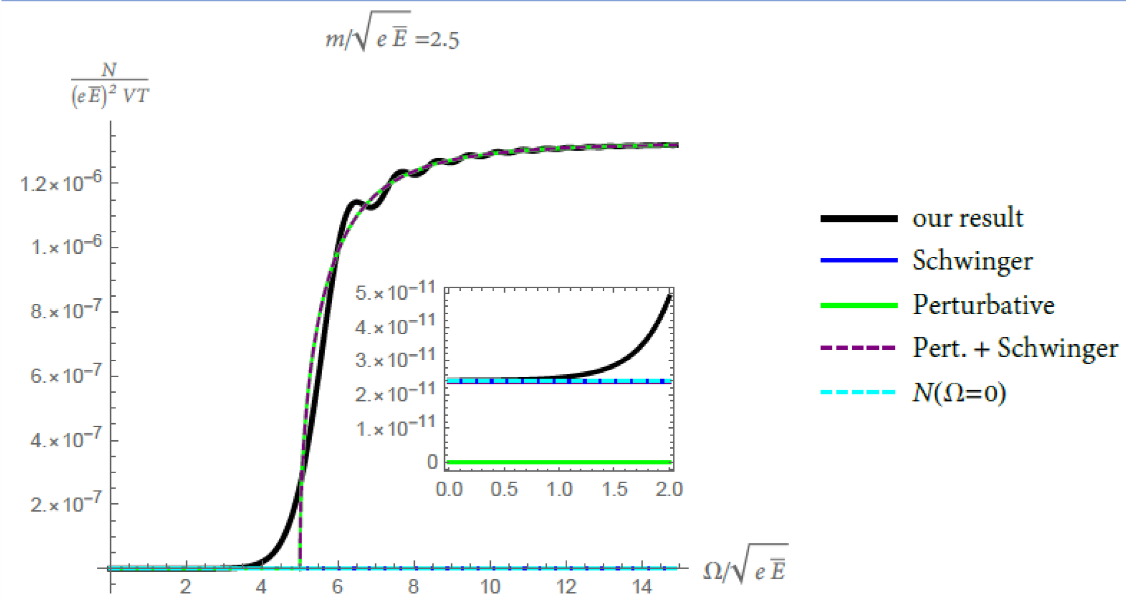
$$n^{(\mp)}_{p,s} \sim \frac{V}{(2\pi)^3}\frac{1}{4}\frac{m^2+p_\perp^2}{\omega_p^2}\frac{|\tilde{\mathcal{E}}(2\omega_p)|^2}{\omega_p^2}$$

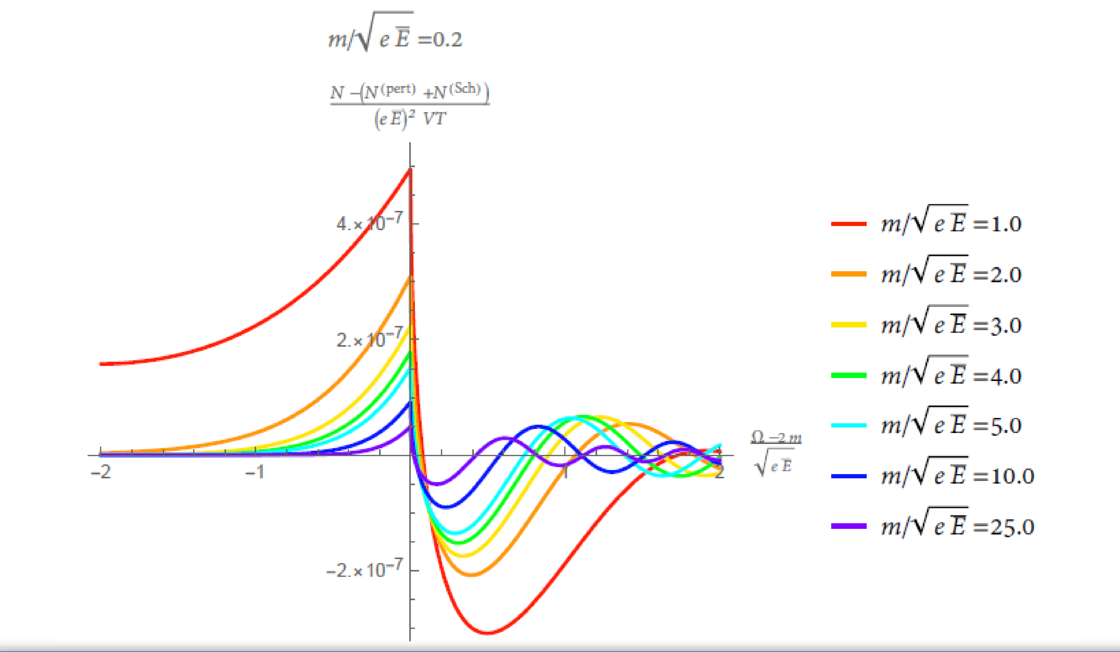
3. RESULTS

3-1. Setup

✔ A demonstration for $\mathcal{E}(t) = \mathcal{E}_0 \cos\Omega t$ with $\mathcal{E}_0 = 0.01\bar{E}$

3-2. Total production number $N^{(\mp)}_{p,s} = \int d^3p\, n^{(\mp)}_{p,s}$

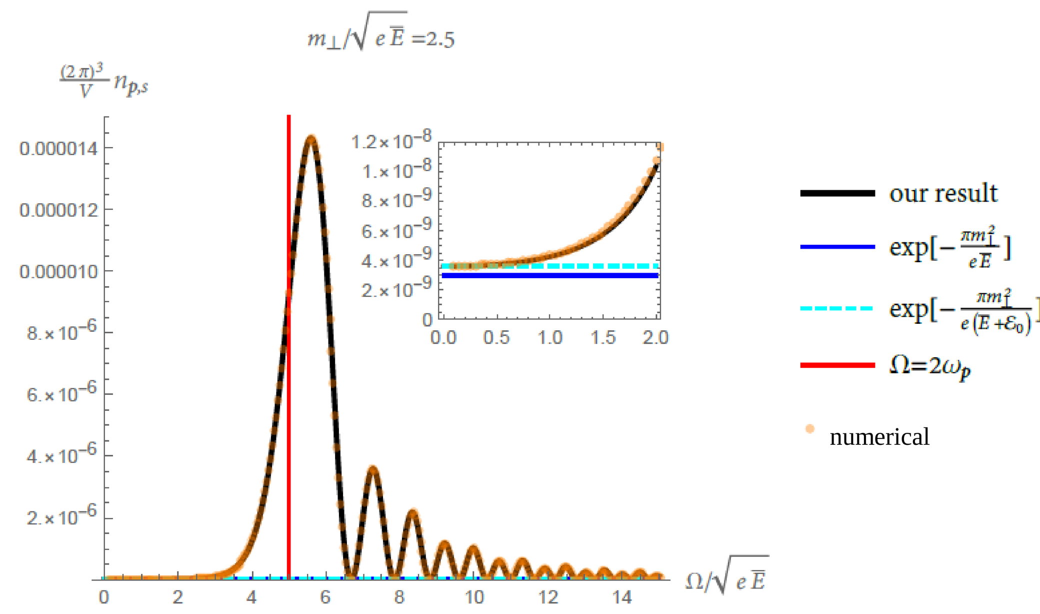




Exactly the same behavior as FK effect in cond-mat !!

- ✔ **FK effect equally occurs in QED** with strong slow E-field + weak fast E-field
- ✔ dyn. assisted Schwinger mech. can be understood as a low-Ω behavior of FK effect
- ✔ an oscillating behavior appears for large Ω (FK oscillation)
  - ➡ non-negligible (~10% effect)
- ✔ a sharp peak in the difference e.g.) modulation spectroscopy in cond-mat

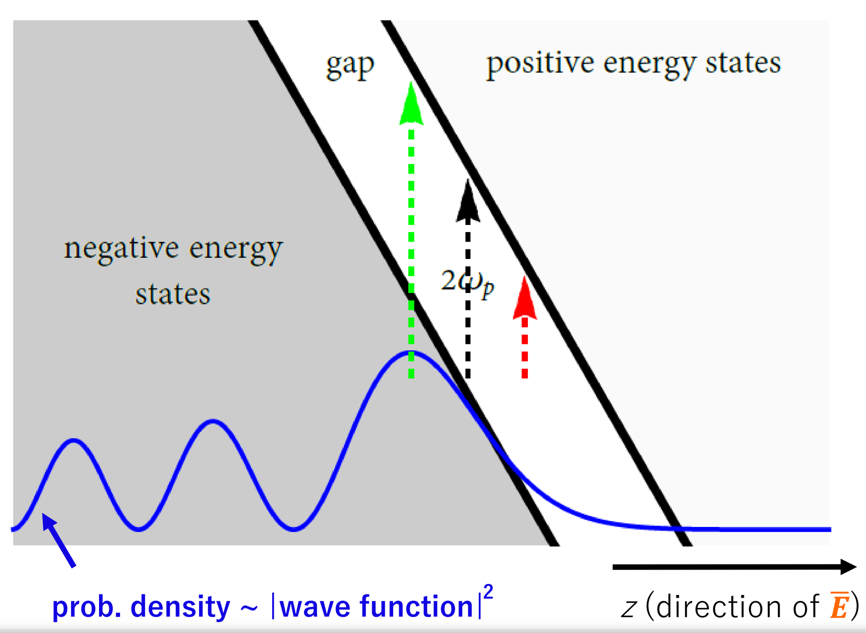
3-3. Momentum distribution $n^{(\mp)}_{p,s}$



FK effect also modifies the momentum distribution

- ✔ FK effect (dyn. assisted Schwinger mech.) enhances the production for low Ω
- ✔ FK effect oscillates the distribution for large Ω
- ✔ The peak position is lowered
- ✔ an excellent agreement with the numerical result obtained by directly solving the Dirac eq. w/o any approx

3-4. Interpretation



Not only tunneling, but also reflection is important

- ✔ Tunneling
  - ➡ wavefunc. get inside of the gap
  - ➡ enhancement in low Ω (dyn. assisted Schwinger mech.)
- ✔ Reflection
  - ➡ interference b/w wavefunc. heading to and reflected by the gap
  - ➡ oscillation & peak shift

4. SUMMARY

What I did

I studied spontaneous particle production from the vacuum (the Schwinger mechanism) in the presence of a slow strong electric field superimposed by a fast weak electric field

What we obtained/learned

- ✔ Analytical formula for the production in strong slow E-field superimposed by a weak fast E-field is obtained based on the pert. theory in the Furry picture
- ✔ A QED analog of the Franz-Keldysh effect occurs, which
  - (i) enhances the production below the threshold (dyn. ass. Schwinger mech.);
  - (ii) shifts the perturbative peak to a lower frequency;
  - (iii) results in an oscillating pattern above the threshold
- ✔ Not only quantum tunneling, but also reflection plays an important role

What to do

- ✔ Application to phenomenology e.g.) particle production in glasma
- ✔ Generalization: spatial inhomogeneity; inclusion of magnetic fields; higher order calculations, regularization of IR divergence...
- ✔ Apply ideas of cond-mat to the Schwinger mechanism